Responses of sub-ice platelet layer thickening rate and frazil-ice concentration to variations in ice-shelf water supercooling in McMurdo Sound, Antarctica

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Abstract. Persistent outflow of supercooled ice-shelf water (ISW) from beneath McMurdo Ice Shelf creates a rapidly growing sub-ice platelet layer (SIPL) with a unique crystallographic structure under the sea ice in McMurdo Sound, Antarctica. A vertically modified frazil-ice-laden ISW plume model that encapsulates the combined non-linear effects of the vertical distributions of supercooling and frazil concentration on frazil ice growth is applied to McMurdo Sound and is shown to reproduce the observed ISW supercooling and SIPL distributions. Using this model, the dependence of the SIPL thickening rate and depth-averaged frazil-ice concentration on ISW supercooling in McMurdo Sound is investigated and found to be predominantly controlled by the vertical distribution of frazil concentration. The complex dependence on frazil concentration highlights the need to improve frazil-ice observations within the sea-ice–ocean boundary layer in McMurdo Sound.

1 Introduction

Ice shelf basal melting removes more mass from the Antarctic Ice Sheet than iceberg calving does, but the three largest ice shelves, Filchner-Ronne, Ross, and Amery, contribute only 18% of the net meltwater flux (Rignot et al., 2013). That is because the seawater-filled cavities beneath those ice shelves are dominated by high-salinity shelf water that has a potential temperature at or near the surface freezing point. Ice shelf basal melting occurs at depth because the freezing point temperature is lower under elevated pressure, and results in the formation of ice-shelf water (ISW), characterized by potential temperatures below the surface freezing point. When the buoyant ISW ascends along the ice-shelf base, the pressure relief causes it to become supercooled in situ, a necessary condition for ice crystals to persist in suspension. Those disk-shaped frazil-ice crystals accumulate under the ice shelves, leading to the formation of marine ice that is thicker and more localized than would be possible through direct freezing at the ice-shelf base (Morgan, 1972; Oerter et al., 1992; Fricker et al., 2001; Holland et al., 2007, 2009). Occasionally, frazil-ice crystals bathed in supercooled ISW are also carried out beyond the ice-shelf front and precipitated under adjacent sea ice, forming an unconsolidated, porous, sub-ice platelet layer (SIPL) (Gow et al., 1998; Hunkeler et al., 2016; Langhorne et al., 2015; Leonard et al., 2006; Robinson et al., 2014). SIPL not only harbours some of the highest concentrations of sea-ice algae on Earth (Arrigo et al., 2010) but also contributes to the sea-ice thickness when the water within the pores of SIPL freezes, due to heat loss to the atmosphere, to become incorporated platelet ice (Smith et al., 2001). Therefore, SIPL should not be ignored when investigating sea-ice thickness near an ice-shelf front.

Owing to the paucity of direct observation, our understanding of the evolution of frazil-ice-laden ISW relies heavily on numerical models. Those models are mostly derived...
from plume theory (Holland and Feltham, 2006; Jenkins and Bomboksh, 1995; Rees Jones and Wells, 2018; Smedsrud and Jenkins, 2004) but include three-dimensional ocean circulation models (Galton-Fenzi et al., 2012) and have been widely applied to assess the marine ice beneath Filchner-Ronne (Bomboksh and Jenkins, 1995; Holland et al., 2007; Smedsrud and Jenkins, 2004), Larsen (Holland et al., 2009) and Amery ice shelves (Galton-Fenzi et al., 2012), and SIPL under the sea ice in McMurdo Sound (Hughes et al., 2014, hereinafter HU14). To date, all the ISW plume models mentioned above have been depth-integrated, and all the scalar quantities, i.e. potential temperature, salinity, and frazil concentration in those models, are treated as vertically uniform. The well-mixed potential temperature and salinity have been validated by borehole observations beneath the Amery Ice Shelf (Herraiz-Borreguero et al., 2013) and under the sea ice in McMurdo Sound (Robinson et al., 2014; HU14). Although there are no observations of the vertical profile of frazil-ice concentration, it is unlikely to be vertically uniform because the buoyant rise of the crystals will counteract the turbulent diffusion that tends to homogenize the other properties. Recently, Cheng et al. (2017) showed that adopting an approach in which the frazil-ice growth is calculated using a vertically uniform frazil concentration results in substantial underestimation of marine-ice production underneath the western side of Ronne Ice Shelf. Idealized one-dimensional models confirm that the vertical distribution of frazil concentration cannot remain well mixed in the upper layers of the ocean (Svensson and Omstedt, 1998) and beneath the ice shelves (Holland and Feltham, 2005). Consequently, earlier assessments of either marine-ice or SIPL production in the aforementioned areas may need to be re-evaluated.

