Multi-Objective Design Optimisation of Standalone Hybrid Wind-PV-Diesel Systems under Uncertainties

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Abstract
Optimal design of a standalone wind-PV-diesel hybrid system is a multi-objective optimisation problem with conflicting objectives of cost and reliability. Uncertainties in renewable resources, demand load and power modelling make deterministic methods of multi-objective optimisation fall short in optimal design of standalone hybrid renewable energy systems (HRES). Firstly, deterministic methods of analysis, even in the absence of uncertainties in cost modelling, do not predict the levelised cost of energy accurately. Secondly, since these methods ignore the random variations in parameters, they cannot be used to quantify the second objective, reliability of the system in supplying power. It is shown that for a given site and uncertainties profile, there exist an optimum margin of safety, applicable to the peak load, which can be used to size the diesel generator towards designing a cost-effective and reliable system. However, this optimum value is problem dependent and cannot be obtained deterministically. For two design scenarios, namely, finding the most reliable system subject to a constraint on the cost and finding the most cost-effective system subject to constraints on reliability measures, two algorithms are proposed to find the optimum margin of safety. The robustness of the proposed design methodology is shown through carrying out two design case studies.

Keywords: design under uncertainties; hybrid renewable energy systems; wind-PV-diesel; probabilistic reliability analysis; multiobjective optimisation

1 Introduction
In optimal design of standalone hybrid renewable energy systems (HRES), reliability of the system in supplying power for a demand load is as important as the levelised cost of energy (LCE) produced by the system. Reliability of a standalone HRES in supplying power depends on various parameters, including, system configuration (e.g. wind-PV-battery, wind-diesel, etc), size of its components, reliability of each component in terms of operation and the availability of renewable resources. The availability of resources has the major influence on the reliability of a standalone HRES as stochastic nature of renewable resources imposes a great deal of uncertainty to the system operation and the power produced. Stochastic nature of renewable resource makes the reliability analysis of a standalone HRES impossible without employing probabilistic methods of analysis. In other words, multi-objective optimisation of standalone HRES (with cost and reliability as two objectives) cannot be performed deterministically.

Results of probabilistic analyses have random errors that can be reduced by increasing the size of sampling space. In order to achieve a desired level of accuracy in the results of probabilistic methods of analysis high computational time is required. This becomes a major concern within a design process, as evaluation of design candidates with respect to their cost and reliability becomes highly time-consuming. In practice, to circumvent this problem, adopting a deterministic approach, design of standalone HRES is carried out for a worst-case-scenario, while applying a load factor on the demand load. All calculations are based on the averaged values and the stochastic nature of demand load and renewable resources as well as the possible errors in the results due to employing low fidelity models are ignored. No reliability measure is calculated as part of the design candidate assessment. It is assumed that a suitable selection of the worst-case-scenario and safety factors will lead to reliable solutions. In fact, the multi-objective optimisation problem with two objectives of reliability and cost is reduced to a single-objective optimisation problem with the objective of cost.
only. In practice, normally, the size of the storage or backup/auxiliary components are determined based on a suitable worst-case-scenario to achieve a level of confidence in the expected power supply, while the remaining components are optimised for minimising the cost. After sizing the storage or backup/auxiliary components a single-objective optimisation search can be carried out to find the optimum size of the renewable components. Most of the literature on design of standalone HRES adopt this approach; for instance see [1-10].

In deterministic optimal sizing of a standalone wind-PV-diesel hybrid system, the margin of safety applied on the demand load affects the nominal size of the diesel generator and consequently the reliability of the power supply and the levelised cost of produced energy. Adopting high-enough margins of safety leads to reliable systems. However, as mentioned above since in deterministic design methods no actual reliability measure is calculated as part of the design candidate assessment, these methods cannot be used for quantifying the optimum value for margin of safety. A procedure including both deterministic and probabilistic analyses is required to find the margin of safety which corresponds to a desired reliability with minimal cost.

More recently, recognising the shortfall of deterministic methods in design of reliable and cost-effective standalone HRES, development of robust nondeterministic design methods has received increasing attention from the research community [11, 12]. The aim of the present study is to develop a robust method of design under uncertainties for wind-PV-diesel configuration with minimal number of probabilistic analysis. Section 2 begins with definition of reliability measures used in this study, and then elaborates on power and cost modelling. Section 3 explains the fundamentals of the proposed design methodology and its development steps. Section 4 details two algorithms proposed for performing two design scenarios and the results of case studies delivered using the proposed design methodology.

2 Reliability assessment and system modelling

2.1 Reliability assessment measures
Performance of a standalone HRES in supplying power can be evaluated against different assessment criteria, amongst them total unmet load, blackout duration distribution and the mean-time between failures. For a standalone HRES the total unmet load is defined as:

$$U_i = \int_0^T (L(t) - P_a(t))dt$$

where, $P_a$ and $L$ are, respectively, the usable available power and the demand load ($0 \leq P_a \leq L$). Usable available power is defined as:

$$P_a = \min\{P_{i,a}, L\}$$

in which, $P_{i,a}$ stands for the total renewable and non-renewable available power. Using hourly-averaged load ($\bar{L}_h$) and hourly-averaged useable available power ($\bar{P}_{h,a}$), and a period of analysis of $T = 1$ year = 8760 h, Equations (1) can be rewritten as

$$U = \sum_{i=1}^{8760} (\bar{L}_h - \bar{P}_{h,a})$$

Total, maximum and average blackout durations are three parameters which indicate the system downtime periods due to power deficiency irrespective of the amount of power deficiency. In
contrast to the unmet load, assessment of design candidates based on blackout duration allows performing customer-need driven designs. Using hourly-averaged data, total blackout duration is defined as:

$$BO_i = \sum_{i=1}^{8760} \left[ \left( t - \frac{P_{h,a}}{L_h} \right) \right]$$

where, pair of square brackets stands for the integer value function. The information that can be extracted from the blackout distribution, such as the maximum blackout duration (the longest continuous blackout) $$BO_{max}$$ and the average blackout duration $$BO_{av}$$ (the average duration of each blackout), also can play an important role in evaluation of the system performance.

