A Task Allocation Algorithm for Profit Maximization in NFC-RAN

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Abstract—In this paper, we study a general Near-Far Computing Enhanced C-RAN (NFC-RAN), in which users can offload the tasks to the near edge cloud (NEC) or the far edge cloud (FEC). We aim to propose a profit-aware task allocation model by maximizing the profit of the edge cloud operators. We first prove that this problem can be transformed to a Multiple-Choice Multi-Dimensional 0-1 Knapsack Problem (MMKP), which is NP-hard. Then, we solve it by using a low complexity heuristic algorithm. The simulation results show that the proposed algorithm achieves a good tradeoff between the performance and the complexity compared with the benchmark algorithm.

Index Terms—mobile edge computing, task allocation algorithm, profit maximization

I. INTRODUCTION

With the rapid development of mobile terminals, the mobile services are increasing, such as augmented reality (AR) and virtual reality (VR) services. These mobile services require high data transmission and huge computing resource, and also may have the strict delay requirement [1]. In some scenarios, the strict requirements can be satisfied by increasing the communication bandwidth. However, the bandwidth is limited in practice [2]. In addition, simply using local processing resources does not satisfy the requirements of the mobile service [3]–[4]. Thus, the cloud computing resources are normally applied to improve the computational ability of mobile services. Mobile edge computing (MEC), as an example, can allow terminals to offload the tasks to the network edge [5]–[7], which can increase the computing capacity and reduce the energy consumption of the mobile terminals.

The cloud radio access networks (C-RAN) architecture is one of the typical wireless networks, which provides cloud resources to signal processing of the mobile terminals [8]. Then, the Near-Far computing enhanced radio access networks (NFC-RAN) goes a step further, which assumes that the computing resource can be provided to the mobile user [9]. In the NFC-RAN architecture, the tasks offloaded from the users can be allocated to either near edge cloud (NEC) or far edge cloud (FEC). Moreover, when dealing with the task allocation problem in NFC-RAN, the network operators may consider the profit as well as the limited amount of the computing resource in the network architecture.

To consider above background, in this paper, we study the profit maximization problem in the NFC-RAN architecture considering the task allocation. We assume the task allocation is related to the pricing scheme, which influences the net income of the operator. We also consider the transmitting capacity of the links in NEC and FEC. Then, we propose a new tasks allocation model aiming at maximizing the profit of the operator. Also, we propose a low complexity heuristic algorithm to deal with this problem.

The rest of the paper is organized as follows. Section II describes the NFC-RAN architecture as well as the system model. Section III introduces how our proposed problem can be transformed to a Multiple-Choice Multi-Dimensional 0-1 Knapsack Problem (MMKP) and then the low complexity heuristic algorithm. Section IV gives the simulation results while Section V concludes the paper.

II. PROFIT MAXIMIZATION PROBLEM

A. NFC-RAN Architecture

In this section, similar to [9], we applied the NFC-RAN architecture, where it is composed of baseband unit (BBU) pool, Remote Radio Head (RRH), NEC and FEC, as shown in Fig.1. The BBU is responsible for signal processing related tasks such as receiving the task data from users and transmitting the results back to the users. The NEC denotes the edge computing resource located next to RRH, which is close to the user, while the FEC is located next to the BBU, which can be seen as the far edge cloud. The users can offload the tasks to either NEC or FEC for processing. If the task is allocated to FEC, the data has to be transmitted via wireless access links first and then the fronthaul link, resulting in more cost.

B. System model

Based on the above architecture, we assume there are $M$ cells, where the $j$-th cell has one NEC, one RRH and $N_j$ users. Also, we assume there is one FEC and one BBU in the remote center. Assume each user has a task $U_{ij}$, $i = 1, 2, \ldots, N_j$ ($i$-th user in $j$-th cell) with the feature $U_{ij} = (F_{ij}, D_{ij}, T_{ij}, P_{cu_{ij}})$, $\forall i \in N_j, \forall j \in M$, where $F_{ij}$ is the computation requirement of the task, $D_{ij}$ is the data needed to be transmitted to the NEC or the FEC, $T_{ij}$ is the delay requirement of the task, and $P_{cu_{ij}}$ is the predefined cost budget. Also, assume that the task can not be processed in the user due to the limited computing resource locally. Assume $a_{ij}$, $b_{ij}$, $\forall i \in N_j, \forall j \in M$ as the
binary variables to indicate where the task is conducted, in which if $a_{ij} = 1$, the $i$-th task is allocated to the NEC, whereas if $a_{ij} = 1$, the $i$-th task is allocated to the FEC. As the task can only be executed in FEC or NEC, one can have $a_{ij} + b_{ij} \leq 1$.