McMurdo Sound, located in the south-western Ross Sea (Fig. 1), is characterized by significant ISW outflow, arguably one of the most comprehensively observed ISW plumes available (HU14; Langhorne et al., 2015; Robinson et al., 2014). A prominent SIPL forms in the central-western sound (Dempsey et al., 2010); the maximum (area-averaged) observational first-year sea ice and SIPL are 2.5 (2) and 8 (3) m as determined from drill-hole measurements adjacent to the McMurdo Ice Shelf front between late November and early December in 2011 (Fig. 9 in HU14). The thin (∼20 m) McMurdo Ice Shelf front allows the ISW outflow to be delivered to the ocean surface without mixing with warmer ambient waters (Robinson et al., 2014). The study documented in HU14 was the first to apply the steady, one-dimensional frazil-ice-laden ISW plume model developed by Smedsrud and Jenkins (2004) to McMurdo Sound, although a constant ISW plume thickness was used. McMurdo Sound therefore seems an ideal setting in which to apply and evaluate the new vertically modified ISW plume model proposed by Cheng et al. (2017), which includes time dependence and two horizontal dimensions. The main objective is to explore the possibility of finding the quantitative relationship between the SIPL thickening rate and ISW supercooling. Establishing such a relationship is of significance to the assessment of total sea-ice thickness and thus the oceanic heat flux associated with SIPL in McMurdo Sound and elsewhere.

Here we first analyze the combined non-linear effects of the vertical distributions of supercooling and frazil concentration on the suspended frazil-ice growth rate in a supercooled ISW plume and compare results with those obtained with a commonly used, depth-averaged formulation. Then, we evaluate the performance of the vertically modified ISW plume model in reproducing the observed ISW supercooling and SIPL distribution to show the importance of considering the combined non-linear effects. Finally, we conduct 211 sensitivity simulations with the purpose of quantitatively establishing the response of the SIPL thickening rate as well as the frazil-ice concentration to variations in ISW supercooling in McMurdo Sound.

2 Physically based formulation for frazil-ice growth rate

The growth rate of suspended frazil-ice controls both the dynamic and thermodynamic evolution of ISW plumes and the accretion of ice crystals beneath ice shelves (Cheng et al., 2017; Holland and Feltham, 2006; Smedsrud and Jenkins, 2004) and sea ice (HU14). The frazil-ice growth rate is found to be proportional to the following integral expression once a number of physical parameters within the commonly used formulation of Jenkins and Beldomsh (1995) are merged:

\[
    I_{gr} = \int_{0}^{1} T_{SC} c_{i}(\sigma) \, d\sigma, \quad T_{SC} = T_{f}(\sigma, S) - T,
\]

where \( \sigma \in [0, 1] \) is the relative vertical coordinate, with 0 and 1 respectively corresponding to the upper-ice-plume and lower-plume–ambient-water interfaces. \( T \) and \( S \) are respectively the plume’s potential temperature and salinity, vertically well mixed within the plume, \( c_{i} \) is the vertically distributed (in this study) volumetric frazil concentration beneath the ice shelves, and \( T_{SC} \) and \( T_{f} \) are respectively the supercooling level (positive for supercooling) and local freezing point. Because of the well-known linear decrease in \( T_{f} \) with increasing water depth, \( T_{SC} \) also varies linearly with depth, transitioning from supercooling to overheating as \( \sigma \) increases (Figs. 2a and 3). The corresponding transition height at which \( T_{SC} = 0 \) is defined by supercooled thickness, \( D_{SC} = \sigma_{SC} D \), where \( \sigma_{SC} \) and \( D \) are respectively the supercooled fraction and total ISW plume thickness.

In earlier ISW plume models, because \( c_{i} \) is treated as vertically uniform, the integral of Eq. (1) can be represented by the product of the depth-averaged values \( \int_{0}^{D_{SC}} c_{i}^{0.5} \) \((0.5 \text{ means at mid-depth})\) and \( C_{i} \). Thus, we refer to these ISW plume models as vertically uniform (VU). It is worth mentioning that, in order to take the supercooling into account when \( \sigma_{SC} < 0.5 \), HU14 integrated \( T_{SC} \) over the supercooled part only without

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introducing any frazil-ice melting. However, in this study, we will demonstrate that the important role of frazil-ice melting in the lower, overheated part of the plume cannot be ignored.