Mean time between failures (MTBF) is defined as the duration of the successful system operation over a period of time divided by the number of failures during that period. If the successful system operation is defined as the case when available usable power is greater than or equal to the load ($$P_a \geq L$$), using hourly-averaged quantities, the MTBF can be defined as:

$$MTBF = \frac{8760 - \sum_{i=1}^{8760} \left[ \left( t - \frac{P_{h,a}}{L_h} \right) \right]}{n_{fail}}$$

where $$n_{fail}$$ is the number of blackout occurrences during period $$T = 8760h$$.

### 2.2 Power modelling and dispatch strategies

The power produced by a wind turbine is given by:

$$P_{WT} = \frac{1}{2} \rho V_{hub}^3 A_{WT} C_p \eta_{EG}$$

in which $$\rho$$ is the air density, $$V_{hub}$$ is the wind speed at hub elevation, $$A_{WT}$$ is the rotor area, $$\eta_{EG}$$ is the overall efficiency of the electrical components and the gearbox, and $$C_p$$ is the rotor power coefficient given by:

$$C_p = -2.025 \times 10^{-7} V_{hub}^6 + 1.926 \times 10^{-5} V_{hub}^5$$

$$- 7.421 \times 10^{-4} V_{hub}^4 + 1.483 \times 10^{-2} V_{hub}^3 - 0.162 V_{hub}^2 + 0.887 V_{hub} - 1.508$$

This model is extracted via curve fitting and using the power coefficient data of about 60 wind turbines within the range of 10-500 kW. The wind turbines used for developing this model are of both types of constant and variable speeds and also both types of pitch controlled and stall regulated. This model has a maximum relative error of 7% for the range of 3 \( \leq V_{hub} \leq 25 \text{ m/s} \).

Given wind speed $$V_{ref}$$ at elevation $$h_{ref}$$, the wind speed at the hub elevation can be calculated by the logarithmic law:

$$V_{hub} = V_{ref} \ln \left( \frac{h_{hub}}{z_0} \right) / \ln \left( \frac{h_{ref}}{z_0} \right)$$
in which, \( z_0 \) stands for the site surface roughness length. The hub height \( h_{hub} \) depends on the size of the wind turbine, which is unknown prior to the design. For small to medium size wind turbines the hub height can be estimated via the rule of thumb:

\[
h_{hub} = \max \{ h_c + R, 2R \}
\] (9)

where \( h_c \) is the minimum blade tip-ground clearance and \( R \) is the rotor radius.

Power produced by PV panels is given by

\[
P_{PV} = IA_{pv}\eta_{pv}
\] (10)

in which, \( I \) stands for the solar irradiance, \( A_{pv} \) is the PV panel area and \( \eta_{pv} \) is the overall PV unit efficiency.

In this study, using hourly-averaged data, the following diesel dispatch strategy is used:

- Excess power \( \bar{P}_{h,R} - \bar{L}_h \geq 0 \): No need for diesel generator power \( \bar{P}_{h,D} = 0 \).
- Power deficit less than the nominal power of the diesel generator \( 0 \leq \bar{L}_h - \bar{P}_{h,R} \leq P_{D,nom} \): The power deficit is compensated by the diesel generator \( \bar{P}_{h,D} = \bar{L}_h - \bar{P}_{h,R} \).
- Power deficit greater than the nominal power of the diesel generator \( \bar{L}_h - \bar{P}_{h,R} > P_{D,nom} \):
  - Blackout; The diesel generator works at its nominal power \( \bar{P}_{h,D} = P_{D,nom} \).

Parameters \( \bar{P}_{h,D} \) and \( \bar{P}_{h,R} \), respectively, stand for the hourly-averaged diesel and renewable power and \( P_{D,nom} \) stands for the diesel generator nominal power.

### 2.3 Cost modelling

Using levelised cost of energy allows design alternatives to be compared when different scales of operation and investment exist. For systems with constant annual output over the life-span of the system LCE, \( C_t \), can be calculated as follows:

\[
C_t = \frac{C_a}{P_t}
\] (11)

where \( P_t \) denotes the annual energy output and \( C_a \) stands for the annualised cost. Since the power produced by a standalone HRES excess to the demand load is dumped, in Equation (11), the usable amount of produced energy should be used instead of the system total energy output:

\[
P_t = \sum_{j=1}^{8760} \min \{ \bar{P}_h, \bar{L}_h \}_j
\] (12)

The annualised cost \( C_a \) is given by [13]:
\( C_a = C_i UCRF \) (13)

Parameters \( C_i \) and \( UCRF \) in Equation (12) are, respectively, total life-span cost (TLSC) and uniform capital recovery factor, given by:

\[
UCRF = \frac{d(1+d)^{N_s}}{(1+d)^{N_s} - 1}
\] (14)

in which, \( d \) is the annual discount rate and \( N_s \) represents the life-span of the system in years. Assuming there is no escalation in the price of the components, the formula for calculating the present value of TLSC is as follows:

\[
C_i = \sum_{j=0}^{N_s} \frac{C_j}{(1+d)^j}
\] (15)

where \( C_j \) is the cost in year \( j \) including capital cost \( C_c \), fixed operation and maintenance (O&M) costs \( C_{O&M,F} \), variable O&M costs \( C_{O&M,V} \), and the replacement cost \( C_r \). Case \( j = 0 \) represents the beginning of the life span with its corresponding cost, \( C_0 \), standing for the capital cost only. The capital cost of the system (including installation cost) is given by:

\[
C_c = \sum_{comp} C_{u,comp} S_{comp} (1 + \alpha_{ins,comp})
\] (16)

in which \( S \) is the size of the component, \( C_u \) is the unit cost and \( \alpha_{ins} \) is the installation cost as a fraction of the total cost of the component. Cost estimation at conceptual design phase of HRES can be based on either cost per unit of nominal power production or cost per unit of size. To be consistent with the power models, for wind turbine and PV array the cost per unit size is used whilst for the diesel generator the cost per nominal output power is used. The O&M cost includes fixed and variable parts:

\[
C_{O&M} = \sum_{comp} C_{O&M,F,comp} + \sum_{comp} C_{O&M,V,comp}
\] (17)

