Quadratic Function based Price Adjustment Strategy on Monitoring Process of Power Consumption Load in Smart Grid

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Abstract: Based on the data collected from smart meters, electricity pricing models can be developed to balance power supply and demand in each time slot and obtain the optimal consumption loads and prices. However, in real life, users' reserved consumption requirement loads sometimes deviate significantly from the optimal consumption loads obtained from models, which results in overloaded power systems or even power cuts. To address this issue, an Engineering Process Control monitoring strategy has been proposed in this paper to minimize the difference between the optimal and the users' reserved consumption requirement loads. We proposed an exponential weighted moving average model to predict the load difference in future time slots, and also developed a novel quadratic function based demand response mechanism to adjust the power price for power providers. The demand response mechanism can be used to adjust the price in the future time slots when the predicted demand exceeds the upper or lower boundary. Simulation results indicate that the quadratic function adjustment strategy has excellent performance in a practical power market in Singapore. Compared with the linear function based adjustment method, the proposed quadratic function based adjustment method decreases the adjustment times and standard errors of residuals, and increases the social welfare and power suppliers' profits under the same boundary conditions. In addition, the performance of the proposed strategy demonstrated its competency in peak-cutting and valley-filling and balancing energy provision with demands.

Keywords: Smart grid; power load monitoring; engineering process control; exponential weighted moving average

1. Introduction

The development of urbanization has improved life quality dramatically, although some issues arose as a result, including environmental pollution. Currently, around 50% of the global population live in modern cities. Although cities only account for 2% land surface, their inhabitants are responsible for 75% of total energy consumption and 80% of CO_2 emission"[1]. This has caught more and more attention in the research community, where urban transformation and energy sustainability have been brought to the forefront. When it comes to energy consumption, a growing number of people recommend using renewable and clean energy, such as electricity, to replace traditional coal fossil energy. There is no doubt that electricity is convenient. However, it is important to note the limitations of the rigid architecture of the traditional power grid , for instance the lack of flexibility to connect with new energy, the delay of information transmission caused by the drawbacks of communication network, and so on.

With the development of artificial intelligence (AI), IBM in the USA proposed a smart grid (SG) in 2006 which is known as "the future energy grid" [2]. Comparing to the previous generation of grids, its safety, reliability, clean, high efficiency and sustainability have been significantly improved. America, the European Union and China have chosen pilot cities where SGs[3] are trailed.

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Different from the traditional grid, the SG can induce more suitable uses of the electrical facilities, ensure a stable and reliable energy load and a supply-demand balance. As a part of an SG system, smart meters with implemented novel sensor technology make it possible to exchange data between users and energy providers in real time.

Owing to the advancing communication technology, the development of smart meters is rapid. Many countries are installing smart meters on distribution networks in large numbers, such as in the U.S., the European Union, Netherlands and Asia-Pacific region [4]. Apart from having the measurement function of a traditional meter, the smart meter also comprises of a data transmission function - it is able to monitor, analyze, predict and manage the consumption load. This leads to the development trend of real-time pricing (RTP) in real life[5]. Yu et al. presented a Stackelberg game and proposed a RTP demand-response (DR) algorithm to induce less consumption during high electricity price[6]. In contrast to conventional pricing structures, such as fixed pricing, critical peak load pricing and time of use pricing, an appropriate RTP can balance energy provision and maximize the suppliers' and the demanders' satisfaction level with its flexibility and intelligence[7].

The main target of the worldwide scholars on RTP research is to maximize social welfare. In order to achieve this, Samadi et al. proposed a real-time pricing model and Lagrange dual algorithm to make the RTP strategy satisfying for the energy providers and users [8]. Wang et al. considered a social welfare maximization model based on demand-response, described a complementarity problem based on the Karush-Kuhn-Tuker condition, and determined the basic electricity price with Lagrange multiplier in the dual method [9]. Samadi et al. developed a Vickrey-Clarke-Groves mechanism for Demand Side Management which aims to maximize the social welfare in the future smart grid [10]. Samadi et al. researched a pricing algorithm to minimize the peak-to-average ratio in aggregate load demand with an iterative stochastic approximation approach [11]. Zhu et al. proposed an expectation social welfare maximization model, considering the classification of the smart home appliances and the correlation of power consumption of multi-time slots[12].

Lately, RTP models have been prosperously developed and widely applied in model establishment and algorithm improvement. Under a hierarchical market framework between the power supplier and multi-microgrids, Yuan et al. came up with a real-time pricing model, and they solved the model with a hybrid algorithm combining the particle swarm optimization (PSO) and the branch and bound algorithm (BBA) [13]. Chiu et al. put forward an energy sale and redemption pricing framework that exploits a time-dependent pricing strategy [14]. Tao et al. took the effect of the random fluctuation of electricity consumption into consideration when establishing a distributed genetic RTP scheme for smart grid with multiple utility companies and users based on expectation bilevel programming. Then they solved the problem with a distributed genetic algorithm[15].