The vertical distribution of frazil concentration, in reality, much like the concentration of suspended sediment (Cheng et al., 2013, 2016), should be vertically non-uniform, with higher concentrations near the ice-shelf/sea-ice base. Considering only the balance between the buoyant-rise-induced vertical advection and turbulent diffusion terms, the governing equation for frazil concentration can be written as

\[ \frac{d}{d\sigma} \left( K \frac{dc_i}{d\sigma} \right) + w_i \frac{dc_i}{d\sigma} = 0, \]

where \( w_i \) is the frazil-ice rise velocity, determined by ice crystal size. \( K \) is the vertical frazil concentration diffusion coefficient, which can be parameterized as vertically constant (Cheng et al., 2013, 2016):

\[ K = \frac{1}{6} \kappa u^*_u D, \]

where \( \kappa = 0.4 \) is von Karman’s constant, \( u^*_u = \sqrt{C_d U} \) is the friction velocity, related to the turbulent intensity within the ISW plume, \( C_d \) is the basal drag coefficient, \( U = \sqrt{(U_p + U_a)^2 + (V_p + V_a)^2 + U_t^2} \) is the total flow speed, \( U_p (U_a) \) and \( V_p (V_a) \) are the depth-averaged ISW plume (ambient current) speed in the \( x \) and \( y \) directions respectively, \( U_t \) is the root-mean square tidal speed. Using a zero net flux condition in the equilibrium state at the lower boundary of the plume, i.e.

\[ K \frac{dc_i}{d\sigma} + w_i c_i = 0, \quad \text{for} \ \sigma = 1 \]

and a Dirichlet boundary condition at the upper boundary, i.e.

\[ c_i = c_{i,b}, \quad \text{for} \ \sigma = 0, \]

where \( c_{i,b} \) is the frazil concentration at the ice-shelf/sea-ice base, the vertical exponential profile for the equilibrium frazil concentration can be written as

\[ c_i = c_{i,b} e^{-\beta \sigma}, \]

where \( \beta = \frac{K}{w_i} \).
concentration can be readily obtained (Cheng et al., 2017):

\[
\frac{c_i(\sigma)}{c_{i,b}} = \exp(-6Z_s),
\]

where \( Z_s = w_i/\kappa u_s \) is the suspension index, otherwise known as the Rouse number. Integrating this exponential profile from \( \sigma = 0 \) to 1, we finally obtain the relation between \( c_i(\sigma) \) and \( C_i \) as

\[
\frac{c_i(\sigma)}{C_i} = \frac{6Z_s \exp(-6Z_s \sigma)}{1 - \exp(-6Z_s)}.
\]

As shown in Fig. 2a, the vertical distribution of frazil concentration is strongly controlled by \( Z_s \). The gradient of the vertical distribution becomes greater with increasing \( Z_s \), and a vertically uniform frazil concentration distribution can only be achieved as \( Z_s \) approaches 0. While low values of \( Z_s \) are attainable with strong currents, those conditions also reduce the tendency for frazil to precipitate and contribute to SIPL formation (see Eq. 3 below). Therefore, we expect a non-uniform vertical distribution of frazil wherever there is active formation of SIPL. Accordingly, Cheng et al. (2017) introduced Eq. (2) into Eq. (1), and as a result, significantly improved the simulated pattern of marine-ice growth under the western side of Ronne Ice Shelf, compared with the VU and satellite-derived (Joughin and Padman, 2003) results. Here-
reasonable and desirable characteristics that are absent from growth rate distributions in the VM model show physically part of the plume. Overall, the frazil concentration and frazil do not exceed the frazil concentration in the lower overheated portion of the plume; therefore 6–30 times larger than that typically applied in ice–water interaction models (Robinson et al., 2017). The parameterization of the sea-ice thermodynamics, the assumption of no entrainment of ambient water into the ISW plume, and the boundary conditions at the ISW outflow follow HU14.

The initial thickness of the ISW outflow (indicated by blue arrow in Fig. 1) from underneath McMurdo Ice Shelf is set equal to that of the supercooled layer; i.e. $D = D_{SC}$, and the discharge per unit width is set to $0.02 \text{ m}^2 \text{s}^{-1}$. The addition of both an ambient circulation and tides follow HU14: the former, which represented the only source of momentum in the study of HU14, is assumed to be parallel to the Victoria Land coast, in the negative $y$ direction, and to be constant throughout the model domain; the latter is calculated using root-mean square tidal speeds from Padman and Erofeeva (2005). Because ISW persists in McMurdo Sound for at least the 8–9 months of the ice growth season (Robinson et al., 2014), all runs are integrated for 240 days. The model resolution and time step ($t_i$) are 1 km and 25 s, respectively. The frazil-ice size distribution is represented by five crystal size classes, and the transfer processes, induced by frazil freezing and melting, between different size classes are calculated using the scheme proposed by Smedsrud and Jenk-
ins (2004). Sensitivity experiments with more crystal size classes yielded qualitatively similar results. The ice concentration at the ISW outflow is evenly distributed among the classes (Holland and Feltham, 2005, 2006; Smelssrud and Jenkins, 2004).

