**Enabling Electricity Access: Revisiting Load Models for AC Grid Operation**

**Part I**

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Abstract: Meeting electricity demand in remote communities and non-electrified regions in the poor developing world is a challenge. Power generation is in shortage compared to electricity demand. Electric utilities either would enforce grid’s zonal load curtailment or not electrify regions. Controlling electricity demand can play a vital role in enabling electricity access; however, weather uncertainty drives electricity demand variability. This paper provides an overview of current demand side management research, identify research gaps and propose a more promising approach to enable electricity access. Also, it proposes manipulating appliances models to fit their operation in applications where power supply shortage is an issue such. The proposed work considers the effect of the probabilistic nature of weather and meeting AC grid codes of operation.

1. Introduction

Power generation capacity in many developing countries cannot accommodate electricity consumption growth [1]. Electrifying many regions to accommodate them in the grid is a challenge for electric utilities. For instance, the demand is higher than the generation and the gap is estimated to be 7GW in Pakistan [2]. Electric utilities under such a condition would either prevent electrification [3] or would enforce a zonal feeder curtailment [4]. The latter would prevent electricity access for long hours of the day (12hours/day) [1, 5]. Some researchers suggested off grid electrification as an alternative option [6]; however, renewable energy technologies and projects [7, 8] necessitate the availability of capital fund [9] which is beyond consumers’ income [10].

Residential appliances’ demand is a component of the total electricity demand. Therefore, demand side management (DSM) might enhance electricity access under power generation deficiency. This paper will provide an overview of DSM research and will focus on identifying research gaps. Then, it proposes an operational planning model for appliances’ control to facilitate electricity accessibility in developing countries. The proposed model accounts for modelling electricity status in appliance load models accounting for consumers’ comfort, preferences, working hours... etc. Also, the model is ensured not to interfere with AC electric grid codes of operation (voltage limits and power flow in feeders).

2. An overview of DSM research

DSM research topics were focused on peak load curtailment, peak to average ratio minimization, peak load reduction, and load shifting and shedding as various approaches to manage the demand [11]. This section will provide an overview of DSM research topics and its relation to appliances control, and will distinguish it from the proposed work by identifying research gaps.

2.1. Load Curtailment

Load curtailment research was visible in literature in various perspectives. The load curtailment for peak demand reduction was selected in [12] as an alternative solution to a power line curtailment during power generation shortage (PGS). On the other hand, scheduling houses’ total demand curtailment was achieved in [1] through a multi-stage optimization approach to address the PGS problem. Load curtailment through a fuzzy decision tree and that could be run online was proposed in [13] to enhance the security of the system. The same objective was targeted in [14], but the approach was to minimize the load curtailment through the optimization approach to address the PGS problem. Load curtailment minimization is also addressed in [15] through the utilization of power flow controllers. The bus scheduling problem was investigated in [17] for the purpose of scheduling transmission outages. At the transmission level too, FACTS were proposed to be used to reduce the load curtailment [18].

2.2. Peak Shaving and Load Shedding

Peak shaving as a tool for DSM was applied in [19, 20], while load shifting and shedding research was noticed in [21-23]. For example, both load shifting and shedding in a nonlinear programming problem were achieved in [21] toward maximizing consumers’ satisfaction in an incentive based demand response approach. Moreover, the incentive concept was shown in [22] as a part of the definition of demand flexibility where the problem was formulated as a linear problem with the objective to reduce the aggregator...
cost for electricity demand bid in a wholesale market. Also, the feasibility of a direct residential appliance control shaving the demand through a duty cycle approach was shown in [23]. Peak shaving through the use of DSM and multi-agent system was presented in [20] indicating a demand reduction of 20%. The peak load shaving and cost minimization were the objectives of [24] where modelling habits through Markov chain and user location are considered. Load management in [25] was guaranteed through an optimal allocation of energy storage system.

Shaving the peak load and smoothing the load curve in a nonlinear programming optimization problem for a system incorporating renewable power generation were achieved in [26] by involving a battery energy storage system. When renewables join the power system, unbalance is created in the system [27]. Thus, battery energy storage system was involved in [28] to shave the peak in power system including wind and diesel power generation. Nevertheless, a hybrid energy system was used in [27] to achieve peak shaving rather than dealing with the peak demand shaving through demand response.

2.3. Peak to Average Ratio Minimization

The topic of peak to average ratio reduction through appliance scheduling was investigated in literature through various approaches [29-32]. The topic was discussed in [29] through a human expert based methodology. In [30], a multi-objective function of a mixed integer linear programming (MILP) problem was applied for peak load and electricity cost reductions in a goal programming approach. A similar appliance scheduling problem was presented in [31] where the minimized objective consisted of two parts: electricity cost and peak to average ratio. Also, the peak to average ratio and energy consumption minimizations were both achieved in [32] in an environment enabling pricing at the end of the day. The problem was subject to a predefined equipment rating, preferable time interval and operating period. In [33], a peak to average ratio minimization was achieved by demand side management and considering cloud computing. In [34], the cost of peak to average ratio was minimized considering energy scheduling through a game theory.

2.4. Peak Load Reduction

The peak load reduction topic was investigated in [35-37]. In [35], a MILP problem accounting for hydrogen energy storage systems and the generated power from a local PV system was applied. Furthermore, the topic of peak load reduction for evening time was presented in [36]. In such a work, smart washing machines were scheduled in Dutch houses supported by energy management system and solar PV system based on a dynamic pricing system. Furthermore, the dynamic pricing system was used in [37] to schedule appliances in a Belgium pilot project. In [38], a model based on heat, ventilation and air conditioning energy consumption data history to reduce peak load and achieve energy savings of 6% was developed.

