Finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory

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Abstract

Finite element model for vibration and buckling of functionally graded sandwich beams based on a refined shear deformation theory is presented. The core of sandwich beam is fully metal or ceramic and skins are composed of a functionally graded material across the depth. Governing equations of motion and boundary conditions are derived from the Hamilton's principle. Effects of power-law index, span-to-height ratio, core thickness and boundary conditions on the natural frequencies, critical buckling loads and load-frequency curves of sandwich beams are discussed. Numerical results show that the above-mentioned effects play very important role on the vibration and buckling analysis of functionally graded sandwich beams.

Keywords: Functionally graded sandwich beams; vibration; buckling; finite element

1. Introduction

In recent years, the application of functionally graded (FG) sandwich structures in aerospace, marine, civil construction is growing rapidly due to their high strength-to-weight ratio. There exist two common types: sandwich structures with FG core and sandwich structures with FG skins. With the wide application of FG sandwich structures, understanding vibration and buckling of FG sandwich structures becomes an important task. Based on the different shear deformation theories, though many works on these problems for FG beams are available ([1]-[13]), research on vibration and buckling of FG sandwich beams is a few in number. Di Sciuva and Gherlone [14] developed finite element formulations of the Hermitian Zig-Zag model to investigate the static and dynamic analyses of sandwich beams.

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Bhangale and Ganesan [15] derived finite element model to study thermal buckling and vibration analysis of a FG sandwich beam having constrained viscoelastic layer in thermal environment. Amirani et al. [16] used the element free Galerkin method for free vibration analysis of sandwich beam with FG core. Bui et al. [17] investigated transient responses and natural frequencies of sandwich beams with inhomogeneous FG core using a truly meshfree radial point interpolation method.

In this paper, which is extended from the previous work [18], finite element model for vibration and buckling of FG sandwich beams is presented. The developed theory accounts for parabolical variation of the transverse shear strain and stress through the beam depth, and satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam. The core of sandwich beam is fully metal or ceramic and skins are composed of a FG material across the beam depth. Governing equations of motion and boundary conditions are derived from the Hamilton's principle. Effects of power-law index, span-to-height ratio, core thickness and boundary conditions on the natural frequencies, critical buckling loads and load-frequency curves of sandwich beams are discussed. Numerical results show that the above-mentioned effects play very important role on the vibration and buckling analysis of FG sandwich beams.

2. Kinematics

Consider a FG sandwich beam, composed of "Layer 1", "Layer 2", and "Layer 3", as shown in Fig. 1. The x-, y-, and z-axes are taken along the length (L), width (b), and height (h) of the beam, respectively. The core of sandwich beam is fully metal or ceramic and skins are composed of a FG material across the beam depth. The vertical positions of the bottom and top, and of the two interfaces between the layers are denoted by $h_0 = -\frac{h}{2}$, h_1, h_2, $h_3 = \frac{h}{2}$, respectively. The effective material properties for each layer, like Young's modulus E and mass density ρ, can be expressed as:

$$P^{(j)}(z) = (P_b - P_t)V_b^{(j)}(z) + P_t \tag{1}$$

where P_t and P_b denote the material property located at the skins and at the core, respectively. The volume fraction function $V_b^{(j)}$ defined by the power-law form [19] as follows:

$$\begin{cases} V_b^{(1)}(z) = \left(\frac{z-h_o}{h_1-h_0}\right)^k & \text{for} \quad z \in [h_0, h_1] \\ \\ V_b^{(2)}(z) = 1 & \text{for} \quad z \in [h_1, h_2] \\ \\ V_b^{(3)}(z) = \left(\frac{z-h_3}{h_2-h_3}\right)^k & \text{for} \quad z \in [h_2, h_3] \end{cases} \tag{2}$$

where k is a power-law index which is positive.

The displacement field of the present theory, based on Reddy-Bickford beam theory ([20],[21]), can be obtained as:

$$
\begin{aligned}
U(x,z,t) &= u(x,t) - z w_b'(x,t) - \frac{4z^3}{3h^2} w_s'(x,t) && \text{(3a)}\\
W(x,z,t) &= w_b(x,t) + w_s(x,t) && \text{(3b)}
\end{aligned}
$$

where u is the axial displacement, w_b and w_s are the bending and shear components of transverse displacement along the mid-plane of the beam. The superscript prime ($'$) denotes the partial derivatives with respect to the x-axis.

The non-zero strains are given by:

$$
\begin{aligned}
\epsilon_x &= \frac{\partial U}{\partial x} = \epsilon_x^\circ + z\kappa_x^b + f\kappa_x^s && \text{(4a)}\\
\gamma_{xz} &= \frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} = [1 - \frac{df}{dz}]\gamma_{xz}^\circ = g\gamma_{xz}^\circ && \text{(4b)}
\end{aligned}
$$

where

$$
\begin{aligned}
f(z) &= \frac{4z^3}{3h^2} && \text{(5a)}\\
g(z) &= 1 - \frac{df}{dz} = 1 - \frac{4z^2}{h^2} && \text{(5b)}
\end{aligned}
$$

and $\epsilon_x^\circ, \gamma_{xz}^\circ, \kappa_x^b$ and κ_x^s are the strains and curvatures in the beam, defined as:

$$
\begin{aligned}
\epsilon_x^\circ &= u' && \text{(6a)}\\
\gamma_{xz}^\circ &= w_s' && \text{(6b)}\\
\kappa_x^b &= -w_b'' && \text{(6c)}\\
\kappa_x^s &= -w_s'' && \text{(6d)}
\end{aligned}
$$

In Eq. (5a), $f(z)$ denotes the distribution of the transverse shear strains and stress through the beam depth. This function is chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam.

3. Variational Formulation

In order to derive the equations of motion, Hamilton's principle is used:

$$
\int_{t_1}^{t_2} (\delta\mathcal{K} - \delta\mathcal{U} - \delta\mathcal{V})dt = 0 \qquad (7)
$$

where $\delta\mathcal{U}, \delta\mathcal{K}$ and $\delta\mathcal{V}$ denote the virtual variation of the strain energy, kinetic energy and potential energy, respectively.

The variation of the strain energy can be stated as:

$$\begin{aligned} \delta\mathcal{U} &= \int_0^l \int_0^b \left[\sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} (\sigma_x^{(n)} \delta\epsilon_x + \sigma_{xz}^{(n)} \delta\gamma_{xz}) dz \right] dydx \\ &= \int_0^l (N_x \delta\epsilon_z^\circ + M_x^b \delta\kappa_x^b + M_x^s \delta\kappa_x^s + Q_{xz} \delta\gamma_{xz}^\circ) dx \end{aligned} \tag{8}$$

where N_x, M_x^b, M_x^s and Q_{xz} are the stress resultants, defined as:

$$N_x = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \sigma_x^{(n)} bdz \tag{9a}$$

$$M_x^b = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \sigma_x^{(n)} zbdz \tag{9b}$$

$$M_x^s = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \sigma_x^{(n)} fbdz \tag{9c}$$

$$Q_{xz} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \sigma_{xz}^{(n)} gbdz \tag{9d}$$

The variation of the potential energy by the axial force P_0 can be written as:

$$\delta\mathcal{V} = -\int_0^l P_0 \Big[\delta w_b'(w_b' + w_s') + \delta w_s'(w_b' + w_s') \Big] dx \tag{10}$$

The variation of the kinetic energy can be expressed as:

$$\begin{aligned} \delta\mathcal{K} &= \int_0^l \int_0^b \left[\sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \rho^{(n)} (\dot{U}\delta\dot{U} + \dot{W}\delta\dot{W}) dz \right] dydx \\ &= \int_0^l \Big[\delta\dot{u}(m_0\dot{u} - m_1\dot{w}_b{}' - m_f\dot{w}_s{}') + \delta\dot{w}_b m_0(\dot{w}_b + \dot{w}_s) + \delta\dot{w}_b{}'(-m_1\dot{u} + m_2\dot{w}_b{}' + m_{fz}\dot{w}_s{}') \\ &+ \delta\dot{w}_s m_0(\dot{w}_b + \dot{w}_s) + \delta\dot{w}_s{}'(-m_f\dot{u} + m_{fz}\dot{w}_b{}' + m_{f^2}\dot{w}_s{}') \Big] dx \end{aligned} \tag{11}$$

where dot-superscript prime indicates the differentiation with respect to the time t; and $m_0, m_1, m_2, m_f, m_{fz}$ and m_{f^2} are the mass inertias, defined by:

$$(m_0, m_1, m_2) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \rho^{(n)} (1, z, z^2) bdz \tag{12a}$$

$$(m_f, m_{fz}, m_{f^2}) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \rho^{(n)} (f, fz, f^2) bdz \tag{12b}$$

By substituting Eqs. (8), (10) and (11) into Eq. (7), the following weak statement is obtained:

$$
\begin{aligned}
0 &= \int_{t_1}^{t_2}\int_0^l \Big[\delta\dot{u}(m_0\dot{u} - m_1\dot{w}_b{}' - m_f\dot{w}_s{}') + \delta\dot{w}_b m_0(\dot{w}_b + \dot{w}_s) + \delta\dot{w}_b{}'(-m_1\dot{u} + m_2\dot{w}_b{}' + m_{fz}\dot{w}_s{}') \\
&+ \delta\dot{w}_s m_0(\dot{w}_b + \dot{w}_s) + \delta\dot{w}_s{}'(-m_f\dot{u} + m_{fz}\dot{w}_b{}' + m_{f^2}\dot{w}_s{}') \\
&+ P_0\big[\delta w_b'(w_b' + w_s') + \delta w_s'(w_b' + w_s')\big] - N_x\delta u' + M_x^b\delta w_b'' + M_x^s\delta w_s'' - Q_{xz}\delta w_s'\Big]dxdt
\end{aligned} \tag{13}
$$

4. Constitutive Equations

The linear constitutive relations of a FG sandwich beam can be written as:

$$
\sigma_x^{(n)} = E^{(n)}\epsilon_x \tag{14a}
$$

$$
\sigma_{xz}^{(n)} = \frac{E^{(n)}}{2\big[1+\nu^{(n)}\big]}\gamma_{xz} = G^{(n)}\gamma_{xz} \tag{14b}
$$

By using Eqs. (4), (9) and (14), the constitutive equations for stress resultants and strains are obtained :

$$
\begin{Bmatrix} N_x \\ M_x^b \\ M_x^s \\ Q_{xz} \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & 0 \\ & R_{22} & R_{23} & 0 \\ & & R_{33} & 0 \\ \text{sym.} & & & R_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_x^\circ \\ \kappa_x^b \\ \kappa_x^s \\ \gamma_{xz}^\circ \end{Bmatrix} \tag{15}
$$

where R_{ij} are the stiffnesses of FG sandwich beams and given by:

$$
R_{11} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} E^{(n)}bdz \tag{16a}
$$

$$
R_{12} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} zE^{(n)}bdz \tag{16b}
$$

$$
R_{13} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} fE^{(n)}bdz \tag{16c}
$$

$$
R_{22} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} z^2E^{(n)}bdz \tag{16d}
$$

$$
R_{23} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} zfE^{(n)}bdz \tag{16e}
$$

$$
R_{33} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} f^2E^{(n)}bdz \tag{16f}
$$

$$
R_{44} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} g^2G^{(n)}bdz \tag{16g}
$$