We treat the frazil-ice precipitation rate \( p' \) as inverted sedimentation and follow the parameterization of McCave and Swift (1976):

\[
p' = w_i C_i \left(1 - \frac{U^2}{U_c^2}\right) \times He \left(1 - \frac{U^2}{U_c^2}\right),
\]

where \( U_c \) is a critical velocity, above which precipitation cannot occur, determined by Jenkins and Bombosch (1995):

\[
U_c^2 = \frac{\theta_i (\rho_0 - \rho_i) g 2r_e}{\rho_0 C_p},
\]

where \( \theta_i \) is the Shields criterion, \( \rho_0 \) and \( \rho_i \) are reference seawater and ice densities, respectively, \( g \) is gravity, and \( r_e \) is the equivalent radius of a sphere with the same volume as the frazil disk. The frazil-ice rise velocity, \( w_i \), is calculated by Morse and Richard (2009):

\[
w_i = \begin{cases} 
2.025 D_i^{1.621} & \text{if } D_i \leq 1.27 \text{ mm} \\
-0.103 D_i^2 + 4.069 D_i - 2.024 & \text{if } 1.27 < D_i \leq 7 \text{ mm}
\end{cases}
\]

where \( D_i = 2r_i \) is the diameter of a frazil crystal in millimetres. The inclusion of the Heaviside function \( He \) means that negative precipitation (i.e. erosion of previously deposited frazil ice) is not permitted. Because we have no idea about how cohesive the ice crystals are once they have settled, the estimation of an erosion rate would entail additional uncertainties.

The complex processes after the frazil-ice precipitates onto the sea-ice base are simplified in our model. In order to calculate SIPL thickness \( D_p \) at the \( n \)th time interval, we adopt the assumptions of HU14 that solid ice fraction within SIPL in McMurdo Sound is 0.25 based on the observational estimation from Gough et al. (2012) and that the ice crystals, on average, double in volume after precipitation:

\[
D_p = \frac{1}{0.25} \times 2 \times \sum_{k=1}^{n} (p'_k \times t).
\]

It should be noted that the volume change factor is a broad estimate, with almost no supporting evidence in the literature to guide it. Coupling our VM model with a model focusing on the processes associated with platelet ice accretion within the sea ice (Buffo et al., 2018) would be necessary to improve on that rough estimate but is beyond the scope of the present study.

4 Results

4.1 Standard model run

The performance of the VU and VM models in reproducing the ISW supercooling and SIPL pattern in McMurdo Sound is evaluated by comparing the results with observational data. To our knowledge, the data reported by HU14 are the most comprehensive available to evaluate our model, including both oceanographic and drill-hole measurements in two horizontal dimensions adjacent to McMurdo Ice Shelf. As this study represents the first application of a two-dimensional ISW plume model to the McMurdo Sound region, extensive tuning of the least constrained model parameters, including the ISW outflow properties, SIPL basal drag coefficient, frazil-ice crystal size distribution, ambient current speed, and Shields criterion was required to produce the distributions of ISW properties and SIPL thickness shown in Figs. 4a and 6, respectively. Despite the limited observational constraints on many of these parameters we do find support in the literature for our adopted values: ISW outflow properties are consistent with those reported by HU14, and the corresponding thickness of the supercooled layer is within the observed range (60–70 m) given in both HU14 and Robinson et al. (2014); the basal drag coefficient fits appropriately within the range identified by Robinson et al. (2017), while the ambient current speed is consistent with the lowest speeds reported in that study; we used five crystal size classes, as did Galton-Fenzi et al. (2012), although our sizes are slightly larger; we used a larger Shields criterion than the middle (0.05) of the observed range, although there is considerable scatter amongst the individual results reported from sedimentary experiments. Table 1 summarizes all the values adopted for the key parameters. Model results are evaluated by means of skill metrics: root-mean-square error (RMSE), correlation coefficient (CC), and skill score (SS), given by

\[
\text{RMSE} = \sqrt{\frac{\sum (X_{\text{cal}} - X_{\text{obs}})^2}{M}},
\]

\[
\text{CC} = \frac{\sum (X_{\text{cal}} - \bar{X}_{\text{cal}}) (X_{\text{obs}} - \bar{X}_{\text{obs}})}{\sqrt{\sum (X_{\text{cal}} - \bar{X}_{\text{cal}})^2 \sum (X_{\text{obs}} - \bar{X}_{\text{obs}})^2}},
\]

\[
\text{SS} = 1 - \frac{\sum (X_{\text{cal}} - X_{\text{obs}})^2}{\sum (X_{\text{obs}} - \bar{X}_{\text{obs}})^2},
\]

where \( X \) is the variable being evaluated, \( M \) is the number of data points, and the overbar denotes the arithmetic mean. The performance of each model is indicated by SS: > 0.65 is excellent, 0.65–0.5 is very good, 0.5–0.2 is good, and < 0.2 is poor (Luo et al., 2017; Ralston et al., 2010; Song and Wang, 2013).

