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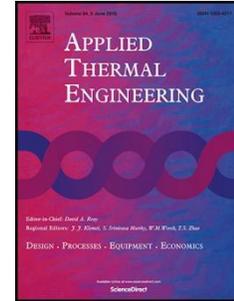
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1 **A Two-dimensional Analytical Model for Prediction of the** 2 **Radiation Heat Transfer in Open-cell Metal Foams**

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9

10 **Highlights:**

11

12 ● A new 2D explicit analytical model is developed to study the radiation heat
13 transfer.

14

15 ● A new correction factor is introduced to correct the deviation of specific area.

16

17 ● The present model has a reasonable precision by comparing with published data.

18

19 ● The effects of different control factors on the radiation characteristics are
20 evaluated.

21

22

23 **Abstract**

24 In this article, a new two-dimensional (2D) explicit analytical model for the

25 evaluation of the radiation heat transfer in highly porous open-cell metal foams is

26 formulated and validated. A correction factor, C , is introduced to correct the deviation

27 of the specific area in a simplified manner. Numerical results are compared with the

28 published experimental data and three-dimensional (3D) model proposed in previous

29 works. It reveals that the present two-dimensional model is proved to be relatively

30 accurate in estimating the radiative conductivity for all the investigated structures. In

31 the current work, the effects of the control parameters, such as the number of order in
 32 the iterative procedure, solid emissivity, the temperature difference, shape of solid
 33 particle and correction factor on the predictions of radiation characteristics are well
 34 discussed.

35

36 **Keywords:** Modelling; Thermal radiation; Porous medium; Open-cell metal foam;
 37 Radiation heat transfer.

Nomenclature

a	side length [m]	X, Y	Cartesian coordinates [-]
A_{sf}	specific area [m^{-1}]	Greek symbols	
b	bottom face of the unit cell [-]	α_i	dimensional coefficient [-]
C	correction factor [-]	β_i	dimensional coefficient [-]
d	side length of the unit cell [m]	ε	solid emissivity[-]
d_f	diameter of strut [m]	ρ	solid reflectance [-]
d_p	characteristic cell size [m]	σ	Stefan-Boltzmann constant[W/ m^2K^4]
F	configuration factor [-]	ϕ	porosity [-]
H	foam sample thickness [m]	Subscripts	
i	sequence of the unit cell [-]	bt	void face b to void face t
J	irradiation from void face[W/m^2]	bj	void face b to solid particle j
k_r	radiative conductivity [$W/m K$]	c	cold side
l_b	length of bottom void face [m]	h	hot side
l_j	length of solid particle [m]	jk	solid particle j to solid particle k
l_s	length of side void face [m]	jt	solid particle j to void face t
l_t	length of top void face [m]	kt	solid particle k to void face t

N_c	total number of cells [-]	sj	void face s to solid particle j
q_r	radiation heat flux [W/m^2]	st	void face s to void face t
Q_r	irradiation [m s^{-2}]	<i>Superscripts</i>	
s	side face of the unit cell [-]	-	negative direction
t	top face of the unit cell [-]		
T	temperature [K]		

38

39

40 **1. Introduction**

41 Metal foams are extensively used for many industrial applications involving
 42 numerous technological fields over more than 50 years due to their attractive physical
 43 properties such as, high porosity, large specific surface, flow mixing enhancement,
 44 attractive stiffness properties and low cost [1]. Their averaged thermo-physical
 45 properties are also important for many applications, e.g., compact heat exchangers [2],
 46 solar receivers [3], and catalytic reactors [4]. The main characteristic of the heat
 47 transfer in metal foams is dictated by the enhanced effective thermal conductivity
 48 (ETC). The ETC used to quantify the magnitude of the heat conduction in metal
 49 foams is studied through model prediction [5-13], numerical simulation [14-16] and
 50 experimental research [16-18].

51 Previous publications reported on the thermal properties of the metal foams at high
 52 temperature where conduction and radiation heat transfer may occur are relatively
 53 weak [19]. To overcome the experimental difficulties, Coquard et al. [20] proposed an

54 innovative method to evaluate the conduction and radiation contribution in metal
55 foams. They developed an identification method using thermograms obtained from
56 laser-FLASH measurements to minimize the discrepancy between experimental and
57 theoretical thermograms. Coquard et al. [19], afterwards, presented a detailed review
58 on the radiation and conduction heat transfer from ambient to high temperature. They
59 also proposed an analytical model for the real foams to predict the conduction and
60 radiation heat transfer at high temperature. Their predicted results agreed well with
61 the experimental results [20].