5. Governing Equations of Motion

The equilibrium equations of the present study can be obtained by integrating the derivatives of the varied quantities by parts and collecting the coefficients of $\delta u, \delta w_b$ and δw_s:

$$\begin{aligned} N_x' &= m_0\ddot{u} - m_1\ddot{w}_b{}' - m_f\ddot{w}_s{}' && (17a) \\ M_x^{b}{}'' - P_0(w_b'' + w_s'') &= m_0(\ddot{w}_b + \ddot{w}_s) + m_1\ddot{u}' - m_2\ddot{w}_b{}'' - m_{fz}\ddot{w}_s{}'' && (17b) \\ M_x^{s}{}'' + Q_{xz}' - P_0(w_b'' + w_s'') &= m_0(\ddot{w}_b + \ddot{w}_s) + m_f\ddot{u}' - m_{fz}\ddot{w}_b{}'' - m_{f^2}\ddot{w}_s{}'' && (17c) \end{aligned}$$

The natural boundary conditions are of the form:

$$\begin{aligned} \delta u &: N_x && (18a) \\ \delta w_b &: M_x^{b}{}' - P_0(w_b' + w_s') - m_1\ddot{u} + m_2\ddot{w}_b{}' + m_{fz}\ddot{w}_s{}' && (18b) \\ \delta w_b' &: M_x^b && (18c) \\ \delta w_s &: M_x^{s}{}' + Q_{xz} - P_0(w_b' + w_s') - m_f\ddot{u} + m_{fz}\ddot{w}_b{}' + m_{f^2}\ddot{w}_s{}' && (18d) \\ \delta w_s' &: M_x^s && (18e) \end{aligned}$$

By substituting Eqs. (6) and (15) into Eq. (17), the explicit form of the governing equations of motion can be expressed with respect to the stiffnesses R_{ij}:

$$\begin{aligned} R_{11}u'' - R_{12}w_b''' - R_{13}w_s''' &= m_0\ddot{u} - m_1\ddot{w}_b{}' - m_f\ddot{w}_s{}' && (19a) \\ R_{12}u''' - R_{22}w_b^{iv} - R_{23}w_s^{iv} - P_0(w_b'' + w_s'') &= m_0(\ddot{w}_b + \ddot{w}_s) + m_1\ddot{u}' \\ &- m_2\ddot{w}_b{}'' - m_{fz}\ddot{w}_s{}'' && (19b) \\ R_{13}u''' - R_{23}w_b^{iv} - R_{33}w_s^{iv} + R_{44}w_s'' - P_0(w_b'' + w_s'') &= m_0(\ddot{w}_b + \ddot{w}_s) + m_f\ddot{u}' \\ &- m_{fz}\ddot{w}_b{}'' - m_{f^2}\ddot{w}_s{}'' && (19c) \end{aligned}$$

6. Finite Element Formulation

The present theory for FG sandwich beams described in the previous section was implemented via a displacement based finite element method. The variational statement in Eq. (13) requires that the bending and shear components of transverse displacement w_b and w_s be twice differentiable and C^1-continuous, whereas the axial displacement u must be only once differentiable and C^0-continuous. The generalized displacements are expressed over each element as a combination of the linear interpolation function Ψ_j for u and Hermite-cubic interpolation function ψ_j for w_b and w_s associated with node j

and the nodal values:

$$u = \sum_{j=1}^{2} u_j \Psi_j \tag{20a}$$

$$w_b = \sum_{j=1}^{4} w_{bj} \psi_j \tag{20b}$$

$$w_s = \sum_{j=1}^{4} w_{sj} \psi_j \tag{20c}$$

Substituting these expressions in Eq. (20) into the corresponding weak statement in Eq. (13), the finite element model of a typical element can be expressed as the standard eigenvalue problem:

$$([K] - P_0[G] - \omega^2[M])\{\Delta\} = \{0\} \tag{21}$$

where $[K], [G]$ and $[M]$ are the element stiffness matrix, element geometric stiffness matrix and element mass matrix, respectively. The explicit forms of them are given by:

$$K_{ij}^{11} = \int_0^l R_{11} \Psi_i' \Psi_j' dx \tag{22a}$$

$$K_{ij}^{12} = -\int_0^l R_{12} \Psi_i' \psi_j'' dx \tag{22b}$$

$$K_{ij}^{13} = -\int_0^l R_{13} \Psi_i' \psi_j'' dx \tag{22c}$$

$$K_{ij}^{22} = \int_0^l R_{22} \psi_i'' \psi_j'' dx \tag{22d}$$

$$K_{ij}^{23} = \int_0^l R_{23} \psi_i'' \psi_j'' dx \tag{22e}$$

$$K_{ij}^{33} = \int_0^l (R_{33} \psi_i'' \psi_j'' + R_{44} \psi_i' \psi_j') dx \tag{22f}$$

$$G_{ij}^{22} = \int_0^l \psi_i' \psi_j' dx \tag{22g}$$

$$G_{ij}^{23} = \int_0^l \psi_i' \psi_j' dx \tag{22h}$$

$$G_{ij}^{33} = \int_0^l \psi_i' \psi_j' dx \tag{22i}$$

$$M_{ij}^{11} = \int_0^l m_0 \Psi_i \Psi_j dx \tag{22j}$$

$$M_{ij}^{12} = -\int_0^l m_1 \Psi_i \psi_j' dx \tag{22k}$$

$$M_{ij}^{13} = -\int_0^l m_f \Psi_i \psi_j' dx \tag{22l}$$

$$M_{ij}^{22} = \int_0^l (m_0\psi_i\psi_j + m_2\psi_i'\psi_j')dx \tag{22m}$$

$$M_{ij}^{23} = \int_0^l (m_0\psi_i\psi_j + m_{fz}\psi_i'\psi_j')dx \tag{22n}$$

$$M_{ij}^{33} = \int_0^l (m_0\psi_i\psi_j + m_{f^2}\psi_i'\psi_j')dx \tag{22o}$$

In Eq. (21), $\{\Delta\}$ is the eigenvector of nodal displacements corresponding to an eigenvalue:

$$\{\Delta\} = \{u\ w_b\ w_s\}^{\mathrm{T}} \tag{23}$$

7. Numerical Examples

For verification purpose, the fundamental natural frequencies and critical buckling loads of FG beams with different values of span-to-height ratio for three boundary conditions, which are clamped-clamped (C-C), clamped-free (C-F) and simply-supported (S-S) are given in Tables 1-4. FG material properties are assumed to be [6]: Aluminum (Al: $E_m = 70\text{GPa}, \nu_m = 0.3, \rho_m = 2702\text{kg/m}^3$) and Alumina ($Al_2O_3$: $E_c = 380\text{GPa}, \nu_c = 0.3, \rho_c = 3960\text{kg/m}^3$). For buckling analysis, Li and Batra [11] used $\nu_m = \nu_c = 0.23$ in their research. The following non-dimensional natural frequencies and critical buckling loads are used in this study: $\overline{\omega} = \frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$, $\overline{P}_{cr} = P_{cr}\frac{12L^2}{E_m h^3}$. A mesh convergence study for clamped-clamped FG beams with the power-law index $k = 0$ and 5 is carried out and plotted in Fig. 2. It can be seen that 20 Hermitian beam elements with 105 degrees of freedom are sufficient to obtain an accurate solution. Therefore, this number of elements is used throughout the numerical examples. The results obtained from the present theory are compared with those of Li and Batra [11] and Nguyen et al. [12] based on the first-order beam theory (FOBT), Simsek [6] and Thai and Vo [8] based on the higher-order beam theory (HOBT). It should be noted that in previous research, Thai and Vo [8] used the Navier procedure to derive the analytical solution for a simply-supported FG beam only. In the case of the FOBT, the shear correction factor is taken to equal 5/6. As expected, an increase of the power-law index makes FG beams more flexible, which leads to a reduction in natural frequencies and buckling loads. This holds irrespective of the consideration of shear effects. It is observed that the present results are in good agreement with the solutions in earlier works, thus accuracy of the present model is established.

In order to investigate the effects of the power-law index and span-to-height ratio on the natural frequencies and critical buckling loads, seven different types of symmetric and non-symmetric FG sandwich beams for various boundary conditions are considered. Unless mentioned otherwise, two cases of FG sandwich beams with two values of span-to-height ratio, $L/h = 5$ and 20, are examined:

- Hardcore: homogeneous core with Al_2O_3 ($E_b = E_c$, $\nu_b = \nu_c$, $\rho_b = \rho_c$) and FG faces with top and bottom surfaces made of Al ($E_t = E_m$, $\nu_t = \nu_m$, $\rho_t = \rho_m$)

- Softcore: homogeneous core with Al ($E_b = E_m$, $\nu_b = \nu_m$, $\rho_b = \rho_m$) and FG faces with top and bottom surfaces made of Al_2O_3 ($E_t = E_c$, $\nu_t = \nu_c$, $\rho_t = \rho_c$)

Numerical results are given in Tables 5-14 and plotted in Figs. 3-8. The fundamental natural frequencies and critical buckling loads of (1-1-1) sandwich beam with respect to the power-law index are plotted in Figs. 3 and 4. When comparing the results between $L/h = 5$ and 20, it can be seen that they nearly coincide for C-F beam and become more discrepancy for S-S and C-C one. It is clear that shear effects are more pronounced on C-C beam than others and stronger for softcore than hardcore. Figs. 5 and 6 illustrated the fundamental natural frequencies and critical buckling loads versus the span-to-height ratios of (1-0-1) and (1-8-1) clamped-clamped sandwich beams. It is shown that the effect of power-law index on sandwich beam without core (1-0-1), is greater than that of sandwich beam with hardcore (1-8-1), and this effect on softcore is greater than that with hardcore for natural frequencies (Fig. 5b). In general, as the power-law index increases and core thickness decreases, the natural frequencies and critical buckling loads decrease for sandwich beams with hardcore and increase for sandwich beams with softcore.

For the sake of completeness, the first fourth natural frequencies of symmetric (1-2-1) and non-symmetric (2-2-1) sandwich beams are given in Tables 13 and 14. In this case, the results of non-symmetric sandwich beam are smaller than those of symmetric one with hardcore and vice versa with softcore. A clamped-clamped sandwich beam is chosen to investigate the vibration mode shapes with the power-law index $k = 5$ in Fig. 7. It can be seen that for symmetric sandwich beam, the coupling stiffnesses R_{12} and R_{13} in governing equations (Eq. 19) become zero, therefore, the first, second and fourth modes exhibit double coupled vibration (w_b and w_s), whereas, the third mode exhibits axial vibration (u). However, for non-symmetric one, all the coupling stiffnesses do not vanish, thus, all three modes (the first, second and fourth modes) display triply coupled vibration (u, w_b and w_s). These modes also highlight the main difference between non-symmetric and symmetric sandwich beam.

Finally, the fundamental natural frequencies of symmetric (2-1-2) and non-symmetric (2-1-1) sandwich beams under axial force with the power-law index $k = 10$ is plotted in Fig. 8. As expected, the smallest group is for C-F beam and the largest one is for the C-C beam. These load-frequency curves explain the duality between the critical buckling load and fundamental natural frequency. Thus, for (2-1-1) sandwich beam with hardcore (Fig. 8a), the critical buckling loads are 42.193, 11.837 and 3.053 for C-C, S-S and C-F beam, respectively.