Although RTP offers the optimal pricing policy and the corresponding theoretical consumption load with stable and reliable features, it ignores the practical application. The RTP model aims to balance the consumption load by adjusting the real-time price. However, the fact is that most users will be apathetic about the pricing changes if they have to continuously adjust the consumption load to achieve the changed price[3]. As a result, the actual consumption load becomes out of control again, which may cause the SG to lose its stability and reliability or even result in blackouts.

In order to maintain the safety, stability and reliability of the SG, we should work out a strategy to monitor the users' electricity consumption load before the abnormal consumption load occurs based on the users' demand response. It can make users' actual consumption load as close as possible to the theoretical one achieved by inducing changes in the users' electricity-usage behavior. The

users can have a reasonable power usage plan with the help of smart appliances' reserving function. What's more, for this plan, they can also use the intelligent terminal equipment such as escalators, which link the smart appliances with the intelligent terminal equipment through the network. In fact, there have been quite a few experts who are doing research on how to monitor and control the consumption load in SG, but they rarely consider reducing the number of price adjustments.

He et al. proposed a bounded linear adjustment strategy to monitor users' reserved consumption requirements with an EWMA model to forecast the load differences in the next time slots. They took the number of adjustments into consideration, but the number of adjustments is not small enough, and the social welfare and profit are not discussed [16]. Leite et al. used a multivariate control chart to monitor the measured variance to detect nontechnical losses and applied a path finding procedure to locate the consumption point with the non-technical loss[17]. Mortaji et al. proposed a load shedding autoregressive integrated moving average time-series prediction model to reduce customers' power outage[18]. Wan et al. constructed an integration process of abnormal electricity based on statistical process control and fuzzy diagnosis technologies to find the reason of the abnormal of electricity[19]. Xie et al. designed a wide-area monitoring and early-warning subsynchronous oscillation system to keep the system stability and equipment safety[20]. Liu et al. solved the simultaneous action problems under real time non-intrusive load monitoring framework applying a robust real time monitoring approach[21].

To summarise, the Engineering Process Control (EPC) strategy can avoid the frequent price adjustment and obtain a stable consumption load in SG by monitoring the difference between the theoretical consumption load and the reserved one in future time slots. Box used the EPC strategy to monitor the product measurement. Adjustment will take place only when the monitored process exceeds the given upper or lower boundaries [22]. In another words, the adjustment is a "trend" rather than a "point". This ensure the number of the adjustment was decreased and the product measurement value sit within a certain range. Li et al. took advantage of the EPC strategy to adjust the number of staff when monitoring the service level in call center. Their purpose was to acquire a better service level [23]. Govind et al. proposed a multi-variable process adjustment state-space model when the adjustment cost is fixed [24]. To detect a range of shifts in the location parameter, Liu et al. provided a sequential rank-based adaptive nonparametric cumulative sum control chart, the chart efficiently detected various magnitudes of shifts[25]. Wang et al. designed a control chart to improve the efficiency of detecting dependence shifts in mixed type data in industrial engineering[26]. Wohlers et al. explored the KPI concepts and monitored them in the mechatronic system to evaluate them for a manufacturing process[27]. While EPC strategy is a widely studied topic in products, manufacture and service fields, few research could be found in the SG.

In this paper, we propose an EPC monitoring and quadratic function adjustment strategy to balance the consumption load by adjusting the real-time price less frequently rather than the linear function adjustment strategy proposed in Ref. [16] in the SG system. In the SG system, the users can reserve power in advance for one and more days through smart meters after getting the real-time price from the energy providers. Then the energy providers monitor the reserved consumption load of users and calculate the difference between the theoretical consumption load and the reserved one. This paper predicts the difference between the theoretical consumption load and the reserved one resulting from the smart meters with the exponential weighted moving average (EWMA) in the next time slots. Energy providers monitor the power consumption process by observing the change of the EWMA values. When the EWMA value exceeds the upper or lower boundaries in any time

slots, the energy providers will adjust the electricity price using the quadratic function adjustment strategy to induce the users to consume electricity reasonably. This can lead to less frequent adjustments and a more stable power consumption load than those by applying the linear one as stated in Ref. [16]. The EWMA has been studied by the scholars for many years. Zaman et al. proposed an EWMA scheme for adapting CUSUM accumulation error from monitoring the process location [28]. Many experts put forward an EWMA control chart catering to different statistical characteristics. Jiang et al. researched abnormal distributed EWMA control chart to monitor and detect the abnormal quality[29]. Malela et al. developed an EWMA control chart based on the Wilcoxon rank-sumstatistic using repetitive sampling to improve the sensitivity of the EWMA control chart to process mean shifts [30]. To monitor both the small and the large shifts simultaneously, Zaman et al. designed an adaptive EWMA for dispersion parameter in connection with Huber and Tukey's bisquare functions[31].

The main contributions of this paper are as follows.

1) A novel quadratic function bounded adjustment strategy has been proposed in this paper to monitor the difference between the users' optimal and reserved consumption requirement loads in power system.
2) The price is adjusted by the power providers in the next time slots only when the predicted load difference value exceeds the upper or lower boundary by using a demand response mechanism for the power price. The price demand response mechanism can induce the users to adjust their consumption requirement.
3) Our proposed quadratic function adjustment method outperforms the linear function adjustment method in terms of price adjustment times, standard error, social welfare and suppliers' profit. The less frequently the price changes, the more feasible the application is.
4) The proposed strategy has good performance in cutting peak and filling valley, balancing energy provision and preventing of power system blackouts.