2.5. Peak Load Shifting

Shifting ON peak demand to OFF peak period under different energy pricing schemes in a net zero energy building was achieved in [39] for measuring savings in renewable energy credits. In [40], a Pareto algorithm was applied for a multi-objective appliance scheduling considering their safe operation. Electricity cost and appliance delays were among the topics discussed in [40]. Peak load shifting was achieved in [41] considering both time of use tariff and energy storage system to manage building energy demand. In [42], buildings’ mechanical pre-cooling strategies to shift at least 50% of the cooling demand away from the peak load were presented. An optimization of energy cost through load shifting was met in [43] through power pinch analysis. Load shifting was achieved in [44] through game theory and prospectus theory.

2.6. An Appliance Perspective of DSM

Residential appliances contribute significantly to electricity demand. Scheduling such a demand is an aspect of DSM research. For example, the work in [45] considered consumers’ electricity bills to be minimized through a comprehensive appliance scheduling in a MILP problem.

In [46], authors focused on quantifying consumer’s satisfaction to be maximized at a minimum cost subject to his budget through a genetic algorithm. Alternatively, reference [47] focused on quantifying the effect of energy consumption of circulating pumps on the power system. However, the study required energy consumption data for many meters for 20 months and showed that the energy consumption of pumps based on aggregated measurements was preferable over the temperature based energy disaggregated approach.

Appliance control through a smart TV or a web server was achieved by the Telecommunication Institute in Korea [48], while household energy patterns could be identified through the grid reader in [49].

In [50], operating water pumps in a water-energy nexus to improve cost efficiency of a co-optimization model was presented. The work was expanded in [51] to use the water network to provide demand response services to the electric network by transferring the mixed integer nonlinear programming problem into a mixed integer programming problem considering a convex hull relaxation.

3. Appliance coordination for electricity access: gaps and prospects

Planning electricity access from an appliance and a grid prospective might be a promising solution. The proposed work targets planning appliance electricity access considering weather uncertainty and subject to operational grid codes. The work can serve as an alternative option to enhance electricity access in developing countries and it is distinguished from previous literature as discussed next.

3.1. Gaps to Overcome and Contribution

Electric utilities’ practical approaches to deal with the problem of PGS [1, 3-5] were not very effective in meeting consumers’ needs. Many consumers’ in developing countries are living without an access to elementary electricity needs. Also, many consumers are undergoing frequent power cuts restricting education and economic development.

From the research perspective, the appliance scheduling problem targeted in [30] was not addressed from the grid operational side. Meeting grid codes and permissible operational voltage levels of the distribution network is necessary to avoid power system collapse. In comparison, the work in [30] fits more developed countries as the user’s preferences definition in scheduling a device embedded in a smart meter of a smart home controller was assumed. Such an
equipment is not a visible ownership option for low income communities [10].

By reviewing the work in [31], it was visible that it was limited to shiftable and non-shiftable appliances where each appliance has a specific watt consumption rather than an operational load model (a function of electricity availability, temperature...etc) as will be presented in this paper. The scheduling problem in [31] was handled through heuristic techniques without incorporating AC power flow to verify the scheduled pattern as with the grid operational codes. The latter was also not accounted for in [33].

Although the work in [16] considered the limit on load priority and its location when performing load shedding, it did not deal with the concept of appliance and factors demanding their electricity consumption patterns, while the application of load shedding in [17] took another perspective toward scheduling outages at the transmission side. References [14, 15] necessitated placing power system devices to minimize load curtailment. Such a consideration may add costs into the system and may not be favored as an alternative in a deficient grid. In [18], the curtailment reduction was addressed from a transmission level point of view besides using grid FACTS, while this paper is to look at the low voltage side consumer and to reduce the curtailment by an optimal planning of electricity access in comparison to utilities’ practical approaches [1, 3-5].

The work in [19] fits an alternative environment where smoothing the load is an objective rather than enabling the electricity. The peak shaving problem through the multi-agent system in [20] was limited to specific type of appliances. In [20], neither a comprehensive analysis of other appliances, nor an optimization model was presented to assess how the operation would impact other devices at the end user scale or the operation at the grid level from the perspective of AC power flow. That negligence also applies for [34, 44]. The work in [24, 25] did not present appliance operational models; Aside from water pumps in [50, 51], no other appliances were modeled. The problem in [24] was subject to the utilization of energy storage system and reducing the cost by minimizing energy purchase from the grid. However, such a concept does not fit the work in this paper where power generation is in deficiency and consumers’ benefits from accessing electricity services can be maximized regardless of the cost.

In [26], the addressed peak shaving and load curve smoothing problem did not incorporate the appliance concept. Besides that, the problems in [26] included battery energy system which is a high cost element not favored if no renewable energy systems are involved or the capital cost is lacked. Even though energy storage systems were involved in [27, 28], the unbalance between power generation and demand created due to wind penetration is a reason for considering peak shaving in an alternative context.

The peak demand shifting in [41] was subject to the time of use tariff and such a concept is not necessary applied in poor developing countries where electricity is priced based on the unit of energy according to [45]. Also, the application of the proposed work in [41] is not for an area with 24 hours of energy deficiency. Shifting cooling loads in [42] can be an option when dealing with peak loads; however, in some developing countries where power generation shortage is continuous over the day, such a concept may not be the best option. Load shifting for electricity cost minimization was the objective in [43] rather than dealing with maximizing consumers’ access to electricity under deficient power generation status as the cost may not be the most critical point in that context. To meet the peak demand reduction in [38], energy consumption historical demand data and monitoring devices were needed which might not be still available when long term planning of electricity access for a region in the poor developing world is considered.