The fixed part can be represented by

\[
C_{O&M,F,comp} = \alpha_{O&M,comp} C_{c,comp}
\] (18)

The variable part of the O&M cost for wind turbine and PV panel is zero. Using hourly-averaged data, the annual variable part of the O&M cost for diesel generator (the cost of consumed fuel) is given by [14]

\[
C_{O&M,V,D} = \frac{0.246 \sum_{i=1}^{8760} P_{h,D,i} + 0.08145 P_{D,nom} T_D}{1000} C_{fuel}
\] (19)

in which \( T_D \) stands for the total number of hours that the diesel generator operates, \( \bar{P}_{h,D} \) is the hourly-averaged diesel power and \( C_{fuel} \) is the fuel price.
For each component the replacement cost is given by:

\[ C_r = \sum_{\text{comp}} n_{r,\text{comp}} C_{r,\text{comp}} \]  \hspace{1cm} (20)

where \( n_r \) is the number of replacements during the life-span of the system. Having the nominal life of system \( N_S \), wind turbine \( N_{\text{nom,WT}} \) and PV panel \( N_{\text{nom,PV}} \) in years and the nominal life of diesel generator \( N_{\text{nom,D}} \) in hours of operation, the following equations can be used to find the number of replacements of these components.

\[ n_{r,\text{comp}} = \left[ \frac{N_S}{N_{\text{nom,comp}}} \right] \]  \hspace{1cm} \text{for wind turbine and PV panel} \hspace{1cm} (21)

\[ n_{r,D} = \left[ \frac{N_S T_D}{N_{\text{nom,D}}} \right] \]  \hspace{1cm} \text{for diesel generator} \hspace{1cm} (22)

In this study the following parameters are used: air density \( \rho = 1.225 \text{kg/m}^3 \); wind turbine electrical and gearbox efficiency \( \eta_{EG} = 0.9 \); surface roughness length \( z_0 = 0.03 \); minimum blade tip-ground clearance \( h_c = 8 \text{m} \); overall PV unit efficiency \( \eta_{PV} = 12\% \); the life-span of the system \( N = 20 \text{years} \) and the real discount rate \( d = 4\% \). Table (1) summarises other parameters required for the cost analysis.

<table>
<thead>
<tr>
<th>Table 1-Cost modelling parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind turbine</td>
</tr>
<tr>
<td>( S )</td>
</tr>
<tr>
<td>( C_u )</td>
</tr>
</tbody>
</table>
| \( \alpha_{\text{ini}} \) | 0.2 | 0.4 | 0.4$/W_{\text{nom}}$
| \( \alpha_{\text{D&M}} \) | 0.03 | 0.01 | 0.15 |
| \( N_{\text{nom}} \) | 20 years | 20 years | 15000 hours |
| \( C_{\text{O&M,V}} \) | 0 | 0 | See Equation (19) |
| \( C_{\text{fuel}} \) | 1$/l$

3 Design methodology development

Probabilistic analyses are highly time-consuming. A robust design method must include minimal number of probabilistic analyses. In order to develop such a method, the effect of margin of safety (MoS) used in the deterministic design method on the reliability measures is first investigated. The deterministic design method encompasses two steps. In the first step, size of diesel generator is found assuming that the diesel generator can cover the maximum peak load with a reasonable margin of safety MoS without any contribution from the renewable resources. Using hourly-averaged data the nominal size of the diesel generator \( P_{D,\text{nom}} \) is obtained by:

\[ P_{D,\text{nom}} = \bar{L}_{h,\text{max}} (1 + \text{MoS}) \]  \hspace{1cm} (23)
in which $\overline{L}_{h,max}$ stands for the maximum hourly-averaged demand load. In the second step of the deterministic design method, using a single-objective optimisation, the size of wind turbine and PV panel which minimise LCE are determined. Using this method, for different margins of safety the optimal size of wind-PV-diesel components are obtained.

A genetic algorithm (GA) was developed to find the optimal size of components. The solution space for hybrid systems is clustered with multiple local optima. This can impact the search performance of an ordinary GA. Special care has been therefore made in design of reproduction operators for the developed GA. In order to increase the exploratory behaviour of the GA, avoiding stagnation in local optima, a dynamic mutation operator combined with a mixed parent selection strategy has been used. At earlier generations, identified by $\overline{fit}_{av}/\overline{fit}_{max} \leq 0.9$, the GA explores the design space towards finding the cluster of the global optima by using a high mutation rate ($P_m = 0.7$) and a random parent selection strategy (irrespective of the individual fitness). At latest generations ($\overline{fit}_{av}/\overline{fit}_{max} > 0.9$) when the GA has found the cluster of the global optima, the algorithm exploits the design space towards finding the global optima itself by adopting a parent selection based on the individual fitness. In this stage still a high mutation rate is used but the mutation effect is limited. The random perturbation of the $i$-th design variable $x_i$ is selected from a shrinking interval

$$I_{i,m} = \frac{1 - \overline{fit}_{av}}{\overline{fit}_{max}} (x_{i,u} - x_{i,l})\,,$$

where $x_{i,l}$ and $x_{i,u}$ are, respectively, the lower and the upper limit of design variable $x_i$. This is aimed at a refine search in the vicinity of the global optima. Individual fitness in this algorithm is defined as the reciprocal of individual LCE. In the developed GA an arithmetic crossover operator is used. The infeasible solutions are defined as those with nonzero total blackout duration and are rejected on creation. The algorithm terminates when $\overline{fit}_{max} - \overline{fit}_{av} \leq 1 \times 10^{-5}$.

For each deterministic design case, employing the Monte Carlo simulation method of Algorithm 1 below, the reliability of the system is evaluated.

