**Viscoelastic shear deformable microplates: nonlinear forced resonant characteristics**

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**Abstract**

A viscoelastic model for a shear-deformable microplate is developed in this paper while accounting for geometric nonlinearities. Nonlinear numerical solutions are conducted to examine the resonant oscillations of the microsystem. The geometrically nonlinear theoretical model is developed utilising the Kelvin-Voigt viscoelastic model, to account for nonlinear dissipation, the modified version of the couple-stress theory, to account for small-scale characteristics, and the third-order deformation theory, to account for shear stress. The constitutive relations for both the classical and higher-order stress tensors are constructed and are divided into elastic and viscous components. The elastic components are used to develop the potential strain energy and the viscous components are employed to model the virtual work of damping (energy dissipation). Additionally, the microplate motion energy is developed while accounting for all in-plane, out-of-plane, and rotational motions. A distributed harmonic load, as the representative an external force, is applied to the microsystem and the corresponding virtual work is obtained. The generalised Hamilton’s principle is applied to the virtual works and variations of the energy terms, resulting in the nonlinear equations of motion of the microsystem. Being of partial differential type, the equations of motion are discretised into a set of nonlinearly coupled ordinary differential equations consisting of nonlinear geometric and nonlinear damping terms. A solution procedure for the forced oscillation analysis of the microsystem is developed using a continuation method. Different diagrams are constructed to examine the nonlinear resonant characteristics of the viscoelastic shear deformable microplate and to highlight the nonlinear dependency of the Kelvin-Voigt viscoelastic damping mechanism on the oscillation amplitude, for a geometrically nonlinear model.

*keywords*: *viscoelastic shear deformable microplates; Kelvin–Voigt model*; *resonant oscillations; nonlinear damping*

**1. Introduction**

The bending and oscillations of microscale beams and plates are commonly used to measure external parameters in different microelectromechanical systems (MEMS) [[1-4](#_ENREF_1)] such as microscale sensors, resonators, energy harvesters, and accelerometers [[5-16](#_ENREF_5)]. It is shown that microresonators display nonlinear energy dissipation and that the internal energy dissipation plays an important role in the forced or free motions of microscale structures. In the presence of geometric nonlinearities, the material damping mechanisms such as Kelvin-Voigt, Zener, and Maxwell, become nonlinearly amplitude-dependent and hence are capable of capturing the damping nonlinearities associated with large-amplitude oscillations. In this paper, Kevin-Voigt material damping is utilised to highlight the effect of damping nonlinearities in the nonlinear forced resonant oscillations of a shear deformable microplate. Furthermore, it has been discovered experimentally [[17-19](#_ENREF_17)] that when a structure becomes small, the size affects the motion/bending behaviour of the system which is characterised by an additional stiffness in a linear sense. An advanced continuum mechanics theory should be employed to capture the size effects [[20](#_ENREF_20), [21](#_ENREF_21)]; this paper utilises the modified couple stress (MCS) theory [[22-27](#_ENREF_22)].

The number of studies on the *elastic* models of microplates incorporating linear damping mechanism is quite large, which includes both geometrically linear and nonlinear studies. For instance, using the modified strain-gradient theory, Ashoori et al. [[28](#_ENREF_28)] obtained the linear out-of-plane equation of motion of a microscale plate. Hashemi and Samaei [[29](#_ENREF_29)] employed the nonlocal Mindlin plate theory to conduct a linear buckling investigation on micro/nonoplates under in-plane forces. Further investigation was conducted by Wang et al. [[30](#_ENREF_30)], who employed the strain gradient theory in conjunction with the Kirchhoff plate theory, so as to study the linear size-dependent behaviour of microplates. Jomehzadeh et al. [[31](#_ENREF_31)] conduced a linear vibration analysis on a microplate making use of the MCS theory. The investigations were continued by Nabian et al. [[32](#_ENREF_32)], who analysed the stability of a functionally graded microplate under electrostatic and hydrostatic pressure. Employing a meshless method together with MCS theory, Roque et al. [[33](#_ENREF_33)] examined the bending characteristics of a shear deformable microplate. Based on the nonlocal Eringen theory, Farajpour et al. [[34](#_ENREF_34)] investigated the linear buckling response of a graphene plate. Apart from the linear studies in the literature, there are also several studies of microplates utilising geometrically nonlinear models. For instance, employing the MCS theory, Asghari [[35](#_ENREF_35)] developed the geometrically nonlinear size-dependent equations of motion of a microplate. Thai and Choi [[36](#_ENREF_36)] developed size-dependent nonlinear functionally graded Kirchhoff and Mindlin plate models on the basis of the MCS theory.