In the NFC-RAN architecture, assume the computation capacity of the $j$-th NEC as $F_{NE}^C$, the computation capacity of the FEC as $F_{FE}^C$, the transmit capacity of the fronthaul as $R_j$. Also, assume the computation capability allocated to the task $U_{ij}$ as $j_{ij}^{NE}$ in NEC or $j_{ij}^{FE}$ in FEC, the transmit capacity allocated to the task $U_{ij}$ as $r_{ij}^{W}$ in the wireless link or $r_{ij}^{F}$ in the fronthaul link.

We also set the price and the cost of the task processing in NEC as $P_{w}^C$ and $C_{w}^C$, respectively, the task processing in NEC as $P_{NE}^C$ and $C_{NE}^C$, respectively, the transmit price and cost in wireless link as $P_{w}^R$, $C_{w}^R$ respectively, whereas price and cost for the fronthaul is as $P_{F}^R$, $C_{F}^R$, respectively. Thus, one can have the overall cost for each task allocated to FEC as $\frac{D_{ij}^{F}}{r_{ij}^{F}} P_{w}^R + \frac{D_{ij}^{F}}{r_{ij}^{F}} P_{F}^R + \frac{F_{ij}^C}{f_{ij}^{F}} P_{FE}^C$, whereas the overall cost for the task allocated to NEC as $\frac{D_{ij}^{N}}{r_{ij}^{N}} P_{w}^R + \frac{F_{ij}^C}{f_{ij}^{N}} P_{NE}^C$. Then the profit for the task allocated to FEC and NEC can be given respectively as

$$P_{ij}^F = \frac{D_{ij}^{F}}{r_{ij}^{F}} (P_{w}^R - C_{w}^R) + \frac{D_{ij}^{F}}{r_{ij}^{F}} (P_{F}^R - C_{F}^R) + \frac{F_{ij}^C}{f_{ij}^{F}} (P_{FE}^C - C_{FE}^C)$$

$$P_{ij}^N = \frac{D_{ij}^{N}}{r_{ij}^{N}} (P_{w}^R - C_{w}^R) + \frac{F_{ij}^C}{f_{ij}^{N}} (P_{NE}^C - C_{NE}^C)$$

Therefore, our profit maximization problem can be given as Eq.(3). In Eq.(3), the $(3b)$ is the latency requirement of each task, $(3c)$ is the computation resource constraint in FEC, $(3d)$ is the computation resource constraint in $j$-th NEC, $(3e)$ is the processing constraint for $j$-th fronthaul, $(3f)$ is the cost constraint/budget for each task, $(3g)$ and $(3h)$ indicate that each task can only be executed at NEC or FEC.

$$\max_{a_{ij}, b_{ij}} \sum_{i \in N_j} \sum_{j \in M} (a_{ij} P_{ij}^F + b_{ij} P_{ij}^N)$$

subject to:

$$a_{ij} \left( \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{F_{ij}^C}{f_{ij}^{F}} \right) + b_{ij} \left( \frac{D_{ij}^{N}}{r_{ij}^{N}} + \frac{F_{ij}^C}{f_{ij}^{N}} \right) \leq T_{ij}$$

$$\sum_{i \in N_j} \sum_{j \in M} a_{ij} f_{ij}^{F} \leq F_{j}^{FE}$$

$$\sum_{i \in N_j} b_{ij} f_{ij}^{N} \leq F_{j}^{NE}$$

$$\sum_{i \in N_j} a_{ij} r_{ij}^{F} \leq R_j$$

$$a_{ij} + b_{ij} \leq 1$$

$$a_{ij}, b_{ij} = \{0, 1\}, \forall i \in N_j, \forall j \in M$$

### III. Proposed Algorithm

In this section, we will first prove that the proposed problem is NP-hard, and then apply an low complexity heuristic algorithm, which can achieve a good trade-off between performance and complexity compared with the benchmark algorithm.

