Short-wavelength analysis of magnetorotational instability of resistive MHD flows

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Abstract

Local stability analysis is made of axisymmetric rotating flows of a perfectly conducting fluid and resistive flows with viscosity, subjected to external azimuthal magnetic field B_θ to non-axisymmetric as well as axisymmetric perturbations. For perfectly conducting fluid (ideal MHD), we use the Hain-Lüst equation, capable of dealing with perturbations over a wide range of the axial wavenumber k to take short wavelength approximation. When the magnetic field is sufficiently weak, the maximum growth rate is given by the Oort A-value $|Ro|$, where $\Omega(r)$ is the angular velocity of the rotating flow as a function only of r, the distance from the axis of symmetry, and the prime designates the derivative in r. As the magnetic field is increased, the keplerian flow becomes unstable to waves of short axial wavelength when $Rb = r^2(B_\theta/r)'/(2B_\theta) > -3/4$ with growth rate proportional to $|B_\theta|$. We also incorporate the effect of the viscosity and the electric resistivity and apply the WKB method in the same way as we do to the perfectly conducting fluid. In the inductionless limit,i.e. when the magnetic diffusivity is much larger than the viscosity, Keplerian-rotation flow of arbitrary distributions of the magnetic field, including the Liu limit, becomes unstable.

1 Introduction

Since rediscovery of Velikhov and Chandrasekhar's result [1, 2] by Balbus and Hawley [3], the magnetorotational instability (MRI) has attracted great attention as a plausible mechanism for triggering turbulence in the flow of an accretion disk, for promoting outward transport of angular momentum, while the matter accretes the center. There is a well known Rayleigh's criterion for stability of a rotating flow of circular streamlines [4]. Given the angular velocity $\Omega(r)$ as a function only of the distance r from the rotation axis, define the local Rossby number by $Ro = \frac{1}{2}\mathrm{d}\log\Omega/\mathrm{d}\log r = r\Omega'/(2\Omega)$ [5, 6]. Here the prime designates the derivative with respect to r. In terms of the epicyclic frequency $\kappa^2 = (\Omega^2 r^4)'/r^3$, it is expressed as $Ro = \kappa^2/(4\Omega^2) - 1$. If $\kappa^2 \geq 0$ or $Ro \geq -1$ everywhere, such a rotating flow is linearly stable against axisymmetric disturbances [4, 6]. For an accretion disk, the angular velocity could satisfy the Keplerian law: $\Omega(r)^2 r = -\nabla\Phi$; $\Phi = 1/r$, for which $Ro = -3/4$. Rayleigh's criterion may suggest that Keplerian rotation $\Omega \propto r^{-3/2}$ is hydrodynamically stable.

The magnetic field parallel to the rotation axis drastically alters the stability characteristics. If the axial magnetic field is applied, however weak it is, a rotating flow suffers from instability if $Ro < 0$ [1, 2], implying that an accretion disk with Keplerian flow is unstable. We refer this instability to the standard magnetorotational instability (SMRI). The maximum growth rate at a local portion was found to be $3|\Omega|/4$ for a Keplerian rotation [7]. For a general rotating flow of differential rotation $\Omega(r)$ satisfying $Ro < 0$, the most unstable local instability mode of the SMRI is the axisymmetric one, with the maximum growth rate being $\nu_A/\Omega = \frac{1}{2}|\mathrm{d}\log\Omega/\mathrm{d}\log r|$, the Oort A-value [7]. A distinguishing feature is that this growth rate is independent of the applied field strength.

When the magnetic field is frozen into the fluid, the differential rotation of the flow generates the azimuthal component of the magnetic field once the magnetic field acquires the radial component which is possibly created by perturbing the axial field [8, 9]. Hence, it is worthwhile to look into the stability of a rotating flow applied by the azimuthal magnetic field and by a combination of the azimuthal and the axial magnetic field. The instability of the former case is called the azimuthal MRI or the AMRI, and the latter is called the helical MRI or the HMRI [5]. The HMRI has been extensively studied for a fluid of very low conductivity, called the inductionless limit [10, 5], because this is relevant to the experimental setting of using a liquid metal of very low conductivity [11]. Recently, an elaborate analysis has also been made for the AMRI in the regime of very low conductivity [12, 13].