**Algorithm 1- Monte Carlo simulation for reliability and cost analysis**

**Given:**

- $x_i = \tilde{x}_i + \hat{x}_i;\,\, i = 1,2,...,n_u$ the set of $n_u$ uncertain parameters and their range and form of distributions ($\tilde{x}_i$ stands for the known mean value of parameter $x_i$ and $\hat{x}_i$ is the random variation of $x_i$ with known distribution).
- The desired level of confidence (LOC) corresponding to each one of the evaluated reliability assessment criteria $\{BO_f, BO_{av}, BO_{max}, MTBF, U_j\}$ and LCE.
- The design candidate $\{A_{WT}, A_{PV}, P_{D,nom}\}$ to be assessed

1. For $j = 1,2,...,n_{sim}$
   1.1. For each $x_i;\,\, i = 1,2,...,n_u$, select a random value $\hat{x}_i$ in the range consistent with its corresponding distribution.
   1.2. Find the value of the assessment measures $\{BO_f, BO_{av}, BO_{max}, MTBF, U_j\}$ and LCE.$j$. 
2. For each assessment criterion
   2.1. Using a histogram, find the probability of failure distribution.
   2.2. Find the value of assessment measure corresponding to the probability of failure of $PF = 1 - LOC$
Figure 1 illustrates how Step 2 of Algorithm 1 is carried out to find the assessment measures at a given LOC (here total unmet load, $U_r$, at LOC 99.9%): First the range of the unmet load is divided into $n_{seg}$ segments (here, $n_{seg} = 1000$). Then, for each segment $k = 1, 2, ..., n_{seg}$ the probability of failure is found: $PF_k = \text{Probability of having a } U_r \text{ greater than or equal to } U_{r,k} = \text{the total number of counts to the right of } U_{r,k} \text{ divided by } n_{sim}$ (for $MTBF_k = \text{Probability of having a } MTBF \text{ less than or equal to } MTBF_k = \text{the total number of counts to the right of } MTBF_k \text{ divided by } n_{sim}$). In this study $n_{sim} = 10^4$ is used.

In reliability analysis, uncertainties in resources (wind speed and solar irradiance), demand load and modelling (wind turbine power coefficient $P_{C}$ and PV array efficiency) are considered. Table (2) shows two cases considered in this study. In this table $\delta$ represent the variation limit as a fraction of the mean value. In this study two sets of resource and demand load data are used. Table (3) compares the site data for these two sites.

![System Configuration](image)

Figure 1-Illustrative example of finding reliability measures at a given level of confidence.
and reliability analysis for site S1

Table 2-Uncertainties in resources, demand load and modelling

<table>
<thead>
<tr>
<th>Parameter/Model</th>
<th>Distribution Case U1</th>
<th>Distribution Case U2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind speed</td>
<td>Uniform (δ = ±0.15)</td>
<td>Uniform (δ = ±0.30)</td>
</tr>
<tr>
<td>Solar irradiance</td>
<td>Uniform (δ = ±0.05)</td>
<td>Uniform (δ = ±0.10)</td>
</tr>
<tr>
<td>Demand load</td>
<td>Uniform (δ = ±0.10)</td>
<td>Uniform (δ = ±0.20)</td>
</tr>
<tr>
<td>$C_p$ model</td>
<td>Uniform (δ = ±0.07)</td>
<td>Uniform (δ = ±0.07)</td>
</tr>
<tr>
<td>PV array efficiency</td>
<td>Uniform (δ = ±0.05)</td>
<td>Uniform (δ = ±0.05)</td>
</tr>
</tbody>
</table>

Table 3-Resources and demand load

<table>
<thead>
<tr>
<th>Site</th>
<th>Wind speed, $V_{ref}$</th>
<th>Solar irradiance, $I$</th>
<th>Demand load, $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site S1</td>
<td>Wind speed as in [15], ($h_{ref} = 3m$)</td>
<td>Solar irradiance as in [15]</td>
<td>Three times of the demand load of [15]</td>
</tr>
<tr>
<td>Site S2</td>
<td>¾ of the wind speed of [15], ($h_{ref} = 3m$)</td>
<td>Solar irradiance as in [15]</td>
<td>Three times of the demand load of [15]</td>
</tr>
</tbody>
</table>

Tables (4) and (5) show the results of deterministic designs for different margins of safety as well as the results of probabilistic reliability analysis. The last row of these tables includes the results of optimisation without considering a margin of safety, in which the size of the diesel generator is determined along with the other design variables.

Table 4-Results of deterministic designs for different MoS and reliability analysis for site S1

<table>
<thead>
<tr>
<th>Design case</th>
<th>MoS</th>
<th>WT rotor radius (m)</th>
<th>PV panel area (m²)</th>
<th>Diesel nom. Power (kW)</th>
<th>Penetration (%)</th>
<th>TLSC ($)</th>
<th>LCE (cent/kWh)</th>
<th>Total BO (h)</th>
<th>Maximum BO (h)</th>
<th>Average BO (h)</th>
<th>LCE (cent/kWh)</th>
<th>Total BO (h)</th>
<th>Maximum BO (h)</th>
<th>Average BO (h)</th>
<th>MTBF (h)</th>
<th>LCE (cent/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>0.00</td>
<td>6.3</td>
<td>0</td>
<td>15</td>
<td>126</td>
<td>239,200</td>
<td>41.3</td>
<td>32</td>
<td>1</td>
<td>1</td>
<td>5.0</td>
<td>274</td>
<td>42.9</td>
<td>51</td>
<td>1</td>
<td>18.7</td>
</tr>
<tr>
<td>D2</td>
<td>0.05</td>
<td>7.0</td>
<td>0</td>
<td>15.75</td>
<td>160</td>
<td>243,850</td>
<td>42.1</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>.5</td>
<td>1096</td>
<td>43.6</td>
<td>34</td>
<td>1</td>
<td>8.7</td>
</tr>
<tr>
<td>D3</td>
<td>0.10</td>
<td>7.0</td>
<td>0</td>
<td>16.5</td>
<td>160</td>
<td>247,800</td>
<td>42.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8760</td>
<td>44.3</td>
<td>20</td>
<td>1</td>
<td>3.6</td>
</tr>
<tr>
<td>D4</td>
<td>0.20</td>
<td>7.0</td>
<td>0</td>
<td>18</td>
<td>160</td>
<td>255,720</td>
<td>44.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8760</td>
<td>45.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D5</td>
<td>0.50</td>
<td>7.0</td>
<td>0</td>
<td>22.5</td>
<td>160</td>
<td>279,450</td>
<td>48.3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8760</td>
<td>50.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D6</td>
<td>1.00</td>
<td>7.0</td>
<td>0</td>
<td>30</td>
<td>209</td>
<td>313,020</td>
<td>54.1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8760</td>
<td>56.7</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D7</td>
<td>2.00</td>
<td>8.1</td>
<td>0</td>
<td>45</td>
<td>221</td>
<td>373,980</td>
<td>64.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8760</td>
<td>68.8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D8</td>
<td>N/A</td>
<td>6.3</td>
<td>0</td>
<td>14.2</td>
<td>126</td>
<td>234,150</td>
<td>40.4</td>
<td>62</td>
<td>1</td>
<td>1</td>
<td>16.1</td>
<td>140</td>
<td>42.0</td>
<td>71</td>
<td>1</td>
<td>30.1</td>
</tr>
</tbody>
</table>
Table 5-Results of deterministic designs for different \( MoS \) and reliability analysis for site S2

