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Multi-Objective Design under Uncertainties of Hybrid Renewable Energy System Using NSGA-II and Chance Constrained Programming

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Abstract

The optimum design of Hybrid Renewable Energy Systems (HRES) depends on different economical, environmental and performance related criteria which are often conflicting objectives. The Non-dominated Sorting Genetic Algorithm (NSGA-II) provides a decision support mechanism in solving multi-objective problems and providing a set of non-dominated solutions where finding an absolute optimum solution is not possible. The present study uses NSGA-II algorithm in the design of a standalone HRES comprising wind turbine, PV panel and battery bank with the (economic) objective of minimum system total cost and (performance) objective of maximum reliability. To address the uncertainties in renewable resources (wind speed and solar irradiance), an innovative method is proposed which is based on Chance Constrained Programming (CCP). A case study is used to validate the proposed method, where the results obtained are compared with the conventional method of incorporating uncertainties using Monte Carlo simulation.

Keywords: multi-objective optimisation; NSGA-II; standalone hybrid wind-PV-battery; reliability; chance constrained programming; design under uncertainties

1. INTRODUCTION

Decision making problems can be categorized in two classes based on the number of objective functions that are involved in the problem; single objective and multi-objective. In a single objective problem, the aim is to identify the best solution corresponding to minimising or maximising a single objective function. However, in real life, the decision making process usually involves more than one objective function. Multi-objective problems do not have a single optimal solution but they have a set of compromised solutions between different objective functions known as Pareto set.

In optimal sizing of HRESs, there are normally more than one objective function to be considered. Two important objective functions in the design of a HRES are cost and reliability. Since these objectives are contradicting, a single optimal solution cannot be found (with minimum cost and maximum reliability) and a multi-objective optimisation is needed to find a trade-off; Pareto set solutions. Several studies have been reported in multi-objective optimisation of HRES considering different objection functions and using various optimisation techniques.
Kaviani et al. [1] used a PSO to optimize a hybrid wind-photovoltaic-fuel cell generation system with the objective of minimizing the annual cost of the hybrid system subject to reliable supply to meet the demand. Diaf et al. [2] analysed the optimum configuration of a stand-alone hybrid photovoltaic-wind system that guarantees the energy autonomy of a typical remote consumer with the lowest levelized cost of energy. The search method was used to analyze different combinations. Kaabeche et al. [3] recommended an optimisation model based on iterative technique to optimise the size of a hybrid wind/photovoltaic system combined with a battery bank with the objective being to minimise the deficiency of power supply and levelized unit of electricity cost.

Bernal-Agustín et al. [4] applied the Multi-Objective Evolutionary Algorithm (MOEA) to the multi-objective design of isolated hybrid systems (photovoltaic–wind–diesel) where the objectives were minimizing the total cost and pollutant emissions during useful life of the installation.

Giannakoudis et al. [5] proposed an optimisation method based on stochastic optimization algorithms (such as simulated annealing) for the design and operation of a hybrid power generation system that consists of PV panels, wind generators, accumulators, an electrolysis apparatus, hydrogen storage tanks, a compressor, a fuel cell and a diesel generator. Genetic Algorithms (GA) proved to be popular in solving optimisation problems. Ould [6] proposed a Pareto-based multi-objective GA for sizing a hybrid solar–wind-battery system with the aim of minimizing the annualized cost and minimizing the probability of loss of power supply.

Montoya et al. [7] presented a hybrid Pareto-based multi-objective meta-heuristic approach to minimize voltage deviations and power losses in power networks, which can be extended to hybrid systems. Yang et al. [8] proposed a GA based optimal sizing technique using typical meteorological yearly data. The proposed optimisation model determines the system optimum configuration which is able to provide the desired Loss of Power Supply Probability (LPSP) with minimum Annualized Cost. The Non-dominated Sorting Genetic Algorithm (NSGA-II) was proposed [9] to perform multi-objective evolutionary algorithms (MOEA) in which an elite-preserving operator gives the best individuals the opportunity to be directly transferred to the next generation. By doing so, a ‘good’ solution which is found in early generations is never removed from the population unless a better solution is discovered. Katsigiannis [10] used the NSGA-II to design a small autonomous hybrid power system that contained both renewable and conventional power sources with the objectives of minimizing the energy cost of the system and total greenhouse gas emission during the system life time. However, the effects of uncertainties in renewable energy generation were not considered in this study.

