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The new type of large-scale vortex structures of dispersionless Alfvén waves in collisionless plasma is investigated. It is shown that Alfvén waves can propagate in the form of Alfvén vortices of finite characteristic radius and characterised by magnetic flux ropes carrying orbital angular momentum. The structure of the toroidal and radial velocity, fluid and magnetic field vorticity, the longitudinal electric current in the plane orthogonal to the external magnetic field are discussed. [http://dx.doi.org/10.1063/1.4936978]

I. INTRODUCTION

Three-dimensional nonlinear structures of Alfvén waves in the form of vortices extended along the external magnetic field were substantially investigated between the 1970s and 1990s.[1–6] In these vortices, the dispersion effects play a key role. The condition of the existence of these quasi-stationary structures was compensation of the dispersion and "vector" nonlinearity effects. The dispersion effects of shear Alfvén waves depend on values of plasma β, namely $\beta_e \equiv \mu_0 n T_e/B_0^2$ and $\beta_i \equiv \mu_0 n T_i/B_0^2$, where μ_0 is the permeability of free space, n is the plasma number density, T_e and T_i are the electron and ion temperature, respectively, and B_0 is the modulus of the external magnetic field $\mathbf{B}_0$. The dispersion relation for shear Alfvén waves in low-β plasma, $\beta_e \ll m_e/m_i$, where m_e and m_i are the electron and ion masses, respectively, takes the form

$$\omega^2 = k_z^2 v_A^2/(1 + k_\perp^2 \lambda_e^2), \tag{1}$$

where ω is the wave frequency, k_z and $k_\perp$ are longitudinal and transverse wave numbers, respectively, $v_A = B_0/(\mu_0 m_i n)^{1/2}$ is the Alfvén velocity, $\lambda_e = c/\omega_{pe}$ is the electron collisionless skin depth, c is the speed of light, and ω_{pe} is the electron plasma frequency. These types of waves are usually called inertial Alfvén waves. The dispersion relation of the shear Alfvén wave of intermediate-β plasma, $m_e/m_i \ll \beta_e \ll 1$, has the form

$$\omega^2 = k_z^2 v_A^2[1 + k_\perp^2 \rho_s^2 + (3/4)k_\perp^2 \rho_i^2], \tag{2}$$

where $\rho_i = (T_i/m_i)^{1/2}/\omega_i$ is the ion Larmor radius, $\rho_s = (T_e/m_i)^{1/2}/\omega_i$ is the ion acoustic radius or the ion Larmor radius calculated at the electron temperature, $\omega_{ci} = qB_0/m_i$ is the ion cyclotron frequency, and q is the magnitude of the electron charge. These types of waves refer to the so-called kinetic Alfvén waves. Dispersion relation of the shear Alfvén wave in high-β plasma, $\beta_i \gg 1$, has the form[4]

$$\omega^2 = k_z^2 v_A^2[1 + k_\perp^2 \rho_*^2 + (3/4)k_\perp^2 \rho_i^2], \tag{3}$$

where $\rho_*^2 = \rho_s^2/(1 + T_e/T_i)$.

It was shown (e.g., reviews 3, 5, and 6 and Ref. 4) that nonlinear Alfvén wave structures can exist in the form of dipole vortices. The characteristic transverse scales of these vortices are comparable with the dispersion scales λ_e, ρ_i, ρ_s, or ρ_*. The nonlinear Alfvén waves play an important role in the coupling between the ionosphere and magnetosphere. They may even determine the fine structure of many auroral forms. This was clearly demonstrated by observations on board Intercosmos-Bulgaria-1300[7] satellite. Several new nonlinear Alfvén wave structures, such as vortex streets, have also been found. However, for a number of problems of solar physics, space physics, geophysics, and astrophysics, Refs. 8–11 are of interest to the study large-scale vortex structures with characteristic transverse scale which by many orders greater than the ion Larmor radius (or collisionless electron skin-depth) when dispersion effects can be neglected. In what follows, in contrast to the previous studies,[1–6] we will investigate the nonlinear structures of dispersionless Alfvén waves.