All the aforementioned valuable studies analysed the linear/nonlinear behaviour of microplates through use of elasticmodels with linear damping mechanism. The present study analyses for the first time the *forced* *nonlinear resonant oscillations of a shear deformable viscoelastic microplate* while accounting for *stiffness* and *damping* nonlinearities. The nonlinear equations of motion of the microplate are derived through use of the (i) MCS theory, (ii) the third-order shear deformable plate theory, (iii) the Kelvin–Voigt viscoelastic material damping model, and (iv) generalised Hamilton’s principle. Furthermore, von-Kármán strain-displacement nonlinearities are accounted for, which due to employment of a Kelvin-Voigt material damping model, results in both geometric and damping nonlinearities. Though use of a double-dimensional Galerkin technique and via incorporating basis functions consistent with the fully clamped boundary conditions, the partial differential equations of motion are reduced and transformed into equations of ordinary differential type. Extensive numerical calculations are then conduced employing a continuation technique. The nonlinear resonant characteristics of the viscoelastic shear deformable microplate are examined while highlighting the nonlinear dependency of the damping to oscillation amplitude.

**2. Model development for a viscoelastic microplate**

A geometrically nonlinear model of the third-order shear deformable microplate is developed in this section taking into account two different damping mechanisms, i.e. the linear viscous damping mechanism and the Kelvin-Voigt viscoelastic internal damping mechanism which consists of linear and nonlinear parts due to presence of geometric nonlinearities. The reason for the presence of two damping mechanisms is to be able to compare them in the numerical simulations. The considered microplate is shown in Fig. 1 within a Cartesian coordinate system (*x*,*y*,*z*). The microplate’s dimensions in the *x* and *y* directions are denoted by *a* and *b*, respectively, while its dimension in the *z* direction (i.e. thickness) is shown by *h*. The microplate is under a distributed excitation load of  with *F*1 and being the forcing amplitude and frequency, respectively, and *t* being time. The employed third-order shear deformation theory contains five independent variables, i.e. three displacements and two rotations. The mid-plane displacement components are denoted by *u*, *v*, and *w*, in the *x*, *y*, and *z* directions, respectively. The rotations of the transverse normal at *z*=0 are shown by  and . Furthermore, the components of the displacement vector are denoted by *ux*, *uy*, and *uz*.

In what follows, the third-order sear deformation theory, the modified couple stress theory, the Kelvin-Voigt viscoelastic model, and the generalised Hamilton’s principle are employed to develop a discretised nonlinear model for the microplate.

Taking into account the von Kármán strain nonlinearities, the strain tensor components for a third-order shear deformable microplate can be written as [[37](#_ENREF_37)]





The symmetric rotation gradient tensor  can be formulated as [[38](#_ENREF_38)]



where the rotation vector can be formulated in terms of the displacement vector **u** as . Hence, the components of the symmetric rotation gradient tensor can be derived as [[37](#_ENREF_37)]





Based on the Kelvin-Voigt viscoelastic model, the components of the stress tensor () are given by



where  and *E* stand for Poisson’s ratio and Young’s modulus, respectively, and *η* represents the material viscosity. Additionally, the subscripts (*v*) and (*e*) denote viscous and elastic, respectively.