Different methods to include the uncertainties in renewable resources in the design of HRES have been reported. Giannakoudis et al. [11] considered adding a known disturbance to the design inputs to maintain optimum mix of renewable resources. Nandi et al. [12] assumed that wind speed variation follows the Weibull distribution. Lujano-Rojas el al. [13] used time series theory to simulate the uncertainties in wind speed in the design of small wind/battery systems. Usually, the Monte Carlo simulation approach is used in solving probabilistic problems. Given a significantly large sample size, this method can provide highly accurate results. However, the main drawback is the computational burden associated with the large number of repeated calculations [14]. The Chance Constrained Programming (CCP) approach, first introduced by Charnes and Cooper [15] in 1959, is now popular method in solving problems that include random parameters. Its main feature is that it ensures the probability of the resulting decision to comply with the specified constraints [16]. The CCP method has been widely applied in different disciplines for optimisation under uncertainty[17], but very few studies are reported on using this
method for the design of HRES. Arun et al. [18] used the CCP approach in the design of a PV-battery system to deal with the uncertainties in the solar radiation. Seeraj et al. [19] used this method to find the battery bank size when renewable energy resource availability, ratings and load demand were assumed to be known.

This paper presents the results of a multi-optimisation NSGA-II based approach for the design of a standalone HRES, shown in Figure 1, considering uncertainties in the resources available. The approach employs the chance constrained programming to deal with the effects of uncertainties in renewable resources instead of common approach of using Monte Carlo simulation. Authors in [20] have shown that chance constrained programming can result in optimum solution for a predefined reliability in a single-objective optimisation problem in design of HRES, however in a multi-objective optimisation problem where there is no predefined reliability, conventionally Monte Carlo simulation is employed. This study proposes a novel method in employing chance constrained programming in multi-objective problems as a substitute of Monte Carlo simulation. The study proposes a method in which chance constrained programing is used as a tool in estimating the expected value of the objective function which is affected by the uncertainties, in other words instead of finding the optimum solution for a predefined value of reliability, chance constrained programing is used to estimate the expected value of the reliability of the design candidates in a multi-objective optimisation problem. To evaluate the performance of the proposed method, the results obtained are compared with those obtained by employing the Monte Carlo simulation.

The outline of this paper is as follows:
The components modelling and cost modelling are presented in sections 2.
Problem formulation and design methodology are presented in section 3.
A case study is described in section 4. Results and discussion are described in section 5 and finally conclusions are presented in section 6.

II. COMPONENTS AND COST MODELLING

The HRES design is crucially dependent on the performance of its individual components. Different mathematical models have been proposed to estimate the output power of wind turbine, photovoltaic panel and batteries (considered in this work). The models implemented in this study are chosen with consideration of giving a realistic estimation of the output of each component.
The mathematical models of components used in this study are briefly presented in Appendices A. More details on components modelling is discussed in [20].

In this study the total cost of the system $TC$ (of the design candidates) is calculated as the economical measure taking into account the initial capital cost ($C_{IC}$), replacement cost ($C_{replacement}$) and present value of maintenance cost ($C_{O&M}$). That is:

$$TC = C_{IC} + C_{replacement} + C_{O&M}$$  (1)

A. Initial Capital Cost

The initial capital cost consists of the components cost and their installation cost.

$$C_{IC} = (A_{PV} P_{PV, Nom} C_{Unit, PV}) + (A_{WT} C_{Unit, WT}) + (N_{Bat} C_{Bat, Bat}) + C_0$$  (2)

$C_0$; the total installation constant cost including the cost of installation of the wind turbine and PV panels and is considered to be 20% of the component cost of the wind turbine and 40% of the component cost of the PV system [3].