The paper is structured as follows: In Sec. II, we derive the reduced magnetohydrodynamic (MHD) equations for shear dispersionless Alfvén waves in a plasma of arbitrary β. Sec. III presents the solutions in the form of Alfvén vortices with finite radius. Our discussion and conclusions are found in Sec. IV. It will be shown that large-scale three-dimensional Alfvén waves can propagate as a twisted vortex beam or in the form of a magnetic field whirl that can transport magneto-convective energy in magnetized plasmas from one region to another.

II. REDUCED MHD EQUATIONS

In order to describe the vortex motion in Alfvén waves, we make use of a cylindrical coordinate system (r, ϕ, z). The

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magnetic field is considered as $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_\perp$, where $\mathbf{B}_0 = \mathbf{e}_z B_0$ is the external magnetic field which, for simplicity, is assumed to be uniform and $\mathbf{B}_\perp = (B_r, B_\phi, 0)$ is the perpendicular part of the wave magnetic field. The field-aligned magnetic field perturbation B_z in Alfvén waves is assumed to be negligibly small. Furthermore, $\mathbf{e}_z$ is the unit vector along the z-axis. The electric field is given as $\mathbf{E} = \mathbf{E}_\perp + \mathbf{e}_z E_z = (E_r, E_\phi, E_z)$. For convenience we introduce the scalar potential φ and the z-component of the vector potential A, so that $E_z = -\partial_t A - \partial_z \varphi$, $\mathbf{E}_\perp = -\nabla_\perp \varphi$, and $\mathbf{B}_\perp = \nabla A \times \mathbf{e}_z$. Here, $\partial_t \equiv \partial/\partial t$ and $\partial_z \equiv \partial/\partial z$; the subscripts z and $\perp$ denote the components along and perpendicular to $\mathbf{B}_0$, respectively.

If one neglects the electron density compressibility and the inertial effects in the electron motion along the total magnetic field the field-aligned electric field vanishes, i.e., $E_\parallel = E_z + B_0^{-1}\mathbf{B}_\perp \cdot \mathbf{E}_\perp = 0$, or

$$\frac{\partial A}{\partial t} + \nabla_\parallel \varphi = 0, \tag{4}$$

where the operator $\nabla_\parallel$ is given by

$$\nabla_\parallel = \frac{\mathbf{B}\cdot\nabla}{B_0} = \frac{\partial}{\partial z} + \frac{1}{B_0 r}\frac{\partial A}{\partial \phi}\frac{\partial}{\partial r} - \frac{1}{B_0 r}\frac{\partial A}{\partial r}\frac{\partial}{\partial \phi}. \tag{5}$$

The second equation connecting φ and A is the current closure equation

$$\nabla_\perp \cdot \mathbf{j}_\perp + \nabla_\parallel j_z = 0, \tag{6}$$

where $\mathbf{j}_\perp$ and j_z are the perpendicular and parallel components of the plasma current density, respectively. If the finite ion Larmor radius effects are neglected the perpendicular current is determined by the inertial (polarisational) part of the ion velocity

$$\mathbf{j}_\perp = qn\mathbf{V}_I. \tag{7}$$

Here,

$$\mathbf{V}_I = -\frac{1}{B_0 \omega_{ci}}\frac{d_0}{dt}\nabla_\perp \varphi, \tag{8}$$

and

$$\frac{d_0}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla = \frac{\partial}{\partial t} + \frac{1}{B_0}(\nabla\varphi \times \nabla)_z, \tag{9}$$

where $\mathbf{v}_E = B_0^{-1}\mathbf{e}_z \times \nabla\varphi$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity. From the z-component of the Ampere's law, we have

$$j_z = -(1/\mu_0)\Delta_\perp A. \tag{10}$$

Here, the operator $\Delta_\perp$ is denoted by

$$\Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}. \tag{11}$$

Making use of (7)–(11), Eq. (6) reduces to

$$\frac{d_0}{dt}\Delta_\perp \varphi + v_A^2 \nabla_\parallel \Delta_\perp A = 0. \tag{12}$$

