**Nonlinear mechanics of nanotubes conveying fluid**

Ali Farajpour a, Hamed Farokhi b, Mergen H. Ghayesh a,\*, Shahid Hussain c

*a School of Mechanical Engineering, University of Adelaide, South Australia 5005, Australia*

*b Department of Mechanical and Construction Engineering, Northumbria University, Newcastle upon Tyne NE1 8ST, UK*

*c Faculty of Science and Technology, University of Canberra, Canberra, ACT, Australia*

*\*Corresponding author: mergen.ghayesh@adelaide.edu.au*

**Abstract**

A nonlocal strain gradient elasticity approach is proposed for the mechanical behaviour of fluid-conveying nanotubes; a nonlinear analysis, incorporating stretching, is conducted for a model based on both a nonlocal theory along with a strain gradient one. A clamped-clamped nanotube conveying fluid, as a conservative gyroscopic nanosystem, is considered and the motion energy and size-dependent potential energy are developed via use of constitutive and strain-displacement relations. An energy minimisation is conducted via Hamilton’s method for an oscillating nanotube subject to external forces. This gives the nonlinear equation of the motion which is reduced to a high DOF system via Galerkin’s technique. As many nanodevices operate near resonance, the resonant motions are obtained using a frequency-continuation method. The effect of different nanosystem/fluid parameters, including fluid/solid interface and the flow speed, on the nonlinear resonance is analysed.

*Keywords:* *Viscoelastic nanotubes; Fluid; Nonlinear mechanical analysis; Nonlocal strain gradient effects*

**1. Introduction**

Mechanical analysis of Micro/Nanoelectromechanical systems (MEMS/NEMS) is an area of research that has received a great deal of attention recently [1-6]. In some promising nanoelectromechanical systems (NEMSs), there is significant interaction between the solid parts and the liquid ones. A salient example of these NEMS devices is nanofluidics-based systems which have a wide range of applications in different areas of nanotechnology, especially nanomedicine. To have a better performance for fluid-conveying nanostructures understanding the effects of fluid-solid interactions on the mechanical response such as dynamic response is important for an appropriate design since these nanostructures usually operate under applied loads. Size dependence [7-19] is an important factor to be incorporated in the theoretical modelling and simulations of MEMS/NEMS applications [20-41].

Some size-dependent mathematical formulation has been recently developed in order to study the mechanics of fluid-conveying nanostructures using various modified elasticity models such as the couple stress [42, 43], nonlocal elasticity [44, 45] and surface elasticity [46]. Wang [47] investigated the vibration of nanoscale tubes containing flowing fluid via a mathematical surface model. Lee and Chang [48] developed a nonlocal model to study the time-dependent deformation of single-walled carbon nanotubes (SWCNTs) conveying fluid; they only considered linear terms in their analysis. Wang et al. [49] also examined the features of wave propagations in a double-walled nanoscale tube containing flowing fluid. In another paper, Soltani et al. [50] examined the effect of stress nonlocality on the instability of a viscous-fluid-conveying SWCNT surrounded by a viscoelastic medium. Zeighampour and Beni [51] proposed a couple stress model to explore the vibration of fluid-conveying nanotubes; they studied vibration characteristics due to small time-dependent deflections. In another study, thermal effects on the stability responses of fluid-conveying nanoscale tubes were investigated by Zhen et al. [52]. Moreover, Khodami Maraghi et al. [53] employed the nonlocal Euler–Bernoulli beam theory to describe the thermo-electro-mechanical nonlinear mechanics of double-walled boron nitride nanoscale tubes conveying fluid. Liang and Su [54] also developed a nonlocal continuum formulation for the stability of a SWCNT conveying pulsating fluid; the effect of being small was ignored for the nanoscale fluid. In another paper, a nonlinear analysis was performed by Askari and Esmailzadeh [55] to study the large amplitude vibration of fluid conveying nanotubes; only one scale parameter was considered to describe the size effects. Oveissi et al. [56] also studied the axial vibration of nanoscale tubes conveying fluid using the nonlocal theory. Ghasemi et al. [57] performed an analytical analysis to examine the post-buckling behaviour of multi-walled carbon nanotubes containing flowing fluid; they employed a nonlocal continuum mechanics for size effects. The effect of an applied magnetic field on the nonlocal instability of fluid-conveying nanotubes was also studied by Bahaadini and Hosseini [58]. More recently, a linear wave propagation analysis has been reported on the vibration of viscoelastic nanotubes including fluid-solid interactions [59].