8. Conclusions

Based on refined shear deformation theory, vibration and buckling of FG sandwich beams is presented. Governing equations of motion and boundary conditions are derived from the Hamilton's principle. Finite element model is developed to determine the natural frequencies, critical buckling loads and load-frequency curves as well as corresponding mode shapes of FG sandwich beam with homogeneous hardcore and softcore. Effects of power-law index, span-to-height ratio, core thickness and boundary conditions are discussed. The present model can provide accurate and reliable results in analysing vibration and buckling problem of FG sandwich beams.

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Figure 1: Geometry and coordinate of a FG sandwich beam.

Figure 2: A mesh convergence study for clamped-clamped FG beams (L/h=5, 10 and 20).

Figure 3: Effect of the power-law index on the non-dimensional fundamental natural frequencies of (1-1-1) FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions ($L/h = 5$ and 20).

Figure 4: Effect of the power-law index on the non-dimensional critical buckling loads of (1-1-1) FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions ($L/h = 5$ and 20).

Figure 5: Effect of span-to-height ratio on the non-dimensional fundamental natural frequencies of (1-0-1) and (1-8-1) clamped-clamped FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions.

Figure 6: Effect of span-to-height ratio on the non-dimensional critical buckling loads of (1-0-1) and (1-8-1) clamped-clamped FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions.

Figure 7: Vibration mode shapes of (1-2-1) and (2-2-1) clamped-clamped FG sandwich beams with homogeneous hardcore ($L/h = 5$).

Figure 8: Effect of the axial force on the fundamental natural frequencies of (1-2-1) and (2-2-1) FG sandwich beams ($L/h = 5$) with homogeneous hardcore and softcore for various boundary conditions.

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Table 3: Comparison of the non-dimensional critical buckling loads of FG beams with various boundary conditions ($L/h = 5$).

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Table 5: Non-dimensional fundamental natural frequencies of FG sandwich beams with homogeneous hardcore for various boundary conditions ($L/h = 5$).

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Table 10: Non-dimensional critical buckling loads of FG sandwich beams with homogeneous hardcore for various boundary conditions ($L/h = 20$).

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Table 13: The first three non-dimensional natural frequencies of FG sandwich beams with homogeneous hardcore for various boundary conditions (L/h=5).

Table 14: The first three non-dimensional natural frequencies of FG sandwich beams with homogeneous softcore for various boundary conditions (L/h=5).

Table 1: Comparison of the non-dimensional fundamental natural frequencies of FG beams with various boundary conditions (L/h=5).

Theory	BC	Reference	k = 0	k = 0.5	k = 1	k = 2	k = 5	k = 10
FOBT	C-C	Simsek [6]	10.0344	8.7005	7.9253	7.2113	6.6676	6.3406
		Present	9.9984	8.6717	7.9015	7.1901	6.6447	6.3161
	S-S	Simsek [6]	5.1525	4.4083	3.9902	3.6344	3.4312	3.3134
		Nguyen et al. [12]	5.1525	4.4075	3.9902	3.6344	3.4312	3.3135
		Present	5.1526	4.3990	3.9711	3.6050	3.4025	3.2963
	C-F	Simsek [6]	1.8948	1.6174	1.4630	1.3338	1.2645	1.2240
		Present	1.8944	1.6169	1.4628	1.3336	1.2642	1.2237
HOBT	C-C	Simsek [6]	10.0705	8.7467	7.9503	7.1767	6.4935	6.1652
		Present	10.0678	8.7457	7.9522	7.1801	6.4961	6.1662
	S-S	Simsek [6]	5.1527	4.4111	3.9904	3.6264	3.4012	3.2816
		Thai & Vo [8]	5.1527	4.4107	3.9904	3.6264	3.4012	3.2816
		Present	5.1528	4.4019	3.9716	3.5979	3.3743	3.2653
	C-F	Simsek [6]	1.8952	1.6182	1.4633	1.3325	1.2592	1.2183
		Present	1.8952	1.6180	1.4633	1.3326	1.2592	1.2184

Table 2: Comparison of the non-dimensional fundamental natural frequencies of FG beams with various boundary conditions (L/h=20).

Theory	BC	Reference	k = 0	k = 0.5	k = 1	k = 2	k = 5	k = 10
FOBT	C-C	Simsek [6]	12.2235	10.4263	9.4314	8.6040	8.1699	7.9128
		Present	12.2202	10.4238	9.4311	8.6047	8.1698	7.9115
	S-S	Simsek [6]	5.4603	4.6514	4.2051	3.8368	3.6509	3.5416
		Nguyen et al. [12]	5.4603	4.6504	4.2051	3.8368	3.6509	3.5416
		Present	5.4603	4.6504	4.2039	3.8349	3.6490	3.5405
	C-F	Simsek [6]	1.9496	1.6604	1.5010	1.3697	1.3038	1.2650
		Present	1.9496	1.6603	1.5011	1.3697	1.3038	1.2650
HOBT	C-C	Simsek [6]	12.2238	10.4287	9.43158	8.59751	8.1446	7.8858
		Present	12.2228	10.4279	9.4328	8.5994	8.1460	7.8862
	S-S	Simsek [6]	5.4603	4.6516	4.2050	3.8361	3.6485	3.5390
		Thai & Vo [8]	5.4603	4.6511	4.2051	3.8361	3.6485	3.5390
		Present	5.4603	4.6506	4.2039	3.8343	3.6466	3.5379
	C-F	Simsek [6]	1.9495	1.6605	1.5011	1.3696	1.3033	1.2645
		Present	1.9496	1.6603	1.5011	1.3696	1.3034	1.2645

Table 3: Comparison of the non-dimensional critical buckling loads of FG beams with various boundary conditions (L/h=5).

Theory	BC	Reference	k = 0	k = 0.5	k = 1	k = 2	k = 5	k = 10
FOBT	C-C	Li & Batra [11]	154.3500	103.2200	80.4980	62.6140	50.3840	44.2670
		Present	154.4150	103.2750	80.5480	62.6616	50.4207	44.2946
	S-S	Li & Batra [11]	48.8350	31.9670	24.6870	19.2450	16.0240	14.4270
		Nguyen et al. [12]	48.8350	31.9610	24.6870	19.2450	16.0240	14.4270
		Present	48.8372	31.9695	24.6898	19.2479	16.0263	14.4286
	C-F	Li & Batra [11]	13.2130	8.5782	6.6002	5.1495	4.3445	3.9501
		Present	13.0770	8.4992	6.5428	5.1042	4.2986	3.9031
HOBT	C-C	Present	154.5500	103.7490	80.6087	61.7925	47.7562	41.8042
	S-S	Present	48.8401	32.0094	24.6911	19.1605	15.7400	14.1468
	C-F	Present	13.0771	8.5020	6.5428	5.0979	4.2776	3.8821

Table 4: Comparison of the non-dimensional critical buckling loads of FG beams with various boundary conditions (L/h=10).

Theory	BC	Reference	k = 0	k = 0.5	k = 1	k = 2	k = 5	k = 10
FOBT	C-C	Li & Batra [11]	195.3400	127.8700	98.7490	76.9800	64.0960	57.7080
		Present	195.3730	127.9050	98.7923	77.0261	64.1324	57.7329
	S-S	Li & Batra [11]	52.3090	33.9960	26.1710	20.4160	17.1920	15.6120
		Nguyen et al. [12]	52.3080	33.9890	26.1710	20.4160	17.1940	15.6120
		Present	52.3085	33.9981	26.1728	20.4187	17.1959	15.6134
	C-F	Li & Batra [11]	13.2130	8.5666	6.6570	5.1944	4.3903	3.9969
		Present	13.3137	8.6363	6.6425	5.1830	4.3785	3.9850
HOBT	C-C	Present	195.3610	128.0500	98.7868	76.6677	62.9786	56.5971
	S-S	Present	52.3082	34.0087	26.1727	20.3936	17.1118	15.5291
	C-F	Present	13.3742	8.6714	6.6680	5.2027	4.3976	4.0046

Table 5: Non-dimensional fundamental natural frequencies of FG sandwich beams with homogeneous hardcore for various boundary conditions (L/h=5).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	10.0678	10.0678	10.0678	10.0678	10.0678	10.0678	10.0678
	0.5	8.3600	8.5720	8.6673	8.7423	8.8648	8.9942	9.5731
	1	7.3661	7.6865	7.8390	7.9580	8.1554	8.3705	9.3076
	2	6.4095	6.7826	6.9908	7.1373	7.4105	7.7114	9.0343
	5	5.7264	6.0293	6.2737	6.3889	6.7188	7.0691	8.7605
	10	5.5375	5.8059	6.0527	6.1240	6.4641	6.8087	8.6391
S-S	0	5.1528	5.1528	5.1528	5.1528	5.1528	5.1528	5.1528
	0.5	4.1268	4.2351	4.2945	4.3303	4.4051	4.4798	4.8422
	1	3.5735	3.7298	3.8187	3.8755	3.9896	4.1105	4.6795
	2	3.0680	3.2365	3.3514	3.4190	3.5692	3.7334	4.5142
	5	2.7446	2.8439	2.9746	3.0181	3.1928	3.3771	4.3501
	10	2.6932	2.7355	2.8669	2.8808	3.0588	3.2356	4.2776
C-F	0	1.8952	1.8952	1.8952	1.8952	1.8952	1.8952	1.8952
	0.5	1.5069	1.5466	1.5696	1.5821	1.6108	1.6384	1.7764
	1	1.3007	1.3575	1.3918	1.4115	1.4549	1.4992	1.7145
	2	1.1143	1.1746	1.2188	1.2416	1.2986	1.3582	1.6518
	5	0.9973	1.0303	1.0806	1.0935	1.1597	1.2257	1.5897
	10	0.9812	0.9909	1.0416	1.0431	1.1106	1.1734	1.5624

Table 6: Non-dimensional fundamental natural frequencies of FG sandwich beams with homogeneous hardcore for various boundary conditions (L/h=20).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	12.2228	12.2228	12.2228	12.2228	12.2228	12.2228	12.2228
	0.5	9.6942	9.9501	10.1001	10.1800	10.3668	10.5460	11.4459
	1	8.3594	8.7241	8.9474	9.0722	9.3550	9.6411	11.0421
	2	7.1563	7.5417	7.8293	7.9727	8.3430	8.7262	10.6336
	5	6.4064	6.6116	6.9389	7.0170	7.4461	7.8692	10.2298
	10	6.3086	6.3590	6.6889	6.6924	7.1296	7.5311	10.0519
S-S	0	5.4603	5.4603	5.4603	5.4603	5.4603	5.4603	5.4603
	0.5	4.3148	4.4290	4.4970	4.5324	4.6170	4.6979	5.1067
	1	3.7147	3.8768	3.9774	4.0328	4.1602	4.2889	4.9233
	2	3.1764	3.3465	3.4754	3.5389	3.7049	3.8769	4.7382
	5	2.8439	2.9310	3.0773	3.1111	3.3028	3.4921	4.5554
	10	2.8041	2.8188	2.9662	2.9662	3.1613	3.3406	4.4749
C-F	0	1.9496	1.9496	1.9496	1.9496	1.9496	1.9496	1.9496
	0.5	1.5397	1.5805	1.6048	1.6175	1.6477	1.6766	1.8229
	1	1.3253	1.3831	1.4191	1.4388	1.4844	1.5304	1.7573
	2	1.1330	1.1937	1.2398	1.2623	1.3217	1.3831	1.6911
	5	1.0145	1.0453	1.0977	1.1096	1.1781	1.2456	1.6257
	10	1.0005	1.0053	1.0581	1.0578	1.1276	1.1915	1.5969