The rest of this paper is organized as follows. Section 2 introduces the real-time pricing model. In Section 3, we offer the detailed EPC strategy monitoring change of the differences between the theoretical consumption load and the reserved one within different time slots. Section 4 includes the model simulation and result analysis. In Section 5, we not only discuss the effects of the different boundaries, targets and adjustment methods for the monitoring and adjustment process, but also compare the performance between the proposed strategy and the liner one. Besides, the practical case study is discussed. Section 6 is the conclusion and observation.

2. Real-time pricing model

2.1 Model assumption

This paper regards some class of users as the object of research. We consider an SG system consisting of an electricity company, a few users installing smart meters and a regulatory authority. The electricity company collects the minimum and maximum consumption requirement load from every user in future time slots through smart meters. The electricity company and the users can exchange real-time price information in the current and next time slots. After receiving the price information shown by the smart meters, the users can use electricity more reasonably. In the system, there are N users (N defines the number of the users), where $N=|\mathrm{N}|$, $\mathrm{N}=\{1,2,\cdots,N\}$, $\square$ is the set of the users. The time period operating the users' electricity consumption load is divided into T time slots (T defines the number of time slots), where $T\square|\mathrm{T}|$, $\mathrm{T}=\{1,2,\cdots,T\}$, T is the set of all time slots. For each user $i\in\square$, let x_i^t denote the amount of power consumed by the user i in time slot $t\in\mathrm{T}$. Based on the data provided by the smart meters, the electricity company obtains the minimum and

maximum power requirements of every user $i \in \square$ in every time slot $t \in \mathrm{T}$ respectively, namely m_i^t and M_i^t. The consumed power x_i^t has to satisfy $m_i^t \le x_i^t \le M_i^t$.

2.2 Utility function of some class of users

From the microeconomics point of view, we adopt Von Neumann-Morgenstern's utility function for decreasing risk aversion $U(x,\omega)$ to show the satisfactory level of the users' power consumption. x is the consumption load, and the parameter $\omega \in (1,4)$ shows the users' power consumption intention, which varies in different time slots and for different users. Assume that there is no satisfaction when the users do not consume power, which means there is no utility. The utility function has three main properties: I. Utility function is non-decreasing, which means that the users will not stop increasing the consumption load until it reaches the maximum. II. The marginal benefit of the users is a non-increasing function. That shows that the utility will not increase any more if the maximum consumption level, i.e. the saturation phenomenon appears. This indicates that the utility function is a convex function. III. The monotonic increases in the parameter ω. In other words, the utility increases with the increase of parameter ω [8].

We choose logarithmic functions for some class of users as follows[32]:

$$U\left(x_i^t,\omega\right) = \begin{cases} \omega \ln(x_i^t+1), & \text{if} \quad x_i^t \ge 0 \\ 0, & \text{if} \quad x_i^t < 0. \end{cases} \tag{1}$$

The first partial derivative of (1) to consumption load x is shown as follows:

$$\frac{\partial U}{\partial x} = \frac{\omega}{x+1} > 0 . \tag{2}$$

It is in accordance with Property I.

The marginal benefit of the users is (2), and we have:

$$\frac{\partial^2 U}{\partial x^2} = -\frac{\omega}{(x+1)^2} < 0 . \tag{3}$$

It is a non-increasing function and meets Property II.

The first partial derivative of (1) to the parameter ω is demonstrated as follows:

$$\frac{\partial U}{\partial \omega} = \ln\left(x+1\right) \ge 0 . \tag{4}$$

When the consumption load $x \ge 0$, it satisfies Property III.

In order to research the users' consumption load, apart from establishing the utility function, we also consider the consumption cost. Suppose the electricity providers offer the directing electricity price through the smart meters as p dollars/kWh, the consumption cost is px dollars. The users' welfare function is as follows[8]:

$$W(x_i^t, \omega_i^t) = U(x_i^t, \omega_i^t) - p_t x_i^t \tag{5}$$

where $W(x_i^t, \omega_i^t)$ is the user i's welfare function in time slot t. Assume that all users aim to get the maximum welfare, i.e., they try to achieve the maximum utility and the minimum cost.

2.3 Cost function of the electricity company

We denote L_t as the generation capacity of the electricity company in t time slot and assume the maximum and the minimum generation capacity as respectively $L_t^{\max}$ and $L_t^{\min}$. The maximum generation capacity from electricity company is the sum of the users' maximum consumption loads, and the minimum generation capacity from electricity company is the sum of the users' minimum consumption loads in t time slot, i.e.,

$$L_t^{\max} @ \sum_{i=1}^{N} M_i^t , \tag{6}$$

$$L_t^{\min} @ \sum_{i=1}^{N} m_i^t . \tag{7}$$

If the users reserve the consumption load before a week and the electricity company supplies them in accordance with the reservation, the blackout will not happen resulting from insufficient power supply in SG.