It can be seen that at the end of the simulations both VM and VU models reproduce the observed reduction in ISW supercooling at the sea-ice base \( (T_{\text{SC}}^0) \). Superscript "0" denotes
the sea-ice base) in the cross- and long-sound directions, in spite of some evident model discrepancies (Fig. 4a) that may result from the limitations in our model setup: both the ambient current and tides are treated as temporally and spatially constant; there are no long-term observations of ISW outflow to provide reliable boundary conditions; we use a constant drag coefficient, ignoring the spatio-temporal evolution of the sea-ice basal form characterized by SIPL. We also ignore the impact on ISW properties of brine drainage from the upper SIPL as it is incorporated into the sea ice by the freeze-up of interstitial water, driven by heat loss to the atmosphere. Including such processes would require coupling with a sea-ice model such as that of Buffo et al. (2018), mentioned above. Nevertheless, the SS of $T_{SC}^0$ calculated using VM and VU models are 0.56 and 0.58, respectively, and the CC and RMSE are also reasonable (Table 2). There are only small differences throughout the time series of $T_{SC}^0$ simulated by the VM and VU models (Fig. 4) and the final distributions of both total ISW plume thickness and supercooled thickness are also very similar (see Fig. 5a–d). A comprehensive comparison of $T_{SC}^0$ calculated by the VM and VU models in an extensive set of sensitivity experiments will be discussed later. Finally, it can be seen that in both models the ISW plume flow is predominantly governed by a geostrophic balance (Fig. 5a–d).

In contrast, the frazil concentration (red lines in Fig. 4b, Fig. 5e and f) and SIPL thickness (green lines in Figs. 4b, 6b and c) are both underestimated by the VU model, compared with the results of the VM model, throughout the time series. Given the small differences in $T_{SC}^0$ calculated by VM and VU models, this result demonstrates that the vertical distribution of frazil concentration within the ISW plume plays a critical role.
Table 1. List of parameters used in standard model run.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>$-1.4244 \times 10^{-4}$ s$^{-1}$</td>
<td>Coriolis parameter</td>
</tr>
<tr>
<td>$D_{\text{ini}}$ ($D_{\text{ini}}^{\text{SC}}$)</td>
<td>78 m</td>
<td>Constant ISW plume outflow thickness (constant outflow supercooled layer thickness)</td>
</tr>
<tr>
<td>$W_{\text{ini}}$</td>
<td>3 km</td>
<td>ISW plume outflow width with constant $D_{\text{ini}}$ ($D_{\text{ini}}^{\text{SC}}$)</td>
</tr>
<tr>
<td>$c_{\text{ini}}$</td>
<td>$1 \times 10^{-6}$</td>
<td>Depth-averaged volumetric frazil concentration in outflow</td>
</tr>
<tr>
<td>$N_{\text{ice}}$</td>
<td>5</td>
<td>Number of frazil-ice sizes</td>
</tr>
<tr>
<td>$r_1, r_2, r_3, r_4, r_5$</td>
<td>0.2, 0.6, 0.9, 1.2, 1.5 mm</td>
<td>Frazil ice radii for each class</td>
</tr>
<tr>
<td>$\alpha_e$</td>
<td>0.02</td>
<td>Aspect ratio of frazil discs</td>
</tr>
<tr>
<td>$\bar{n}$</td>
<td>$1 \times 10^3$ m$^{-3}$</td>
<td>Average number of frazil crystals in all size classes per unit volume</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.02</td>
<td>SIPL basal drag coefficient</td>
</tr>
<tr>
<td>$V_a$</td>
<td>$-0.01$ m s$^{-1}$</td>
<td>Ambient flow speed</td>
</tr>
<tr>
<td>$A_H$</td>
<td>100 m$^2$ s$^{-1}$</td>
<td>Horizontal eddy viscosity</td>
</tr>
<tr>
<td>$K_H$</td>
<td>20 m$^2$ s$^{-1}$</td>
<td>Horizontal turbulent diffusivity</td>
</tr>
<tr>
<td>$S_{\text{ini}}$</td>
<td>34.59 psu</td>
<td>ISW plume outflow salinity</td>
</tr>
<tr>
<td>$T_{\text{ini}}$</td>
<td>$-0.0573 \times S_{\text{ini}} + 0.0832 - 7.61 \times 10^{-4} D_{\text{ini}}$</td>
<td>Potential temperature of ISW plume outflow</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.075</td>
<td>Shields criterion number</td>
</tr>
</tbody>
</table>

Table 2. List of calculated skill metrics for the results of VM and VU standard model runs.