The components of the deviatoric part of the symmetric couple stress tensor (**m**) can be similarly formulated as



in which *l* denotes the characteristic length-scale parameter; here again subscripts (*v*) and (*e*) denote viscous and elastic, respectively.

The variation of the potential strain energy of the microsystem can be written as



The variation of the microplate kinetic energy is given by



The virtual work of the Kelvin-Voigt (KV) internal damping can be formulated as



The virtual work of the viscous linear damping (LD) mechanism is given by



where *c* is the linear damping coefficient.

The virtual work of the external distributed dynamic load exerted on the microplate in the *z* direction is expressed as [[10](#_ENREF_10), [39](#_ENREF_39)]



Substituting Eqs. 8-12 into generalised Hamilton’s principle and substituting the instances of  by  and similarly those of  by , the equations of motion of the viscoelastic third-order shear deformable microplate can be derived in a compact form, for *u*, *v*, *w*, , and  motions, as:











Expanding these equations and writing them in terms of the mid-plane displacements and rotations results in the final equations of motion of the microsystem as













Equations (18)-(22) must be reduced into a set of nonlinear ordinary differential equations (ODEs) in order to be able to conduct numerical simulations. A double-dimensional Galerkin technique is utilised to construct the reduced-order model of the viscoelastic third-order shear deformable microplate. First, the mid-plane displacements and rotations are expanded as



where , , , , and  represent the time-dependent generalised coordinates to be calculated numerically. and  are eigenfunctions for the longitudinal and transverse displacements of a linear doubly-clamped beam. The formulations for ,, and  are given by



where , and  is the *m*th root of the characteristic frequency equation of a doubly-clamped beam. Using hyperbolic trial functions, i.e.  and , increases the computational costs, but yields more accurate results since they satisfy both geometric and dynamic boundary conditions of a fully clamped microplate. Application of the double-dimensional Galerkin technique yields a set of coupled equations consisting of linear terms as well as quadratic and cubic nonlinear terms. Due to presence of linear, quadratic, and cubic stiffness and damping terms, the discretised equations of motion can be written in a compact matrix form as



in which **M** is the linear *non-diagonal* mass matrix and **F** is the harmonic forcing amplitude vector; additionally, **C**1, **C**2, and **C**3 denote the linear, quadratic, and cubic damping matrices, respectively, while **K**1, **K**2, and **K**3 represent the linear, quadratic, and cubic stiffness matrices, respectively. Furthermore, **r** is a vector of the generalised coordinates given by



To write the discretised equations in a form suitable for numerical simulations, the discretised set should be decoupled inertially by pre-multiplying both sides of Eq. (25) by **M**-1 as



The discretised set of equations given in Eq. (27) is then solved numerically using a continuation technique [[40](#_ENREF_40), [41](#_ENREF_41)]. In this study, a 40-degree-of-freedom (DOF) reduced-order model is constructed by retaining the generalised coordinates *u*(2,1), *u*(4,1), *u*(6,1), *u*(2,3), *u*(4,3), *u*(2,5), *u*(8,1), *u*(6,3), *u*(4,5), *u*(2,7), *u*(10,1), *v*(1,2), *v*(1,4), *v*(1,6), *v*(3,2), *v*(3,4), *v*(5,2), *v*(1,8), *v*(3,6), *v*(5,4), *v*(7,2) and *v*(1,10) for the in-plane motions, *w*(1,1), *w*(3,1), *w*(5,1), *w*(1,3), *w*(3,3), and *w*(1,5) for the out-of-plane motion, and , , , , , , , , , , , and  for the rotational motions.