<table>
<thead>
<tr>
<th>Design case</th>
<th>( MoS )</th>
<th>WT rotor radius (m)</th>
<th>PV panel area (m(^2))</th>
<th>Diesel nom. Power (kW)</th>
<th>Penetration(%)</th>
<th>TLSC ($)</th>
<th>LCE (cent/kWh)</th>
<th>Total BO (h)</th>
<th>Maximum BO (h)</th>
<th>Average BO (h)</th>
<th>( U_t ) (kWh)</th>
<th>MTBF (h)</th>
<th>LCE (cent/kWh)</th>
<th>Total BO (h)</th>
<th>Maximum BO (h)</th>
<th>Average BO (h)</th>
<th>( U_t ) (kWh)</th>
<th>MTBF (h)</th>
<th>LCE (cent/kWh)</th>
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<td>15</td>
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<td>1</td>
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<td>69</td>
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<td>0</td>
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<td>1</td>
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<td>0</td>
<td>0</td>
<td>8760</td>
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<td>65</td>
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<td>1</td>
<td>1</td>
<td>37.1</td>
<td>83</td>
<td>61.8</td>
</tr>
</tbody>
</table>

Figures (2) through (4) show three reliability measures: total unmet load, mean time between failures and total blackout duration against \( MoS \). Figures (5) and (6) show trends of the variations of reliability measures with respect to \( MoS \) versus \( MoS \). Figure (7) shows LCE obtained deterministically and the LCE obtained using Monte Carlo simulation @ 99.99% LOC versus \( MoS \). Solution spaces in two planes of LCE-total unmet load and LCE-total blackout duration are shown in Figures (8) and (9).

These figures show

(i) Strong dependency of the reliability measures on the site data and their associated uncertainties (Figures 1 through 6).

(ii) Regardless of the site data and their associated uncertainties, using a large-enough \( MoS \) leads to reliable designs (Figures 1 through 4). That is, optimisation for reliability is equivalent to maximisation of \( MoS \).

(iii) Probabilistic LCE deviates from deterministic LCE and this deviation increases with \( MoS \) (Figure 7). In other words, the LCE calculated using deterministic methods is not accurate and should be found via probabilistic methods.

(iv) Parameter \( MoS \) used in deterministic design has significant effect on the LCE, and that both deterministic and probabilistic LCE vary linearly with \( MoS \) (Figure 7). In other words, optimisation for cost is equivalent to minimisation of \( MoS \).

(v) The LCE calculated using probabilistic methods depends on both site data and uncertainties profile (Figure 7).

(vi) Predictable effect of increasing/decreasing \( MoS \) on the direction of forming Pareto Front in 2D solution space (Figures 8 and 9).

Observations (ii), (iv) and (vi) lead us to the conclusion that \( MoS \) used in deterministic design is a key design parameter which can be used for directing the design towards solutions with desired reliability or cost. However, referring to observation (i), this key parameter is highly problem dependent and cannot be obtained deterministically. Moreover, according to observation (iii) and
(v), even in the absent of uncertainties in cost modelling, design candidate assessment with respect to cost must be based on probabilistic cost analysis.

In summary, for each design problem, there exists an optimum $MoS$ that can be used to produce a Pareto solution. Hence, the original multi-objective optimisation problem in which the optimum size of the system components are to be found through probabilistic analysis, can be reduced to a single-objective problem in which the optimum $MoS$ is to be determined via probabilistic analysis and a single-objective optimisation in which the optimum size of system components are to be found deterministically.

Figure 2-Total unmet load versus $MoS$.

Figure 3-Mean time between failures versus $MoS$. 
Figure 4-Total blackout duration versus $MoS$.

Figure 5-Variation of total unmet load with respect to $MoS$ versus $MoS$.

Figure 6-Variation of total blackout duration with respect to $MoS$ versus $MoS$. 
Figure 7- LCE versus MoS.

Figure 8-Solution space in plane of LCE-Total blackout duration (solid markers represent Pareto solutions).

Figure 9-Solution space in plane of LCE-Total unmet load (solid markers represent Pareto solutions).
4 Design scenarios
There are three main approaches being adopted in performing a multi-objective optimisation. In the first approach, known as a priori method, a multi-objective optimisation problem is transformed to a single objective problem by combining all design objectives using a weighting system and forming a single aggregate or cost function. Weighting systems comprise of a set of weighting factors and/or tuning exponents representing the relative degree of importance of design objectives. At the end of a successful search process, the design alternative that minimises the cost function is entitled the optimum solution. This solution is a single point on the Pareto frontier of the corresponding original problem. In the second approach, known as a posteriori methods, no weighting system is used and the search process forms the Pareto frontier itself, or its best viable approximation. Here the first goal is to find Pareto front solutions. The designer evaluates the generated design alternatives against the assessment criteria and looks for trade-off solution. This is the chief advantage of this method compared to the first approach. However, the high computational time required to produce enough uniformly distributed Pareto solutions is the main drawback of this approach when adopted for design optimisation problems including probabilistic analyses. In the third approach of multi-objective optimisation, by treating all-but-one design objectives as constraints, the multi-objective optimisation problem is transformed to a single objective one. This method is most suited for cases in which one objective is dominant and other objectives either have known target values or have known upper and/or lower bounds. In case of conflicting objectives, solution obtained by this method is again a single point on the Pareto frontier of the original problem, while unlike the first approach the designer actually directly imposes constraints on the locus of the solution prior to commencing the optimisation. Adopting the third approach, the following two design scenarios are developed.