The set of Equations (4) and (12) coincides with known[13,14] equations for dispersionless shear Alfvén waves. The set of these equations possesses the integral of energy conservation

$$W = \int [(\nabla_\perp \varphi)^2 + v_A^2 (\nabla_\perp A)^2] d\mathbf{r}, \tag{13}$$

with $\partial W/\partial t = 0$ that can be considered as the wave energy per unit length in the z-direction. It should be noted that in Ref. 12 an interesting generalization of Eqs. (4) and (12) (or Eqs. (30) and (31) in terms of numeration of this paper) has been suggested by including the gravitational force into consideration.

III. LARGE-SCALE ALFVÉN VORTICES

For the case of steady-state waves travelling along the external magnetic field with velocity v_A according to Refs. 1 and 2, we introduce the standard automodel variable $\eta = z - v_A t$. In this case, the set (4) and (12) reduces to

$$-v_A \frac{\partial}{\partial \eta} A + \frac{\partial}{\partial \eta}\varphi - J(A, \varphi) = 0, \tag{14}$$

and

$$-v_A \frac{\partial}{\partial \eta}\Delta_\perp \varphi + \frac{1}{B_0} J(\varphi, \Delta_\perp \varphi) + v_A^2 \frac{\partial}{\partial \eta}\Delta_\perp A - \frac{v_A^2}{B_0} J(A, \Delta_\perp A) = 0, \tag{15}$$

where

$$J(a,b) \equiv \frac{\partial a}{\partial r}\frac{1}{r}\frac{\partial b}{\partial \phi} - \frac{\partial b}{\partial r}\frac{1}{r}\frac{\partial a}{\partial \phi}$$

is the Jacobian. One can see that this system satisfies the solution

$$\varphi = v_A A. \tag{16}$$

In this case in the waves, not only $E_\parallel = 0$, but also $E_z = 0$.

We consider that the scalar potential φ can be represented in the form

$$\varphi = F(r, \phi)\Phi(\eta). \tag{17}$$

As a function $\Phi(\eta)$, one can choose the periodic function $\exp(i\eta/L_z)$, where L_z is the characteristic longitudinal wave scale. The function F describes the wave structure in the (r, ϕ) plane in the moving coordinate system with constant η. Therefore, it is convenient to find self-similar solution, i.e., below we investigate the structures in the moving coordinate system. The wave structure with such a function F should satisfy the necessary conditions: the limited energy W, see Eq. (13), and the condition of electroneutrality $\int(\nabla\cdot\mathbf{E})d\mathbf{r} = 0$, or

$$\int \Delta_\perp \varphi d\mathbf{r} = 0. \tag{18}$$

Furthermore, we assume that F is a periodic function of the angle ϕ. These conditions are satisfied if it has the form

$$F(r,\phi) = F_1 = \alpha B_0 v_A \frac{r^{2n}}{2r_0^{2n-1}} \exp\left(-\frac{r^2}{r_0^2} \pm il\phi\right). \quad (19)$$

Here, $0 < |\alpha| < 1$ is the constant value and l is integer. Taking into account that the integral of energy W reaches a minimum value when $n=1$, we accept

$$F(r,\phi) = \alpha B_0 v_A \frac{r^2}{2r_0} \exp\left(-\frac{r^2}{r_0^2} \pm il\phi\right). \quad (20)$$

Making use of (16) and (20) from (13), we obtain that

$$W = \frac{\alpha^2 \pi r_0^2 v_A^2 B_0^2}{4}\left(1 + \frac{l^2}{2}\right). \quad (21)$$