B. The Present Value of Replacement Cost

In this study the only component which needs to be replaced during life time of the HRES is assumed to be the battery bank so this cost is only calculated when the battery bank exists in the configuration.

The replacement cost of the battery bank can be calculated as [3]:

$$C_{replacement} = N_{Bat} C_{Bat, Bat} \sum_{i=1}^{N_{op}} \left( \frac{1+f}{1+k_d^i} \right)^{i/N_{op}}$$  (3)

C. The Present Value of Operation and maintenance Cost

The present value of operation and maintenance cost of the hybrid system is expressed as[3]:

$$C_{O&M, HRES} = \begin{cases} C_{O&M, h} \left( \frac{1+f}{k_d^f - f} \right)^{ \left( \frac{1+f}{1+k_d^f} \right)^Y} & k_d \neq f \\ C_{O&M, h} Y, & k_d = f \end{cases}$$  (4)

$C_{O&M, h}$; the operation and maintenance cost in the first year and can be given as a fraction ($k$) of the initial capital cost $C_{IC}$ as:

$$C_{O&M, 0} = k C_{IC}$$  (5)

The value of $k$ is assumed to be 1% for the PV system, 3% for wind turbine and 1% for battery bank [3].

III. PROBLEM FORMULATION AND DESIGN METHODOLOGY

The proposed technique adopts the non-dominated sorting genetic algorithm (NSGA-II) [9] in combination with the chance constrained programming (CCP) [15] to effectively solve the multi-objective optimisation problem of design of a HRES under uncertainties.
The aim is to find the Pareto set solutions based on the desired objective functions using NSGA-II. The NSGA-II provides a very efficient procedure in keeping the elitism optimisation process as well as preserving the diversity which assures a good convergence towards the Pareto-optimal front without losing the solution diversity [21].

The following steps are implemented in the NSGA-II algorithm.
1: Initial population is generated based on defined decision variables and number of populations.
2: Evaluation of each chromosome in terms of defined objective functions. The adopted methods in evaluation the objective functions affected by uncertainties are explained in sub-section (III-A) and (III-B).
3: Set the generation count
4: Prepare the mating pool
5: Perform crossover and mutation operators
6: Evaluation of new offspring in terms of defined objective functions.
7: Perform non-dominated sorting
8: Calculate the crowding distance
9: Perform the selection based on rank. If individuals with the same rank are encountered, crowding distance is compared. A lower rank and higher crowding distance is the selection criteria.
10: Increment the generation count and repeat steps 4 to 9 until the counter reaches the maximum number of generation

The decision variables are the wind turbine rotor swept area (\( A_{WT} \)), the PV panel area (\( A_{PV} \)) and the number of batteries (\( N_{Bat} \)).

The optimisation problem can be defined as:

\[
\min_{A_{WT}, A_{PV}, N_{Bat}} \{ TC, DPSP \}
\]

Subject to

\[
SOC \geq SOC_{\text{min}}
\]

where

\[
TC = C_{IC} + C_{O&M} + C_{\text{replacement}}
\]

\[
DPSP = \frac{\sum_{i=1}^{h} DPS_i}{\sum_{i=1}^{h} Demand_i} \times 100
\]

As Equation 6 shows, two objective functions have been considered associated with both minimisation of the system total cost (\( TC \)) and the deficiency of power supply probability (\( DPSP \)); where \( DPS \) is the amount of power shortage at each hour and \( h \) is the total hours under study. Since different applications of HRES need different reliability requirements the trade-off between reliability and cost of the designed system expectedly would result in obtaining different HRES with
reliability values between 0 and 100% and that would be the decision makers choice to choose the design option that suits the application.

The energy balance of the system can be modelled as:

\[
P_{\text{HRES}} = \begin{cases} 
P_{\text{WT}} + P_{\text{PV}}, & (a) \\
P_{\text{WT}} + P_{\text{PV}} + P_{\text{Bat}}, & (b) 
\end{cases}
\]  

(10)

(a) if total power generated by the wind turbine and PV is sufficient to cover the load demand, otherwise 

(b) \(P_{\text{WT}} + P_{\text{PV}}\) is not sufficient to meet the demand and the battery has to supply the difference.