#### A. Problem Analysis

We first prove that Eq.(3) is NP-hard. To do this, we simplify it at first. We relax and rewrite the $(3b)$ and $(3f)$ as following formulations:

$$a_{ij} \left( \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{F_{ij}^C}{f_{ij}^{F}} \right) \leq T_{ij}$$

$$b_{ij} \left( \frac{D_{ij}^{N}}{r_{ij}^{N}} + \frac{F_{ij}^C}{f_{ij}^{N}} \right) \leq T_{ij}$$

$$a_{ij} \left( \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{F_{ij}^C}{f_{ij}^{F}} \right) \leq P_{CU}$$

$$b_{ij} \left( \frac{D_{ij}^{N}}{r_{ij}^{N}} + \frac{F_{ij}^C}{f_{ij}^{N}} \right) \leq P_{CU}$$

Then, we put the Eq.(4), Eq.(6), (3c) and (3e) together to form the holistic constraints in the FEC. Next, we sum all the constraints related to FEC. For example, based on Eq.(4), we sum all the latency constraints as following:

$$\sum_{j=1}^{M} \sum_{i=1}^{N_j} \left( \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{D_{ij}^{F}}{r_{ij}^{F}} + \frac{F_{ij}^C}{f_{ij}^{F}} \right) a_{ij} \leq \sum_{j=1}^{M} \sum_{i=1}^{N_j} T_{ij}$$
Then it can be written as:

\[
\sum_{j=1}^{M} \sum_{i=1}^{N_j} w_{ij}^1 a_{ij} \leq C_1^1
\]  

where the \( w_{ij}^1 \) is \( \left( \frac{D_{ij}}{r_{ij}} + \frac{D_{ij}}{r_{ij}} + \frac{F_{ij}^k}{r_{ij}} \right) \), the \( C_1^1 \) is \( \sum_{j=1}^{M} \sum_{i=1}^{N_j} T_{ij} \).

One can also have: \( w_{ij}^2 = f_{ij} F_{ij}, w_{ij}^3 = r_{ij} F_{ij}, w_{ij}^4 = \frac{D_{ij}}{r_{ij}} p_{w_j} + \frac{D_{ij}}{r_{ij}} F_{ij} + \frac{F_{ij}^k}{r_{ij}} F_{ij} \), \( C_1^2 = F_{ij}, C_1^3 = \sum_{j=1}^{M} R_{ij} \), and \( C_1^4 = P_{ci} u_{ij} \).

Similarly, we can simplify the constraints in the NEC.

Finally we can simplify and rewrite the problem (3) as follows:

\[
\max_{a_{ij}, b_{ij}} \sum_{j=1}^{M} \sum_{i=1}^{N_j} a_{ij} P_{ij}^F + b_{ij} P_{ij}^N
\]

subject to:

\[
\sum_{j=1}^{M} \sum_{i=1}^{N_j} w_{ij}^k a_{ij} \leq C_1^k
\]

\[
\sum_{j=1}^{M} \sum_{i=1}^{N_j} w_{ij}^k b_{ij} \leq C_2^k
\]

\[
a_{ij} + b_{ij} \leq 1, i \in \{1, ..., N_j\}, j \in \{1, ..., M\}
\]

\[
b_{ij} \in \{0, 1\}, a_{ij} \in \{0, 1\}
\]

Let \( W_{1ij} = (w_{1ij}^1, ..., w_{1ij}^k) \) be a resource vector. Then, the overall amount of the resource is given by \( C_1 = (C_1^1, ..., C_1^k) \) in FEC. Let \( W_{2ij} = (w_{2ij}^1, ..., w_{2ij}^k) \) be a resource vector. Then, the overall amount of the resource is given by \( C_2 = (C_2^1, ..., C_2^k) \) in the NEC.