We define the energy supply cost $C(L_t)$ of the electricity company in time slot t as follows[8]:

$$C\left(L_t\right) = aL_t^{\ 2} + bL_t + c\ , \tag{8}$$

where $a > 0, b, c \geq 0$ are pre-determined cost parameters. The revenue function of the electricity company is $p_t L_t$. Then the profit function of the electricity company in time slot t is

$$P(L_t) = p_t L_t - C(L_t)\ . \tag{9}$$

2.4 The real-time pricing model

In the SG system, we formulate the optimization problem. We pursue the maximum utility of the users and the minimum cost of electricity providers. Meanwhile, the total consumed power cannot exceed the available capacity to avoid the blackout caused by the insufficient electricity supply . Thus the RTP model is as follows[8]:

$$\max_{x_i^t, L_t} \sum_{i=1}^{N} U\left(x_i^t, \omega_i^t\right) - C\left(L_t\right), \tag{10}$$

$$s.t. \quad \sum_{i=1}^{N} x_i^t \leq L_t, i \in \square, t \in \mathrm{T},, \tag{11}$$

$$m_i^t \ £\ x_i^t \ £\ M_i^t, \tag{12}$$

$$L_t^{\min} \ £\ L_t \ £\ L_t^{\max} \tag{13}$$

where x_i^t refers to user i's consumption load in time slot t and L_t refers to the generation capacity in time slot t.

Because the objective function (10) of the model is a concave function, and its constraint condition (11) is a linear function, the problem (10)-(13) is a convex optimization problem. Thus, some convex programming methods, such as interior point algorithm, can be employed to obtain the consumption load and generation capacity. But these methods cannot obtain the real-time price, which is an important basis for monitoring and adjusting the consumption load in this paper. Therefore, this paper transforms the problem into a Lagrange dual problem with the Lagrange dual method in each time slot $t \in \mathrm{T}$. Then we can obtain the optimal Lagrange multiplier and theoretical power consumption load of the users. The optimal Lagrange multiplier is exactly the electricity price in that time slot. The detailed derivation process is available in Appendix A.

3. EPC adjustment strategy

Solving the problem (A4) (in Appendix A), we can get the optimal price p_t^* in t time slot and theoretical optimal consumption load x_t^*. The energy provider can get a stable and reliable consumption load according to x_t^*. The users consume reasonably electricity with the optimal price p_t^*. But it only happens under an ideal condition. In most cases, the reserved consumption load, on which the feedback to energy provider through the smart meters, is far different from the optimal consumption load x_t^*. Even in extreme cases, the reserved consumption to the users offered by the energy provider may cause the overload or blackout at peak time. To solve the overload, the energy provider has to increase the generation capacity which will make the energy costfar bigger than the profits. Therefore,the energy provider always try to keep away from this scenario. The best way to avoid this is to induce the users to consume power properly.

The EPC strategy can help the energy provider obtain stable consumption load in real life. Considering the demand response mechanism for the power price to the users, we calculate the

difference between the theoretical consumption loads with the RTP model and the reserved one. After that, we adjust the difference when it exceeds the upper or lower boundaries with the EPC adjustment strategy. The energy provider makes users adjust their actual consumption loads by changing the price. As a result, the reserved consumption load is close to the theoretical consumption load. What's more, we obtain the least adjustments with the EPC strategy. Because the fluctuation range of consumption load difference is limited, this paper predicts the next consumption load difference with the EWMA estimation model.

3.1 EWMA estimation model

The users reserved the consumption load of the next time slot (one day or even one week) via smart meters, and the reservation has an important reference value for exactly adjusting the consumption requirement load and making a reasonable price.

For exactly getting the degree of difference between the reserved consumption load x_t of users in time slot t and the theoretical one x_t^*, we define the difference as y_t [16]:

$$y_t = x_t - x_t^* \tag{14}$$

We predict the difference value y_{t+1} in the next time slot $t+1$ with the EWMA value of past adjusted difference values. The detailed operational process is as follows.

The time series of original difference value is set as $y_l, l = t, t-1, \cdots, 1$, and the time series of the adjusted one is $y_l', l = t, t-1, \cdots, 1$. Then the EWMA estimate value $\bar{y}_{t+1}$ for the difference value y_{t+1} in the next time slot $t+1$ is[16]:

$$\bar{y}_{t+1} = \mu(y_t' + \theta y_{t-1}' + \theta^2 y_{t-2}' + \cdots), \quad 0 \le \theta \le 1 \tag{15}$$

where the parameter $\mu = 1-\theta$ is the instability parameter, and θ is the smoothing constant. Simplifying (15), we get

$$\bar{y}_{t+1} = \mu y_t' + \theta \bar{y}_t . \tag{16}$$

3.2 EPC adjustment strategy

Now we consider how to make the EPC strategy to let the deviation from a target load difference S become minimal. We will not adjust the price until EWMA value $\bar{y}_{t+1}$ exceeds the boundary, i.e.,

$$\bar{y}_{t+1} \ge B_1 \text{ or } \bar{y}_{t+1} \le B_2, B_1 \ge 0, B_2 \le 0 , \tag{17}$$

where B_1 and B_2 are the preset regulated upper and lower boundaries. During the monitoring, if $\bar{y}_{t+1}$ satisfies (17), it means that the EWMA value $\bar{y}_{t+1}$ exceeds the boundary and needs to adjust to a target S . The simulation results show that it is worth discussing to achieve a stable adjustment system by finding a way to set the target parameter $E_1 \ge 0$ and $E_2 \le 0$ properly. Section 5 will discuss this issue in detail.