In order to compare the performance of the proposed method, the NSGA-II algorithm objectives affected by uncertainties are evaluated with CCP (explained in section III-A) as well as a conventional method based on Monte Carlo simulation (explained in section III-B).

A. Optimal Estimation of the Objective Functions Affected by Uncertainties Using CCP

Each design candidate in the main optimisation process needs to be evaluated in terms of the desired objective functions; here these are the total cost (TC) and deficiency of power supply probability (DPSP). As uncertainties with renewable resources have direct effects on the second objective function (DPSP); finding an exact value for DPSP is not realistically possible. Therefore, this objective function needs to be estimated using stochastic methods, in this study using chance constrained programming.

As DPSP_\text{estimated} is completely dependent on the correct estimation of uncertain variables, here the aim would be to estimate the hourly values of \(P_{\text{WT,estimated}}\) and \(P_{\text{PV,estimated}}\). The estimation problem of \(P_{\text{WT,estimated}}\) and \(P_{\text{PV,estimated}}\) can be written as a chance constrained problem. The aim of this problem would be to estimate the hourly values of \(P_{\text{WT,estimated}}\) and \(P_{\text{PV,estimated}}\) in such way that their sum would have a value with a desired confidence level \(\alpha\). The estimation problem can be described as a chance constrained problem, as:

\[
\Pr(P_{\text{WT,estimated}} + P_{\text{PV,estimated}} \geq \text{DPSP}_\text{estimated}) \geq \alpha
\]  

(11)

Following the method proposed by the authors [20], the hourly values of \(P_{\text{WT,estimated}}\) and \(P_{\text{PV,estimated}}\) are extracted and then used to calculate the \(\text{DPSP}_\text{estimated}\) . As shown in the case study (section IV), this method requires considerably shorter process time as compared with the conventional Monte Carlo simulation.

B. Monte Carlo Simulation

Monte Carlo simulation is conventionally used to estimate the expected value of the parameters with uncertainties. The performance of the Monte Carlo simulation is dependent on the accurate modelling of uncertainties in the wind speed and solar irradiance. Different approaches are used to model the renewable sources. One of the common approaches is by fitting the uncertainties to known distributions such as Weibull or Beta distributions [22]. However, research show that for some locations (e.g. in the UK), using predefined distributions may not simulate the weather data properly[23]. Erken [24] used different distributions to find the best fitted distribution for each hourly meteorological data. Another method in considering uncertainties is adding a random disturbance to the average values of wind speed and solar irradiance variations. To obtain accurate modelling of
wind speed and solar irradiance variations, two methods are used to correlate historical data to known distributions and time series analysis using autoregressive moving average models (ARMA) [26]. Based on the location of the desired site, the performance of different modelling methods should be investigated and the most suitable model selected as the random data generator to model the uncertainties in Monte Carlo simulation. Using these random data generators, the Monte Carlo simulation is repeated enough for each configuration until the expected values of the objective function; here \( E[DPSE] \), is calculated with the confidence level of %90 and variation value of less than %3.

The flowchart of the implemented NSGA-II using adopted methods in evaluation the objective functions affected by uncertainties are explained in sub-section (III-A) and (III-B) is presented in Figure 2.

### IV. CASE STUDY

The proposed method is used to design a standalone HRES for a household in Kent, UK. The input data for the design are historical hourly data (2000-2008) of wind speed and solar irradiance for 12 months of the year together with typical summer and winter load profiles shown in Figure 3. The load profile is a typical load profile in the UK which is adopted from [27].

Details of technical and economical characteristics of the system components are given in Table I.