4.1 Design Scenario 1
In this design scenario the most reliable hybrid wind-PV-diesel system subject to the constraint \( LCE \leq LCE_g \) is obtained. Here, \( LCE \) is calculated using the probabilistic analysis method of Algorithm 1 and therefore a LOC must be associated to \( LCE_g \). Algorithm 2 below details the design method for this design scenario. The optimum \( MoS \) which maximises the reliability subject to the constraint \( LCE \leq LCE_g \) is represented by \( MoS_g \) and is calculated through Steps 1 and 2 of this algorithm.

Algorithm 2-Most reliable system subject to a constraint on the cost
Given:
- Goal levelised cost of energy \( LCE_g \) and its corresponding LOC
- Tolerance \( \varepsilon : LCE \leq LCE_g + \varepsilon ; \varepsilon \geq 0 \)
- Site data
- The set of uncertain parameters and their range and form of distributions (\( x_i = \tilde{x}_i + \tilde{x}_i ; i = 1,2,\ldots,n_u \))

Step 1. For two arbitrary \( MoS_1 \) and \( MoS_2 \) do:
1.1. Using Equation (23), calculate the nominal size of diesel generator \( P_{D,\text{nom}} \).
1.2. Use a deterministic optimisation method to find the optimum size of other components.
1.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding \( LCE \).

Step 2. Calculate the corresponding \( MoS \) to the goal \( LCE \) using Equation (24)

\[
MoS_g = c_1 LCE_g + c_2
\]  \hspace{1cm} (24)
Step 3. For \( MoS = MoS_g \) do:

3.1. Employ Equation (23) to calculate the nominal size of diesel generator \( P_{D,nom} \).

3.2. Use a deterministic optimisation method to find the optimum size of the other components.

3.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding \( LCE \) and reliability measures.

3.4. If \( LCE \leq LCE_g + \varepsilon \) stop the search; otherwise: update coefficients \( c_1 \) and \( c_2 \); go to Step 2.

For the first time in Step 2 parameters \( c_1 \) and \( c_2 \) are found using two points \((MoS_1, LCE_1)\) and \((MoS_2, LCE_2)\) in \( MoS - LCE \) plane:

\[
\begin{align*}
    c_1 &= \frac{MoS_2 - MoS_1}{LCE_2 - LCE_1} \quad (25.a) \\
    c_2 &= \frac{MoS_1 LCE_2 - MoS_2 LCE_1}{LCE_2 - LCE_1} \quad (25.b)
\end{align*}
\]

Updating coefficients \( c_1 \) and \( c_2 \) in Step 3.4 can be carried out either via Equations (25) by using the new point \((MoS, LCE)\) from latest iteration and one of the previous points or via data regression (e.g. least square method) using all points. It should be noted that in case of a perfect linear correlation between probabilistic \( LCE \) and \( MoS \), the first iteration should lead to the final solution.

**Case study 1**

It is desired to find the most reliable hybrid wind-PV-diesel system for site S1 with uncertainty profile U2 subject to \( LCE \leq 45.5 cent/kWh \) \( @ \ LOC \ 99.99\% \) \((LCE_g = 45.5 cent/kWh)\).

A tolerance of \( \varepsilon = 0.01 cent/kWh \) is used. By selecting \( MoS_1 = 0 \) and \( MoS_2 = 1 \), Step 1 of Algorithm 2 leads to the results shown in the first two rows of Table (6). The genetic algorithm optimisation explained in Section 3 is used for performing the deterministic optimisation of Steps 1.2 and 3.2. Using Equations (24) and (25) the goal \( MoS \) is calculated as: \( MoS_g = 0.036 \). Using this value Step 3 of Algorithm 2 leads to the results shown in the third row of Table (6).

<table>
<thead>
<tr>
<th>MoS</th>
<th>Diesel Nom. Power (kW)</th>
<th>WT rotor radius (m)/ Deterministic optimisation for LCE</th>
<th>PV panel area (m²)/ Deterministic optimisation for LCE</th>
<th>LCE @ 99.99% LOC (cent/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (1st initial point)</td>
<td>15.00</td>
<td>6.3</td>
<td>0</td>
<td>44.9</td>
</tr>
<tr>
<td>1 (2nd initial point)</td>
<td>30.00</td>
<td>7.9</td>
<td>0</td>
<td>61.7</td>
</tr>
<tr>
<td>0.036 (1st iteration)</td>
<td>15.54</td>
<td>6.3</td>
<td>0</td>
<td>45.5</td>
</tr>
</tbody>
</table>

As it can be observed the first iteration leads to the final solution. The reliability measures for this solution are: \( BO_t = 38h \), \( BO_{av} = 1h \), \( BO_{max} = 1h \), \( MTBF = 237h \) and \( U_t = 11.2 kWh \) (all at a LOC of 99.99%).

For this case by performing only three Monte Carlo simulations a multi-objective optimal design under uncertainty is carried out. This highlights the robustness of this design method.
4.2 Design Scenario 2
In this design scenario the most cost-effective hybrid wind-PV-diesel system subject to satisfying some goal reliability measures \( R = \{R_i\} \subseteq \{BO_{t,g}, BO_{av,g}, BO_{\max,g}, MTBF, U_{t,g}\} \) is obtained. Each goal reliability measure considered for the assessment is associated with a LOC. Algorithm 3 details the design method for this design scenario. The optimum \( MoS \) which minimises the LCE subject to the constraints \( R_i \leq R_{t,g} \) is represented by \( MoS_g \) and is calculated through Steps 1 to 3 of this algorithm.