If $\bar{y}_{t+1}$ satisfies (17), it expresses that the reserved consumption load of the users has exceeded the stable boundary. In order to avoid the users' unreasonable consumption, the energy provider takes actions based on a demand response mechanism for the power price. It induces the users to change their electricity consumption to achieve a stable and reliable consumption load. When monitoring the reserved consumption load of the users, we get a time series $\{\bar{y}_t\}_{t=1}^{T}$ of EWMA estimation. On the one hand, if the estimated value $\bar{y}_{t+1}$ exceeds the upper boundary B_1, it indicates that the reserved consumption load is too high. At this moment, the power provider needs to raise

the electricity price to guide the users to reduce the reserved consumption load reasonably. On the other hand, if $\bar{y}_{t+1}$ is under the lower boundary B_2, it shows that the reserved consumption load is too low and there is enough electricity left to use. The provider then encourages the users to increase the reserved consumption load by decreasing the electricity price. The users can even be encouraged to store the power in a user-owned battery, which can save the consumption load when the electricity price rises. By applying this adjustment method, the users are induced to consume reasonably. Consequently, it ensures a stable and reliable generation capacity from the power providers.

The above adjustment strategy needs to quantitatively analyze the relationship between the price changes and the users' consumption load. To illustrate the adjustment process of this strategy, the following theorem provides a price adjustment strategy formulated by the demand response mechanism for the power price. The behavior of the demand response mechanism for the power price can also be accurately modeled by certain demand functions. In this paper, we consider quadratic demand functions.

Theorem 3.1 Assume that the users are sensitive to the electricity price changes and that the demand function is a quadratic function. That is to say, EWMA estimating difference value $\bar{y}_{t+1}$ is quadratic to the corresponding EWMA price with the form $\bar{y}_{t+1} = -k\bar{p}_{t+1}^2$, $k > 0$ is a constant. Then, if $\bar{y}_{t+1} \geq B_1 > 0$, $\bar{y}_{t+1}$ is adjusted to $E_1 \in [0, B_1)$, and the price adjustment is shown in (18),

$$g_{t+1} = \frac{1}{\mu}\left(\sqrt{\bar{p}_{t+1}^2 + \frac{\bar{y}_{t+1} - E_1}{k}} - \bar{p}_{t+1}\right), \tag{18}$$

If $\bar{y}_{t+1} \leq B_2 < 0$, $\bar{y}_{t+1}$ is adjusted to $E_2 \in (B_2, 0]$, and the price adjustment is shown in (19),

$$g_{t+1} = \frac{1}{\mu}\left(\sqrt{\bar{p}_{t+1}^2 + \frac{\bar{y}_{t+1} - E_2}{k}} - \bar{p}_{t+1}\right). \tag{19}$$

Proof. According to the assumption, in time slot $t+1$, if $\bar{y}_{t+1} \geq B_1$, $\bar{y}_{t+1}$ need to be adjusted to $E_1 \in [0, B_1)$. At this point, the EWMA price shifts from $\bar{p}_{t+1}$ to $\bar{p}'_{t+1}$, where $\bar{p}_{t+1} = \mu p'_t + (1-\mu)\bar{p}_t$ is the EWMA predict value for the reserved electricity price p_{t+1} in $t+1$ time slot, p'_t is the electricity price adjustment value of the current price p_t in time slot $t+1$, and $\bar{p}'_{t+1}$ is the adjusted value of $\bar{p}_{t+1}$ in time slot $t+1$[16]. Hence,

$$\bar{p}'_{t+1} = \mu p''_t + (1-\mu)\bar{p}_t. \tag{20}$$

where the price p''_t means that if the EWMA value of the price $\bar{p}'_{t+1}$ is taken and the price must be adjusted to p''_t in time slot t. Under the assumed condition $E_1 = -k\bar{p}'^2_{t+1}$, we have that

$$E_1 - \bar{y}_{t+1} = -k\left(\bar{p}'^2_{t+1} - \bar{p}^2_{t+1}\right), \tag{21}$$

Substitute $\bar{p}_{t+1}$ and $\bar{p}'_{t+1}$ into (21), we have that

$$\begin{aligned}
E_1 - \bar{y}_{t+1} &= -k\left[(\mu p''_t + (1-\mu)\bar{p}_t)^2 - (\mu p'_t + (1-\mu)\bar{p}_t)^2\right] \\
&= -k\left[\mu^2 p''^2_t + 2\mu p''_t(1-\mu)\bar{p}_t + (1-\mu)^2\bar{p}_t^{\,2} - \mu^2 p'^2_t - 2\mu p'_t(1-\mu)\bar{p}_t - (1-\mu)^2\bar{p}_t^{\,2}\right] \\
&= -k[\mu^2 p''^2_t + 2\mu p''_t(1-\mu)\bar{p}_t - \mu^2 p'^2_t - 2\mu p'_t(1-\mu)\bar{p}_t] \\
&= k[\mu^2(p'^2_t - p''^2_t) + 2\mu(1-\mu)\bar{p}_t(p'_t - p''_t)] \\
&= k[\mu^2(p'_t + p''_t)(p'_t - p''_t) + 2\mu(1-\mu)\bar{p}_t(p'_t - p''_t)] \\
&= k\mu(p'_t - p''_t)[\mu(p'_t + p''_t) + 2(1-\mu)\bar{p}_t]
\end{aligned}, \tag{22}$$