For [36, 39], the conducted analysis in such work were limited rather than including a comprehensive number of appliances as done in this paper. In the proposed work, the type of appliances expected to be owned by consumers in a region was based on [45]. Also, the work in [21, 35, 46] treated the electric load as a fixed load while the only flexible load in [35] was the electric vehicle. On the other hand, the work in [32] did not show how demand uncertainty was linked to the probabilistic nature of weather as will be incorporated in the model of this paper. Incorporating such an uncertainty is essential in planning electricity access.

In [22, 33, 37], the problem formulation did not incorporate the idea of enabling electricity access to non-electrified regions. Also, the work in [22] would not fit a region with underfunded power generation capacity where consumers are not currently electricity enabled since it discussed the idea of DSM considering the concept of incentives. In comparison to this concept, consumers in regions with PGS do not receive monetary for demand reduction.

In [40], manually operated appliances were not considered part of the energy management system and such a work did not reflect the electricity status and how the probabilistic nature of weather would affect enabling electricity access to non-electrified regions. Also, references [7, 11, 19] did not look at the case where electricity is in shortage and could not accommodate the demand. Even though the approach in [12] seems to be a future solution, the gap in it can be summarized as follows: First, the predefined load threshold was a consideration for appliance classification and might cause consumers with high demand and high energy consuming appliances to be out of supply all the time. Second, the queuing of the demand for an appliance at a next hour based on the current demand for it was not shown. Third, the obtained results were not verified at the grid level to avoid violation of the AC power flow and voltage constraints.

The contribution in the proposed work can be highlighted as follows: This paper presents operational planning model for appliances’ control when planning electricity access to non-electrified regions in the developing countries and when power generation is in shortage. The planning problem is to meet AC grid’s operational codes under such power generation deficiency. The proposed work accounts for two atmospheric factors: climate and weather uncertainty where climate would be represented by an expected temperature value if long term planning is needed and that fits more the feasibility stage of an electrification project; while the weather uncertainty consideration can improve the accuracy of operational planning problem and serves as a good fit for various planning horizons (short or medium). Appliance load models presented in literature do not fit an environment where power generation shortage is an issue. Therefore, such models are not valid and cannot be utilized to plan electricity access under power generation shortage. In this paper, reformulated and linearized appliance...
operational load models to account for new decision variables to indicate electricity status availability/absence are to be presented. The reformulation measures the effect of such a condition on the linkage between consumers’ comfort and preferences from one hour to another, current decisions and anticipates future actions under the probabilistic nature of the weather condition and under the climate concept. Also, the reformulation accounts for the linkage between one device operations to another. In Part II of this work, a multi-stage optimization framework that first considers the models presented in this paper, second accounts for all possible scenarios of appliance scheduling as driven by weather uncertainty in the first stage of the framework, third validates the scheduling plan with respect to AC grid codes of operation in the second stage, and fourth considers rescheduling when violations exists in the second stage such that the problem is tracked in the form of a cycle until both stages are satisfied will be presented in more details. To elaborate more, the models presented in this paper are constraints of the multi-stage optimization framework of Part II and the optimization problem would be investigated under all post the scenarios of appliance ownership and preference of devices’ accessibility. A comprehensive analysis and comparisons are conducted in Part II to demonstrate the effectiveness of the proposed work in this paper with respect to the traditional work in literature from perspectives of improving electricity access, energy efficiency and computation time.

3.2. Prospects

Planning electricity access in developing countries to accommodate non-electrified off grid regions through appliance scheduling can be a promising solution. Such a planning problem would necessitate an understanding of four factors: 1) generation/demand status, 2) weather variability and attitudes governing demand patterns, 3) type of loads and their corresponding operational models, and 4) operational grid codes.

The solution is promising because the proposed model focuses first on showing how such appliances could be scheduled by accounting for electricity availability/absence as binary decision variables within load operational models, and defining hourly electricity services to the new region of the grid under the probabilistic nature of weather temperature while satisfying the grid codes.

Second, the consequent effect of such decision variables on appliance operation considers the continuous calculation of houses’ conditions such as the current room temperature and the future room temperature as governed by electricity availability/absence.

Third, although electricity might or might not be available, the load operational models to be presented accounts for a range of comfort level such that it links consumer’s demand (subject to their comfort) with the status of electricity. If electricity is not available and consumers’ comfort cannot be met, the models predict their future demand to temperature dependent appliances by taking a continuous queue of previous, current and future probabilistic nature of room temperatures as well as consumers’ tendency toward using appliances at such time slots. It is important to emphasize that even though there can be a demand for a temperature dependent appliance at the instant of time when electricity is not available, this does not necessarily mean that consumers will demand such an appliance at the next instant of time when electricity is available. The room temperature could reach the comfort level due to an increase in the outdoor temperature affecting the indoor room temperature.