For the perfectly conducting case, the AMRI and the HMRI to three-dimensional disturbances of short wavelength were both examined numerically by Balbus and Hawley [8]. They showed occurrence of the instability by conducting numerical computation of linearized equations made simplified by leaving out, by a physical intuition, terms which appeared to be less important when the spatial variation of the basic magnetic field is slow. But they did not give the values of the growth rate or the parameter region for instability. Define magnetic Rossby number $Rb = r^2(B_\theta/r)'/(2B_\theta)$ [14]. Ogilvie and Pringle [15] addressed three-dimensional AMRI by not only the short-wavelength but also the global analyses. By the former analysis, they showed that, in the limit of the axial wavenumber $k \to \infty$, the maximum growth rate approaches the Oort A-value in the weak-field regime, while that, in the same limit, the instability occurs for magnetic Rossby number $Rb > -3/4$ in the strong-field regime. We point out that the traditional treatment of the short-wave stability analysis is liable to miss some terms if a WKB-form of the solution is substituted at an early stage. For a circular symmetric flow, the equation for the radial displacement field is known as the Hain-Lüst equation [16, 17]. We resort to the Hain-Lüst equation, as augmented with the terms of the basic flow, in its full form, for the AMRI to non-axisymmetric as well as axisymmetric disturbances. With this equation at hand, we are capable of exploring the local instability over a wide range of k. And the same idea is used when the viscosity and electric resistivity are included.

2 Equations and short-wavelength approximation

We consider a circular symmetric flow of an incompressible inviscid fluid with infinite electric conductivity, subjected to a steady external magnetic field, and the linear stability of a localized disturbance along one of the streamlines. We assume that the radial wavelength is much small compared with the radius r of the streamline, being a sort of the WKB approximation. We introduce global cylindrical coordinates (r,θ,z) with the z-axis lying on the symmetric axis. The basic state is a rotating flow in equilibrium, with the angular velocity $\Omega(r)$, subject to a magnetic field having the azimuthal and the axial components $B_\theta(r) = r\mu(r)$ and B_z.

$$\boldsymbol{U} = r\Omega(r)\boldsymbol{e}_\theta, \ \boldsymbol{B} = r\mu(r)\boldsymbol{e}_\theta + B_z\boldsymbol{e}_z, \tag{1}$$

where $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_z$ are the unit vectors in the azimuthal and the axial directions, respectively. We mainly focus on the azimuthal field.

Denote $\tilde{\lambda} = \lambda + im\Omega$. Assume disturbance of velocity, magnetic field and pressure to be $\tilde{\boldsymbol{u}}$, $\tilde{\boldsymbol{B}}$ and $\tilde{p}$ and a new variable $\chi = -ru_r/\tilde{\lambda}$ associated with the radial Lagrangian displacement [16]. We have the Hain-Lüst equation [17],

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(f\frac{\mathrm{d}\chi}{\mathrm{d}r}\right)=g\chi, \tag{2}$$

where, by use of the definition $h^2=m^2/r^2+k^2$,

$$\begin{aligned} f &= \frac{1}{h^2 r}\left(\tilde{\lambda}^2+\frac{F^2}{\rho\mu_0}\right), \\ g &= \frac{\mathrm{d}}{\mathrm{d}r}\left[\frac{2im}{h^2r^2}\left(\Omega\tilde{\lambda}-\frac{i\mu F}{\rho\mu_0}\right)\right]+\frac{1}{r}\left(\tilde{\lambda}^2+\frac{F^2}{\rho\mu_0}\right) \\ &\quad +\frac{\mathrm{d}\Omega^2}{\mathrm{d}r}-\frac{1}{\rho\mu_0}\frac{\mathrm{d}\mu^2}{\mathrm{d}r}+\frac{4k^2\left(\Omega\tilde{\lambda}-\mu iF/(\rho\mu_0)\right)^2}{h^2r(\tilde{\lambda}^2+F^2/(\rho\mu_0))}, \end{aligned} \tag{3}$$

Here the magnetic permeability μ_0 the density ρ are assumed to be constant, and $F=m\mu+B_zk$. We seek the solution of (2) in the WKB approximation. For this purpose, we substitute into (2) the form $\chi(r)=p(r)\exp[i\int q(r)\mathrm{d}r]$ with the constraint that the radial wavelength $2\pi/q$ is assumed to be much shorter than the characteristic length L, a measure for radial inhomogeneity, namely, $qL\gg 1$. Neglecting the second-order terms in $qL\gg 1$, the dispersion relation is gained from (2) as $q^2=-g/f$, producing

$$\begin{aligned} &(h^2+q^2)\left(\tilde{\lambda}^2+\frac{F^2}{\rho\mu_0}\right)^2+4k^2\left(\Omega\tilde{\lambda}-\frac{i\mu F}{\rho\mu_0}\right)^2 \\ &+4h^2\left[\frac{imr}{2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Omega\tilde{\lambda}-\frac{i\mu F}{\rho\mu_0}}{h^2r^2}\right)+\Omega^2Ro-\frac{\mu^2}{\rho\mu_0}Rb\right]\times\left(\tilde{\lambda}^2+\frac{F^2}{\rho\mu_0}\right)=0. \end{aligned} \tag{4}$$