Suppose the price adjustment as $g_{t+1} := p''_t - p'_t$, we have that $p''_t + p'_t = p''_t - p'_t + 2p'_t = g_{t+1} + 2p'_t$. Hence,

$$\begin{aligned}\bar{y}_{t+1}-E_1 &= k\mu g_{t+1}[\mu(g_{t+1}+2p_t')+2(1-\mu)\bar{p}_t] \\ &= k\mu g_{t+1}[\mu g_{t+1}+2\mu p_t'+2(1-\mu)\bar{p}_t] \\ &= k\mu g_{t+1}[\mu g_{t+1}+2(\mu p_t'+(1-\mu)\bar{p}_t)], \\ &= k\mu g_{t+1}(\mu g_{t+1}+2\bar{p}_{t+1}) \\ &= k\mu(\mu g_{t+1}^2+2\bar{p}_{t+1}g_{t+1})\end{aligned} \tag{23}$$

$$\mu g_{t+1}^2+2\bar{p}_{t+1}g_{t+1}-\frac{\bar{y}_{t+1}-E_1}{k\mu}=0. \tag{24}$$

Then the effect g_{t+1} of price adjustment is

$$g_{t+1}=\frac{1}{\mu}\left(\sqrt{\bar{p}_{t+1}^2+\frac{\bar{y}_{t+1}-E_1}{k}}-\bar{p}_{t+1}\right) \text{or } g_{t+1}=\frac{1}{\mu}\left(-\sqrt{\bar{p}_{t+1}^2+\frac{\bar{y}_{t+1}-E_1}{k}}-\bar{p}_{t+1}\right), \tag{25}$$

We decrease the EWMA estimate value $\bar{y}_{t+1}$ by increasing the price, and achieve the positive root of the price adjustment

$$g_{t+1}=\frac{1}{\mu}\left(\sqrt{\bar{p}_{t+1}^2+\frac{\bar{y}_{t+1}-E_1}{k}}-\bar{p}_{t+1}\right). \tag{26}$$

Similarly, if $\bar{y}_{t+1}\le B_2$, $\bar{y}_{t+1}$ is adjusted to $E_2\in(B_2,0]$, the formula (19) holds.

The proof is completed.

□

3.3 Consumption load monitoring and price adjustment algorithm

Given the basic principle of EPC, we can assume that in the continuous monitoring consumption load process, once the price is adjusted in a certain time slot, the price in subsequent time slots will also undergo the same adjustment. Therefore, in the actual adjustment process, the cumulative price adjustment v_{t+1} is as follows[16]:

$$v_{t+1}=\sum_{i=1}^{t+1}g_i=v_t+g_{t+1}. \tag{27}$$

Under the demand response mechanism for the power price, according to the assumption of theorem 3.1, the adjustment value of consumption load is quadratic with the cumulative price adjustment value. This means that when the EWMA estimate difference value $\bar{y}_{t+1}$ satisfies Eqs. (17), the real-time price is adjusted to change the users' reserved consumption load x_{t+1}' to achieve the balanced state. On the one hand, when the rising price appears in cumulative price adjustment quantity, i.e. $v_{t+1}\ge 0$, the reserved consumption load of users will decrease. On the other hand, when the decreasing price appears in cumulative price adjustment quantity, i.e. $v_{t+1}\le 0$, the reserved consumption load of users will increase. Then we have

$$x_{t+1}'-x_{t+1}=\begin{cases}-kv_{t+1}^2, & v_{t+1}\ge 0\\ kv_{t+1}^2, & v_{t+1}<0\end{cases}. \tag{28}$$

The adjusted value of the difference value y_{t+1}' between the adjusted reserved consumption load x_{t+1}' and the theoretical load x_{t+1}^* is

$$y_{t+1}'=x_{t+1}'-x_{t+1}^*. \tag{29}$$

Then,

$$
\begin{aligned}
y'_{t+1} - y_{t+1} &= x'_{t+1} - x^*_{t+1} - y_{t+1} \\
&= x'_{t+1} - x^*_{t+1} - (x_{t+1} - x^*_{t+1}) \\
&= x'_{t+1} - x_{t+1} \\
&= \begin{cases} -kv^2_{t+1}, & v_{t+1} \geq 0 \\ kv^2_{t+1}, & v_{t+1} < 0 \end{cases}
\end{aligned} . \tag{30}
$$

Based on the EPC strategy, the algorithm of consumption load monitoring and price adjustment is as follows.