Table 7: Non-dimensional fundamental natural frequencies of FG sandwich beams with homogeneous softcore for various boundary conditions (L/h=5).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	5.2311	5.2311	5.2311	5.2311	5.2311	5.2311	5.2311
	0.5	8.0509	7.6627	7.5623	7.3914	7.2456	7.0539	6.3333
	1	8.8221	8.3354	8.2273	7.9726	7.8056	7.5187	6.6705
	2	9.4121	8.8948	8.7823	8.4655	8.2835	7.8960	6.9233
	5	9.8336	9.3724	9.2560	8.9201	8.7255	8.2498	7.1155
	10	9.9640	9.5608	9.4430	9.1193	8.9195	8.4162	7.1884
S-S	0	2.6773	2.6773	2.6773	2.6773	2.6773	2.6773	2.6773
	0.5	4.4427	4.3046	4.1960	4.1839	4.0504	3.9921	3.4342
	1	4.8525	4.7178	4.5916	4.5858	4.4270	4.3663	3.7065
	2	5.0945	4.9970	4.8668	4.8740	4.7047	4.6459	3.9303
	5	5.1880	5.1603	5.0399	5.0703	4.9038	4.8564	4.1139
	10	5.1848	5.1966	5.0866	5.1301	4.9700	4.9326	4.1855
C-F	0	0.9847	0.9847	0.9847	0.9847	0.9847	0.9847	0.9847
	0.5	1.6615	1.6168	1.5728	1.5744	1.5211	1.5020	1.2790
	1	1.8135	1.7756	1.7238	1.7330	1.6691	1.6539	1.3883
	2	1.8969	1.8772	1.8239	1.8425	1.7745	1.7662	1.4798
	5	1.9206	1.9291	1.8804	1.9104	1.8444	1.8466	1.5562
	10	1.9137	1.9368	1.8928	1.9280	1.8652	1.8734	1.5861

Table 8: Non-dimensional fundamental natural frequencies of FG sandwich beams with homogeneous softcore for various boundary conditions (L/h=20).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	6.3509	6.3509	6.3509	6.3509	6.3509	6.3509	6.3509
	0.5	10.7743	10.4993	10.2050	10.2298	9.8754	9.7587	8.2818
	1	11.7579	11.5383	11.1911	11.2767	10.8502	10.7706	9.0064
	2	12.2833	12.1920	11.8341	11.9911	11.5374	11.5168	9.6175
	5	12.4132	12.5088	12.1839	12.4199	11.9817	12.0423	10.1297
	10	12.3564	12.5460	12.2541	12.5230	12.1079	12.2122	10.3315
S-S	0	2.8371	2.8371	2.8371	2.8371	2.8371	2.8371	2.8371
	0.5	4.8579	4.7460	4.6050	4.6294	4.4611	4.4160	3.7255
	1	5.2990	5.2217	5.0541	5.1160	4.9121	4.8938	4.0648
	2	5.5239	5.5113	5.3390	5.4410	5.2242	5.2445	4.3542
	5	5.5645	5.6382	5.4834	5.6242	5.4166	5.4843	4.5991
	10	5.5302	5.6452	5.5073	5.6621	5.4667	5.5575	4.6960
C-F	0	1.0130	1.0130	1.0130	1.0130	1.0130	1.0130	1.0130
	0.5	1.7368	1.6974	1.6467	1.6560	1.5955	1.5796	1.3315
	1	1.8944	1.8679	1.8076	1.8308	1.7574	1.7516	1.4535
	2	1.9742	1.9712	1.9091	1.9471	1.8691	1.8778	1.5576
	5	1.9878	2.0157	1.9601	2.0121	1.9375	1.9636	1.6459
	10	1.9750	2.0177	1.9681	2.0252	1.9551	1.9896	1.6809

Table 9: Non-dimensional critical buckling loads of FG sandwich beams with homogeneous hardcore for various boundary conditions (L/h=5).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	152.1470	152.1470	152.1470	152.1470	152.1470	152.1470	152.1470
	0.5	92.8833	99.9860	102.9120	105.6790	109.6030	114.1710	134.2870
	1	67.4983	76.2634	80.1670	83.8177	89.2208	95.7287	125.3860
	2	47.7010	56.2057	60.6056	64.4229	70.7563	78.5608	116.6580
	5	35.5493	42.0033	46.3743	49.2763	55.8271	63.7824	108.2970
	10	32.3019	37.9944	42.1935	44.3374	50.7315	58.2461	104.6920
S-S	0	48.5959	48.5959	48.5959	48.5959	48.5959	48.5959	48.5959
	0.5	27.8574	30.0301	31.0728	31.8784	33.2536	34.7653	41.9897
	1	19.6525	22.2108	23.5246	24.5596	26.3611	28.4447	38.7838
	2	13.5801	15.9152	17.3249	18.3587	20.3750	22.7863	35.6914
	5	10.1460	11.6676	13.0270	13.7212	15.7307	18.0914	32.7725
	10	9.4515	10.5348	11.8370	12.2605	14.1995	16.3783	31.5265
C-F	0	13.0594	13.0594	13.0594	13.0594	13.0594	13.0594	13.0594
	0.5	7.3314	7.9068	8.1951	8.4051	8.7839	9.1940	11.2021
	1	5.1245	5.7921	6.1490	6.4166	6.9050	7.4639	10.3093
	2	3.5173	4.1156	4.4927	4.7564	5.2952	5.9348	9.4531
	5	2.6298	3.0004	3.3609	3.5310	4.0620	4.6806	8.6493
	10	2.4683	2.7077	3.0527	3.1488	3.6595	4.2267	8.3073

Table 10: Non-dimensional critical buckling loads of FG sandwich beams with homogeneous hardcore for various boundary conditions (L/h=20).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	208.9510	208.9510	208.9510	208.9510	208.9510	208.9510	208.9510
	0.5	117.3030	126.5080	131.1240	134.4810	140.5450	147.1040	179.2350
	1	81.9927	92.6741	98.3880	102.6650	110.4830	119.4220	164.9490
	2	56.2773	65.8489	71.8900	76.1020	84.7291	94.9563	151.2500
	5	42.0775	48.0070	53.7820	56.4958	65.0007	74.8903	138.3880
	10	39.4930	43.3233	48.8510	50.3811	58.5607	67.6270	132.9170
S-S	0	53.2364	53.2364	53.2364	53.2364	53.2364	53.2364	53.2364
	0.5	29.7175	32.2629	33.2376	34.0862	35.6405	37.3159	45.5742
	1	20.7212	23.4211	24.8796	25.9588	27.9540	30.2307	41.9004
	2	14.1973	16.6050	18.1404	19.3116	21.3927	23.9900	38.3831
	5	10.6171	12.0883	13.5523	14.2284	16.3834	18.8874	35.0856
	10	9.9847	10.9075	12.3084	12.6819	14.7525	17.0443	33.6843
C-F	0	13.3730	13.3730	13.3730	13.3730	13.3730	13.3730	13.3730
	0.5	7.4543	8.0405	8.3385	8.5512	8.9422	9.3634	11.4424
	1	5.1944	5.8713	6.2378	6.5083	7.0096	7.5815	10.5174
	2	3.5574	4.1603	4.5457	4.8110	5.3615	6.0134	9.6321
	5	2.6605	3.0275	3.3948	3.5637	4.1043	4.7323	8.8025
	10	2.5032	2.7317	3.0832	3.1759	3.6952	4.2698	8.4500

Table 11: Non-dimensional critical buckling loads of FG sandwich beams with homogeneous softcore for various boundary conditions (L/h=5).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	28.0272	28.0272	28.0272	28.0272	28.0272	28.0272	28.0272
	0.5	77.9961	68.9991	66.5929	63.1171	59.9391	56.1398	42.8635
	1	99.9200	86.3184	83.0703	77.1502	72.7727	66.4523	48.5321
	2	120.5040	103.3130	99.2457	90.9271	85.3675	76.0626	53.3211
	5	138.5140	119.9170	115.0350	105.0250	98.2566	85.8462	57.3987
	10	145.3360	127.1130	121.8940	111.5780	104.2590	90.5815	59.0677
S-S	0	8.9519	8.9519	8.9519	8.9519	8.9519	8.9519	8.9519
	0.5	28.4280	25.9503	24.5423	24.0540	22.3861	21.3821	15.1589
	1	36.2103	32.8974	30.9311	30.2449	27.8873	26.4801	17.9093
	2	42.4501	38.8589	36.4842	35.7058	32.7904	31.0152	20.4222
	5	46.6504	43.5338	40.9813	40.3235	37.0356	35.0357	22.6881
	10	47.7825	45.1141	42.6000	42.0693	38.7018	36.6874	23.6329
C-F	0	2.4057	2.4057	2.4057	2.4057	2.4057	2.4057	2.4057
	0.5	8.0335	7.4323	6.9678	6.9293	6.3920	6.1578	4.2316
	1	10.2081	9.4771	8.8199	8.8253	8.0532	7.7858	5.0876
	2	11.8304	11.1295	10.3464	10.4268	9.4787	9.2209	5.8960
	5	12.7779	12.2818	11.4718	11.6547	10.6163	10.4208	6.6450
	10	12.9729	12.6107	11.8343	12.0619	11.0206	10.8716	6.9614

Table 12: Non-dimensional critical buckling loads of FG sandwich beams with homogeneous softcore for various boundary conditions (L/h=20).

BC	k	1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1	1-8-1
C-C	0	38.4910	38.4910	38.4910	38.4910	38.4910	38.4910	38.4910
	0.5	128.5360	118.9170	111.4880	110.8690	102.2750	98.5240	67.7055
	1	163.3300	151.6340	141.1240	141.2050	128.8570	124.5720	81.4018
	2	189.2860	178.0720	165.5480	166.8290	151.6660	147.5350	94.3363
	5	204.4470	196.5090	183.5550	186.4750	169.8680	166.7330	106.3210
	10	207.5660	201.7710	189.3530	192.9900	176.3370	173.9460	111.3830
S-S	0	9.8067	9.8067	9.8067	9.8067	9.8067	9.8067	9.8067
	0.5	33.2187	30.8546	28.8514	28.8167	26.5120	25.6086	17.4355
	1	42.1810	39.4124	36.5675	36.8445	33.5153	32.5803	21.0698
	2	48.7215	46.2035	42.8267	43.5408	39.4599	38.7192	24.5356
	5	52.3655	50.7608	47.3056	48.5163	44.0843	43.7637	27.7736
	10	53.0331	51.9804	48.6930	50.0902	45.6732	45.6040	29.1471
C-F	0	2.4635	2.4635	2.4635	2.4635	2.4635	2.4635	2.4635
	0.5	8.3754	7.7874	7.2768	7.2764	6.6897	6.4654	4.3919
	1	10.6331	9.9519	9.2260	9.3130	8.4644	8.2402	5.3144
	2	12.2712	11.6613	10.8006	11.0062	9.9665	9.8018	6.1964
	5	13.1722	12.7966	11.9184	12.2538	11.1270	11.0792	7.0223
	10	8.3754	7.7874	7.2768	7.2764	6.6897	6.4654	4.3919

Table 13: The first four non-dimensional natural frequencies of FG sandwich beams with homogeneous hardcore for various boundary conditions (L/h=5).