To clarify the above concept, the following example is presented. If the demand for a space heater (SH) based on the comfort level exists at time $t$ under weather scenario (SEC$_t$) and if electricity is available in such a case, then it will be supplied. Then, the temperature inside the house at $t+1$ is dependent on the temperature at the time instant $t$ under such a scenario. Hence, it is possible that the temperature at $t+1$ will meet the comfort level as the SH was ON at the previous hour, and the outdoor temperature (affecting the indoor temperature through the house thermodynamic model) did not decrease enough to interfere with the customers’ comfort at time $t+1$. Thus, the SH will be OFF at $t+1$. In this case, the energy dedicated to warm the home at $t+1$ using the traditional approaches [1, 3, 4] can be dedicated toward an alternative device that the consumer would demand during that time, or to supply an alternative consumer who is in demand for this energy rather than overheating the home and resulting in customers’ discomfort. On the other hand, if there is a demand for the SH at time $t$ under weather scenario (SEC$_t$) and the electricity is not available to meet it, then the device will not be supplied. This operation can be translated into one of two scenarios: 1) a degradation in the indoor temperature if the outdoor temperature has not risen enough to rise the home indoor temperature to be in the zone of residential comfort and therefore necessitating the operation of the SH at $t+1$, or 2) an increase in the indoor temperature at $t+1$ because the outdoor temperature has increased. In the latter case, there is no need for the SH to be ON at $t+1$. The effect of electricity absence or availability on the appliance operation is taken in a queue for next hours.

For other home appliances which are not weather dependent, such a concept can still be applied as consumers’ demand for such appliances subjects to working hours, consumers’ hourly preferences, and bedtimes as detailed in Section 5.

4. Generation-demand status

For the first factor (generation/demand status), non-electrified regions can be on grid regions without influencing electricity access of other grid zones by using the left-over supply (regardless the amount) that is not sufficient to enable electricity access to such a regionug for 24 hours. Such a deficient supply is to permit the off grid region to join the grid by scheduling appliances with respect to the four factors.

Consumers of the region to be electrified are expected to own the following appliances if electricity is enabled: space heater, water heater (WH), washing machine (WM), well pump (WP), and light bulbs. Such an ownership of devices is predicted by considering appliances owned by other consumers in similar regions that are electricity enabled as further discussed in [45]. Other devices, such as stove and fridge...etc. are not considered in the analysis as they may not be owned by many consumers. For instance, many consumers might be dependent on wood for cooking.
Electricity demand of individual consumers is highly driven by weather temperature, sunrise and sunset hours, or working hours (agricultural activities). Moreover, consumers’ tendencies toward utilizing their appliances are governed by the electricity availability and preferences. Since high power consumption appliances are driven by weather temperature and consumers’ comfort, the uncertainty/randomness nature in the weather temperature is a significant factor influencing the variability of electricity demand. Also, it can influence the electricity access of such consumers since the power generation capacity is deficient over the 24 hours of the day. Therefore, such a factor is essential in various planning stages of electrification. In more details, climate consideration is beneficial when electrification is considered on the long-term planning perspective and especially in the feasibility stage of an electrification project. In this case, a typical hourly temperature data can be a representative set of such temperature for any day. Thus, average values of hourly temperature data can be sufficient to model the climate under such a consideration. Nevertheless, weather uncertainty is preferable for a better prediction of demand variability and can be used at various stages of an operational planning problem (short or medium term planning). For this purpose, modeling hourly weather temperature data can be achieved through both probability paper plots (PPPs) and Monte-Carlo simulation (MCS) to enhance the accuracy of demand prediction under the uncertainty of weather. This is

Fig. 1. Aggregated electricity demand of the houses in the new region under uncertainty of weather temperature represented by different weather cenarios (SEC c) with different probabilities
recommended as the smart appliance control is not an option in the case of poor developing countries.

PPP is an approach to derive the best probabilistic distribution function (pdf) that describes the outdoor ambient air temperature historical data and the corresponding parameters of the distribution. More description on how such an approach is applied to derive the pdf of the hourly outdoor temperature can be found in [45]. In this case, 24 pdfs are obtained as further described in [1, 45] based on historical weather data from an Indian city. The data is collected from [52]. The selected probabilistic distribution functions include normal, log-normal, Weibull and exponential pdfs.

The inverse of the cumulative distribution function is then used in the MCS for data training and thus accounting for all possible scenarios of weather ambient air temperature for every hour of the day. Data validation in such a simulation was achieved by errors criteria. Since the parameters of the hourly probabilistic distributions of weather temperature obtained through PPP are what characterizes them, and typically the mean and standard deviation are functions of these parameters, errors in both the means and standard deviation between those of the distributions and the data generated from MCS are set as the criteria to stop MCS. The stopping criteria was based on 0.1% error tolerance in each criterion. More emphasis on equations utilized and criteria applied in such a simulation can be found in [1].

Since the MCS generated data set can be huge, data clustering is essential to simplify the analysis and to have a smaller typical data sets representing the large sample size of temperature data for every hour of the day. Clustering can be applied to the generated data such that these data sharing common/relative attributes are set in a group. K-means clustering is a preferred approach for an analysis of a limited number of data rather than dealing with the entire population, and it is recognized for its simplicity and popularity [53]. The approach in [53] can be applied for the weather temperature states such that they are described by the intervals (boundaries) of the clusters. The centroid of the cluster is selected to represent the state, while the probability (prob) corresponding to the state is determined as per the following equation where s1 and s2 are the cluster bounds [1]:

$$\text{prob} \left( \text{To\text{ut}} \right) = \int_{s1}^{s2} \text{pdf} \left( \text{To\text{ut}} \right) \, \text{d} \left( \text{To\text{ut}} \right) \quad (1)$$

Then, a data matrix of temperature sets is generated given that every row indicates a temperature state at an hour of the day and every column indicates a weather scenario (SEC) resembling the variability in the weather nature. Each temperature state would be assigned a probability (prob) as per the clustering algorithm such that the sum of probabilities of temperature states over the row in the data matrix is one.