Now we introduce the Multiple-Choice Multi-Dimensional 0-1 Knapsack Problem (MMKP) to prove that the above proposed optimization can be re-written to a MMKP, which is NP-hard. In a MMKP, one can assume that there are \( S \) classes of the items, where each class, \( s = 1, ..., S \), has \( H_s \) items. Each item \( h, h = 1, ..., H_s \), of the group \( s \) has the profit value \( v_{sh} \). Thus, the MMKP problem can be defined as follow:

\[
\max_{x_{sh}} \sum_{s=1}^{S} \sum_{h=1}^{H_s} v_{sh} x_{sh}
\]

subject to:

\[
\sum_{s=1}^{S} \sum_{h=1}^{H_s} w_{sh} x_{sh} \leq C_k, k \in \{1, ..., m\}
\]

\[
\sum_{h=1}^{H_s} x_{sh} = 1, s \in \{1, ..., S\}
\]

\[
x_{sh} \in \{0, 1\}, s \in \{1, ..., S\}, h \in \{1, ..., H_s\}
\]

Considering the above \((10d)\) is an inequality constraint, we can add an auxiliary variable \( c_{ij}, c_{ij} \in \{0, 1\} \) to \((10d)\) such that \( a_{ij} + b_{ij} + c_{ij} = 1 \). If the \( c_{ij} = 1 \), the user is not allocated, if the \( c_{ij} = 0 \), the user is allocated to the NEC or the FEC.

After adding this auxiliary variable, \((10d)\) can be written as \((11c)\). The \( x_{sh} \) is the vector combined by \( (a_{ij}, b_{ij}, c_{ij}) \) and the profit \( v_{sh} \) is the profit vector combined by \( (P_{ij}^F, P_{ij}^N, 0) \).

Also, we have \( S = \sum_{j=1}^{M} N_j \). As for the resource required, we can express the \((11b)\) as \( \text{WX} \leq \text{C} \). We can combine the \( W_{1ij}, W_{2ij}, W_{3ij} \) to form the \( W_{ij} \), setting \( W_{3ij} = 0 \). The elements in \( \text{C} \) is same as the \( W_{ij} \), thus, we can combine the \( C_1, C_2, C_3 \) to form the \( \text{C} \), where \( C_3 \) is a zero vector 0. Then, one can see that the proposed optimization problem can be transformed to a MMKP problem \((11)\), which is NP-hard and very difficult to solve.

Next, we apply the following heuristic (HEU) algorithm.

**B. Proposed Algorithm**

In this subsection, we use the HEU to solve the MMKP, similar to [10]. We first calculate the aggregate resource value \( \text{aggre} \) of the each item and sort the items according to the non-decreasing order of this value. The \( \text{aggre}_{ij} \) can be calculated as follows:

\[
\text{aggre}_{ij} = \frac{\sum_{k=1}^{4} r_{ij}^k C^k / |C|}{|C|}
\]

where \( C^k \) is the \( k \) th element of constraint vector \( C \) in the proposed optimization problem, \( |\cdot| \) denotes the inner product, and \( r_{ij}^k \) is obtained by mapping the resource vector \( w_{ij}^k \) to the interval \([0, 1]\) and the \( \max(w_{ij}^k) \) denotes the maximal value of \( w_{ij}^k, i = 1, ..., N_j, j = 1, ..., M \). Similarly, we can get the \( \min(w_{ij}^k) \). Thus, the \( r_{ij}^k \) can be given as follows:

\[
r_{ij}^k = \begin{cases} 
\frac{\max(w_{ij}^k) - w_{ij}^k}{\max(w_{ij}^k) - \min(w_{ij}^k)} & \text{if } \max(w_{ij}^k) - \min(w_{ij}^k) \neq 0 \\
1 & \text{if } \max(w_{ij}^k) - \min(w_{ij}^k) = 0
\end{cases}
\]