62 Several studies have been devoted to the radiation heat transfer in metal foam
63 [21-24]. Coquard et al. [21] modelled the radiation heat transfer in open cell metal
64 foams and closed cell polymer foams utilizing two approaches, i.e., Homogeneous
65 Phase Approach (HPA) and Multi-Phase Approach (MPA). The radiation heat
66 transfer of these two types of foams was investigated using three-dimensional (3D)
67 tomographic images. The calculated results were compared with the results of direct
68 Monte Carlo (MC) simulations and the suitability of the two approaches was then
69 evaluated. Tancrez et al. [22] developed a general method with direct identification of
70 the radiation properties, i.e., absorption, scattering coefficients and phase function of
71 porous medium using Monte Carlo (MC). This method was applied to both sets of
72 Dispersed radius Overlapping Opaque Spheres (DOOS) in a transparent fluid phase
73 and Dispersed radius Overlapping Transparent Spheres (DOTS) in an opaque solid
74 phase. Zhao et al. [23] measured the ETC of metal foams with a range of pore sizes

75 and porosities between 300 and 800 K. The radiative conductivity was decoupled
76 from the equivalent conductivity due to conduction. As for the equivalent
77 conductivity due to conduction contribution alone, the model proposed in [6] was
78 used. At the same time, Zhao et al. [24] used the Rosseland equation to calculate the
79 equivalent radiative conductivity based on the experimentally obtained spectral
80 transmittance and reflectance. The calculated results were found to be in satisfactory
81 agreement with the experimental data [23].

82 Although many significant results in the modelling radiation heat transfer of
83 open-cell metal foam have already been obtained, the aforementioned approaches are
84 not quite suitable for engineering applications. Thus, Zhao et al. [25] proposed an
85 explicit analytical model based on the simplified cubic structure. In this model, the
86 fundamental foam parameters and the emission and reflectance in metal foam
87 structure were considered to establish functional relationships between the structure
88 and the radiation characteristics of open-cell metal foams. The calculated equivalent
89 radiative conductivity showed that in general there was a good agreement between the
90 predicted and experimental data. Most recently, as an extension of the simplified
91 analytical approach of [25], Contento et al. [26] made further improvements by
92 recalculating the configuration factors that involved in the dimensionless coefficients
93 and a close agreement between predicted result and measured data was achieved. As
94 the same time, Contento et al. [27] developed a new radiative heat transfer model
95 based on a more realistic Lord Kelvin representation of open cell metal foams instead

96 of the simplified cubic structure using the same analytical approach. This explicit
97 simple approach that initially proposed by Zhao et al. [25] can be relatively suitable
98 for engineering applications.

99 Based on the brief literature review, it can be seen that much effort has been made
100 to develop models for the estimation of the radiation heat transfer in open-cell metal
101 foam. From an engineering perspective, however, due to the complex nature of the
102 configuration factors for implementation in three-dimensional modelling, research on
103 modelling radiation heat transfer has been far from complete. More effort needs to be
104 made in this area. In this study, a newly simplified two-dimensional model is
105 proposed and could serve as an efficient alternative to evaluate the radiative
106 characteristics in porous open-cell metal foams for engineering applications. For the
107 assessment of the new model, the comparisons between numerical predictions with
108 experimental data [23] and previously proposed model [25] are carried out.

109 **2. Model description**

110 *2.1. Structure simplification*

111 The microstructure of the typical open cell metal foam is shown in Fig. 1. Porous
112 medium such as metal foams has a complex microstructure made up of solid
113 ligaments and pores generally filled with fluid. In order to simplify the analysis of the
114 radiation heat transfer in metal foam, the microstructure can be assumed to consist of
115 randomly oriented cells with characteristic size d_p , which are mostly homogeneous in
116 size and shape, whilst the solid of the metal foam can be treated as particles with

117 simple geometry (circle, square and rectangle etc.) distributed in fluid zone regularly
 118 or randomly. For the purpose of simplification, the connection of the solid phase of
 119 the metal foam can be neglected (Due to the large porosity ($\phi \geq 90\%$) of metal foam,
 120 thermal radiation in metal foam mainly passes through the void).