<table>
<thead>
<tr>
<th>Variable</th>
<th>RMSE</th>
<th>CC</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VM</td>
<td>VU</td>
<td>VM</td>
</tr>
<tr>
<td>$T_{0}^{\text{SC}}$</td>
<td>0.0070 °C</td>
<td>0.0069 °C</td>
<td>0.83</td>
</tr>
<tr>
<td>SIPL thickness</td>
<td>1.034 m</td>
<td>2.928 m</td>
<td>0.91</td>
</tr>
</tbody>
</table>

The role in determining the suspended frazil-ice growth (Fig. 2), and thus the frazil concentration and SIPL thickness distributions. The supercooling is utilized more efficiently in the VM model, giving a greater depth-averaged frazil concentration than is produced by the commonly used VU model. The simulated SIPL thickness near the ISW outflow exhibits steeper gradients than are observed (Fig. 6a and b), which probably result from the spatial non-uniformity of ISW plume near the outflow (Fig. 5a and b). That non-uniformity in flow leads to localized non-uniformities in thermodynamics (Fig. 5c and d), frazil concentration (Fig. 5e and f), and thus SIPL thickness (Fig. 6b and c). Moreover, because the sea-ice base is horizontal, there are no changes in the freezing point associated with pressure change, so supercooling is always highest at the ISW outflow (Fig. 5c and d). That results in the greatest frazil concentration (Fig. 5e and f) and SIPL thickness (Fig. 6b and c) near the location of the outflow, and because the outflow is steady in time, spatial gradients in SIPL close to the outflow are enhanced. In reality, temporal changes in the ISW outflow position, width, supercooled layer thickness and duration could lead to a broader region of elevated frazil precipitation and a less peaked distribution of SIPL thickness. In addition, such small-scale features in the SIPL thickness distribution, if present, would not be resolved by the relatively coarse spatial distribution of drill-hole measurements (dots in Fig. 6). Nevertheless, the largest SIPL thickness undoubtedly occurs adjacent to the ISW outflow in McMurdo Sound, and the SIPL thickness calculated by the VM model at drill sites agrees well with the measurements (Fig. 6a), being graded “excellent” in contrast with the “poor” performance of the VU model (Table 2). Despite efforts to tune the VU model to give a better match with the observed SIPL thickness, even a limited expansion of SIPL can only be achieved with a considerable increase in the calculated $T_{0}^{\text{SC}}$, in disagreement with the observations.

For both VM and VU models, the time series of area-averaged $T_{0}^{\text{SC}}$ and $C_i$ (hereafter $T_{0}^{\text{SC}}$ and $C_i$ denote their area-average values), and SIPL thickness indicate respectively two near-constant values and one near-constant growth rate after about the 150th day (Fig. 4b). It is informative to explore how our various assumptions about the vertical distribution of frazil concentration influence the steady-state relationship between those variables in the McMurdo Sound region.
4.2 Dependence of SIPL thickening rate on ISW supercooling

The response of ice-shelf basal melting to variations in ocean temperature has been investigated using satellite altimetry (Rignot and Jacobs, 2002; Shepherd et al., 2004) and numerical models (Grosfeld and Sandhäger, 2004; Holland et al., 2008; Payne et al., 2007; Walker and Holland, 2007; Williams et al., 1998, 2002). In contrast, we know of no studies to date that provide a quantitative relationship between marine-ice (or SIPL) thickening rate beneath ice shelves (or sea ice) and ISW supercooling. Such a relationship is of potential significance for evaluating the mass balance of deep-draughting ice shelves in cold-water environments and adjacent sea ice subject to climatic variability.

Owing to the number of poorly constrained parameters in the frazil-ice-laden ISW plume model, we conducted 211 comparative sensitivity experiments between VM and VU models, varying both physical and input parameters, including drag coefficient, frazil-ice crystal size configuration, average number of frazil crystals, ambient current speed, width and thickness of the ISW outflow, and frazil concentration within the outflow (see Table 3). For all model runs, we plot the relationship between $T_{0,SC}$ and thickening rate in...
Table 3. Parameter settings for sensitivity runs, indicated by a tick and colour-coded by ISW outflow thickness (bottom row). All other parameters remain as they were for the standard model run.

<table>
<thead>
<tr>
<th>Parameter settings</th>
<th>Drag coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>✓✓✓✓✓✓ ✓✓✓✓✓✓✓✓✓✓✓✓</td>
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<td>✓✓✓✓✓✓ ✓✓✓✓✓✓✓✓✓✓✓✓</td>
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the steady state, using output from the last 30 days of each run (Fig. 7).