Algorithm 3- Most cost-effective system subject to constraints on reliability measures
Given:
- Goal values for a selected subset of the reliability measures
  \( R = \{R_i\} \subseteq \{BO_{t,g}, BO_{av,g}, BO_{\max,g}, MTBF, U_{t,g}\} \) and their corresponding LOC
- Set of tolerance \( \varepsilon = \{\varepsilon_i\}; R_i \leq R_{t,g} + \varepsilon_i \) for each \( R_i \in R \) to be minimised (\( BO_{t,g}, BO_{av,g}, BO_{\max,g}, U_{t,g} \)) and \( R_i \geq R_{t,g} - \varepsilon_i \) for each \( R_i \in R \) to be maximised (\( MTBF \)) (\( \varepsilon_i \geq 0 \))
- Site data
- The set of uncertain parameters and their range and form of distributions (\( x_i = \bar{x}_i + x_i^*; i = 1, 2, ..., n_u \))

Step 1. For three arbitrary \( MoS \) do:
1.1. Using Equation (23), calculate the nominal diesel size \( P_{D,nom} \).
1.2. Use a deterministic optimisation method to find the optimum size of other components.
1.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding LCE and reliability measures.

Step 2. For each \( R_i \in R \), using Equation (26) find its corresponding \( MoS_{g,R_i} \).

\[
MoS_{g,R_i} = c_2 R_i^2 + c_4 R_i + c_5
\] (26)

Step 3. Assign \( MoS_g = \max\{MoS_{g,R_i}\} \).

Step 4. For \( MoS = MoS_g \) do:
4.1. Employ Equation (23) to calculate the nominal diesel size \( P_{D,nom} \).
4.2. Use a deterministic optimisation method to find the optimum size of the other components.
4.3. For the obtained optimal solution run the Monte Carlo simulation of Algorithm 1 to find its corresponding LCE and the set of reliability measures \( R \).
4.4. If desired reliability achieved end the search; otherwise updates parameters \( c_3 \) through \( c_5 \) and go to Step 2.

Calculating/updating coefficients \( c_3 \) through \( c_5 \) is carried out via data regression (e.g. least square method) using all available points \( \{MoS_{g,R_i}, R_i\} \). It should be noted that three arbitrary \( MoS \) of Step 1 should produce at least two distinct points in each \( R_i - MoS \) plane to be able to correlate \( R_i \) to \( MoS \) through Equation (26). Lower \( MoS \) are more likely to produce distinct points.

Case study 2
In this design case study it is desired to design a wind-PV-diesel system for site S2 under uncertainties U2. The reliability measures \( BO_{t,g} \leq 40h, MTBF \geq 200h \) and \( U_t \leq 5kWh \) at a LOC=99.99% are desired.
Tolerances \( \varepsilon = \{1h, 50Wh, 1h\} \) for the reliability measures \( R = \{BO_t, U_t, MTBF\} \) are used. Results are shown in Table (7). The designed system in the second iteration satisfies all constraints within the tolerated margins. Table (8) summarises the results of Steps 2 and 3 leading to \( MoS_s \) for the first and second iterations.

### Table 7-Results of Steps 1 and 4 of Algorithm 3 for case study 2

<table>
<thead>
<tr>
<th>MoS</th>
<th>Diesel Nom. Power (kW)</th>
<th>WT rotor radius (m)/ Deterministic optimisation for LCE</th>
<th>PV panel area (m²)/ Deterministic optimisation for LCE</th>
<th>LCE @ 99.99% LOC (cent/kWh)</th>
<th>BO_t @ 99.99% LOC (h)</th>
<th>U_t @ 99.99% LOC (kWh)</th>
<th>MTBF @ 99.99% LOC (h)</th>
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</thead>
<tbody>
<tr>
<td>0 (1st initial point)</td>
<td>15.0</td>
<td>6.2</td>
<td>26</td>
<td>61.8</td>
<td>104</td>
<td>37.1</td>
<td>83</td>
</tr>
<tr>
<td>0.05 (2nd initial point)</td>
<td>15.8</td>
<td>6.2</td>
<td>26</td>
<td>63.1</td>
<td>73</td>
<td>20.0</td>
<td>119</td>
</tr>
<tr>
<td>0.1 (3rd initial point)</td>
<td>16.5</td>
<td>6.2</td>
<td>41</td>
<td>64.4</td>
<td>46</td>
<td>8.7</td>
<td>190</td>
</tr>
<tr>
<td>0.1215 (1st iteration)</td>
<td>16.8</td>
<td>6.2</td>
<td>41</td>
<td>64.9</td>
<td>37</td>
<td>5.45</td>
<td>243</td>
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<tr>
<td>0.1219 (2nd iteration)</td>
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### Table 8-Results of Steps 2 and 3 of Algorithm 3 for case study 2

<table>
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<tr>
<th>Iteration</th>
<th>( R_i )</th>
<th>([c_3, c_4, c_5] ) as in Eq. (4)</th>
<th>( MoS_{s,i} )</th>
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<td>( U_t(kWh) )</td>
<td>[+5E-5, -0.0059, +0.1477]</td>
<td>0.11945</td>
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<tr>
<td></td>
<td>( MTBF(h) )</td>
<td>[-6E-6, +0.0027, -0.1785]</td>
<td>0.1215</td>
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<tr>
<td>2</td>
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<td>[+3E-6, -0.0021, +0.1914]</td>
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<tr>
<td></td>
<td>( U_t(kWh) )</td>
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<td></td>
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<tr>
<td></td>
<td>( MTBF(h) )</td>
<td>[-4E-6, +0.002, -0.1372]</td>
<td>0.1028</td>
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</tbody>
</table>

Figures (10) through (13) show the histograms and probability of failure distributions obtained via Monte Carlo simulation of Algorithm 1 for four design qualities (three reliability measures and \( LCE \)) of the final design.
Figure 10-Probability of failure distribution of the final solution-design quality: total blackout duration

Figure 11-Probability of failure distribution of the final solution-design quality: total unmet load
Figure 12-Probability of failure distribution of the final solution-design quality: mean time between failures

Figure 13-Probability of failure distribution of the final solution-design quality: levelised cost of energy
5 Summary and Concluding Remark

Optimal design of a standalone wind-PV-diesel HRES is a multi-objective optimisation problem with conflicting objectives of cost and reliability. Due to uncertainties in renewable resources and demand load, probabilistic analysis methods such as Monte Carlo simulation are required to quantify the system reliability. Performing probabilistic analysis within a search process, in which tens of thousands of design candidates are produced and evaluated towards finding the global optima, is highly time-consuming and inefficient.