BC	k	2-2-1				1-2-1			
		ω_1	ω_2	ω_3	ω_4	ω_1	ω_2	ω_3	ω_4
C-C	0	10.0678	24.1007	30.2391	39.0057	10.0678	24.1007	30.2391	39.0057
	0.2	9.5047	22.9421	29.4509	37.4030	9.5616	23.0662	29.5874	37.5844
	0.5	8.8648	21.5905	28.5849	35.5049	8.9942	21.8773	28.8834	35.9269
	1	8.1554	20.0538	27.6279	33.3144	8.3705	20.5397	28.1196	34.0378
	2	7.4105	18.3996	26.5629	30.9184	7.7114	19.0954	27.2873	31.9753
	5	6.7188	16.8234	25.3675	28.5831	7.0691	17.6608	26.3756	29.9060
	10	6.4641	16.2276	24.7734	27.6708	6.8087	17.0706	25.9312	29.0452
S-S	0	5.1528	15.1167	17.8812	34.2097	5.1528	15.1167	17.8812	34.2097
	0.2	4.7951	14.7174	16.8307	32.5001	4.8287	14.7909	16.9320	32.6813
	0.5	4.4051	14.2575	15.6631	30.5118	4.4798	14.4389	15.8764	30.9311
	1	3.9896	13.6661	14.4555	28.2591	4.1105	14.0571	14.7246	28.9684
	2	3.5692	12.6881	13.5145	25.8449	3.7334	13.5149	13.6410	26.8553
	5	3.1928	11.5025	12.8112	23.5593	3.3771	12.3428	13.1852	24.7627
	10	3.0588	11.0454	12.4956	22.7013	3.2356	11.8695	12.9631	23.9043
C-F	0	1.8952	10.2454	15.1167	24.4965	24.4965	1.8952	10.2454	15.1167
	0.2	1.7584	9.6237	14.7227	23.2176	23.2176	1.7710	9.6851	14.7909
	0.5	1.6108	8.9267	14.2902	21.7469	21.7469	1.6384	9.0656	14.4389
	1	1.4549	8.1639	13.8123	20.0962	20.0962	1.4992	8.3933	14.0571
	2	1.2986	7.3726	13.2806	18.3399	18.3399	1.3582	7.6906	13.6410
	5	1.1597	6.6463	12.6835	16.6845	16.6845	1.2257	7.0127	13.1852
	10	1.1106	6.3819	12.3867	16.0653	16.0653	1.1734	6.7397	12.9631

Table 14: The first four non-dimensional natural frequencies of FG sandwich beams with homogeneous softcore for various boundary conditions (L/h=5).

BC	k	2-2-1				1-2-1			
		ω_1	ω_2	ω_3	ω_4	ω_1	ω_2	ω_3	ω_4
C-C	0	5.2311	12.5225	15.7120	20.2670	5.2311	12.5225	15.7120	20.2670
	0.2	6.4965	15.1062	18.4477	23.9261	6.3873	14.8367	18.0374	23.4831
	0.5	7.2456	16.6065	20.6338	26.0691	7.0539	16.0881	19.9545	25.1877
	1	7.8056	17.7749	22.4543	27.7983	7.5187	16.9638	21.5859	26.4095
	2	8.2835	18.8484	24.0103	29.4552	7.8960	17.7202	23.0036	27.5139
	5	8.7255	19.9408	25.3647	31.2259	8.2498	18.5077	24.2544	28.7293
	10	8.9195	20.4603	25.9256	32.1033	8.4162	18.9119	24.7768	29.3794
S-S	0	2.6773	7.8545	9.2909	17.7751	2.6773	7.8545	9.2909	17.7751
	0.2	3.5205	9.2019	11.7128	21.6189	3.4711	9.0170	11.5157	21.2465
	0.5	4.0504	10.2693	13.1601	23.8406	3.9921	9.9753	12.8310	23.1294
	1	4.4270	11.1581	14.2226	25.5494	4.3663	10.7909	13.7464	24.4327
	2	4.7047	11.9220	15.0971	27.0940	4.6459	11.4996	14.4701	25.5391
	5	4.9038	12.5955	15.8642	28.6344	4.8564	12.1248	15.1156	26.6663
	10	4.9700	12.8788	16.1847	29.3549	4.9326	12.3860	15.4047	27.2351
C-F	0	0.9847	5.3234	7.8545	12.7282	12.7282	0.9847	5.3234	7.8545
	0.2	1.3116	6.7549	9.2246	15.6609	15.6609	1.2933	6.6509	9.0170
	0.5	1.5211	7.6171	10.3193	17.3931	17.3931	1.5020	7.4544	9.9753
	1	1.6691	8.2464	11.2290	18.7017	18.7017	1.6539	8.0182	10.7909
	2	1.7745	8.7542	12.0056	19.8353	19.8353	1.7662	8.4585	11.4996
	5	1.8444	9.1855	12.6816	20.9068	20.9068	1.8466	8.8373	12.1248
	10	1.8652	9.3606	12.9618	21.3884	21.3884	1.8734	9.0006	12.3860

CAPTIONS OF FIGURES

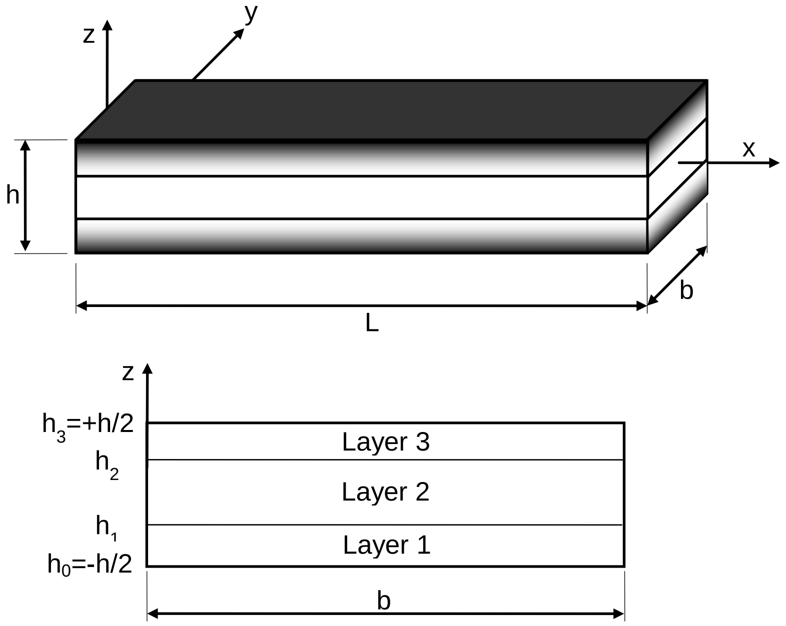
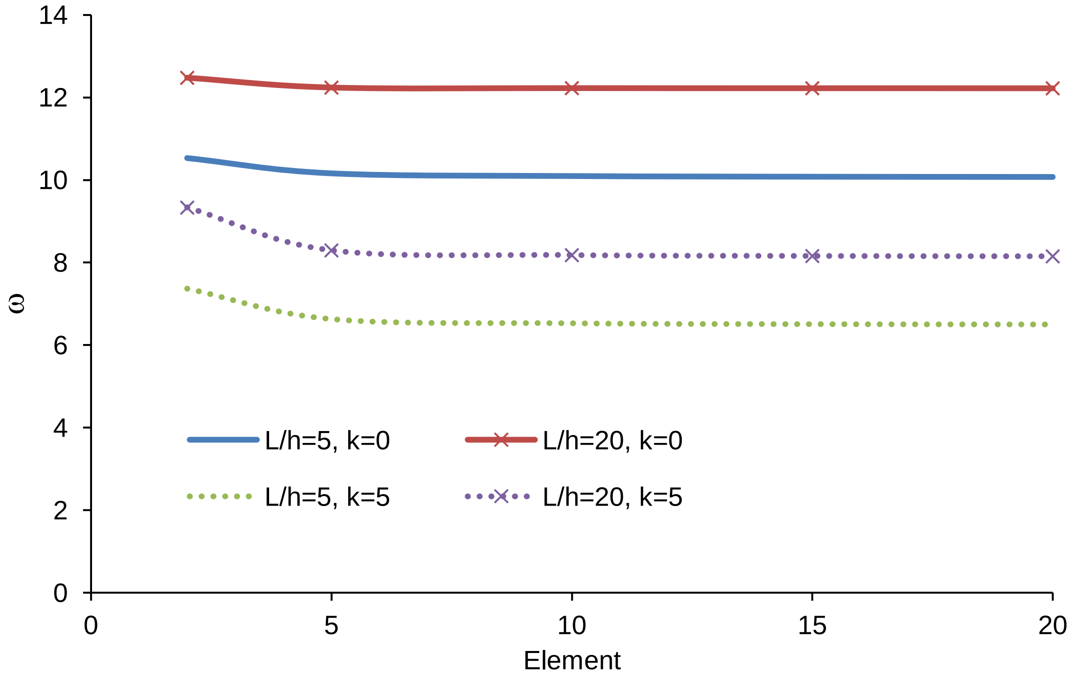
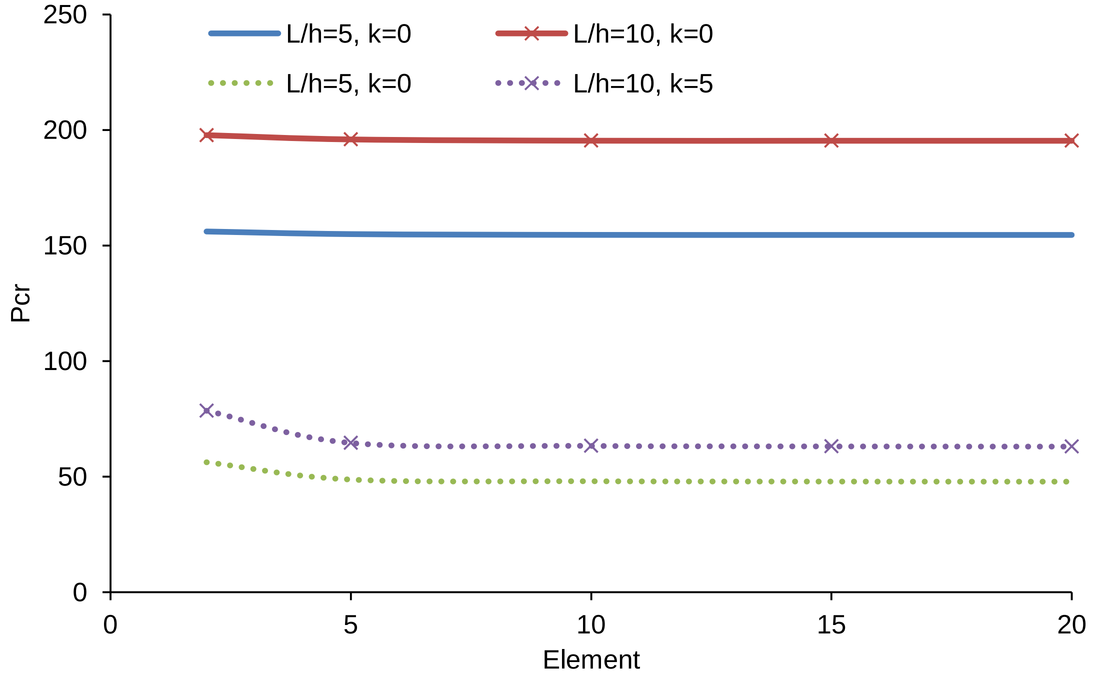