Including kinematic viscosity ν and electric resistivity η, repeating the previous procedure and applying the short-wavelength approximation, we obtain the following algebraic dispersion equation

$$\begin{aligned} &(h^2+q^2)\tilde{\lambda}_\eta^2\Lambda^2+4k^2\left(\Omega\tilde{\lambda}_\eta-\frac{iF\mu}{\rho\mu_0}\right)\times\left[\Omega Ro(\omega_\eta-\omega_\nu)+\left(\Omega\tilde{\lambda}_\eta-\frac{iF\mu}{\rho\mu_0}\right)\right] \\ &+4\Lambda h^2\tilde{\lambda}_\eta\left[(\Omega^2Ro-\frac{\mu^2}{\rho\mu_0}Rb)+\frac{imr}{2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Omega\tilde{\lambda}_\eta-\frac{i\mu F}{\rho\mu_0}}{h^2r^2}\right)\right]=0, \end{aligned} \tag{5}$$

where $\Lambda=\tilde{\lambda}_\nu+F^2/\tilde{\lambda}_\eta$, $\tilde{\lambda}_\nu=\lambda+im\Omega+\omega_\nu$, $\tilde{\lambda}_\eta=\lambda+im\Omega+\omega_\eta$, with use of $\omega_\nu=|\boldsymbol{k}|^2\nu$ and $\omega_\eta=|\boldsymbol{k}|^2\eta$.

For our purpose of stability analysis, it is expedient to define two kinds of Alfvén frequency ω_A and $\omega_{A\theta}$, along with their ratio $\beta(r)$ representing the helical geometry of the magnetic field,

$$\omega_A = \frac{kB_z}{\sqrt{\rho\mu_0}}, \quad \omega_{A\theta} = \frac{\mu}{\sqrt{\rho\mu_0}}, \quad \beta = \frac{\omega_{A\theta}}{\omega_A}. \tag{6}$$

In addition, we introduce three dimensionless parameters, namely, the magnetic Prandtl number Pm, the Reynolds number Re and the Hartmann number Ha

$$Pm = \frac{\omega_\nu}{\omega_\eta}, \quad Re = \frac{\Omega}{\omega_\nu}, \quad Ha = \frac{\omega_A}{\sqrt{\omega_\nu\omega_\eta}}. \tag{7}$$

The dispersion relation for non-dimensional variables, with the derivative terms in (5) being expanded out, leads to

$$\begin{aligned}
&(\Lambda_1\Lambda_2 + \widetilde{Ha}^2)^2 + 4\frac{h^2(\Lambda_1\Lambda_2 + \widetilde{Ha}^2)}{h^2+q^2}(Re^2 PmRo - \beta^2 Ha^2 Rb) \\
&+ \frac{4im(\Lambda_1\Lambda_2 + \widetilde{Ha}^2)}{r^2(h^2+q^2)}\Bigg[ReRo\sqrt{Pm}(\Lambda_2 + imRe\sqrt{Pm}) - i(2m\beta+1)\beta Ha^2 Rb \\
&+ (i\widetilde{Ha}\beta Ha - Re\sqrt{Pm}\Lambda_2)\frac{k^2}{h^2}\Bigg] + 4\alpha^2\Bigg[(Re\Lambda_2\sqrt{Pm} - i\widetilde{Ha}\beta Ha) \\
&\times\Big(Re\Lambda_2\sqrt{Pm} - i\widetilde{Ha}\beta Ha + RoRe(1-Pm)\Big)\Bigg] = 0,
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\Lambda_1 &= \frac{\lambda}{\Omega}Re\sqrt{Pm} + imRe\sqrt{Pm} + \sqrt{Pm}, \\
\Lambda_2 &= \frac{\lambda}{\Omega}Re\sqrt{Pm} + imRe\sqrt{Pm} + \frac{1}{\sqrt{Pm}}, \\
\widetilde{Ha} &= Ha(1+m\beta), \\
\alpha^2 &= \frac{k^2}{h^2+q^2}.
\end{aligned} \tag{9}$$