**3. Numerical results**

Large-amplitude nonlinear resonant characteristics of the size-dependent viscoelastic shear deformable microplate is studied in this section. The nonlinear resonant response of the microsystem is analysed when the excitation frequency is varied in the vicinity of the fundamental natural frequency associated with linear (1,1) transverse vibration mode. A thorough comparison between different damping mechanisms, i.e. Kelvin-Voigt model and linear viscous damping model, is conducted. The small-scale effects on the large-amplitude oscillation of the microsystem are also highlighted. The numerical simulations are carried out for an aluminium microplate of dimensions *h*=3 µm ad *a*=*b*=200 µm, and mechanical properties of *E*=69 GPa, *ρ*=2700 kg/m3, and *ν*=0.33. The characteristic length-scale parameter for aluminium is calculated as *l*=1.0653 µm employing the experimental data reported in Ref. [[42](#_ENREF_42)] as well as the formula suggested in Ref. [[43](#_ENREF_43)]. The following dimensionless quantities are introduced to the numerical calculations



where  and . *cd* is replaced by 2*ζ**ω*1,1 with *ζ* and *ω*1,1 being the modal damping ratio and the fundamental out-of-plane dimensionless natural frequency.

Figure 2 illustrates the frequency-amplitude diagrams of the shear deformable viscoelastic microplate; *ηd*=0.00025, *f*1=35.0, and *cd*=0. For this case, the fundamental out-of-plane dimensionless natural frequency is calculated as *ω*1,1 = 44.1065. Sub-figures (a-d) show the generalised coordinates of the out-of-plane motion, while sub-figures (e) and (f) show the fundamental in-plane and rotational generalised coordinates, respectively. It should be noted that due to equal in-plane dimensions of the microplate, the frequency-amplitude diagrams of *v*(1,2) and  are the same as those of *u*(2,1) and , respectively. As seen, the microsystem shows nonlinear resonant response of hardening type, originating from the nonlinear in-plane tension induced due to mid-plane stretching. The viscoelastic shear deformable microplate undergoes two jumps at points SN1 (Ω*e*/*ω*1,1= 1.2150) and SN2 (Ω*e*/*ω*1,1= 1.0561), characterised by saddle-node bifurcations. The dashed line between the two saddle-node bifurcations indicates that the solution is unstable.

A comparison between the two damping mechanisms, i.e. the viscoelastic nonlinear damping (i.e. nonlinearly amplitude-dependent) and the linear damping, at different forcing amplitudes is shown in Fig. 3. It should be noted that the presence of geometric nonlinearities in the strain-displacement relation renders the Kelvin-Voigt model a geometrically *nonlinear* damping mechanism. Furthermore, for the model with viscoelastic nonlinear damping *ζ* is set to zero while for the model with linear damping *ηd* is set to zero. In order to be able to compare the two damping mechanisms, first a value should be selected for the linear damping coefficient. To do so, the frequency-amplitude diagrams for both viscoelastic nonlinear damping and linear damping models are plotted for a small forcing amplitude (here *f*1=4.0), for which the effect of nonlinear terms is negligible, and the value of *ζ* is chosen such that both curves coincide. Based on the dimensionless material viscosity of *ηd*=0.00025, *ζ* is obtained as 0.0055. Having set the damping coefficients to fixed values, the forcing amplitude is increased to 14.0 and the frequency-amplitude curves are constructed for both models. As seen in the figure, the model with the linear damping mechanism predicts a larger peak amplitude of oscillation compared to the one consisting a viscoelastic nonlinear damping mechanism. The difference between the two models increases even further as the forcing amplitude is set to *f*1=28.0. This is due to the fact that at larger oscillation amplitudes, the geometrically nonlinear damping terms become larger which in turn increases the amount of energy dissipation and hence results in a smaller peak amplitude of oscillations.