Fig. 1 shows electricity demand variation with various temperature scenarios obtained after applying PPP, MCS and data clustering. Although one of temperature scenarios (at time = 24) has a high value for a winter day (39°C), it is important to clarify that such a scenario has a very low probability of occurrence that is 1x10^-4 (tail of the probabilistic distribution function). Also, it can be seen from Fig. 1 that the sum of probabilities for all the clusters of temperature represented by sub-blocks within the main block at an hour is one.

### Table 1 Acronym used in load models

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$P_{th}$</td>
<td>Power consumed by SH (kW).</td>
</tr>
<tr>
<td>$H$</td>
<td>One hour</td>
</tr>
<tr>
<td>$T_{in}$</td>
<td>Temperature in the house (°C)</td>
</tr>
<tr>
<td>$T_{out}$</td>
<td>House outside temperature (°C)</td>
</tr>
<tr>
<td>Cycles</td>
<td>Number of cycles used by the WM</td>
</tr>
<tr>
<td>Capacity</td>
<td>Capacity of the WM</td>
</tr>
<tr>
<td>efficiency</td>
<td>Efficiency of the WM</td>
</tr>
<tr>
<td>EF</td>
<td>WH efficiency</td>
</tr>
<tr>
<td>Therm</td>
<td>WH thermostat set point (°F)</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Temperature of water from a well (°C)</td>
</tr>
<tr>
<td>Vl</td>
<td>Volume of hot water consumption</td>
</tr>
<tr>
<td>n</td>
<td>House number</td>
</tr>
<tr>
<td>c</td>
<td>Weather scenario</td>
</tr>
<tr>
<td>t</td>
<td>Time of the day</td>
</tr>
<tr>
<td>Prating,wh</td>
<td>WH power rating (kW)</td>
</tr>
<tr>
<td>$T_o$</td>
<td>A set seasonal temperature operating point (°C)</td>
</tr>
<tr>
<td>light</td>
<td>Light bulbs</td>
</tr>
</tbody>
</table>

A probabilistic scheduling of demand driven by the probabilistic nature of weather temperature can facilitate electricity access as would be demonstrated in in Part II of this work. Such a consideration can be used at any planning stage as it can provide better accuracy than the other of the climate representation.

## 6. Load operational models

This section introduces the proposed load operational model that incorporates the electricity status as decision variable and the adjustment needed to maintain the linearity of the problem. Marinating this can be useful when considering scheduling of devices to enable electricity access in a mixed integer linear programming optimization problem to guarantee fast computational time and global optimality. The reformulated SH, WP, WM, and WH models are presented in this section accounting for the probabilistic nature of hourly weather condition in various houses of the region to be electrified. Some of the acronyms describing the load models are presented in Table 1.

### 6.1. SH Model

The operation of the SH is driven by two factors: 1) the consumer’s comfort level and 2) the availability of electricity supply at the time of demand as per the first factor. The energy consumed by SH ($Z_{th}$) is given by (2). $X_{th}$ is a binary variable with a value of one when SH is to be turned ON due to a consumer’s demand driven by his comfort level, and with a value of zero otherwise. Also, $\Theta_{th}$ is a binary variable that is one when the electricity is available to supply SH and is zero otherwise.

$$Z_{th} = H * P_{th} * X_{th} * \Theta_{th}$$

For restructuring the problem as a MILP, the term ($X_{th} * \Theta_{th}$) is to be linearized. Such a consideration will result in a reduction in the computation time and can prevent optimization solver failure. Also, it guarantees global optimality when considering optimal scheduling of devices as

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to be shown in Part II of this work. The linearization can be carried out as per rules of [54]. Thus, \( W_{\text{new}} \) (a free variable) is to replace the term \( (X_n \cdot Q_{\alpha}) \). So, constraints (2-6) are required to adjust for this replacement. In this case, \( W_{\text{new}} \) is not to exceed an upper bound (one) on the \( X_n \) and \( Q_{\alpha} \) values. Therefore, (3) and (4) are used. For \( W_{\text{new}} \), to be zero when \( X_n \) or \( Q_{\alpha} \) is zero, (5) is used. Irrespective of \( X_n \) and \( Q_{\alpha} \) values, the maximum value of \( W_{\text{new}} \) is one as shown by (56). In case \((X_n = Q_{\alpha})\), (7) is needed for a correct problem formulation. 

\[
W_{\text{new},n,t} \leq X_{\alpha,n,t} (3)
\]

\[
W_{\text{new},n,t} \leq Q_{\alpha,n,t} (4)
\]

\[
W_{\text{new},n,t} \geq 0 (5)
\]

\[
W_{\text{new},n,t} \leq 1 (6)
\]

\[
W_{\text{new},n,t} \geq X_{\alpha,n,t} + Q_{\alpha,n,t} - 1 (7)
\]

The effect of SH on the indoor temperature is governed by (8) as per the SH model in [25] after modifying the model to incorporate: the electricity status as decision variables, a contribution, to reflect the availability of electricity for such an appliance, and the consumer’s demand for SH as per his comfort. The initial room temperature in (9) is set to (a) at 1 am [45].

\[
P_{\text{sh,loc}} W_{\text{sh,loc}} = 0.36 T_{\text{sh,loc}} - 0.5 T_{\text{sh,sec}} - 0.056 T_{\text{sh,sec}} \forall t \in 24 (8)
\]

The comfort levels based on which SH is demanded are given by (10) and (11) according to the adequate warmth level as declared in [45].