The class of nanotubes conveying fluid is gyroscopic and conservative/non-conservative; a clamped-clamped one considered in this paper is conservative gyroscopic (in the absence of viscosity), meaning that both Coriolis and centrifugal forces are present, the former due to rotations and relative translations, and the latter one due to a curvature and relative fluid translation.

All the above-explained investigations are restricted to either linear models or simple one-parameter size-dependent models. In this paper, for the first time, a nonlinear nonlocal strain gradient technique is presented for a viscoelastic nanotube conveying fluid. Both the effects of nonlocal stresses and the strain gradient are incorporated. The ends of the nanotube are assumed to be clamped, leading to a gyroscopic nanosystem. Using modified constitutive equations and strain-displacement relations, the motion energy and the size-dependent potential energy are developed. Then an energy minimisation is performed by Hamilton’s method, giving the nonlinear equation of the size-dependent motion. As a decomposition approach, Galerkin’s procedure is applied to the derived equation. The resonant response of the fluid-conveying nanotube is determined via a frequency-continuation method. At the end, the influences of various nanotube/nanofluid parameters such as the scale parameters and the speed of the flowing fluid on the nonlinear response are studied.

**2. Effect of slip boundary condition**

At macroscale levels, it is usually assumed that the no-slip boundary condition is valid at the interface between the tube and the fluid of a system containing flowing fluid. However, when the dimensions of a system are reduced to nanometers, the no-slip boundary condition is not valid anymore. To take into account the effect of slip boundary conditions, a non-dimensional parameter termed Knudsen number (*Kn*) is defined as the ratio of the average distance of the molecular free path to an external characteristics dimension of a fluid-conveying system. Using this parameter, the effective viscosity of the fluid () can be defined as [60]

  (1)

where  denotes the bulk viscosity.  is a constant coefficient obtained by

 (2)

where

 (3)

It should be noted that in the above relations, ,  and  are coefficients, which are respectively determined as ,  and . The Navier–Stokes equations are expressed as

 (4)

in which , , ,  and *P* are respectively the fluid density, flow speed vector, Laplacian operator, gradient operator and pressure. Assuming a Newtonian laminar incompressible fluid inside the nanotube (see Fig. 1), the following relation can be obtained for the axial fluid speed

 (5)

Here *C*0 and *C*1 are integration constants. *r* and *x* denote the radial and axial coordinates of the system. It should be noted that the influences of body forces due to gravity and an electromagnetic filed are neglected in this analysis. Since the second term in Eq. (5) takes infinite values at *r*=0, *C*0 must be zero. Based on the Beskok-Karniadaki model, the slip speed can be written as [60]

 (6)

where *R*i is the inner radius of the nanoscale tube;  denotes a coefficient related to the tangential momentum accommodation. A value of 0.7 is usually chosen for this coefficient. Using Eqs. (5) and (6), one obtains

 (7)

Substituting Eq. (7) into Eq. (5), the axial fluid speed is obtained as

 (8)

Now, a correction factor for the average speed of the flowing fluid inside the nanoscale tube is defined as

 (9)

in which  and  are respectively the average fluid speed for slip and no-slip boundary conditions. In view of the above relations (i.e. Eqs. (8) and (9)), the fluid speed correction factor is obtained as

 (10)

From the above relation, it is seen that when the average distance of molecular free path is very small compared to the diameter of the tube (i.e. *Kn*=0), the average fluid speeds for slip and no-slip boundary conditions are the same (i.e. 1).

**3. Fluid-conveying viscoelastic nanotubes**

A nonlinear model incorporating nonlocal strain gradients is developed in this section for the nonlinear mechanics of fluid-conveying viscoelastic nanotubes under an external loading (see Fig. 1). In the nonlocal strain gradient elasticity, there are three types of stresses: 1) the total nonlocal stress , 2) the zeroth-order nonlocal stress , and 3) the first-order nonlocal stress . The relation between these stresses is [61]

 (11)

where  stands for the gradient operator. Each stress component is divided into two parts: 1) the elastic part ****, and 2) the viscoelastic part  in which . These parts are related by the following relations





 (12)