In Fig. 7a, the results of the VM model are grouped by the prescribed supercooled layer thickness $D_{SC}^{ini}$ in the ISW outflow. For $D_{SC}^{ini} < 65$ m there is a relatively consistent increase in thickening rate with increasing $T_{0}^{SI}$, while for $D_{SC}^{ini} \geq 65$ m the thickening rate tends to be much more variable. It is worth mentioning that $D_{SC}^{ini} = 65$ m is the value estimated by HU14 based on the measurements conducted by Lewis and Perkin (1985) and Jones and Hill (2001). For $D_{SC}^{ini} = 78$ m and greater, inflexions emerge separating a region of low thickening rate, where the thickening rate tends to decrease with increasing $T_{0}^{SI}$ from a region of high thickening rate, where there is a very rapid increase in thickening rate with increasing $T_{0}^{SI}$. This complex response of the VM model must result from the consideration of vertical structure in the frazil concentration, controlled by the frazil-ice suspension index $Z_s$ (Fig. 2) in the calculation of frazil-ice growth.

We therefore calculated the weighted average of $Z_s$ at each grid point for the VM model using the following equation:

$$
\overline{Z_s} = \frac{\sum_{k=1}^{n} C_i^k Z_s^k}{\sum_{k=1}^{n} C_i^k} = \frac{\sum_{k=1}^{n} C_i^k Z_s^k}{C_i},
$$

where $C_i^k$ is the frazil concentration of the $k$th size class, and $n$ is the number of size classes used. Then, we took the average of $\overline{Z_s}$ over all the grid points occupied by the plume to give a representative suspension index for the VM runs (hereinafter $\overline{Z_s}$ denotes its area-averaged value). We replotted the VM model results characterized by $\overline{Z_s}$ in Fig. 7b. We find systematic changes in $\overline{Z_s}$ with increasing thickening rate (along the coloured lines in Fig. 7b), particularly for $D_{SC}^{ini} \geq 78$ m, where the inflexions emerge. With decreasing $\overline{Z_s}$, $T_{0}^{SI}$ first decreases and then increases. If $\overline{Z_s}$ is sufficiently large, the suspended frazil crystals deposit out of the ISW plume so rapidly that they cannot efficiently use the
ISW supercooling to grow, leading to the smallest SIPL production for the VM model. For smaller \( Z_s \), the frazil crystals bathed in the supercooled layer of the ISW plume can remain in suspension and grow longer, resulting in a thicker SIPL and less residual supercooling. However, if \( Z_s \) decreases further, higher frazil concentration occurs within the lower, overheated part of the ISW plume, where melting of the crystals can mitigate the release of latent heat (Fig. 2b). That promotes further growth of frazil ice, which can remain in suspension even longer and thus lead to rapid SIPL production. The thickening rate calculated by the VU model is also shown, and is discernibly smaller than that calculated by the VM model. In other words, the relation between thickening rate and \( T_{SC}^0 \) is independent of \( Z_s \) for such small \( D_{ini}^{SC} \). When \( D_{ini}^{SC} \) is within the range of 65 to 78 m, the VM model results are distinguishable, with data points having smaller thickening rate and larger \( T_{SC}^0 \) corresponding to larger \( Z_s \) (Fig. 7b). When \( D_{ini}^{SC} \geq 78 \) m, the inflexions emerge, and the ISW supercooling revives when \( Z_s \) decreases further. Therefore, we conclude that when \( D_{ini}^{SC} \) exceeds a critical value (about 65 m for these McMurdo Sound simulations), the efficiency of converting ISW supercooling into frazil ice growth is controlled by the suspension index.

4.3 Dependence of frazil concentration on ISW supercooling

In view of the correlation between the SIPL thickening rate and frazil concentration shown in Eq. (3) (also see Figs. 5e and f, 6b and c), we will explore the relationship between \( T_{SC}^0 \) and \( C_i \) here. As expected, the complex response of \( C_i \) to variations in \( T_{SC}^0 \) (Fig. 9) is similar to the relationship between \( T_{SC}^0 \) and thickening rate (Fig. 7) in the VM model.

The magnitude of the difference in \( C_i \) calculated by VM and VU models (VM minus VU) is compared in Fig. 10a, where we find that \( C_i \) calculated by the VM model is always larger than that calculated by the VU model. In general, the difference increases with decreasing \( Z_s \), while the sensitivity grows with increasing \( D_{ini}^{SC} \). The dependence on \( Z_s \) is once again due to the impact of the combined thermodynamic processes, i.e. the efficient growth in the upper supercooled part of the plume together with the maintenance of supercooling by melting of frazil in the lower part, discussed above. We also see similar behaviour for the difference in the thickening rate (Fig. 10b).

Figure 7 (Fig. 9) suggests a possible relationship between SIPL thickening rate (frazil concentration) and supercooling in McMurdo Sound, but observations of suspended frazil ice crystal sizes and turbulence within the ISW would be needed to calculate a representative suspension index. To date, there are limited observations of frazil ice in situ, and the majority of the observations made use of instruments not specifically designed for ice crystal detection (Leonard et al., 2006).