Next, we introduce the overall algorithm in the Algorithm 1. We first sort the tasks in each cell in the non-descending order according to their \( \text{aggre}_{ij} \) value. Then in each cell, we compare the \( P_{ij}^F \) with the \( P_{ij}^N \). If the \( P_{ij}^F \leq P_{ij}^N \), the task is allocated to the NEC until the overall constrains are violated. This is because we try to increase our profit as much as possible. After NEC is fully occupied, we check if there are still users left for resource allocation. If so, we sort the rest of the user by calculating their \( P_{ij}^F / \text{aggre}_{ij} \) and the \( P_{ij}^N / \text{aggre}_{ij} \) in non-descending order and then allocate the rest of the users to the FEC according to the sorting. This is because we try to allocate the rest of the user according to the profit per resource unit. By using this algorithm, the tasks can be allocated in a fast manner. Since the problem can also be formulated as a linear integer programming problem (LIP), we can use the LIP solver in Matlab to solve this problem and apply it as
Algorithm 1  The proposed HEU algorithm

1. Input $M$, $N_j$, $r_{ij}^F$, $v_{ij}^w$, $f_{ij}^NE$, $f_{ij}^{NE}$, $P_w$, $C_w$, $P_F$, $C_F$, $P_{FE}$, $C_{FE}$, $P_{NE}$, $C_{NE}$.
2. Output $a_{ij}$, $b_{ij}$.

Initialization
3. Calculate the profit of the user by using Eq.(1) and Eq.(2);
4. Set $a_{ij} = 0$, $b_{ij} = 0$

Begin the Task Allocation
5. For $j=1$ to $M$
6. For $i=1$ to $N_j$
7. Calculate the $r_{ij}$ and $aggre_{ij}$ by using Eq.(12) and Eq.(13), respectively;
8. End for
9. End for
10. For $j=1$ to $M$
11. Sort the users in cell $j$ according the $aggre_{ij}$ in non-descending order;
12. While the constrains are not violated in the NEC
13. If $P_{ij}^F \leq P_{ij}^N$;
14. Task is allocated to NEC and set $b_{ij} = 1$;
15. Else If The constraints in the FEC are not violated
16. Task is allocated to FEC and set $a_{ij} = 1$;
17. End while
18. End For
19. Add the users with $a_{ij} = 0$, $b_{ij} = 0$ to the Rest User Group
20. Sort the users in Rest User Group according to $P_{ij}^F/\text{aggre}_{ij}$ and $P_{ij}^N/\text{aggre}_{ij}$ in the descending order;
21. While the Rest User Group is not empty;
22. If The constraints are not violated;
23. The task is allocated to FEC according to sort results; and remove the user from the Rest User Group;
24. End while

the benchmark in our simulation. The simulation result is presented in the following section.

IV. SIMULATION RESULT

In the simulation, the number of cells is set to $M = 5$, the delay requirement of the task $T_{ij}$ is chosen from $20s$, $25s$, $30s$, we run the simulation with different number of users $N_j = 10, ..., 50$. We repeat each experiment 50 times for each given user number $N_j$. Other parameters are shown in the Table I.

We first compare the performance of proposed HEU with the scheme presented in [9], which aims to maximize the number of offloaded tasks and we denoted the scheme in [9] as TMA. One can see from Figure 2 that HEU can achieve much higher profit than the TMA, as TMA only focuses on maximizing the number of offloaded tasks, without considering the profit from the operator.

Also, as stated before, we compare our algorithm with the LIP solver used in MATLAB R2018a, which can be seen as the optimal solution of this problem. We depict the profit and running time of these two algorithms in Figure 3 and Figure 4, respectively. The simulation is conducted in the computer with 8 gigabytes of memory and the Intel(R) Core(TM)i5-6400 CPU. One can see that our proposed algorithm has the similar performance as the LIP solver but with much less running time.

V. CONCLUSION

In this paper, we have studied the profit maximization problem in the NFC-RAN architecture. We have transformed the proposed problem into a MMKP first, which is proved to be a NP-hard problem. Then, we applied a heuristic algorithm to solve this problem, which is a polynomial time complexity algorithm. The simulation results have shown that the proposed scheme achieved a good tradeoff between the performance and complexity.
ACNOWLEDGEMENTS

This work was supported in part by the Zhongshan City Team Project (Grant No. 180809162197874), National Natural Science Foundation of China (Grant No. 61620106011, 61572389 and U1705263).

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