Using (20), one can find

$$\frac{v_\phi}{v_A} = \frac{1}{v_A B_0}\frac{\partial\varphi}{\partial r} = \alpha\frac{r}{r_0}\left(1 - \frac{r^2}{r_0^2}\right)\exp\left(-\frac{r^2}{r_0^2}\right)\cos(l\phi), \quad (22)$$

and

$$\frac{v_r}{v_A} = -\frac{1}{v_A B_0 r}\frac{\partial\varphi}{\partial\phi} = \alpha l\frac{r}{2r_0}\exp\left(-\frac{r^2}{r_0^2}\right)\sin(l\phi). \quad (23)$$

From the expression $\mathbf{B}_\perp = \nabla A \times \mathbf{e}_z$, we have

$$B_r = \frac{1}{r}\frac{\partial A}{\partial\phi} = -B_0\frac{v_r}{v_A}, \quad (24)$$

and

$$B_\phi = -\frac{\partial}{\partial r}A = -B_0\frac{v_\phi}{v_A}. \quad (25)$$

The vertical fluid vorticity $\omega_z = (\nabla\times\mathbf{v})_z = \Delta_\perp F$, the normalised magnetic field vorticity $\Omega = (v_A/B_0)(\nabla\times\mathbf{B}_\perp)_z$, and normalised longitudinal electric current $J_z = (\mu_0 j_z v_A/B_0)$ are related to each other as

$$\omega_z = -\Omega = -J_z = 2\alpha\frac{v_A}{r_0}\left[\left(1 - \frac{r^2}{r_0^2}\right)^2 - \frac{r^2}{r_0^2} - \frac{l^2}{4}\right] \times \exp\left(-\frac{r^2}{r_0^2}\right)\cos(l\phi). \quad (26)$$

When $r = r_0$ and $l = 0$, the vertical vorticity is $\omega_z = -2\alpha v_A/r_0$. For $l = 0$, such structures correspond to axially symmetric vortices with $v_r = 0$ and $B_r = 0$. Note that for these cases, the nonlinear term in Eq. (14) and both nonlinear terms in Eq. (15) are equal to zero. When $l = 1$, $l = 2$, the Alfvén vortices have a dipole, quadrupole, or multipole structures.

Figs. 1 and 2 illustrate the behaviour of the normalised values of the toroidal speed v_ϕ and vertical vorticity ω_z or the longitudinal electric current j_z in an axially symmetrical vortex as a function on the normalised radius r/r_0. Fig. 1 shows that the toroidal speed v_ϕ changes sign when $r = r_0$ and reaches a maximum and minimum value when $r \simeq 0.5r_0$ and $r \simeq 1.5r_0$, respectively. From Fig. 2 follows that the longitudinal current j_z and vorticity ω_z change the sign when $r \simeq 0.6r_0$ and attain the maximum value at the center of the vortex and for $r \simeq 1.6r_0$. Finally, in Fig. 3, we have shown the normalised vortex velocity field for axially symmetric ($l = 0$), dipole ($l = 1$), and quadrupole ($l = 2$) cases from top to bottom, correspondingly. Accordingly to Eqs. (24) and (25), these plots can also represent the magnetic field distribution, but vector field direction will be opposite.

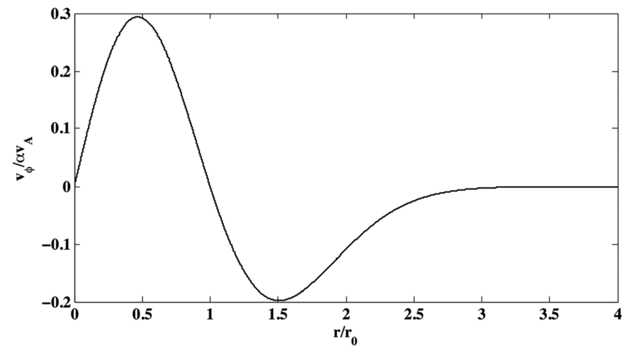

FIG. 1. The normalised value of the toroidal speed v_ϕ as a function of distance from the centre of vortex r/r_0.