3 Axisymmetric perturbations for perfectly conducting fluid

For the SMRI, the dispersion relation (4) simplifies, when $m = 0$, to

$$\frac{\lambda^2}{\Omega^2} + Ro\left(\frac{\lambda^2}{\Omega^2} + \frac{\omega_A^2}{\Omega^2}\right) + \frac{1}{4\alpha^2}\left(\frac{\lambda^2}{\Omega^2} + \frac{\omega_A^2}{\Omega^2}\right)^2 = 0, \tag{10}$$

where $\alpha = k/\sqrt{q^2+k^2}$. We read off from (10) limited to $\lambda = 0$ the stability boundary as

$$Ro_c = -\frac{\omega_A^2}{4\alpha^2\Omega^2}(<0), \quad \text{or} \quad \frac{\omega_A}{\Omega} = 0. \tag{11}$$

For the azimuthal MRI (AMRI), for which the magnetic field has an azimuthal component $\boldsymbol{B} = r\mu(r)\boldsymbol{e}_\theta$ only. For the axisymmetric case ($m = 0$), the growth rate calculated from (4) is

$$\lambda = \pm 2\Omega\alpha\sqrt{-1 - Ro + Rb\,\omega_{A\theta}^2/\Omega^2}, \quad \lambda = 0, \tag{12}$$

where $\omega_{A\theta} = \mu/\sqrt{\rho\mu_0}$, and $\lambda = 0$ is a double root. The instability region is $Ro < Rb\,\omega_{A\theta}^2/\Omega^2 - 1$, i.e., the critical Rossby number $Ro_c = Rb\,\omega_{A\theta}^2/\Omega^2 - 1$, which recovers Michael's criterion [19] (See also refs [2, 20]). Recently, this criterion is extended to include the viscosity and the electric resistivity [13].

4 Non-axisymmetric perturbations: weak magnetic field

Hereafter we restrict to azimuthal magnetic field $\boldsymbol{B} = r\mu(r)\boldsymbol{e}_\theta$. We start with the case of a very weak magnetic field. By trial and error of numerical calculation, it is probable that the maximum growth rate is attained in the limit of $k \to \infty$. The dispersion relation (4) reduces, in the limit of $k^2 + q^2 \to \infty$ and $\omega_A \to 0$, to

$$\begin{aligned} &4(\tilde{\lambda}\Omega - im\omega_{A\theta}^2)^2 + \frac{1}{\alpha^2}(\tilde{\lambda}^2 + m^2\omega_{A\theta}^2)^2 \\ &+(\tilde{\lambda}^2 + m^2\omega_{A\theta}^2)(4\Omega^2 Ro - 4Rb\,\omega_{A\theta}^2) = 0. \end{aligned} \tag{13}$$

Equation (13), which is valid for a strong magnetic field as well, was derived by Ogilvie and Pringle [15], and coincides with the dispersion relation of the work [13] if the viscous and resistive terms are dropped off.

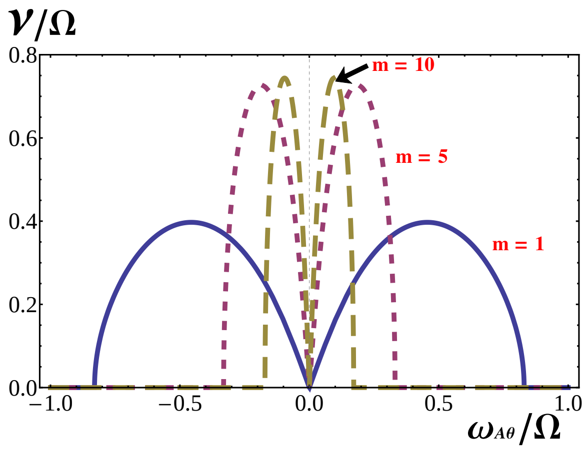

Figure 1: The growth rate, in the limit $k \to \infty$ with fixing $\alpha = 1$, of the non-axisymmetric AMRI versus $\omega_{A\theta}/\Omega$, in the range of small values, for different azimuthal wavenumbers $m = 1$ (solid line), 5 (dashed line) and 10 (long dashed line) for $Ro = -3/4$, a Keplerian rotation. The magnetic Rossby number is $Rb = -1$.