121 Based on the above simplification, a new 2D structure with regularly distributed
 122 square particles with side length of a are selected to develop the analytical model for
 123 analysing the radiation heat transfer, as presented in Fig. 2(a). Since the structure is
 124 periodic, Fig.2 (b) shows the details of two neighbouring square unit cells. Within
 125 each cell, there are four quarters of solid particle at four corners which are labelled
 126 with 1-4 respectively. As for the four faces, two side faces are referred as s , whereas
 127 the top and bottom faces are represented by t and b . The relationship between d and
 128 measured d_p based on the same area is shown as:

$$129 \quad d^2 = \frac{\pi}{4} d_p^2 \quad (1)$$

$$130 \quad d = \frac{\sqrt{\pi}}{2} d_p \quad (2)$$

131 Then, a is obtained based on the porosity for the two-dimensional structure as:

$$132 \quad \frac{4a^2}{d^2} = 1 - \phi \quad (2)$$

$$133 \quad a = \sqrt{\frac{1 - \phi}{4}} d \quad (3)$$

134 here ϕ is the porosity of the metal foam.

135 2.2. Assumptions

136 In order to simplify the heat transfer mechanism in open-cell metal foam, the
137 following major assumptions were made in the derivations of the governing
138 equations:

139 (i) the diffraction is neglected. The characteristic size of the porous medium is
140 considered to be large compared to the heat radiation wavelengths.

141 (ii) the solid particles are assumed as grey and opaque since they are metallic, and the
142 void zone is considered as vacuum.

143 (iii) the surface of the solid particles reflecting diffusely the incident radiation is
144 assumed since the surface roughness at 10 μm scale is being taken into account
145 [26].

146 (iv) steady-state heat flow is assumed in a specific zone of the metal foam
147 sandwiched between two plates with cold boundary temperature (T_c) for the top
148 plate and hot boundary temperature (T_h) for the bottom plate. Sample is thermally
149 insulated at side walls, which means that there exists a radiation heat flux in the
150 positive Y direction.

151 (v) it is assumed that the radiation is decoupled from the conduction and the
152 temperature varies linearly with Y direction [25].

153 (vi) temperature difference within unit cell can be neglected since the porous foam
154 sample is sufficiently thick. This means that each unit cell has a unique value of
155 temperature in the same layer [26].

156 Other simplifications are described in the due course in the rest of the paper.

157 2.3. Mathematical formulations

158 2.3.1 Basic formulations

159 Based on the assumptions, the temperature difference between the two cells in
 160 adjacent planes in Y direction is represented by equation:

$$161 \quad \Delta T = \frac{T_h - T_c}{N_c} \quad (4)$$

162 where ΔT is the temperature difference between the two cells in adjacent planes, N_c
 163 denotes the total number of cells in Y direction which is given by:

$$164 \quad N_c = \frac{H}{d} \quad (5)$$

165 where H is the thickness of the porous medium sample. The temperature of the i th
 166 ($i=1,2,3 \dots N_c$) cell is:

$$167 \quad T[i] = T_h - (i - 1)\Delta T \quad (6)$$

168 Thus, the radiative conductivity k_r can be obtained by:

$$169 \quad k_r = \frac{q_{r,net}}{(T_h - T_c) / H} \quad (7)$$

170 where $q_{r,net}$ is the net radiation heat flux.

171 The net radiation heat flux $q_{r,net}$ will be calculated based on the top void face t of
 172 the i th cell. Since the radiation heat fluxes in both directions are not identical, the net
 173 radiation heat flux can be mathematically expressed by the following equation:

$$174 \quad q_{r,net} = q_r - q_r^- \quad (8)$$

175 where q_r is the radiation heat flux in the positive Y direction and q_r^- is the
 176 radiation heat flux in the negative Y direction, respectively.

177 2.3.2 Formula derivation

178 Firstly, radiation in the positive Y direction is analysed, as radiation in the negative
 179 Y direction is familiar with that in positive Y direction. The total irradiation on the
 180 void face t of the i th cell (Fig. 2(b)) consists of both the emission and reflectance from
 181 the solid particles $1-4$ to the void faces s, b . The total irradiation Q_r on t is given by:

$$182 \quad Q_r = (Q_r)_{emission} + (Q_r)_{reflec\ tan\ ce} \quad (9)$$

183 where,

$$184 \quad (Q_r)_{emission} = \sum_{j=1}^4 l_j F_{jt} \cdot \varepsilon \sigma T^4 + l_b F_{bt} J_b + 2l_s F_{st} J_s \quad (10)$$

185 In Eq. (11):

186 l_j ($j=1,2,3,4$) is the length of the j th solid particle within a unit cell, ε is the solid
 187 emissivity, σ is Stefan-Boltzmann constant equal to 5.669×10^{-8} W/m²K⁴, T is the
 188 temperature of the unit cell, l_b and J_b are the length and irradiation of the void face b ,
 189 l_s and J_s are the length and irradiation of the void faces, F is the configuration factor.