IV. CONCLUSIONS

The properties of nonlinear long-wavelength dispersionless Alfvén waves in a homogeneous plasma are investigated. As the initial equations describing nonlinear dispersionless Alfvén waves, we use the set (4) and (12) for scalar potential φ and the z-component of the vector potential A. It possesses a solution in the form of the product of two functions. One of these functions describes the wave propagation along the external magnetic field with velocity v_A whereas another one describes the spatial structure in perpendicular plane (r,ϕ). It was shown that nonlinear Alfvén waves can propagate in a homogeneous magneto-plasma in the form of the magnetic flux ropes with finite vorticity of the fluid motion or the magnetic field perturbations and can be termed as the large-scale Alfvén vortices. The characteristic radial size of these structures r_0 can be many orders of magnitude larger than the ion Larmor radius or the electron collisionless skin depth. Note that the scalar potential φ of small-scale Alfvén vortices was previously investigated in Refs. 3–6 decreasing in the external vortex region as $\varphi(r) \propto 1/r$ so that the energy of such

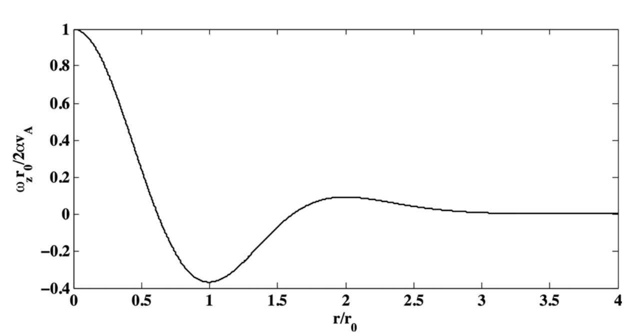

FIG. 2. The normalised value of the vertical vorticity ω_z as a function of distance from the centre of vortex r/r_0.