\[
T_{\text{sh,sec}} < 18 \quad X_{\alpha,n,t} + 27 (1 - X_{\alpha,n,t}) (10)
\]

\[
T_{\alpha,n,t} \geq 18 (1 - X_{\alpha,n,t}) - 30 X_{\alpha,n,t} (11)
\]

6.2. WP and WM Models

The WP and WM scheduling models will follow (2-7) with the following modifications in terms: 1) \( Z_{\text{new}} \) is replaced with \( Z_{\text{new}} \) in the case of WP and \( Z_{\text{wm,sh}} \) in the case of WM, 2) \( P_{\text{sh}} \) is replaced with \( P_{\text{sh}} \) in the case of WP and \( P_{\text{wm,sh}} \) in the case of WM, 3) \( X_{\alpha} \) is replaced with \( X_{\text{sh}} \) in the case of WP and \( X_{\text{wm,sh}} \) in the case of WM, 4) \( \alpha_X \) is replaced with \( \alpha_{\text{sh}} \) in the case of WP and \( \alpha_{\text{wm,sh}} \) in the case of WM, and 5) \( W_{\text{new}} \) is replaced with \( W_{\text{new}} \) in the case of WP and \( W_{\text{new}} \) in the case of WM. Moreover, the following constraints will govern the demand for such devices as driven by consumer’s preferences and working hours.

The WP is to operate for \( K \) hours at any time that is after 6 am and before 7 pm as in (12) and (13).

\[
\sum_{i=0}^{24} X_{\text{wp,sh},i} = K (12)
\]

\[
\sum_{i=0}^{24} X_{\text{wp,sh},i} + 12 X_{\text{wp,sh},24} = 0 (13)
\]

The energy consumed by WM is governed by (14) based on the model in [45]. Such an energy consumption is not to exceed the device power rating as in (15) [45]. The WM is to be used for \( L \) hours/day as in (16) [45].

\[
Z_{\text{sh,wp,sh},t} = H \cdot Q_{\text{sh,wp,sh},t} \cdot C_{\text{sh,wp,sh},t} \cdot \text{efficiency}_{\text{sh}} (14)
\]

\[
Z_{\text{sh,wp,sh},t} \leq H \cdot P_{\text{sh,wp,sh},t} (15)
\]

\[
\sum_{i=0}^{24} W_{\text{sh,wp,sh},i} = L (16)
\]

Although well pumps and washing machines are independent on weather in their operation, the subscript \( c \) is shown in (12-16). The main reason is that under certain ownership scenarios (what devices owned by a house) these appliances are not the first devices to enter the scheduling optimization problem. Thus, the scheduling of such appliances is dependent on the scheduling of earlier weather dependent appliances. Therefore, such a subscript is needed in the definition of constraints.

6.3. WH Model

The electricity consumed by a WH can be decomposed into the electricity consumed due to hot water consumption (\( Z_{\text{wh}} \) in kWh) as in (17) and the electricity consumed to substitute heat losses from the tank surfaces (\( Z_{\text{loss}} \) in kWh) due to weather effect and heat transfer to or from surroundings as in (18) [45].

\[
Z_{\text{wh},t} = \frac{\text{WF}_{t} \cdot \frac{9}{5} \text{Therm}_{t} + 32 - \left( \frac{9}{5} \text{Therm}_{t} + 32 \right) - \frac{9}{5} \text{T}_{\text{sh},t} + 32}{3413} (17)
\]

\[
Z_{\text{loss},t} = \frac{1.3621875 \text{WF}_{t} \cdot \frac{9}{5} \text{Therm}_{t} + 32 - \left( \frac{9}{5} \text{Therm}_{t} + 32 \right) - \frac{9}{5} \text{T}_{\text{sh},t} + 32}{3413} (18)
\]

Although consumers are not yet supplied with electricity, hot water use patterns can be either measured or follow similar regions’ patterns as home appliances are expected to be owned when consumers are electricity enabled. In this work, (17) and (18) will be reformulated, to be suitable for a MILP scheduling problem to be shown in second part of this article. The reformulation will reflect consumers’ decisions to utilize such devices when a demand for hot water exists. It will also reflect actions taken by the electric utility to supply electricity to the device based on the electricity status. The reformulation is expressed by the following constraints (19)-(38):
When (19) was initially formulated, the variable \((D_{n,t,c})\) did not exist. Instead, the term \((\Psi_{n,t,c} \ast \delta_{n,t,c})\) was utilized. \(\Psi\) is a binary variable that is one if the electricity is available to supply the device and that is zero otherwise. Since this term is nonlinear, the positive variable \((D)\) is considered as a replacement. \(D\) is not to exceed an upper limit on either \(\delta\) or \(\Psi\). Thus, (24) and (25) are considered.

\[
\begin{align*}
\Delta_{n,t,c} &= D_{n,t,c} \ast \frac{1}{2} \Psi_{n,t,c} \ast H \\
&+ 1.3621875 \frac{3413}{3413^3} H \left( (\alpha_{n,t,c} - \frac{9}{5} \theta_{n,t,c} - \frac{32}{3} \beta_{n,t,c}) \ast (G_{n,t,c} \ast \frac{9}{5} + D_{n,t,c} \ast \frac{32}{3}) \right) \\
\end{align*}
\]

\[
\begin{align*}
\delta_{n,t,c} + \Delta_{n,t,c} &= V_{n,t,c} \\
\delta_{n,t,c} &\geq 0.0001 \ast \delta_{n,t,c} \\
V_{n,t,c} &\geq V_{n,t,c} + \Lambda_{n,t,c} \\
\delta_{n,t,c} + \Lambda_{n,t,c} &= 1 \\
D_{n,t,c} &\leq \delta_{n,t,c} \\
D_{n,t,c} &\leq \Psi_{n,t,c} \\
\end{align*}
\]