Initialization: By solving the RTP model (A4) (in Appendix A), we can get the optimal price time series $\{p^*_t\}_{t=1}^T$ and theoretical consumption load time series $\{x^*_t\}_{t=1}^T$. According to the feedback data of the smart meters, we can gain the reserved consumption load time series $\{x_t\}_{t=1}^T$ of the users. According to Eqs. (14), we can get the time series $\{y_t\}_{t=1}^T$ of difference between original reserved consumption load x_t and the theoretical one x^*_t. Let the adjustment value of the initial consumption load difference be $y'_1 = y_1$, and the initial adjusted electricity price be $p'_1 = p_1*$. Set the initial EWMA value $\bar{y}_1 = S = 0$, $\bar{p}_1 = 1$, and hence the initial predicted error is $e_1 = y'_1 - \bar{y}_1 = y'_1$. Given the initial price adjustment effect $g_1 = 0$, and hence the cumulative price adjustment effect is $v_1 = 0$. Suppose the parameters $k > 0$, $0 \leq E_1 < B_1$, $B_2 < E_2 \leq 0$, $\mu \in [0,1]$. In time slot $t, t = 1, \cdots, T-1$, we adjust the price with Algorithm 1 to change the consumption load difference.

Algorithm 1 (EPC Adjustment Strategy)

Step 0: Initialization. Set t=1.

Step 1: Set $\bar{y}_{t+1} = \mu y'_t + (1-\mu)\bar{y}_t$, $\bar{p}_{t+1} = \mu p'_t + (1-\mu)\bar{p}_t$.

If Eqs. (17) is not satisfied, $g_{t+1} = 0$, return Step 2. Otherwise, return Step 4.

Step 2: Set $v_{t+1} = v_t + g_{t+1}$, $p'_{t+1} = p_{t+1} + v_{t+1}$. If $v_{t+1} \geq 0$, set $x'_{t+1} = x_{t+1} - kv^2_{t+1}$,

if $v_{t+1} < 0$, set $x'_{t+1} = x_{t+1} + kv^2_{t+1}$, $y'_{t+1} = x'_{t+1} - x^*_{t+1}$, $e_{t+1} = y'_{t+1} - \bar{y}_{t+1}$.

Step 3: Replace t by $t + 1$ and go back to Step 1.

Step 4: If $\bar{y}_{t+1} \geq B_1$, set $g_{t+1} := \frac{1}{\mu}\left(\sqrt{\bar{p}^2_{t+1} + \frac{\bar{y}_{t+1} - E_1}{k}} - \bar{p}_{t+1}\right)$;

If $\bar{y}_{t+1} \leq B_2$, set $g_{t+1} := \frac{1}{\mu}\left(\sqrt{\bar{p}^2_{t+1} + \frac{\bar{y}_{t+1} - E_2}{k}} - \bar{p}_{t+1}\right)$, then return Step 2 .

3.4 Flowchart of the wholesale method

Fig.1 shows the flowchart of the wholesale method. According to Fig.1, we first solve the RTP model. Secondly, we monitor the difference between the reserved consumption load and the theoretical one. Finally, we adjust the price with the quadratic function adjustment strategy if EWMA values exceed the preset upper and lower boundaries.

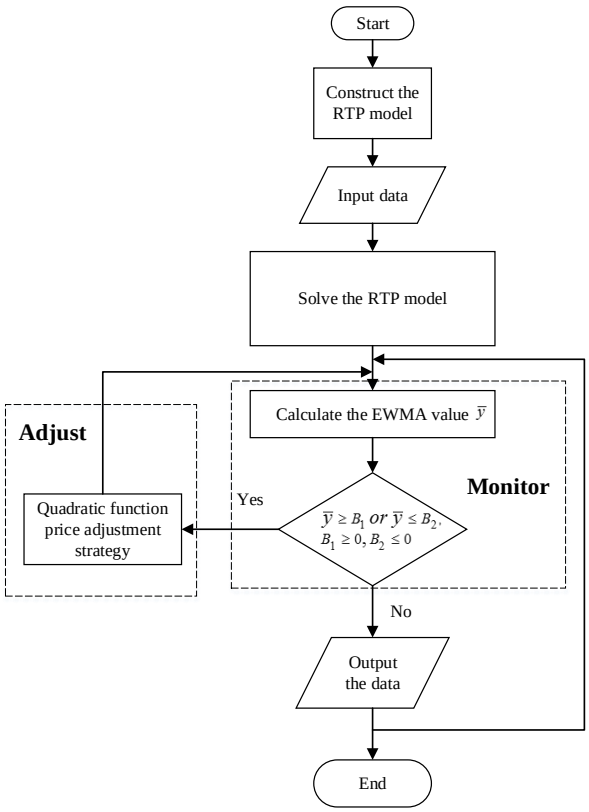

Fig.1. Flowchart of the proposed monitoring and adjustment strategy

4. Result analysis

In this section, we analyze the performance of EPC monitoring and adjustment strategy through the simulation results.

In the RTP model, we suppose that there are 50 users ($N = 50$) and the time cycle (3 days) from Thursday to Saturday is divided into 72 time slots $T = 72$. The power demand parameter of the users $\omega \in (1,4)$ in utility function Eqs. (1) is selected randomly and remains unchanged during the monitoring. We assume the parameters a, b, c in cost function are respectively assumed as $0.01, 0, 0$.