The nonlinear strain component along the *x* axis is given by

 (13)

in which *u* and *w* are the displacements along the axial and transverse axes, respectively. Using Eq. (13), the constitutive equation of the nanotube is obtained as

 (14)

where , , , ,  and  are respectively the Young modulus, viscosity coefficient, nonlocal calibration parameter, internal characteristics length, strain gradient parameter and Laplacian operator. In this study, the stress resultants (*Nxx*,*Mxx*) are defined by integration over the cross-section of the nanotube with area *A* as follows

 (15)

Using Eqs. (14) and (15), *Nxx* and *Mxx* can be formulated as

 (16)

 (17)

in which *I* denotes the inertia moment of the nanotube. For deriving motion equations, first the total kinetic energy (*Ke*), the elastic energy (*Uel*), the external work (*Wext*), and the work due to internal friction (*Wvis*) are formulated. The variation of *Uel* can be expressed as

 (18)

Here *L* is the length of the fluid-conveying nanotube. The variation of *Wvis* is given by

 (19)

Taking into account the effect of slip boundary conditions at the interface, the variation of *Ke* can be written as

 (20)

where *m*, *M* and *U* represent the mass per nanotube length, the mass per fluid length and the speed of the flowing fluid, respectively. Furthermore, the variation of *Wext* is given by [62, 63]

 (21)

in which  and *F* stand for the excitation frequency and the force amplitude. Substituting Eqs. (18)-(21) into the following principle [64, 65]

 (22)

the differential equations of the fluid-conveying system are obtained as

 (23)

 (24)

Employing Eqs. (16), (17), (23) and (24) and ignoring the inertia terms along the longitudinal direction [66], one obtains

 (25)

 (26)

The following relation can be derived from Eq. (25)

 (27)

where *C* denotes an integration coefficient; from the boundary conditions of *u*, this coefficient is obtained as

 (28)

Using Eqs. (16), (26) and (28), the nonlinear nonlocal strain gradient equation of fluid-conveying viscoelastic nanotubes is derived as

 (29)

Using a set of appropriate non-dimensional parameters given by

 (30)

the nonlinear nonlocal strain gradient equation of the fluid-conveying viscoelastic nanotube can be expressed as

 (31)

in which *F(x)=F*; *d*o denotes the outer diameter. In the above equation, asterisk superscripts are no taken into account for the sake of convenience. To discretise Eq. (31), the transverse deflection is expressed as

 (32)

where ,  and  indicate the number of base functions, the generalised coordinate and the transverse base function, respectively. Using Eqs. (31) and (32), one obtains

 (33)

To describe the nonlinear mechanical behaviour of the fluid-conveying viscoelastic nanotube, a frequency-continuation method is employed. For the sake of precision, eight base functions are incorporated.

**4. Results and discussion**

In this section, the scale-dependent frequency response of fluid-conveying viscoelastic nanotubes is investigated. The density, Young’s modulus, Poisson’s ratio, non-dimensional viscosity coefficient, thickness and outer radius of the nanotube are taken as 1024 kg/m3, 1440 MPa, 0.3, 0.0005, 24.4 nm and 248.4 nm, respectively. The non-dimensional nanosystem parameters are set to *Kn*=0.05, =1.4052, =0.8096 and  = 3529.6878. Moreover, the two different size parameters are as  and . In all figures, the non-dimensional excitation frequency is shown by *ω*.

Figure 2 indicates the bifurcation diagram of the nanotube conveying fluid; the transverse deflection at midpoint is plotted versus the fluid speed. The speed correction factor is *κnf*1=1.4052. As the fluid speed increases, the transverse deflection at midpoint is zero until the fluid speed reaches a critical value. The critical fluid flow speed corresponding to the divergence of the nanosystem is obtained as *Ucr* = 3.6506. Both stable and unstable results are plotted in the figure.

Plotted in Fig. 3 is the influence of slip boundary conditions on the bifurcation diagram of the nanotube conveying fluid. *wcentr*e denotesthe transverse displacement at midpoint. The speed correction factor is set to *κnf*1=1.4052 for slip boundary conditions while it is set to *κnf*1=1 for no-slip boundary conditions. It is found that no-slip boundary conditions result in overestimated values for the critical speed. When the slip boundary condition is implemented, the critical fluid speed is *Ucr*=3.6506 while for a fluid-conveying nanotube with no-slip boundary conditions, the critical speed is *Ucr* = 5.1297.