5 Summary and future works

In this study, we demonstrated how the vertical distributions of supercooling and frazil ice concentration within an ISW plume jointly determine the growth of suspended frazil ice, and thus the rate of SIPL formation under sea ice and marine ice beneath ice shelves. A vertically modified, frazil-ice-laden ISW plume model, which encapsulates these combined non-linear effects, was applied to the McMurdo Sound
Figure 7. Relationship between $T_{0}^{\text{SC}}$ and thickening rate classified by (a) outflow supercooled layer thickness $D_{\text{ini}}^{\text{SC}}$ and (b) $Z_{*}$ (colour-coded). Numbers in the legend of panel (a) represent the values of $D_{\text{ini}}^{\text{SC}}$. Solid and hollow dots in panel (b) correspond to the VM and VU model runs, respectively. Coloured lines depict the central trend of the corresponding data points shown in panel (a). The triangle corresponds to the standard run. The results are from the last 30 days of the model runs.

Figure 8. Comparison of $T_{0}^{\text{SC}}$ calculated by the VM and VU models. The triangle corresponds to the standard run. The colour scale of $Z_{*}$ is the same as in Fig. 7b.

region, and reproduced the observed ISW supercooling and SIPL distributions in two horizontal dimensions. Using multiple model runs, the relationship of ISW supercooling to SIPL thickening rate and frazil concentration in McMurdo Sound was explored and shown to be dependent on the suspension index that controls the vertical distribution of frazil concentration within the ISW plume. Moreover, when the thickness of a supercooled layer of ISW is large enough, the efficiency of converting ISW supercooling into frazil concentration, and thus SIPL growth is determined by the suspension index. These findings highlight the need for further observations in McMurdo Sound, particularly focused near the ISW outflow region in the western sound, where the supercooled ISW plume and SIPL are prominent, and more general observations that help to constrain the frazil size spectrum within the sea-ice–ocean boundary layer. In addition, the performance of the VM model in providing reliable estimates of supercooling and frazil ice flux at the SIPL base.
makes it an attractive tool for coupling with sea-ice models, focusing on microscale processes within the bottom layer of the ice (Buffo et al., 2018).

It would be straightforward for the next step to investigate the relationship between supercooling and marine-ice thickening rate underneath ice shelves using the VM model. Quantifying this relationship would be the key to parameterizing the process in more complex three-dimensional, primitive equation ocean models, which frequently neglect details of the ice-shelf–ocean boundary layer and processes associated with an evolving suspension of frazil ice crystals (Liu et al., 2017, 2018; Mueller et al., 2012, 2018). Results may differ from those discussed above, because of the subtly different environments beneath sea ice and ice shelves. Beneath a SIPL, supercooling is produced by the pressure drop experienced by ISW as it emerges from beneath an ice shelf and rises towards the sea surface, while supercooling that drives marine-ice accretion beneath ice shelves is produced as the ISW ascends a very gentle basal slope. The in situ supercooling level beneath ice shelves is therefore likely to be much smaller than that observed in McMurdo Sound, while the differing slopes also yield differing buoyancy forcings on the flow. Furthermore, after experiencing the step change in pressure as it ascends the ice front, the supercooled plume in McMurdo Sound is in the process of adjustment, through the formation of suspended frazil and direct freezing onto the accreted SIPL, towards an equilibrium that is presumably attained beyond the region of observations. At the base of an ice shelf, typically several hundred metres thick, the vertical temperature gradient is comparatively small, so the deposited crystals form a slushy layer (Engelhardt and Dettermann, 1987) that slowly consolidates, possibly as much through compaction as freezing. The ice–ocean interface and the associated drag coefficient are therefore likely to be very different to those observed in McMurdo Sound, where SIPL appears to comprise a more open matrix of ice and water that consolidates by freezing as heat is lost to the atmosphere. In addition, the vastly different timescales over which crystal accretion occurs (about 1–3 years in McMurdo Sound vs. tens of hundreds of years beneath ice shelves) could lead to
Figure 10. Relationship between $Z_\ast$ and differences in (a) $C_i$ and (b) thickening rate calculated by VM and VU models (VM minus VU), classified by outflow supercooled layer thickness $D_{\text{ini}}^{\text{SC}}$. Numbers in the legend represent the values of $D_{\text{ini}}^{\text{SC}}$.

further differences in the internal structure of the crystal layers and hence in the physical boundaries they present to the ISW plume. Therefore, the VM model would need to be re-evaluated against observations of sub-ice-shelf ISW plumes and the ice-shelf–ocean boundary layer. Finally, further process studies, including the influence of the vertical current structure within either the ice-shelf or sea-ice–ocean boundary layer (Jenkins, 2016; Robinson et al., 2017) could also contribute to improving our understanding of marine-ice and SIPL formation.


Author contributions. CC led the study. The simulations were designed by ZW and CC, implemented by CL and RX, and analysed by CC, AJ, and PRH. The paper was written by CC, AJ, and PRH.

Competing interests. The authors declare that they have no conflict of interest.

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