For guaranteeing that \(D\) is zero when either \(\Psi\) or \(\delta\) has a zero value, (26) reflecting the non-negativity is to be incorporated. For \((\Psi_{n,t,c} = \delta_{n,t,c} = 1)\), (27) is added to the problem formulation to guarantee that the value of \(D_{n,t,c}\) is one. It is important to emphasize that the maximum value that the variable \(D\) can have is one irrespective of the values of \(\Psi\) and \(\delta\). For \(D_{n,t,c}\) to be at least one when \((\Psi_{n,t,c} \ast \delta_{n,t,c} = 1)\), (28) is used.

\[
\begin{align*}
D_{n,t,c} &\geq 0 \\
\frac{1}{2} D_{n,t,c} &\leq 1 \\
D_{n,t,c} &\geq \Psi_{n,t,c} + \delta_{n,t,c} - 1 \\
\end{align*}
\]

When the device should be in the OFF mode, it is not to be electricity supplied to substitute energy losses from the device surfaces when a demand for hot water is not available. Therefore, (24-28) are followed similarly except that \(D\) is replaced with the positive variable \(\tilde{\delta}\) and \(\tilde{\Psi}\) is replaced with \(\Lambda\).

In the initial formulation of (19), the positive variable \((G_{n,t,c})\) was \((\Psi_{n,t,c} \ast (1 - \Lambda_{n,t,c}) \ast T_{n,t,c})\). Since the term is nonlinear, it is replaced with \(G_{n,t,c}\). Following the linearization procedure in [30], \(G_{n,t,c}\) should be less than \(T_{n,t,c}\), or equal to it. Thus, constraint (29) is used. Moreover, a variable denoted as \(\beta_{n,t,c}\) is used to replace \((1 - \Lambda_{n,t,c})\).

\[
G_{n,t,c} \leq T_{n,t,c} \\
\]

For the problem formulation to be valid such that the correct operation of the device is ensured, (30) and (31) are used where \(\tilde{e}\) is a parameter with a large value.

\[
\begin{align*}
G_{n,t,c} &\leq \tilde{e} \ast \Psi_{n,t,c} \\
G_{n,t,c} &\leq \tilde{e} \ast \beta_{n,t,c} \\
\end{align*}
\]

If \(\Psi_{n,t,c}\) or \((1 - \Lambda_{n,t,c})\) is zero, \(G_{n,t,c}\) is not to be negative as given by (32). On the other hand, when both are ones, the value of \(G_{n,t,c}\) is the value of \(T_{n,t,c}\). The variable \(G_{n,t,c}\) is further constrained as in (33) regardless of the values of \(\Psi_{n,t,c}\) and \((1 - \Lambda_{n,t,c})\).

\[
\begin{align*}
G_{n,t,c} &\geq 0 \\
G_{n,t,c} &\geq \tilde{e} \ast (\Psi_{n,t,c} + \beta_{n,t,c} - 2) + T_{n,t,c} \\
\end{align*}
\]

### Table 2: AC grid codes of operation: Acronyms' descriptions

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>i and j</td>
<td>buses of an electric feeder</td>
</tr>
<tr>
<td>P_{g}</td>
<td>available real power supply for the region to be electrified in per unit</td>
</tr>
<tr>
<td>Q_{s}</td>
<td>available reactive power supply for the region to be electrified in per unit</td>
</tr>
<tr>
<td>P_{d}</td>
<td>real electricity demand in per unit</td>
</tr>
<tr>
<td>Q_{d}</td>
<td>reactive electricity demand in per unit</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>tangent of the inverse of the cosine function of the power factor at which device (m) is operated as the case with WM and WP</td>
</tr>
<tr>
<td>RTS</td>
<td>hourly variation factor of non-residential loads as used in [1]</td>
</tr>
<tr>
<td>(N)</td>
<td>node at which the off grid region (residential houses region) is to join the grid</td>
</tr>
<tr>
<td>(P_{b})</td>
<td>power consumed by a device (m) when scheduling SH and WP</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>a free variable defined as (W_{a}) when scheduling SH, and (W) when scheduling WP for irrigation</td>
</tr>
<tr>
<td>(n)</td>
<td>total number of houses in the region to be electricity enabled</td>
</tr>
<tr>
<td>(\tilde{\sigma})</td>
<td>bus voltage angle</td>
</tr>
<tr>
<td>(Y)</td>
<td>admittance magnitude</td>
</tr>
<tr>
<td>(\theta)</td>
<td>admittance angle</td>
</tr>
<tr>
<td>(R)</td>
<td>a factor for converting the power demand to the per unit system</td>
</tr>
<tr>
<td>(f)</td>
<td>a binary variable with a value of one when a device (m) schedule of house (n) at time (t) under SEC.</td>
</tr>
<tr>
<td>(P_{low})</td>
<td>the minimum limit on the available real power supply for the region to be electrified in per unit</td>
</tr>
<tr>
<td>(P_{up})</td>
<td>the maximum limit on the available real power supply for the region to be electrified in per unit</td>
</tr>
<tr>
<td>(Q_{low})</td>
<td>the minimum limit on the available reactive power supply for the region to be electrified in per unit</td>
</tr>
<tr>
<td>(Q_{up})</td>
<td>the maximum limit on the available reactive power supply for the region to be electrified in per unit</td>
</tr>
</tbody>
</table>