Given a small value of $|\omega_{A\theta}/\Omega|$, the maximum growth rate increases with $|m|$. Interestingly, the maximum growth rate approaches, as $|m|$ is increased, the same value as that of the SMRI as found by Ogilvie and Pringle [15]. Fig. 1 displays the growth rate $\sigma = \mathrm{Re}[\lambda_{3,4}]$ as functions of the Alfvén frequency $\omega_{A\theta}$ with azimuthal wavenumbers $m = 1$, 5 and 10 for $Ro = -3/4$ and $Rb = -1$. Since the system is Hamiltonian, to each damping perturbation ($\sigma < 0$) corresponds the growing perturbation ($\sigma > 0$) and therefore we display only the solution with positive real part σ. The change of the sign of Rb, namely, the choice of $Rb = 1$, does not change much the growth rate. We observe from Fig. 1 that, as m increases, the maximum growth rate approaches $3|\Omega|/4$, though the width of the instability band in $\omega_{A\theta}/\Omega$ is narrowed with m. Indeed, by taking $m\omega_{A\theta}^2 = 0$ and $Rb\omega_{A\theta}^2 = 0$ in (13) as a limit of small values of $|\omega_{A\theta}/\Omega|$ with maintaining $|m\omega_{A\theta}/\Omega|$ finite, we can show that the maximum growth rate happens to coincides with the Oort A-value $\sigma_{\mathrm{A}}/|\Omega| = |Ro|$.

5 Non-axisymmetric perturbations: strong external field

5.1 Strong fields and ideal AMRI

Here we consider ideal MHD with the case of $|\omega_{A\theta}/\Omega| \sim 1$. In the limit of $k \to \infty$, the maximum value is taken at $|\alpha| = 1$, and at $m = 0$ for $Rb \geq 3/4$, but at $|m| = 1$ for $-3/4 < Rb < 3/4$, with the maximum values

$$\frac{\sigma_{\max}}{\Omega} \approx \begin{cases} 2\sqrt{Rb}|\omega_{A\theta}/\Omega| \\ \quad (Rb \geq 3/4) \\ \\ \sqrt{2Rb - 1 + 2\sqrt{1+Rb^2}}|\omega_{A\theta}/\Omega| \\ \quad (-3/4 < Rb < 3/4). \end{cases} \tag{14}$$

This value decreases to zero as Rb decreases to $-3/4$.

Fig. 2 shows the growth rate, for $m = 1$ in the limit of $k \to \infty$, over a wide range of the Alfvén frequency $\omega_{A\theta}/\Omega$ and for typical values of Rb $(= 0,\ 1,\ 5)$ in the range of $Rb > -3/4$. The flow is Keplerian ($Ro < 0$).

5.2 Strong fields and inductionless AMRI

We are concerned with the inductionless limit and rotating flow. Taking the limit of $Pm \to 0$ and $Ha \to 0$ in (8), we get

$$\begin{aligned} &\hat{\lambda}^2 + \frac{4\hat{\lambda}}{(h^2+q^2)r^2}\left\{Ha_\theta^2\left(2m^2Rb - h^2r^2Rb - \frac{k^2m^2}{h^2}\right) + imRe(Ro - \frac{k^2}{h^2})\right\} \\ &+4\alpha^2(Re - imHa_\theta^2)(Re - imHa_\theta^2 + ReRo) = 0, \end{aligned} \tag{15}$$

where $\hat{\lambda} = 1 + Ha_\theta^2 m^2 + \lambda Re/\Omega + imRe$ and we recall $\alpha^2 = k^2/(k^2+q^2+m^2/r^2)$ and $Ha_\theta = \omega_{A\theta}/\sqrt{\omega_\nu \omega_\eta}$. Taking the limit of $k \to 0$, thus $\alpha \to 0$ and $h \to m/r$, of (15), the eigenvalues become

$$\begin{aligned} \frac{\lambda_1}{\Omega} &= -im - (1 + Ha_\theta^2 m^2)\frac{1}{Re}, \\ \frac{\lambda_2}{\Omega} &= -im(1 + \frac{4Ro}{m^2+q^2r^2}) - \left[1 + Ha_\theta^2 m^2\left(1 + \frac{4Rb}{m^2+q^2r^2}\right)\right]\frac{1}{Re}. \end{aligned} \tag{16}$$