In order to better illustrate the differences between the viscoelastic nonlinear damping mechanism and the linear damping one, the frequency-amplitude diagrams are obtained for the two models when *f*1=1.0, 2.0, …, 30. Afterwards, at each forcing amplitude, the nonlinear resonant frequency and amplitude at drop point are extracted for both models; for convenience, the nonlinear resonant frequency and amplitude at drop point are referred to as *drop frequency* and *drop amplitude*. Figures 4 and 5 are then constructed by plotting the forcing amplitude versus the drop frequency and drop amplitude, respectively. Figure 4 shows the forcing amplitude versus the drop frequency for the two models; as seen in Fig. 4, both models, i.e. the one with the viscoelastic nonlinear damping mechanism and the one including a linear damping mechanism, predict almost the same drop frequency for small forcing amplitudes up to *f*1≈5.0; however, as the forcing amplitude is increased to larger values, the drop frequency predicted by the model with linear damping mechanism starts to deviate from that predicted by the model with viscoelastic nonlinear damping mechanism. Hence, Fig. 4 shows clearly that the geometric nonlinearities associated with damping become more significant at larger forcing amplitudes and large-amplitude oscillations; therefore, the damping nonlinearities should be accounted for when the microsystem undergoes large-amplitude vibrations.

Figure 5 demonstrates the variation of the drop amplitude as a function of the forcing amplitude for the two damping models through plots of generalised coordinates *w*(1,1), *w*(3,1), *w*(3,3), and *w*(5,1). At first glance, it is seen that as the forcing amplitude is increased, the difference between the drop amplitude predicted by the two models, i.e. one consisting of a viscoelastic nonlinear damping mechanism and the other including a linear damping model, increases accordingly. It is interesting to note that for the model with linear damping, the drop amplitudes of *w*(1,1) and *w*(3,1) do not change much at large forcing amplitudes, i.e. near 30; however, a different scenario is observed for *w*(3,3) and *w*(5,1) generalised coordinates whose drop amplitudes increases sharply at large forcing amplitudes. This behaviour is due to presence of modal interactions and internal resonances [[44](#_ENREF_44), [45](#_ENREF_45)] where the energy is transferred between modes of vibration.

A comparison between the nonlinear primary resonant responses of the viscoelastic shear deformable microplate obtained via the MCS and classical continuum theories is depicted in Fig. 6. For both models, *ηd*=0.00025, *f*1=35.0, and *cd*=0. It should be mentioned that when the characteristic material length-scale parameter is set to zero, the model developed in Section 2 based on the MCS theory reduces to a model based on the classical continuum theory. As seen in Fig. 6, both models predict a geometrically hardening-type nonlinear behaviour with similar magnitude of oscillations in the primary out-of-plane and rotational vibration modes; for higher out-of-plane modes as well as the in-plane modes, the MCS theory predicts smaller *magnitude* of oscillation. Furthermore, it is seen that when the characteristic length-scale parameter is accounted for, i.e. when the MCS theory is employed, the primary resonant region is clearly shifted to larger values of Ω*e*, signalling an increase in the fundamental natural frequency of the microsystem.

**4. Conclusions**

The nonlinear small-scale-dependent resonant vibration characteristic of a third-order shear deformable viscoelastic microplate has been investigated numerically while highlighting the nonlinear dependency of the damping to amplitude. To this end, the MCS theory is employed together with the third-order shear deformable plate theory as well as the Kelvin–Voigt viscoelastic damping model to derive the coupled nonlinear equations governing the in-plane, out-of-plane, and rotational motions of the microplate. The presence of von-Kármán strain-displacement nonlinearities renders the Kelvin-Voigt model a geometrically nonlinear damping model. The developed nonlinear continuum model is reduced to a set of discretised equations with the aid of a double-dimensional Galerkin technique, utilising trial functions consistent with the fully clamped boundary conditions of the microplate. The nonlinear resonant behaviour of the viscoelastic shear deformable microplate is investigated numerically via a continuation technique. It was shown that the induced mid-plane tension due to large-amplitude oscillations results in a hardening-type nonlinear behaviour. A comparison between microplate models consisting of viscoelastic damping and linear damping showed that at small forcing levels, both models predict similar resonant response with similar peak oscillation amplitudes; however, as the oscillation amplitude increases (as a result of increased forcing amplitude) the difference between the drop frequencies and drop amplitudes of the two models starts to manifest itself due to the strong dependency of the damping in the viscoelastic model to oscillation amplitude. More specifically, as the forcing amplitude is increased, the model consisting of viscoelastic nonlinear damping mechanism predicts smaller drop frequency and drop amplitude in comparison to the model including linear damping.