\(T_{\phi} = V_{l} \phi \quad 1.3621875 \frac{3413}{3413^3} H \left( (\alpha_{n,t,c} - \frac{9}{5} \theta_{n,t,c} - \frac{32}{3} \beta_{n,t,c}) \ast (G_{n,t,c} \ast \frac{9}{5} + D_{n,t,c} \ast \frac{32}{3}) \right) + \frac{3413}{3413^3} H \left( (\alpha_{n,t,c} - \frac{9}{5} \theta_{n,t,c} + D_{n,t,c}) \ast (G_{n,t,c} \ast \frac{9}{5} + D_{n,t,c} \ast \frac{32}{3}) \right) \)
The temperature of the well’s water entering the tank is given by (34) [45]. $dT_{in,t}$ represents the well water temperature that is lost due to the effect of radiation among two bodies (water and air) as given by (35), and $q$ reflects the energy lost/gain as a result of radiation as in (36) [25]. The initial well’s water temperature is set at the first hour of the day as shown in (37) [45].

$$T_{in,t} = T_{in,1} + dT_{in,t}, \quad \text{for } t = 24$$

$$dT_{in,t} = -0.0099 * q_{in,t}$$

$$q_{in,t} = \{16.91361 + 10^8 * t - 3 * (273 + T_d)^4 + 4 * (273 + T_d)^3 - (273 + T_d)^2) \}$$

(36)

$$T_{in,1} = A$$

(37)

The maximum energy consumed by the device is not to exceed the device power rating as given by (38).

$$Z_{nwhctn} \leq H \times Prating_{wh}$$

(38)

When the WH is the first device prioritized to enter the scheduling optimization problem (part II of this article), the positive variable $W_{nwhctn}$ is used in the SH model is assigned a zero value. Based on this assignment, the indoor temperature is calculated. Conversely, when the SH is to enter the optimization problem earlier than the WH, the hourly indoor temperature is calculated. Such a consideration will establish the link between temperature dependent appliances.

6.4. Light Bulbs

Light bulbs are identified as a minimum basic need. Therefore, they are provided the priority of supply over other appliances. They can be modeled as fixed load. However, if electricity status cannot accommodate such a basic demand into the grid, then the light bulb model can follow the problem formulation of the WP or WM load models.

7. AC grid codes of operation

Appliance management under PGS can enable electricity access to non-electrified regions. However, the aggregated demand is to be confirmed with AC grid codes of operation to avoid any violations of voltage limits. For this purpose, a link is to be established between appliance operation and power system operation as will be further elaborated in Part II of this work.

The power flow equations and grid codes are presented in (39)-(46). In more details, constraints (39) and (40) represent the active and reactive power flow respectively considering the critical and essential load buses, respectively. Given a set of m devices and light bulbs, constraints (41) and (42) represent the active and reactive power flow respectively considering the normal bus if (m = 1) where $s$ is an indication of the device number in the m devices set owned by the consumer and in this case it is the first device to enter the optimization problem as will be further discussed in more details in Part II of this work. Constraints (43) and (44) represent the active and reactive power flow for the normal bus respectively of if $s = 1$. For clarification purposes, the whole term $(P_{nwhctn}, \alpha_n)$ describes $Z_{nwhctn}$ and $Z_{nwhctn} = \frac{1}{\alpha_n}$ per hour when scheduling WH and WM, respectively. $\alpha_n$ represents $W_{nwhctn}$.

and $\varphi$ when scheduling device m (SH and WP), respectively. Voltage limits constraints are presented in (45) and (46). A description of the acronyms is shown in Table 2 and more elaboration on the problem formulation would be provided in Part II of this work.

$$P_{nwhctn} - P_d * \text{RTS}_{nwhctn} = \sum_{j=1}^{m} \left( v_{ij} * v_{ij}^* \right)$$

$$\cos(\hat{\theta}_{ij} - \theta_{ij}^*) \forall i \neq N$$

(39)

$$Q_{nwhctn} - Q_d * \text{RTS}_{nwhctn} = \sum_{j=1}^{m} \left( v_{ij} * v_{ij}^* \right)$$

$$\sin(\hat{\theta}_{ij} - \theta_{ij}^*) \forall i \neq N$$

(40)

$$P_{nwhctn} - \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$- \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$- \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

(41)

$$Q_{nwhctn} - \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

(42)

$$P_{nwhctn} - \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

(43)

$$Q_{nwhctn} - \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

$$= \sum_{j=1}^{m} \left( r^* P_{nwhctn} \right) * a_{ij}$$

(44)

$$- \pi \leq \hat{\theta}_{ij} \leq \pi$$

(45)

$$0.95 pu \leq v_{ij} \leq 1.05 pu$$

(46)

Equations (47) and (48) indicate that no generation sources are installed at any node in the feeder connecting the region to be electrified to the traditional electric grid [1]. At bus $i = 1$, the amount of power flowing to the region to be electrified is described by (49) and (50) [1].

$$P_{nwhctn} = 0$$

(47)

$$Q_{nwhctn} = 0$$

(48)

$$P_{nwhctn} \leq P_{nwhctn}$$

(49)

$$Q_{nwhctn} \leq Q_{nwhctn}$$

(50)

8. Conclusion

This paper presented a comprehensive overview of the current research in DSM and appliance scheduling. The paper proposed an alternative promising approach that overlooks at appliance operational models and restructure them to make their operation fits well for application in developing countries with PGS. Models were structured as a component of the grid so that their operation when scheduling devices can meet operational grid codes as will be presented in Part II of this paper.
9. References


[52] http://freemeteo.in/