190 The three terms on the right side of Eq. (11) are the emission on the void face t
 191 from four solid particles in four corners, bottom void face b and side void faces s ,
 192 respectively.

$$193 \quad (Q_r)_{reflec\ tan\ ce} = \sum_{j=1}^4 \sum_{k=1, j \neq k}^4 \rho l_j F_{jk} F_{kt} \cdot \varepsilon \sigma T^4 + \sum_{j=1}^4 \rho l_b F_{bj} F_{jt} J_b + 2 \sum_{j=1}^4 \rho l_s F_{sj} F_{jt} J_s \quad (11)$$

194 where $\rho = 1 - \varepsilon$ is the solid reflectivity. Similarly, the three terms on the right side of Eq.
 195 (12) represent the reflectance of the incident radiation on the solid particles from each
 196 other, bottom void face and two side faces, respectively.

197 It is noted that the specific surface area in the present 2D model is different
 198 compared with that in 3D structure, this may result in the emission deviation from
 199 solid particles in the calculation of radiation. In order to reduce this deviation, a new
 200 correction factor C is introduced to correct the emission from solid particles, which is
 201 defined as:

$$202 \quad C = \frac{A_{sf,3D}}{A_{sf,2D}} \quad (12)$$

203 where $A_{sf,2D}$, $A_{sf,3D}$ are the specific surface areas in the present simplified 2D
 204 model and 3D model respectively.

205 For the 3D structure of the metal foam, according to [28], the specific surface area
 206 was defined as:

$$207 \quad A_{sf,3D} = \frac{3\pi d_f [1 - e^{-(1-\phi)/0.04}]}{(0.59 d_p)^2} \quad (13)$$

208 where d_f is diameter of the strut.

209 For the present model, the specific surface area can be defined as the ratio of the
 210 total side length of solid particles to the area:

$$211 \quad A_{sf,2D} = \frac{8a}{d^2} \quad (14)$$

212 Thus, the correction factor C can be derived as:

$$213 \quad C = \frac{A_{sf,3D}}{A_{sf,2D}} = \frac{3\pi d_f [1 - e^{-(1-\phi)/0.04}] d^2}{8a(0.59 d_p)^2} = 1.0773 \pi^{2.5} (1-\phi)^{-0.5} [1 - e^{-(1-\phi)/0.04}] \frac{d_f}{d_p} \quad (15)$$

214 Thus, the previous analysis needs to be reconsidered. The proposed correction factor
 215 C is added into the emission radiation term in Eqs. (11-12), then Eqs. (11-12) can be
 216 rewritten as:

$$217 \quad (Q_r)_{emission} = \sum_{j=1}^4 Cl_j F_{jt} \cdot \varepsilon \sigma T^4 + l_b F_{bt} J_b + 2l_s F_{st} J_s \quad (16)$$

$$218 \quad (Q_r)_{reflec\ tan\ ce} = \sum_{j=1}^4 \sum_{k=1, j \neq k}^4 C \rho l_j F_{jk} F_{kt} \cdot \varepsilon \sigma T^4 + \sum_{j=1}^4 \rho l_b F_{bj} F_{jt} J_b + 2 \sum_{j=1}^4 \rho l_s F_{sj} F_{jt} J_s \quad (17)$$

219 Considering the model is two-dimensional, the unit of Q is W/m.

220 For the purpose of convenience, the configuration factors can be analysed

221 geometrically. The following formulations are used:

$$222 \quad F_{12} = F_{21} = F_{13} = F_{31} = F_{34} = F_{43} = F_{24} = F_{42} = F_1 \quad (18)$$

$$223 \quad F_{14} = F_{41} = F_{23} = F_{32} = F_2 \quad (19)$$

$$224 \quad F_{1t} = F_{2t} = F_3 \quad (20)$$

$$225 \quad F_{3t} = F_{4t} = F_4 \quad (21)$$

$$226 \quad l_1 = l_2 = l_3 = l_4 \quad (22)$$

$$227 \quad l_t = l_b = l_s \quad (23)$$

$$228 \quad F_{s1} = F_{s3} = F_{b3} = F_{b4} = \frac{l_1}{l_s} F_3 \quad (24)$$

$$229 \quad F_{s2} = F_{s4} = F_{b1} = F_{b2} = \frac{l_1}{l_s} F_4 \quad (25)$$

230 where l_t is the length of the top void face in the unit cell.