Depicted in Fig. 4 is the frequency-amplitude plots of the viscoelastic nanotube conveying fluid; the non-dimensional parameters are as *U*/*Ucr*=0.7, *F*=3.0, *η*=0.0005, and *κnf*1=1.4052. Three important observations are: (1) the frequency-amplitude response of fluid-conveying viscoelastic nanotubes is of hardening type, (2) two saddle-node bifurcations at *ω*/*ω*1 = 1.2621 and 1.0599 are found for the nanosystem, and (3) strong modal interactions [67, 68] are observed owing to the fluid flow.

Figure 5 illustrates the effects of the slip boundary condition on the frequency-amplitude plots of the fluid-conveying viscoelastic nanotube; the non-dimensional fluid speed, the force amplitude and the viscosity coefficients are set to *U*=2.5, *F*=3.0, and *η*=0.0005, respectively. The speed correction factor is set to *κnf*1=1.4052 for slip boundary conditions while it is set to *κnf*1=1 for no-slip boundary conditions. The resonance frequency is overestimated when the slip boundary condition is not incorporated. Moreover, the fluid-structure interaction model with no-slip boundary conditions cannot predict modal interactions in the size-dependent frequency-amplitude behaviour of fluid-conveying viscoelastic nanotubes.

Illustrated in Fig. 6 is the effect of the fluid speed on frequency-amplitude behaviour of the fluid-conveying viscoelastic nanotube for *F*=3.0, *η*=0.0005, and *κnf*1=1.4052. From this figure, it can be seen that the excitation frequency is lower for higher fluid speeds. However, the peak amplitude is higher for higher fluid speeds. The effect of the fluid speed on the peak amplitude of the nanosystem is more significant for higher generalised coordinates.

**5. Conclusions**

In the present analysis, a nonlocal strain gradient theory incorporating nonlinear stretching has been developed for fluid-conveying viscoelastic nanotubes. Eringen’s nonlocal theory along with a strain gradient model were used to develop the scale-dependent nonlinear model. An energy minimisation was performed based on Hamilton’s method for an oscillating fluid-conveying nanotube under an external excitation. This energy minimisation led to the nonlinear equation of the motion which was discretised via Galerkin’s technique. The bifurcation and frequency-amplitude diagrams of fluid-conveying viscoelastic nanotubes were determined using a flow-speed/frequency-continuation scheme.

It was found that the no-slip boundary condition results in overestimated values for the critical speed of fluid-conveying viscoelastic nanotubes. Moreover, the slip boundary condition predicts a larger buckling amplitude. The scale-dependent nonlinear frequency-amplitude behaviour of the fluid-conveying viscoelastic nanosystem is of hardening type including two saddle-node bifurcations. Moreover, it was found that when the slip boundary condition is not incorporated, the resonance frequency is significantly overestimated. Another drawback observed for the fluid-structure interaction model with no-slip boundary conditions was that it cannot comprehensively predict modal interactions in the size-dependent nonlinear behaviour of the nanosystem. For higher fluid speeds, the excitation frequency is lower while the peak amplitude is higher; in addition, for higher generalised coordinates, the influence of the fluid speed on the peak amplitude is more noticeable.

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Fig. 1. A viscoelastic nanotube conveying fluid.



Fig. 2. Bifurcation diagrams of the nanotube conveying fluid, showing the transverse displacement at midpoint.



Fig. 3. Effect of slip boundary condition on bifurcation diagram of the nanotube conveying fluid for the transverse displacement at midpoint.

|  |
| --- |
| (a) |
| (b) |
| (c) |
| (d) |

Fig. 4. Frequency-amplitude plots of the viscoelastic nanotube conveying fluid; (a-d) the maximum values of *q*1, *q*2, *q*3, and *q*4, respectively.

|  |
| --- |
| (a) |
| (b) |
| (c) |

Fig. 5. Fluid slip boundary condition effect on frequency-amplitude plots of the viscoelastic nanotube conveying fluid; (a-c) the maximum values of *q*1, *q*2, and *q*3, respectively.

|  |
| --- |
| (a) |
| (b) |
| (c) |

Fig. 6. Effect of the fluid speed on frequency-amplitude plots of the viscoelastic nanotube conveying fluid; (a-d) the maximum values of *q*1, *q*2, and *q*3.