Uncertainties in renewable resources, demand load and power modelling make deterministic methods of multi-objective optimisation fall short in optimal design of standalone HRES. Firstly, deterministic methods of analysis, even in the absence of uncertainties in cost modelling, do not predict the LCE accurately. Secondly, since these methods ignore the random variations in parameters, they cannot be used to quantify the second objective, reliability of the system in supplying power. While it is well established that using safety factors and design for worst-case-scenarios leads to reliable solutions, it is also well known that deterministic designs can lead to non-optimal over-designed/under-designed systems as a result of employing improper safety factors.

Parameter $MoS$ used in deterministic sizing of the diesel generator plays the key role in the development of the new design methodology. First it is shown that $MoS$ has a major and predictable influence on both LCE and reliability-related design qualities. It is also shown that in the context of multi-objective optimisation with conflicting objectives of cost and reliability, for each design problem, there exists an optimum $MoS$ that can be used to produce a Pareto solution. Hence, the original multi-objective optimisation problem in which the optimum size of the system components are to be found through tens of thousands of probabilistic analysis, can be reduced to a single-objective problem in which the optimum $MoS$ is to be determined via few probabilistic analysis and a single-objective optimisation in which the optimum size of system components are to be found deterministically. As a result of this the number of probabilistic analysis reduces dramatically.

Optimum $MoS$ depends on: (i) site data, (ii) uncertainties and (iii) desired (goal) design qualities in terms of the system cost and reliability of power supply (e.g. $LCE \leq 45.5\text{cent} / \text{kWh}$, $BO_t \leq 40h$, etc). For a given site and set of uncertainty profiles, different goal design qualities correspond to different optimum $MoS$, and consequently different Pareto solutions.

For two design scenarios, namely, most reliable system subject to a constraint on the cost and most cost-effective system subject to constraints on reliability measures, two algorithms are proposed to find the optimum $MoS$. The robustness of the proposed design methodology is shown through carrying out two design case studies. Design case study 2 also shows that how the proposed design methodology can be employed to design systems compatible with the end-user requirements.

References


Nomenclature

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<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$A$</td>
<td>Area ($m^2$)</td>
</tr>
<tr>
<td>$BO$</td>
<td>Blackout duration</td>
</tr>
<tr>
<td>$C$</td>
<td>Cost ($)</td>
</tr>
<tr>
<td>$C_u$</td>
<td>Unit cost ($/unit)</td>
</tr>
<tr>
<td>$d$</td>
<td>Discount rate (-)</td>
</tr>
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<td>$h_c$</td>
<td>Ground-blade tip clearance ($m$)</td>
</tr>
<tr>
<td>$I$</td>
<td>Solar irradiance ($W/m^2$)</td>
</tr>
<tr>
<td>$L$</td>
<td>Demand load ($W$)</td>
</tr>
<tr>
<td>$MoS$</td>
<td>Margin of safety</td>
</tr>
<tr>
<td>$MTBF$</td>
<td>Mean time to failure</td>
</tr>
<tr>
<td>$N$</td>
<td>Nominal life-span (years; hours of operation)</td>
</tr>
<tr>
<td>$n$</td>
<td>Number</td>
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<tr>
<td>$n_u$</td>
<td>Number of uncertain parameter</td>
</tr>
<tr>
<td>$P$</td>
<td>Power ($W$)</td>
</tr>
<tr>
<td>$PF$</td>
<td>Probability of failure (-)</td>
</tr>
<tr>
<td>$S$</td>
<td>Size (various units)</td>
</tr>
<tr>
<td>$U_t$</td>
<td>Total unmet load ($Wh$)</td>
</tr>
<tr>
<td>$UCRF$</td>
<td>Uniform capital recovery factor</td>
</tr>
<tr>
<td>$z_0$</td>
<td>Site surface roughness ($m$)</td>
</tr>
</tbody>
</table>
\( \alpha \) Cost as a fraction of initial cost (-)  
\( \eta_{PV} \) Overall PV unit efficiency (-)  
\( \rho \) Air density \( (kg/m^3) \)  
\( \eta_{EG} \) Wind turbine electrical and gearbox efficiency (-)  

**Subscripts**  
\( a \) Available; Usable available; Annualised  
\( av \) Average  
\( c \) Capital  
\( comp \) HRES component (WT, PV, D)  
\( D \) Diesel  
\( d \) Daily  
\( F \) Fixed  
\( fail \) Failure  
\( h \) Hourly  
\( hub \) Hub elevation  
\( ins \) Installation  
\( max \) Maximum  
\( min \) Minimum  
\( nom \) Nominal  
\( O \& M \) Operation and maintenance  
\( PV \) Photovoltaic  
\( p \) Performance measures  
\( R \) Renewable  
\( r \) Replacement  
\( S \) System  
\( sim \) Simulation  
\( t \) Total  
\( u \) Unit, Uncertain parameter  
\( V \) Variable  
\( WT \) Wind turbine  

**Symbols**  
\( \bar{\varphi}_T \) Averaged value of quantity \( \varphi \) over time period \( T \)  
\( \tilde{\varphi} \) Mean value of uncertain parameter \( \varphi \)  
\( \hat{\varphi} \) Random part of uncertain parameter \( \varphi \)  
\( [\varphi] \) Integer value of parameter \( \varphi \)  

**Abbreviations**  
HRES Hybrid renewable energy system  
LCE Levelised cost of energy  
LOC Level of confidence  
O& M Operating and maintenance  
TLSC Total life-span cost
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