Figure 1: Geometry and coordinate of a FG sandwich beam

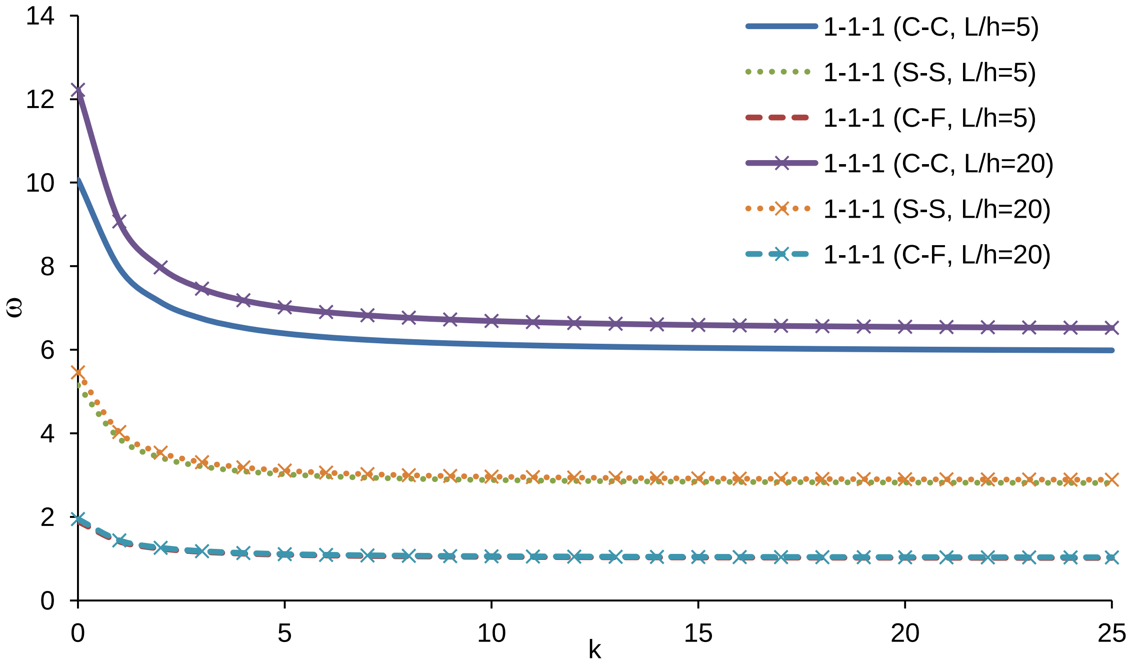


a. Fundamental natural frequencies

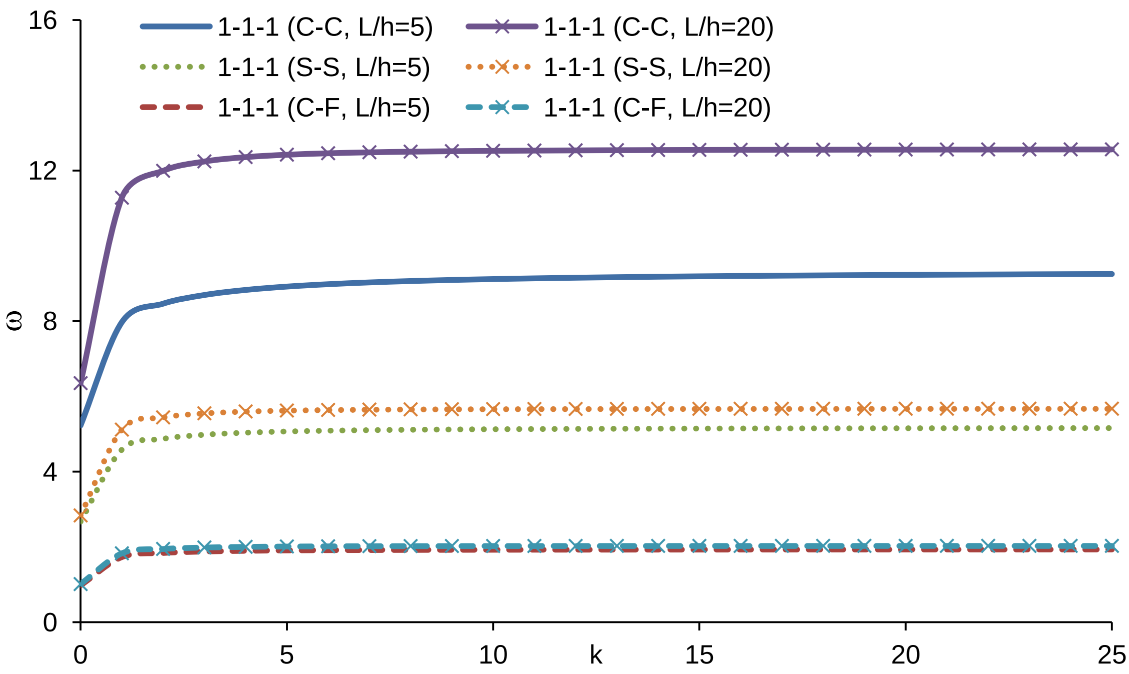


b. Critical buckling loads

Figure 2: A mesh convergence study for clamped-clamped FG beams (L/h=5, 10 and 20).

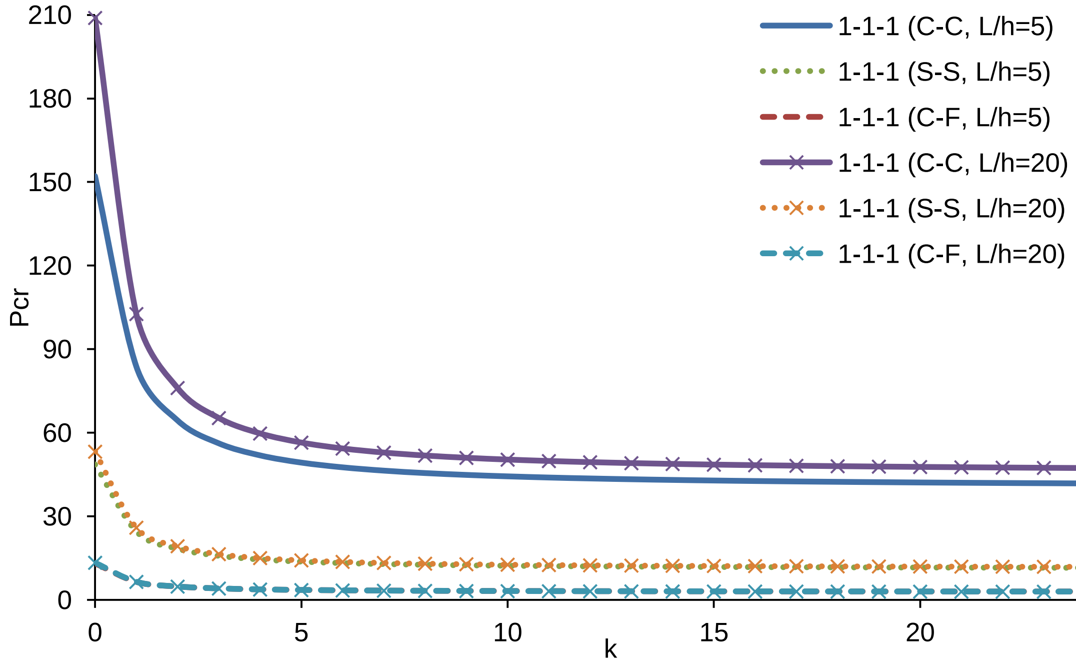


a. Homogeneous hardcore

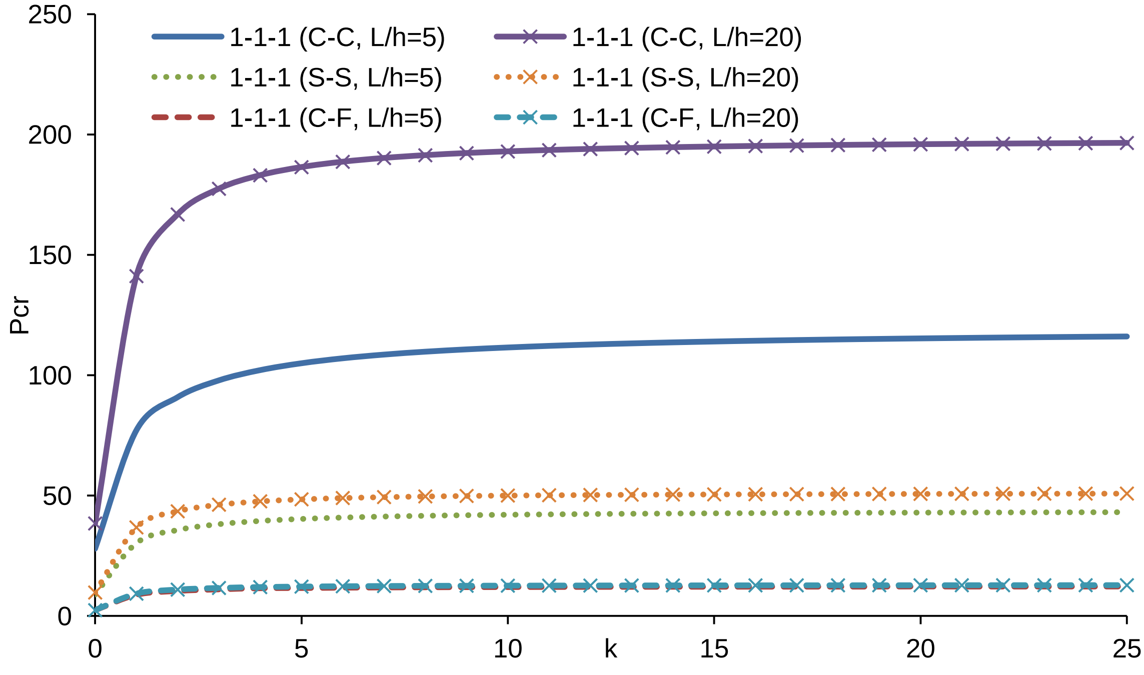


b. Homogeneous softcore

Figure 3: Effect of the power-law index on the non-dimensional fundamental natural frequencies of (1-1-1) FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions (L/h=5 and 20).