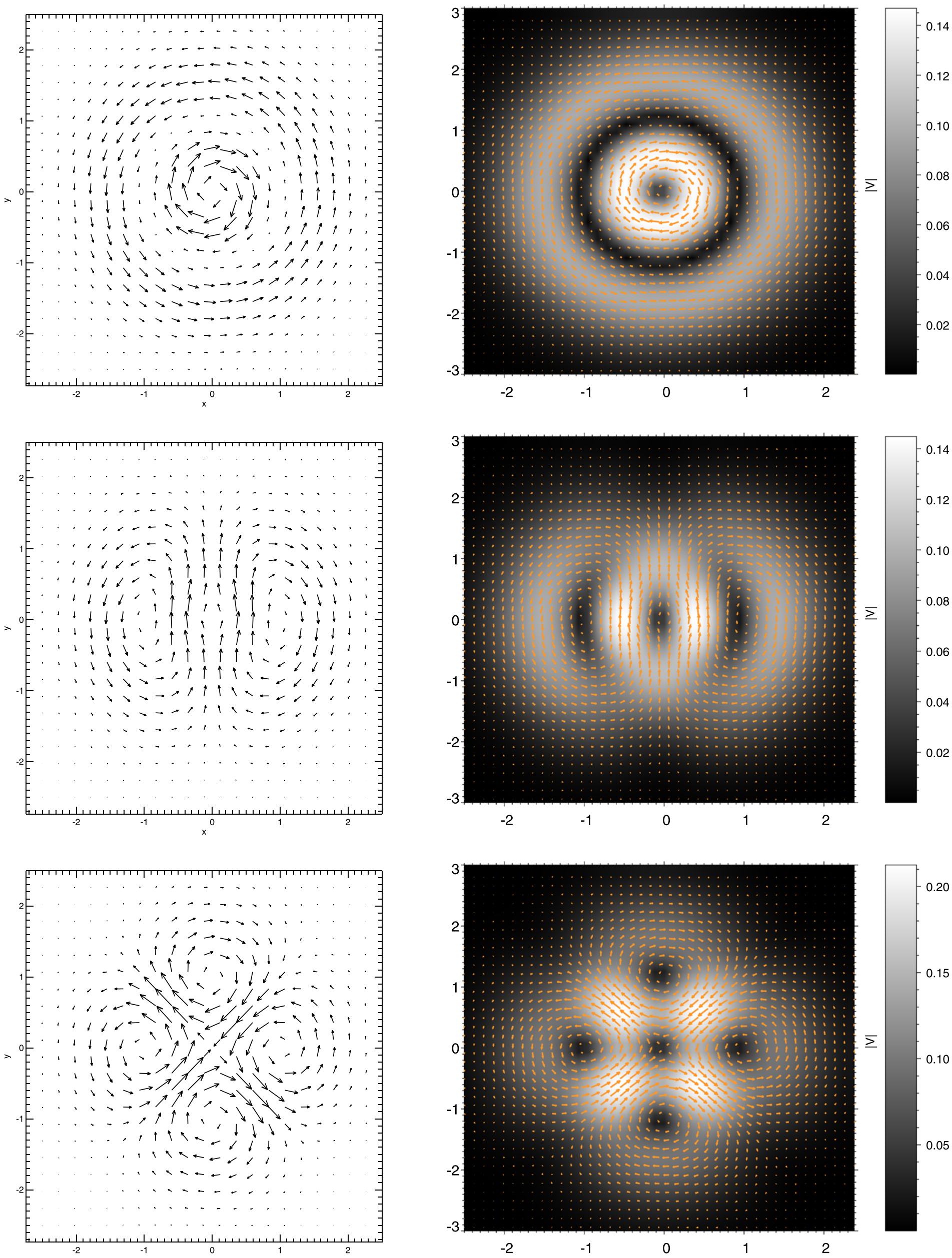

FIG. 3. The horizontal structure of the three different vortex velocity field patterns calculated for $l = 0$, 1, and 2 (from the top to the bottom, correspondingly).

vortices possesses logarithmic divergence while the energy of discussed here large-scale vortices is limited, see Eq. (21).

Understanding the scalable nature of vortex motions in space plasmas will provide unique insights into the association of large- and small-scale structures in the atmosphere of the Sun. Giant tornadoes,[15] the observed analogue to large-scale vortex motions in the solar atmosphere, are recently observed as the rotating legs of large-scale prominence structures. They are thought to be formed in a similar way as with vortex-driven, small-scale magnetic tornadoes[11] and they are often associated with triggering the eruption of prominences into the heliosphere. Large-scale vortex motions could play an important role in the dynamics of the solar transition region.[16,17] Similarities with small-scale vortex motions are present in their measured Doppler velocities (i.e., ± 6 km/s) within the adjacent legs of a quiescent hedgerow prominence.[18] Co-ordinated ground and space-based observations of another giant tornado also exhibited similar opposing Doppler velocity signatures (indicative of rotation of the structure) within the legs of a prominence,[19] although with on-average larger Doppler velocities of ± 25 km/s. However, it is not yet clear whether giant tornadoes actually rotate as a vortex with a central axis or whether this motion is a representation of flows along a stable helical magnetic structure.[20] As future work, it should be possible to spatially identify different oscillatory modes within giant tornadoes in sufficient detail, using current and future solar observational imaging capabilities, in order to confirm the presence of dispersionless Alfvén waves in large-scale vortex structures, as derived here.

Summarising we conclude the following. It was shown that 3D Alfvén waves in a homogeneous plasma can arise in the form of magnetic flux ropes carrying the orbital angular momentum. They can be identified in observations as signatures of rapidly rotating magnetic field structures and swirl motions observed in the solar atmosphere. This can provide an alternative mechanism for channeling the heat fluxes and vorticity from one region to another. The present investigation is thus useful for understanding nonlinear vortex motions observed in space plasmas.

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