231 Radiation in the positive Y direction is given by:

$$232 \quad q_r = \frac{Q_r}{l_t} \quad (26)$$

233 Substitute Eqs. (19-26) into Eq. (27), the radiation in the positive Y direction can be

234 expressed in the following manner:

$$\begin{aligned}
235 \quad q_r &= \frac{Q_r}{l_t} = C \frac{l_1}{l_t} (2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)\varepsilon\sigma T^4 \\
236 \quad &+ (F_{bt} + 4\frac{l_1}{l_t}\rho F_3 F_4)J_b + \left[2F_{st} + 2\frac{l_1}{l_t}\rho(F_3 + F_4)^2 \right] J_s \quad (27)
\end{aligned}$$

237 For reducing Eq. (28), dimensionless coefficients $\beta_1, \beta_2, \beta_3$ are introduced and defined
238 as:

$$239 \quad \beta_1 = Cl_1(2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)/l_t \quad (28)$$

$$240 \quad \beta_2 = F_{bt} + 4l_1\rho F_3 F_4 / l_t \quad (29)$$

$$241 \quad \beta_3 = 2F_{st} + 2l_1\rho(F_3 + F_4)^2 / l_t \quad (30)$$

242 Thus, Eq. (28) can be further reduced to:

$$243 \quad q_r = \beta_1 \varepsilon\sigma T^4 + \beta_2 J_b + \beta_3 J_s \quad (31)$$

244 In order to calculate the radiation in the positive Y direction q_r, J_b and J_s which are in
245 the right side of Eq. (32) should be calculated firstly. Similarly, the irradiation from
246 the void face s, J_s can be analyzed

$$\begin{aligned}
247 \quad J_s &= C \frac{l_1}{l_s} (2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)\varepsilon\sigma T^4 \\
248 \quad &+ (F_{bt} + 4\frac{l_1}{l_s}\rho F_3 F_4)J_s + \left[F_{st} + \frac{l_1}{l_s}\rho(F_3 + F_4)^2 \right] J_b \quad (32)
\end{aligned}$$

249 The quantity of J_s can be calculated from Eq. (33) which is written as following:

$$250 \quad J_s = C \frac{l_1(2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)/l_s}{1 - F_{bt} - 4l_1\rho F_3 F_4 / l_s} \varepsilon\sigma T^4 + \frac{2F_{st} + 2l_1\rho(F_3 + F_4)^2 / l_s}{1 - F_{bt} - 4l_1\rho F_3 F_4 / l_s} J_b \quad (33)$$

251 Eq. (34) can be further written as:

$$252 \quad J_s = \alpha_1 \varepsilon\sigma T^4 + \alpha_2 J_b \quad (34)$$

253 where α_1 and α_2 are the dimensionless coefficients, defined as:

$$254 \quad \alpha_1 = C \frac{l_1(2 + 4\rho F_1 + 2\rho F_2)(F_3 + F_4)/l_s}{1 - F_{bt} - 4l_1\rho F_3 F / l_s} \quad (35)$$

$$255 \quad \alpha_2 = \frac{2F_{st} + 2l_1\rho(F_3 + F_4)^2 / l_s}{1 - F_{bt} - 4l_1\rho F_3 F / l_s} \quad (36)$$

256 Substitute Eq. (35) into Eq. (32), Eq. (32) can be further written as:

$$257 \quad q_r = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma T^4 + (\beta_2 + \beta_3\alpha_2)J_b \quad (37)$$

258 2.3.3 Iteration process

259 For the convenience of iteration process, q_r , T , J_b of the i th unit cell can be

260 rewritten as $q_r[i]$, $T[i]$, $J_b[i]$, thus, Eq.(38) can be rewritten as:

$$261 \quad q_r[i] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i])^4 + (\beta_2 + \beta_3\alpha_2)J_b[i] \quad (38)$$

262 As the bottom face b of the i th unit cell is the top face of the $(i-1)$ th unit cell.

263 Therefore, the Eq. (39) can be expressed as:

$$264 \quad q_r[i] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i])^4 + (\beta_2 + \beta_3\alpha_2)q_r[i-1] \quad (39)$$

265 Similarly,

$$266 \quad q_r[i-1] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i-1])^4 + (\beta_2 + \beta_3\alpha_2)q_r[i-2] \quad (40)$$

$$267 \quad q_r[i-2] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T[i-2])^4 + (\beta_2 + \beta_3\alpha_2)q_r[i-3] \quad (41)$$

268 ...