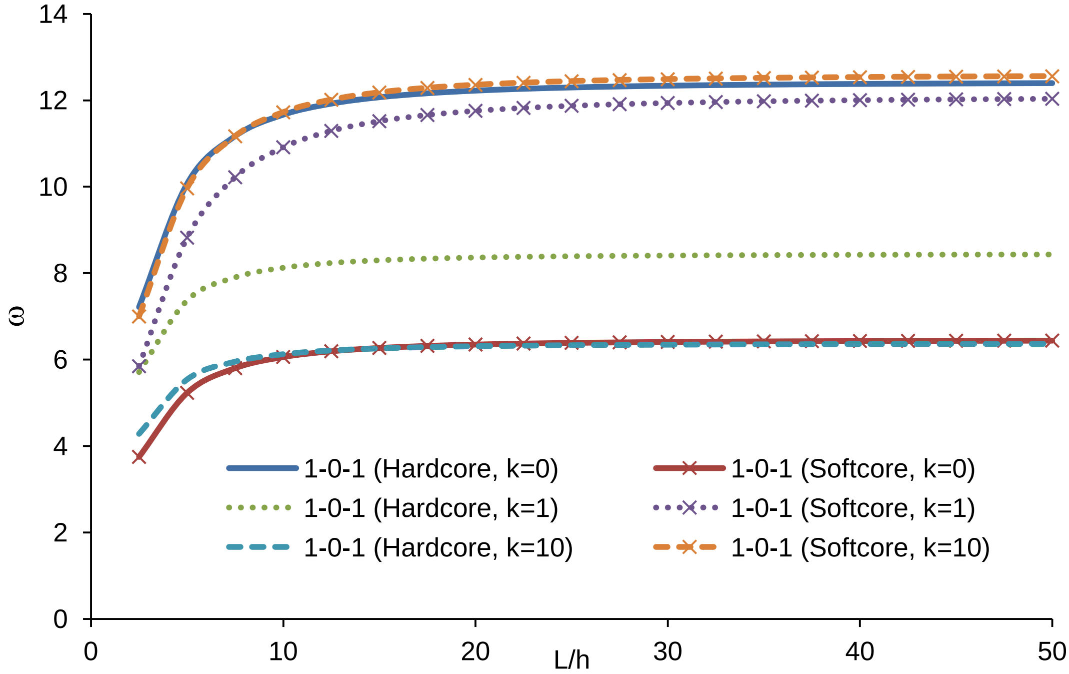


a. Homogeneous hardcore

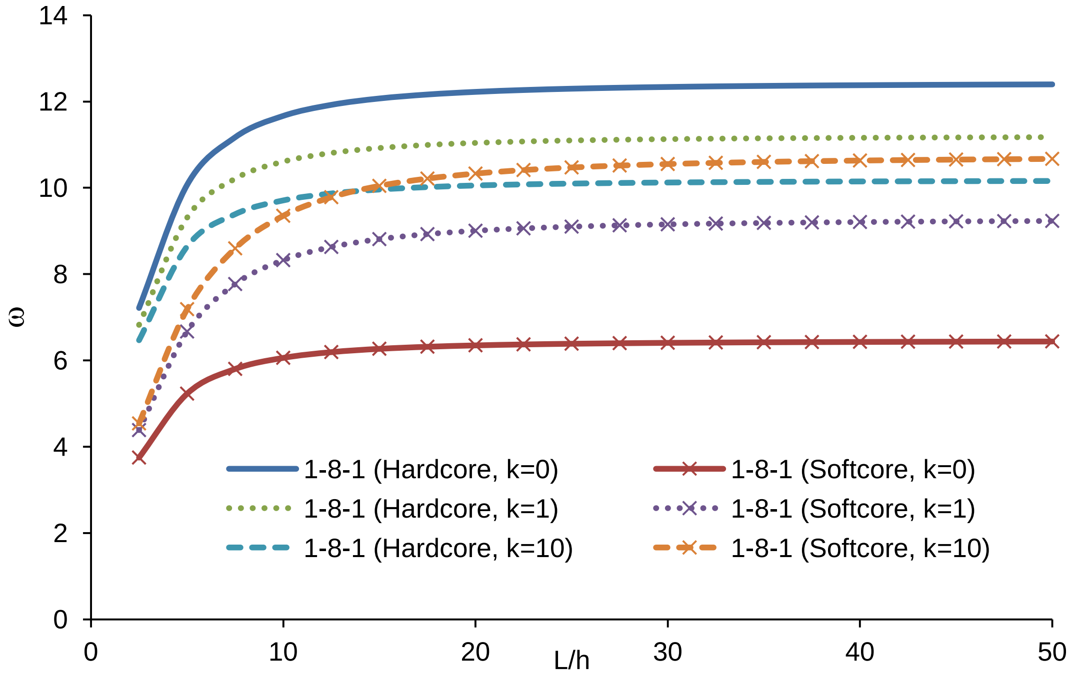


b. Homogeneous softcore

Figure 4: Effect of the power-law index on the non-dimensional critical buckling loads of (1-1-1) FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions (L/h=5 and 20).

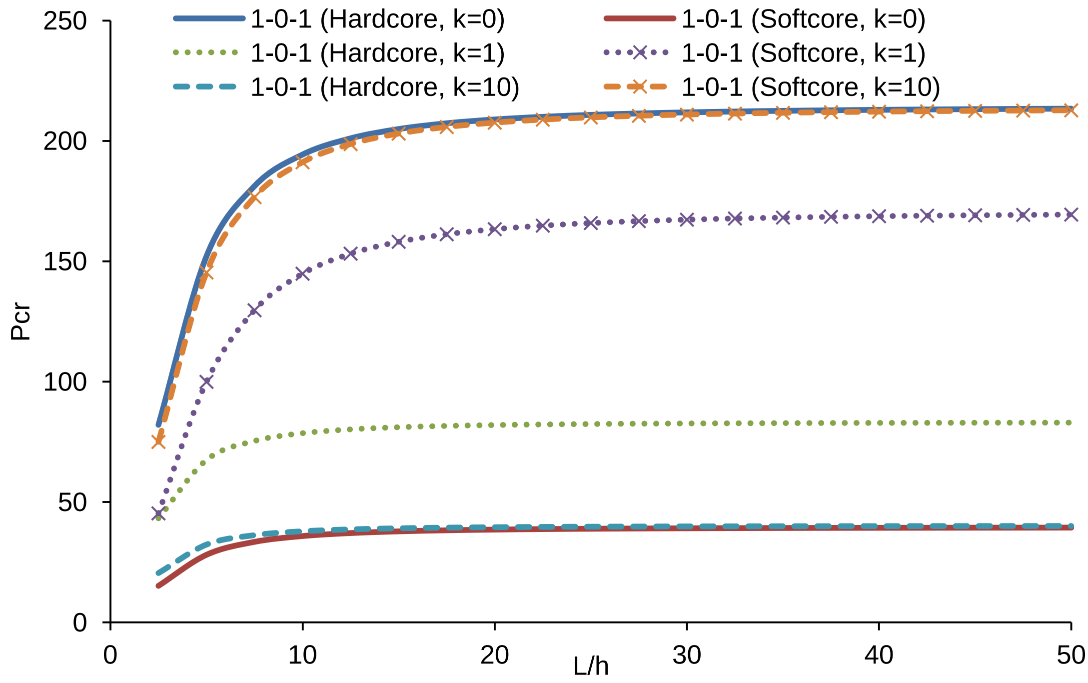


a. (1-0-1) sandwich beam with power-law index k=5

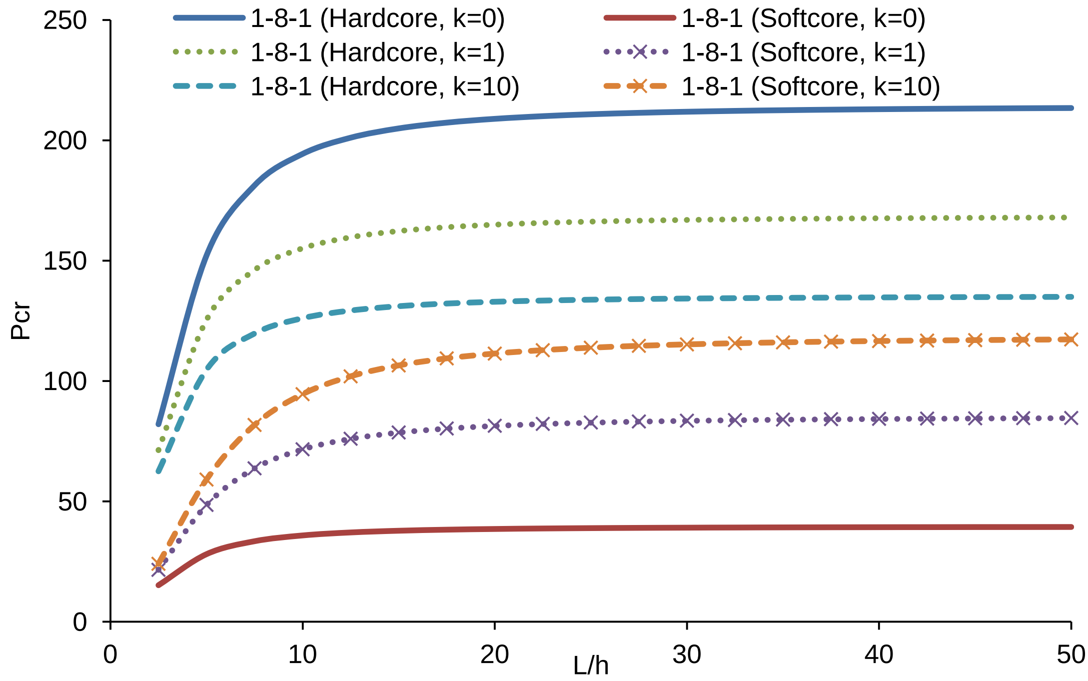


b. (1-8-1) sandwich beam with power-law index k=5

Figure 5: Effect of span-to-height ratio on the non-dimensional fundamental natural frequencies of (1-0-1) and (1-8-1) clamped-clamped FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions.

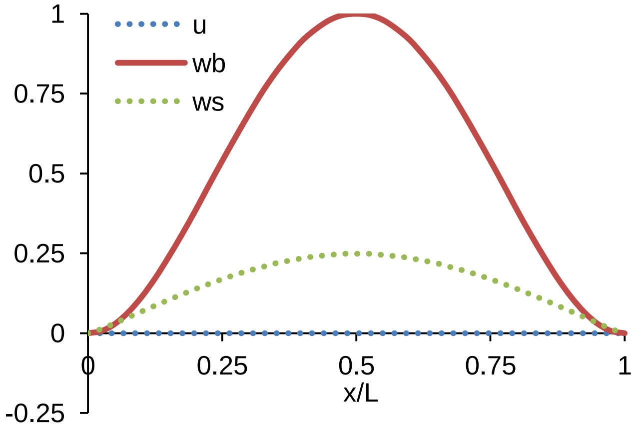


a. (1-0-1) sandwich beam with power-law index k=5

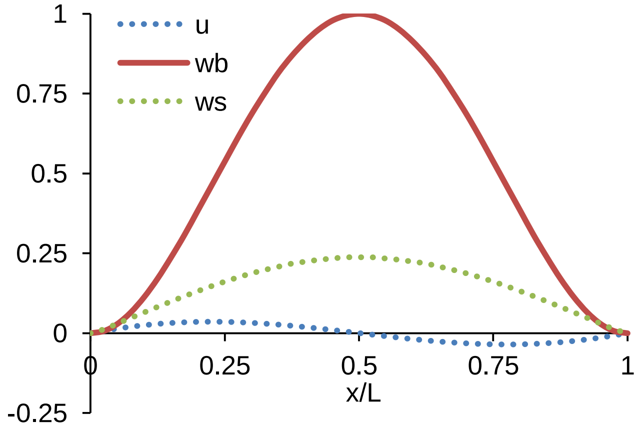


b. (1-8-1) sandwich beam with power-law index k=5

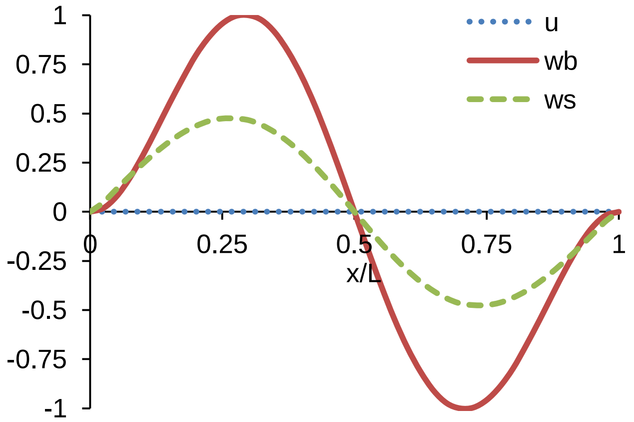
Figure 6: Effect of span-to-height ratio on the non-dimensional critical buckling loads of (1-0-1) and (1-8-1) clamped-clamped FG sandwich beams with homogeneous hardcore and softcore for various boundary conditions.