**Appendix A**

In the present study, 40 generalised coordinates are retained in the discretised model of the viscoelastic shear deformable microplate to ensure reliable results. A convergence analysis is conducted in this section by comparing the frequency-amplitude diagram of *w*(1,1) obtained based on the 40-DOF model to that of obtained using 5-DOF, 13-DOF, and 24-DOF models of the microsystem. The retained generalised coordinates are: *u*(2,1), *v*(1,2), *w*(1,1), , and  for the 5-DOF model, *u*(2,1), *u*(4,1), *v*(1,2), *v*(1,4), *w*(1,1), *w*(3,1), *w*(1,3), , , , , , and  for the 13-DOF model, and *u*(2,1), *u*(4,1), *u*(6,1), *u*(2,3), *u*(4,3), *u*(2,5), *v*(1,2), *v*(1,4), *v*(1,6), *v*(3,2), *v*(3,4), *v*(5,2), *w*(1,1), *w*(3,1), *w*(1,3), *w*(3,3), , , , , , , , and  for the 24-DOF model. As seen in Fig. 7, the 5-DOF and 13-DOF discretised models do not predict converged results. Going from 24-DOF to 40-DOF results in only slight change in drop frequency and amplitude, indicating that the 40-DOF model employed in this study gives accurate results.

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| (a) |
| (b) |

Figure 1: (a) Schematic representation of a viscoelastic shear deformable microplate under distributed load; (b) deformed configuration of the microplate.

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| (a) |
| (b) |
| (c) |
| (d) |
| (e) |
| (f) |

Figure 2: Nonlinear frequency-amplitude diagrams of the shear deformable viscoelastic microplate: (a-d) the maximum amplitudes of *w*(1,1), *w*(3,1), *w*(3,3), and *w*(5,1), respectively; (e) the minimum amplitude of *u*(2,1); (f) the maximum amplitude of φ1 (1,1).

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| (a)  F:\AUTO_RUNS\1_MicroPlate_Visco_ThirdOrder\4ViscoEls_3f\plot4\w11.wmf |
| (b) |

Figure 3: Nonlinear frequency-amplitude diagrams of the shear deformable microplate obtained employing linear and nonlinear damping mechanisms: (a,b) the maximum amplitudes of *w*(1,1) and *w*(3,1), respectively.



Figure 4: Forcing amplitude versus the drop frequency obtained employing linear and nonlinear damping mechanisms.

|  |
| --- |
| (a) |
| (b) |
| (c) |
| (d) |

Figure 5: Forcing amplitude versus the drop amplitude obtained employing linear and nonlinear damping mechanisms: (a-d) the drop amplitudes of *w*(1,1), *w*(3,1), *w*(3,3), and *w*(5,1), respectively.

|  |
| --- |
| (a) |
| (b) |
| (c) |
| (d) |

Figure 6: Nonlinear frequency-amplitude diagrams of the shear deformable microplate obtained employing the MCS and classical theories: (a,b) the maximum amplitudes of *w*(1,1) and *w*(3,1), respectively; (c) the minimum amplitude of *u*(2,1); (d) the maximum amplitude of φ1 (1,1).

C:\MinGW\msys\1.0\home\HM\1_MicroPlate_Visco_ThirdOrder\5ConvergenceTest\plot5\w11.wmf

Fig. 7. Nonlinear resonant response of the viscoelastic shear deformable microplate based on different discretised models. Thick and thin lines show stable and unstable solutions, respectively.