269 where the bottom face of the first unit cell is the bottom boundary of the porous

270 medium sample with the temperature T_h , thus:

$$271 \quad q_r[1] = (\beta_1 + \beta_3\alpha_1)\varepsilon\sigma(T_h)^4 + (\beta_2 + \beta_3\alpha_2)\varepsilon\sigma T_h^4 \quad (42)$$

272 Thus, the quantity of $q_r[i]$ can be calculated implementing an iterative procedure from

273 the boundary.

274 In the case of the radiation flux in the negative y direction, it can similarly be written
 275 as:

$$276 \quad q_r^- [i] = (\beta_1 + \beta_3 \alpha_1) \varepsilon \sigma (T[i+1])^4 + (\beta_2 + \beta_3 \alpha_2) J_b^- [i] \quad (43)$$

277 where $J_b^- [i]$ is the irradiation on void face t of i th unit cell from the top void face of
 278 the $(i+1)$ th unit cell, as shown in Fig.2(b).

279 Similarly,

$$280 \quad q_r^- [i] = (\beta_1 + \beta_3 \alpha_1) \varepsilon \sigma (T[i+1])^4 + (\beta_2 + \beta_3 \alpha_2) q_r^- [i+1] \quad (44)$$

$$281 \quad q_r^- [i+1] = (\beta_1 + \beta_3 \alpha_1) \varepsilon \sigma (T[i+2])^4 + (\beta_2 + \beta_3 \alpha_2) q_r^- [i+2] \quad (45)$$

282 ...

$$283 \quad q_r^- [N_c] = \varepsilon \sigma T_c^4 \quad (46)$$

284 The determination of $q_r^- [i]$ is the same as that of $q_r [i]$. Then $q_{r,net}$ can be
 285 calculated by Eq. (9). Consequently, the equivalent radiative conductivity is
 286 determined by Eq. (8).

287 3. Determination of coefficients

288 In the analytical solution of the equivalent radiative conductivity, the dimensionless
 289 coefficients, i.e., $\beta_1, \beta_2, \beta_3$ and α_1, α_2 need to be determined. As previously mentioned,
 290 the coefficients are the functions of the configuration factors, geometric parameters
 291 and the solid reflectance according to Eqs. (29-31) and Eqs. (36,37). In order to
 292 determine these coefficients, the configuration factors, $F_1, F_2, F_3, F_4, F_{bt}$ and F_{st} ,
 293 should be firstly determined. The crossed strings method is utilized to calculate the

294 configuration factors for a two-dimensional geometric structure with known
295 geometric parameters of the unit cell.

296 As for the solid reflectance, it is recognized that the solid reflectance is related to
297 the emissivity ($\rho + \varepsilon = 1$ for opaque material). However, the emissivity of a solid
298 material depends on many other factors such as temperature and orientation. The
299 influence of the emissivity on the radiation heat transfer is discussed in next section.

300 **4. Results and discussion**

301 *4.1. Model validation*

302 In the current work, the validation of the model is based on the FeCrAlY (Fe 75%,
303 Cr 20%, Al 5%, Y 2%) metallic foam produced via the sintering route which is
304 studied by Zhao et al. [23] and the test conditions employed for the current simulation
305 are listed in Table 1. Due to the fact that the real values of the geometric parameters
306 of the metal foam usually are different from that supplied by manufacturers, the
307 measured values instead of the nominal values will be considered. The currently
308 developed model will be evaluated through the comparison of the equivalent radiative
309 conductivity between the experimental data [23] and numerical results under the same
310 test conditions that shown in Table 1 based on the previous analytical models [25,26].

311 Figs. 3-6 show the comparison of the radiative conductivity versus temperature at
312 different pores per inch (PPI) and porosity between the present predicted results of
313 corrected model and experimental data [23] as well as numerical results using
314 previous models in [25,26]. The results in Figs. 3-6 clearly show that the proposed

315 model and model from reference [26] perform well in predicting the experimental
316 data in all cases, while the initial model proposed by Zhao et al. [25] performs not
317 well for S2, S4. The differences between the predicted results and experimental data
318 as well as the current prediction results are reported in Table 2. And it is noted that
319 there may have been a slight over-estimation or under-estimation of the radiative
320 conductivity. This could be mainly due to the fact that the current model assumes
321 uniform distribution of the solid particles in the porous media and uses the average
322 particle diameter whereas in the real case the particle size is within a certain range.
323 Despite this, it can be seen that in general there is a good agreement between the
324 currently predicted and the experimental data.