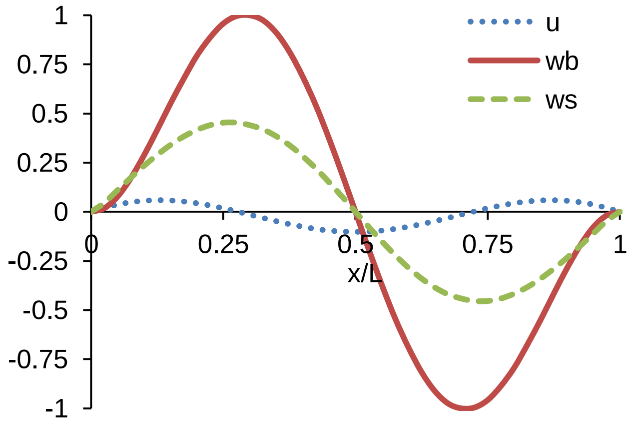
a. Fundamental mode shape ω_1 = 7.0691



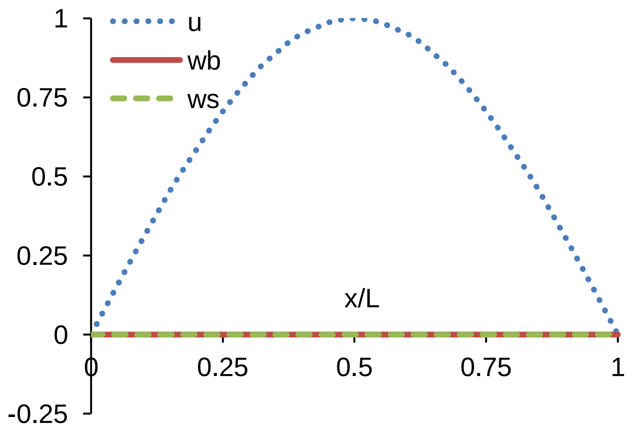
a. Fundamental mode shape ω_1 =6.7188



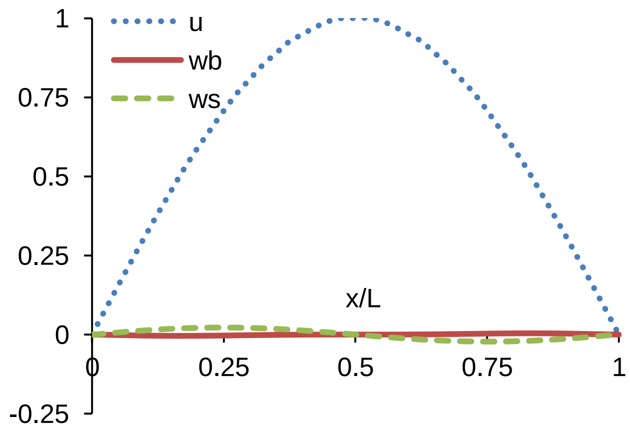
b. Second mode shape ω_2 = 17.6608



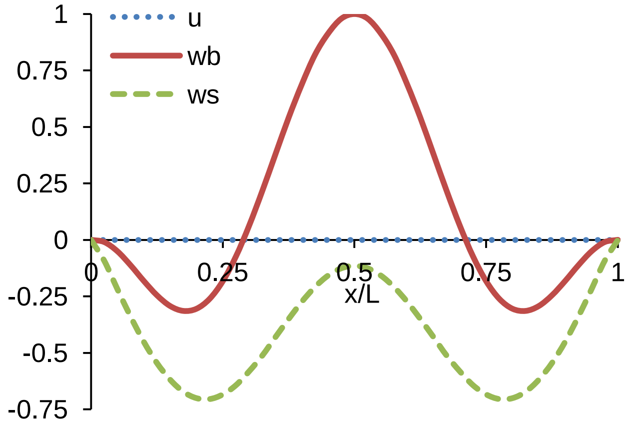
b. Second mode shape ω_2 = 16.8234



c. Third mode shape ω_3 = 26.3756

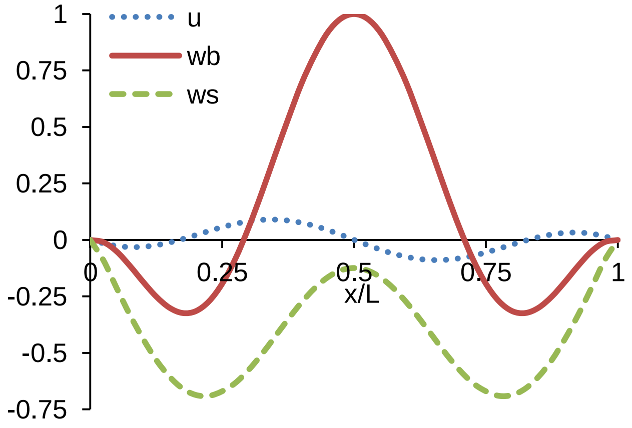


c. Third mode shape ω_3 = 25.3675



d. Fourth mode shape ω_4 = 29.9060

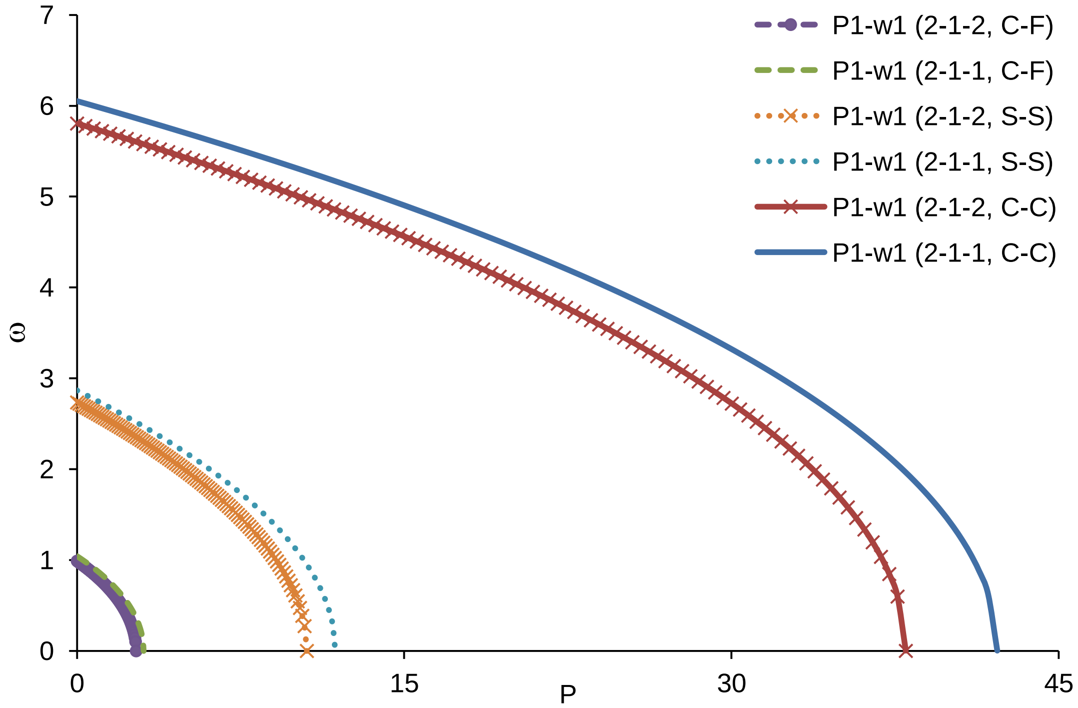
(1-2-1) FG sandwich beam (k=5)



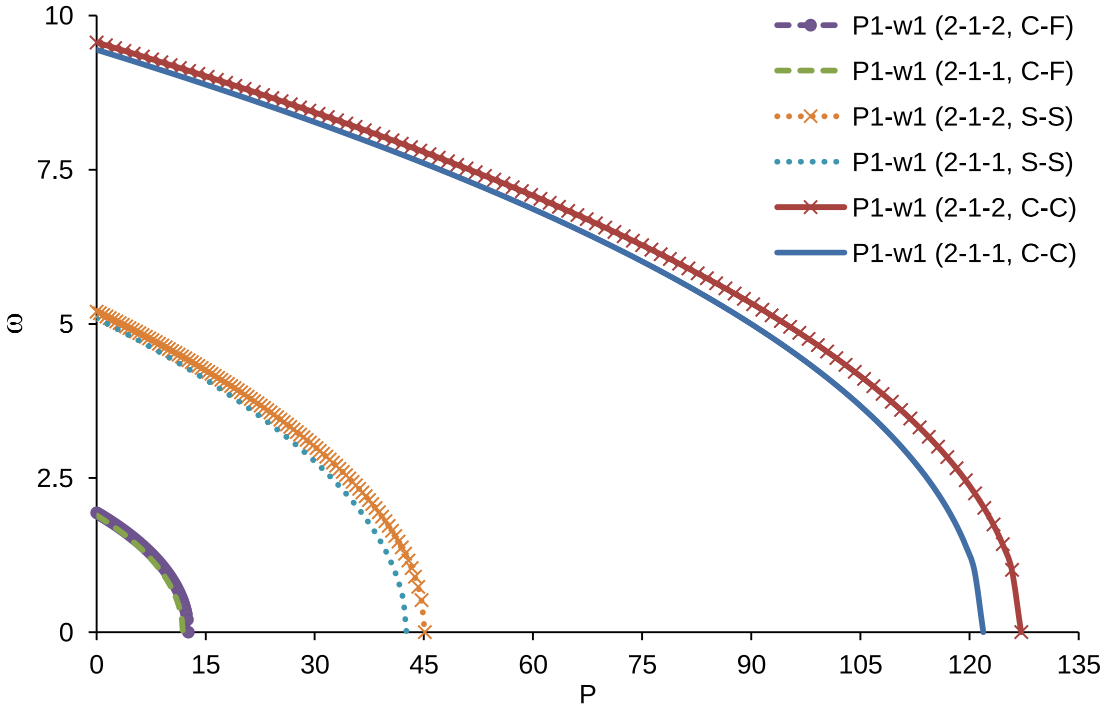
d. Fourth mode shape ω_4 = 28.5831

(2-2-1) FG sandwich beam (k=5)

Figure 7: Vibration mode shapes of (1-2-1) and (2-2-1) clamped-clamped FG sandwich beam with homogeneous hardcore (L/h=5)



a. Homogeneous hardcore with power-law index k=10



b. Homogeneous softcore with power-law index k=10

Figure 8: Effect of the axial force on the fundamental natural frequencies of (2-1-2) and (2-1-1) FG sandwich beams (L/h=5) with homogeneous hardcore and softcore for various boundary conditions.