The system under study consists of a wind turbine, a PV panel and a battery bank. The wind turbine rotor area is varied in the range from 0 to 154 m\(^2\) (in 10 steps), PV panels area is from 0 to 260 m\(^2\) and minimum number of batteries required to meet the probabilistic constraint is determined for each case. The number of batteries is assumed to vary from 0 to 478. The maximum permitted number of batteries in this study is calculated, using Equation 12, considering required storage for one day of autonomy with highest daily load demand; here typical winter load demand is used.

\[
N_{Bat} = \frac{Load S_D}{c_{Bat} V_{Bat} DOD_{max} \eta_{Bat}}
\]

where \( Load \) is maximum daily load (W); \( S_D \) is the number of autonomy or storage days (in this study considered as 1 day); \( V_{Bat} \) is the battery bank voltage in (V); \( DOD_{max} \) is the maximum depth of discharge and \( \eta_{Bat} \) is the battery efficiency.

The NSGA-II algorithm is performed for 250 iterations with a population number of 100, the mating pool size is considered as 0.5 of the population, crossover probability \( pc = 0.9 \) and mutation probability of \( 1/n \); where \( n \) is the number of variables; here 3.

To select the best model for wind speed and solar irradiance in Monte Carlo simulation, the results of ARMA simulation are compared with those obtained from fitting the historical data of wind speed to Weibull distribution and solar irradiance to...
Input Data:
- Historical Meteorological Data (Wind Speed (WS), Solar Irradiance (SI))
- Summer and Winter Load Profiles
- Design Variables

Evaluate the objective functions of each chromosome using CCP (Section III-A)

Calculating Joint CDF of WT power and PV panel power for \( A_{PV} \) and \( A_{WT} \)

Find the values of WT and PV output powers for each hour according to desired \( \alpha = 0.9 \)

Calculate DPSP & TC

Prepare the mating pool

Perform Cross-over & Mutation

Evaluating the offspring using CCP

Perform non-dominated sorting

Calculate the crowding distance

Perform the selection based on rank

Max No. of Generations?

No

Max No. of Generations?

No

Generate the Pareto set

Generate the Pareto set

Evaluating the objective functions of each chromosome using Monte Carlo Simulation (Section III-B)

Simulate hourly WS, SI variation using the identified ARMA model

Simulate hourly WS, SI variation using identified distributions

Compare the results with observed data Choose the best method to simulate hourly WS,

Perform Monte Carlo simulation for required times

Calculate mean value of DPSP and TC

Simulate hourly WS, SI variation using identified distributions

Perform non-dominated sorting

Calculate the crowding distance

Perform the selection based on rank

Max No. of Generations?

Yes

Generate the Pareto set

Yes

Generate the Pareto set

Figure 2. Flowchart of NSGA-II using CCP/ Monte Carlo Simulation
Beta distribution. Based on the results presented in [20] Weibull distribution showed better performance in modelling wind speed variation and ARMA simulation is used to model solar irradiance in the desired site.

V. RESULTS AND DISCUSSION

The system under study was designed based on the methodologies explained in section III and results are presented in this section.

Figures 4-a and 5-a present a comparison between the initial populations and the final Pareto sets of performing NSGA-II in combination with CCP and Monte Carlo simulation. It can be observed that the NSGA-II with CCP produces more conservative results as compared to the other method, as it results in solutions with higher total cost. However, it obtains better results in high reliabilities close to 100%; with lower total cost of the system.

Figures 4-b and 5-b show how the output of each technique converges to its final Pareto set. As can be seen, in both cases (using the CCP and Monte Carlo simulation), the outputs of the NSGA-II converges to the final Pareto set at generation 150. The final Pareto sets of performing optimisation process using proposed NSGA-II algorithm on the site under study; in combination with CCP as well as Monte Carlo simulation are compared; Figure 6. The Pareto sets obtained in both cases of employing NSGA-II are well defined and solutions are spread over the reliability axis. It should be noted that a solution with zero total cost is not feasible.

Although using CCP instead of Monte Carlo simulation results in more conservative set of solutions (as shown in Figure 6); the execution time is significantly lower. The calculation time for evaluating the objective function of each chromosome is 11.44 seconds using CCP which is significantly lower than performing the Monte Carlo simulation, which takes 56.81 seconds for each design candidate.
The Figure 6 also shows that maximum deviation between two Pareto sets happens when the DPSP is between 15% to 35%.