325 Then the effects of the control parameters such as, correction factor, the number of
326 the orders, the solid emissivity, temperature gradient, and the geometry on the
327 radiative conductivity will be examined in detail.

328 *4.2. Effect of correction factor*

329 Fig. 7 shows the predicted radiative conductivity with and without the correction
330 factor for the case of S1. It can be seen clearly that there is a large deviation between
331 experimental data and predicted results for the case without correction factor. Thus,
332 the contribution of the correction is significant. It reveals that the
333 geometrical characteristics needs to be consistent with that in three-dimensional
334 structure of metal foam to ensure the validity of the simplified model.

335 *4.3. Effect of number of orders*

336 As analysed in Section 2, the radiative conductivity is determined by implementing
337 an iterative procedure which takes into account the irradiation from other unit cells up
338 to the ones in contact with the boundaries. We define that the model has first-order
339 accuracy if the $(i-1)$, i , $(i+1)$, $(i+2)$ th unit cells are reserved which implies that the i th
340 cell and $(i+1)$ th cell share the face t that only accounts for the contributions from the
341 adjacent neighbouring cells($(i-1)$ th, $(i+2)$ th) in both directions. Geometrically, the
342 face t is the central face within these four cells along y direction. Thus, the bottom
343 face of the $(i-1)$ th cell and the top face of $(i+2)$ th cell are boundaries. Similarly, for
344 second-order accuracy, one more unit cell in both directions is included in the
345 calculation. For the other numbers of the orders, they can be defined in a same
346 principle. Fig. 8 shows that the radiative conductivity of the sample 1 varies with the
347 number of the order at two different temperatures, i.e. 550 K and 750 K at a solid
348 emissivity of 0.6. It reveals that the numbers of the cells above and below the central
349 face need to be considered to obtain the stable values of the radiative conductivity.
350 Thus, in order to stabilize the calculated values of the radiative conductivity, the
351 number of orders of 25 is used for the current model.

352 4.4. Effect of the solid emissivity

353 As previously mentioned, the effect of the solid emissivity on the radiative
354 conductivity needs to be addressed. Generally, the emissivity of the steel varies
355 between 0.3 and 0.8 [29]. Fig. 9 shows the effect of the solid emissivity on the values
356 of the radiative conductivity at two temperatures of 550 K and 750 K. It is seen that

357 the value of the radiative conductivity increases with increasing solid emissivity even
358 though a large emissivity can lead to a smaller reflectance. It reveals that the
359 proportion of the emission in total radiation is relatively large. In addition, the effect
360 of the solid emissivity on the radiative conductivity is significant at temperature of
361 750 K, while that is relatively mild at temperature of 550 K. The reason is that the
362 emitting radiation is in proportion to the biquadrate of temperature. However, for the
363 purpose of comparison, a solid emissivity of 0.6 is assumed in present work, which is
364 consistent with the previous study of [25] and [26].

365 *4.5. Effect of temperature gradient*

366 For a fixed thickness with the same mean temperature, the effect of the temperature
367 difference on the predicted radiative conductivity at fixed temperature of 750 K is
368 shown in Fig.10. A specific mean temperature can be determined in different
369 temperature difference between the top and bottom boundaries of the foam samples. It
370 can also be concluded from Fig. 10 that the radiative conductivity is not sensitive to
371 the temperature difference. In the current model, therefore, a 10 K temperature
372 difference is used for the iterative procedure.

373 *4.6. Effect of geometry*

374 As mentioned in Section 2.1, the shape of the solid particles can be other simple
375 geometries. In the current study, two shapes, i.e. circle, rhombus are assumed based
376 on the same porosity and characteristic size in order to investigate the effect of the
377 shape of solid particles, as shown in Fig. 11. The calculated results are shown in Fig.

378 12 for the case of S1. It can be seen that the shape of the solid particles has small
379 effect on the thermal radiation in the present model. It is noted that different shapes of
380 the solid particles may lead to different geometry structure, which implies that the
381 configuration factors may be different. However, due to the large porosity of the
382 metal foam, the influence of the different structures is insignificant.