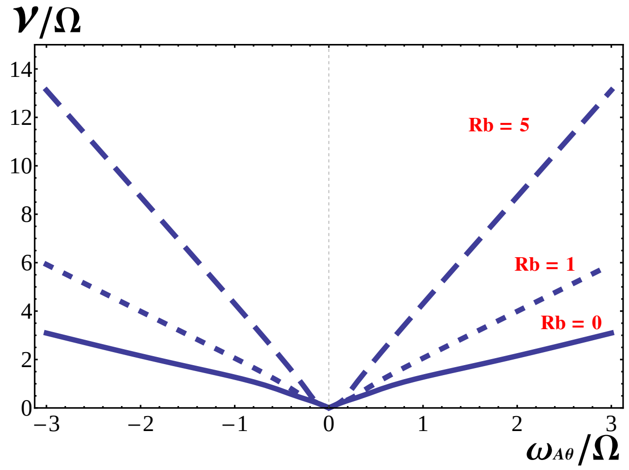

Figure 2: The growth rate, for $m=1$ and $k\to\infty$ with fixing $\alpha=1$, of the non-axisymmetric AMRI over a wide range of $\omega_{A\theta}/\Omega$ for negative $Ro=-3/4$ and different non-negative magnetic Rossby numbers Rb: $Rb=0$ (solid line), 1 (dashed line) and 5 (long dashed line).

We immediately find the instability region as

$$Rb < -\frac{1}{4}(m^2+q^2r^2) \text{ and } Ha_\theta^2 > \frac{1}{m^2\left(\frac{4|Rb|}{m^2+q^2r^2}-1\right)}. \tag{17}$$

We consider the mode of $k\to\infty$, for which the eigenvalues are

$$\frac{\lambda_{1,2}}{\Omega} = \frac{2\alpha^2 Ha_\theta^2 Rb - 1 - m^2 Ha_\theta^2}{Re} - im \pm 2\alpha\left\{\frac{Ha_\theta^4}{Re^2}(m^2+\alpha^2 Rb^2) - (1+Ro) + im\frac{Ha_\theta^2}{Re}(2+Ro)\right\}^{\frac{1}{2}}. \tag{18}$$

The instability occurs when $Ro<-1$ with growthrate $\lambda_R/\Omega \approx 2\alpha\sqrt{-(1+Ro)}$. This mode pertains to the classical Rayleigh instability since no magnetic field is required. When $Ro>-1$, the instability criterion becomes

$$2\alpha^2 Rb - m^2 + \frac{|\alpha m|(2+Ro)}{\sqrt{1+Ro}} > 0,$$

$$\text{and } Ha_\theta^2 > \frac{\sqrt{1+Ro}}{(2\alpha^2 Rb - m^2)\sqrt{1+Ro} + |\alpha m|(2+Ro)}. \tag{19}$$

In the long-wave limit of $k \to 0$, $Rb < -1/4$ is necessary for the instability of m=1 mode as shown by (17), while in the short-wave limit of $k = \infty$, $Rb > -25/32$ is necessary for the instability that is attained at $m/\alpha = \pm 5/4$. Because the later one overlaps with the former one, we conclude that the instability exists for arbitrary magnetic Rossby number. Either the mode of $k \to 0$ or $k \to \infty$ dominate in large range of Rb, and the maximum growth rate is attained at finite value of k in a narrow range of Rb as illustrated by FIG. 3. FIG. 3 draws the growth rate against the magnetic Rossby number Rb for $Re = 10^4$, $Ha_\theta = 100$, $m = 1$ and $Ro = -3/4$. We observe the crossover of the $k = 0,\ q = 0$ mode to the $k = \infty$ mode. The range of small value of Rb is dominated by the $k = 0$ mode and the one of large values of Rb is dominated by the $k \to \infty$ mode.

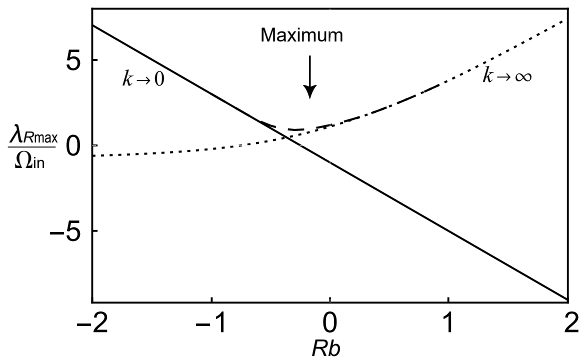

Figure 3: the growth rate to magnetic Rossby number Rb for $Re = 10^4$, $Ha_\theta = 100$, $m = 1$, and $Ro = -3/4$. solid line is $k = 0,\ q = 0$ mode, Dotted one is the $k = \infty$ mode and dashed line stands for the maximum growth rate, whose left part coincide with the $k = 0$ mode and the right part coincides with the $k = \infty,\ \alpha = 1$ mode.

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