To help the decision maker to choose the solution which fits the requirements, the output solution of two design methods for reliability of 80% or DPSP of 20% is studied in detail. Design parameters of CCP and the optimum solutions of two design methods are presented in Table 2. To evaluate the performance of each of the selected solutions; Monte Carlo simulation is performed for the simulation number of 2500 runs. A selection of results is presented in Figures 7-8. Figures 7-a and 8-a present the distribution of different blackout occurrence probabilities along the hours of a year. By comparing these two
graphs one can see that there are more hours in the year with very little chance of having power shortages in solution-1 than in solution-2. It is also observed that the maximum hourly blackout occurrence probability is less in solution-1 than solution-2; 0.6 for solution-1 and around 0.8 for solution-2.

Figures 7-b and 8-b present the average daily probability of blackout occurrence throughout the year. Comparing these to figures shows that in solution-1 the last three months of the year have the highest probability of blackout occurrence which is due higher load demand in winter (see Figure 3) as well as less renewable resources available in these months. However, in solution-2 the second half of the year has higher probability of power loss.

The day that has the largest probability of blackout occurrence in Figures 7-b and 8-b is selected and details of having blackout at each hour of that day is presented in Figures 7-c and 8-c.

Figures 7-d and 8-d show the results of performing Monte Carlo simulation for 2500 times on the hour with most probability of blackout and presents the frequency and load satisfaction percentage for that hour.
Table 2. Optimum solutions of two design methods for reliability=0.8

<table>
<thead>
<tr>
<th>α</th>
<th>NSGA-II and chance constrained programming (Solution-1)</th>
<th>NSGA-II and Monte Carlo simulation (Solution-2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WT Rotor Area (m²)</td>
<td>PV Panel Area (m²)</td>
</tr>
<tr>
<td>0.9</td>
<td>92</td>
<td>26</td>
</tr>
</tbody>
</table>

Figure 7. Monte Carlo simulation results on optimum solution of NSGA-II with chance constrained prog. for reliability=0.8

Figure 8. Monte Carlo simulation results on optimum solution of NSGA-II with Monte Carlo sim. for reliability=0.8
VI. CONCLUSIONS

This paper proposes a multi-objective optimisation algorithm for optimum economic and reliability oriented design of hybrid renewable energy system. The algorithm takes into account the uncertainties in renewable resources. The decision variables are the wind turbine rotor swept area, the PV panel area and the number of batteries. Two conflicting objectives which are total cost and system reliability are considered. A novel method in using chance constrained programming is proposed in this study to estimate the expected value of the objective function; the reliability of design candidate; affected by uncertain values of wind speed and solar irradiance at each hour under study. This reduces the evaluation time of the design candidate and consequently the run time of the NSGA-II program.

The results obtained by using the proposed method are compared with those obtained using a conventional Monte Carlo simulation. The comparison shows that the proposed method yields conservative results in lower reliability values and better results in high reliability values.

ACKNOWLEDGMENT

The authors would like to thank the Synchron Technology Ltd. for their partial financial support of this research.
### Table A-1 Wind Turbine Model

The wind power generated by a wind turbine

\[
P_{WT} = \frac{1}{2} \rho C_p V_w^3 A_{WT}
\]

The wind speed \( V_w \) at the hub height

\[
V_w = V_{ref} \frac{\ln \left( \frac{z_{hub}}{z_0} \right)}{\ln \left( \frac{z_{ref}}{z_0} \right)}
\]

The wind turbine power coefficient, \( C_p \)

\[
C_p = 3.646e^{-3} V_w^{-6} - 2.95e^{-3} V_w^{-5} + 0.9088e^{-3} V_w^{-4} - 0.01295 V_w^{-3} + 0.07874 V_w^{-2} - 0.11 V_w - 0.001027
\]