383 Fig. 13 demonstrates the variation of the radiative conductivity with the change of
384 the PPI for the same porosity of 95%. For comparison purposes, two PPI are used i.e.
385 30 and 60. Comparison shows that the radiative conductivity increases monotonously
386 with decreasing PPI at the same temperature, such a result is due to the smaller PPI
387 results in a bigger pore size. And the bigger pore size would lead to a large
388 “penetration thickness” which implies that more heat can be directly transferred by
389 thermal radiation to a deeper thickness of the foam before it decays to a lower level
390 [25].

391 **5. Conclusions**

392 A newly developed two-dimensional model is employed for the calculation of the
393 radiation heat transfer in highly porous open-cell metal foams and comparing these
394 results with available experimental data as well as three-dimensional numerical
395 solution proposed in the previous work. A new correction factor, C , is introduced for
396 correcting the deviation of specific area between simplified two-dimensional structure
397 and three-dimensional structure. The results demonstrated that using a
398 two-dimensional analytical model instead of a three-dimensional approach leads to a

399 relatively minor discrepancy. Besides, the calculation is simpler than the
400 three-dimensional model because of the simpler determination of configuration
401 factors and coefficients due to the nature of the two-dimensional structure, which is
402 significant for engineering applications. The effect of the solid emissivity on the
403 radiative conductivity is more significant at higher temperature. The radiative
404 conductivity is not sensitive to the temperature difference during the iterative
405 procedure. The effect of the shape of the solid particle is observed and it is relatively
406 small. It is found that the samples with smaller PPI could lead to a higher value of
407 radiative conductivity. The correction factor C is found to be significant for the
408 present model. Overall, the main contribution of the proposed two-dimensional model
409 is the simplicity and convenience of calculation with good accuracy compared with
410 the previous three-dimensional model. In addition, the present model is also suitable
411 for vacuum condition. Future works are still needed to investigate the thermal
412 radiation in metal foam in atmospheric pressure. Besides, more experimental data of
413 different metal foams (material, PPI, porosity etc.) are needed to validate the present
414 model.

415

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506

507 **Fig. 1.** Typical open-cell metallic foam morphology [25].

508

509 **Fig. 2.** (a) Two-dimensional idealized structure of porous medium; (b) Model foam
510 structure and notations.

511

512 **Fig. 3.** Comparison between predicted results of present corrected model and
513 experimental data, results of previous 3D models for S1.

514

515 **Fig. 4.** Comparison between predicted results of present corrected model and
516 experimental data, results of previous 3D models for S2.

517

518 **Fig. 5.** Comparison between predicted results of present corrected model and
519 experimental data, results of previous 3D models for S3.

520

521 **Fig. 6.** Comparison between predicted results of present corrected model and
522 experimental data, results of previous 3D models for S4.

523

524 **Fig. 7.** Effect of correction factor on radiative conductivity for S1.

525

526 **Fig. 8.** Radiative conductivity vs. the number of orders at fixed solid emissivity of 0.6
527 and different temperatures for S1.

528

529 **Fig. 9.** Radiative conductivity vs. solid emissivity at different temperatures for S1.

530

531 **Fig. 10.** Radiative conductivity vs. temperature difference at fixed mean temperature
532 for S1.

533

534 **Fig. 11.** Different shapes of solid particle.

535

536 **Fig. 12.** Effect of shape of solid particle on radiative conductivity for S1.

537

538

Fig. 13. Radiative conductivity vs. temperature at different PPI.

539

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543

544 **Table 1**

545 Geometric properties of different foam samples [26].

546

	Sample			
	S1	S2	S3	S4
Pores per inch (PPI)	30	30	60	60
Nominal porosity (%)	95	90	95	90
Measured porosity (%)	95.9	90.7	94.5	90.8
Nominal cell size(mm)	0.847	0.847	0.423	0.423
Measured cell size(mm)	1.999	2.089	0.975	0.959
Equivalent cell size(mm)	1.772	1.851	0.864	0.850
Measured diameter of the strut(mm)	0.215	0.267	0.124	0.154

547

548

549 **Table 2**

550 Differences between predicted results and experimental data.

551

Sample	Zhao's model [25]	Contento's model [26]	Present corrected model
S1	-48.16%	-17.35%	-12.49%
S2	485.95%	63.37%	35.57%
S3	-19.14%	23.98%	-19.23%
S4	205.50%	-13.17%	-7.07%

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553