### Table A-2 Photovoltaic Panel Model

The power generated by PV

\[
P_{PV} = I A_{PV} \eta_{PV}
\]

### Table A-3 Battery Bank Model

The state of the charge (SOC), during the charging process

\[
SOC_{t+1} = SOC_t (1 - \delta_t) + \frac{I_{Bat_t} \Delta t \eta_{IC}}{c_{Bat}}
\]

The battery current \( I_{Bat} \), during the charging process

\[
I_{Bat_t} = \frac{P_{PV_t} + P_{WT_t} - P_{load_t}}{V_{Bat}}
\]

The state of the charge (SOC), during the discharging process

\[
SOC_{t+1} = SOC_t (1 - \delta_t) - \frac{I_{Bat_t} \Delta t}{c_{Bat}}
\]

The battery current \( I_{Bat} \), during the discharging process

\[
I_{Bat_t} = \frac{P_{load_t} - P_{PV_t} - P_{WT_t}}{V}
\]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{PV}$</td>
<td>PV panel area (m²)</td>
</tr>
<tr>
<td>$A_{WT}$</td>
<td>wind turbine rotor disk area (m²)</td>
</tr>
<tr>
<td>$C_0$</td>
<td>total constant cost including the cost of installation of the wind turbine and PV panels</td>
</tr>
<tr>
<td>$C_{Bat}$</td>
<td>nominal battery bank capacity (Ah)</td>
</tr>
<tr>
<td>$C_{IC}$</td>
<td>the total cost of the system</td>
</tr>
<tr>
<td>$C_{OhM}$</td>
<td>present value of maintenance cost</td>
</tr>
<tr>
<td>$C_{rep}$</td>
<td>the present value of replacement cost</td>
</tr>
<tr>
<td>$C_{Unit, Bat}$</td>
<td>unit Cost of battery bank ($/Ah)</td>
</tr>
<tr>
<td>$C_{Unit, PV}$</td>
<td>unit Cost of PV panel ($/m²)</td>
</tr>
<tr>
<td>$C_{Unit, WT}$</td>
<td>unit cost of the wind turbine($/m²)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>wind turbine power coefficient,</td>
</tr>
<tr>
<td>$f$</td>
<td>the time step (one hour in this study)</td>
</tr>
<tr>
<td>$\eta_{PV}$</td>
<td>efficiency of the PV array and corresponding converters</td>
</tr>
<tr>
<td>$\eta_{Bat}$</td>
<td>battery efficiency</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>battery bank charge efficiency factor</td>
</tr>
<tr>
<td>$f$</td>
<td>inflation rate</td>
</tr>
<tr>
<td>$\eta_{PV}$</td>
<td>efficiency of the PV array and corresponding converters</td>
</tr>
<tr>
<td>$\eta_{Bat}$</td>
<td>battery efficiency</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>battery bank charge efficiency factor</td>
</tr>
<tr>
<td>$f$</td>
<td>inflation rate</td>
</tr>
</tbody>
</table>

**Nomenclature**

- **PV**: Solar panel area (m²)
- **I**: Horizontal solar irradiance in (W/m²)
- **I_{Bat}**: Battery current
- **k_d**: Annual real interest rate
- **L_p**: System life period in years
- **N_{Bat}**: Total number of batteries
- **N_{rep}**: Number of replacements of the battery over the system life period
- **P_{Bat}**: Battery bank available power (W)
- **P_{PV}**: PV array output power (W)
- **P_{WT}**: Wind turbine power (W)
- **P_{PV, Nom}**: PV panel nominal power (W)
- **\rho**: Air density (1.225 Kg/m³)
- **SOC**: State of the charge of the battery
- **TC**: Total cost of the system
- **V_{Bat}**: Battery voltage
- **V_{ref}**: Wind speed at the reference height
- **V_w**: Hourly average wind velocity (m/s)
- **Y**: System life span in (years)
- **z_{hub}**: Wind turbine hub height
- **z_0**: Surface roughness length (m)
- **z_{ref}**: Reference height (m)
REFERENCES