4.1 The optimal consumption load of the RTP Model

Based on the data provided by the smart meters, we obtain the reserved electricity consumption requirement load series $\{x_i^t\}_{t=1}^T$ and the minimum and maximum power requirements $\{m_i^t\}_{t=1}^T$ and $\{M_i^t\}_{t=1}^T$ of every user $i \in \square$. Hence, we gain the total reserved power consumption $x_t = \sum_{i=1}^N x_i^t$ and the aggregated minimum and maximum power requirements $m_t = \sum_{i=1}^N m_i^t$ and $M_t = \sum_{i=1}^N M_i^t$ in each time slot $t \in \mathrm{T}$. With m_t, M_t and the parameters given above, we get the theoretical consumption load of the users x_t^* and optimal price p_t^* by solving the RTP model (A4) (in Appendix A). It is assumed that original reserved price p_t is the optimal price $p_t^*, t \in \mathrm{T}$. The simulation results are

shown in Fig. 2. As illustrated in Fig. 2, the theoretical power consumption load is stable and the reserved power consumption load of the users fluctuates greatly. In order to make the users consume power reasonably, it is necessary to adopt the EPC adjustment strategy to adjust the price by the energy provider. Then this action can cause the adjustment of the actual consumption load of the users.

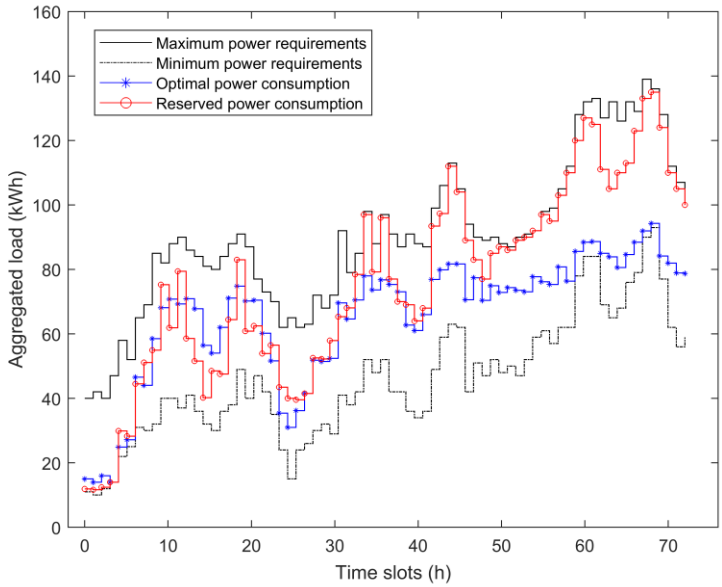

Fig. 2. Comparison of theoretically optimal and actually reserved power consumption.

4.2. Process of EPC monitoring and price adjustment

In order to shift peak load and save energy, we can achieve the stable consumption load of the users by implementing the EPC adjustment strategy. We monitor the EWMA estimate value $\{\bar{y}_t\}_{t=1}^{T}$ for the adjusted difference time series $\{y'_t\}_{t=1}^{T}$, the difference time series $\{y_t\}_{t=1}^{T}$ is generated from the reserved consumption load x_t and the theoretical consumption load x_t^*. The optimal price time series $\{p_t^*\}_{t=1}^{T}$ is the original reserved price. Once a certain $\bar{y}_{t+1}$ is greater than the upper limit B_1 or less than the lower limit B_2, it demonstrates that the time series $\{y'_t\}_{t=1}^{T}$ becomes abnormal again in time slot $t+1$. Thus, according to the Theorem 3.1 and the demand response mechanism for the power price of users, the energy provider adjusts the price to change the time series $\{y'_j\}_{j=t+1}^{T}$ or the time series of the adjusted consumption load $\{x'_j\}_{j=t+1}^{T}$ to a normal range. The $\{y'_j\}_{j=t+1}^{T}$ above is the adjusted time series of the consumption load difference of the users in the subsequent period. The adjusted price sequence $\{p'_j\}_{j=t+1}^{T}$ is the actual price of energy providers.

In Algorithm 1, the parameters are supposed as follows, $k=50$, $\mu=0.2$, $B_1=-B_2=4.2$, $E_1=0$, $E_2=0$. On the one hand, when $\bar{y}_{t+1}$ is greater than the upper limit B_1, the energy providers will adjust the price to make $\bar{y}_{t+1}$ go down to E_1. On the other hand, if $\bar{y}_{t+1}$ is less than the lower limit B_2, the energy providers will change the price to make $\bar{y}_{t+1}$ increase to E_2. The simulation results of the monitoring and adjustment are shown in Fig. 3 and Fig. 4. In Fig. 5, the social welfare and the profits of the electricity company among the EPC adjusted pricing, the original reserved pricing and the fixed-pricing strategies are compared, and the parameter $\omega \in (1,4)$ in utility function Eqs.

(1) is selected randomly.

We can draw a conclusion from Fig.3 that after seven minor changes, the difference between consumption loads is stable and this is distinct from the initial unstable scenario. This is the expected adjustment effect. The empirical average adjustment interval (AAI) is AAI=71/8=8.9, the standard

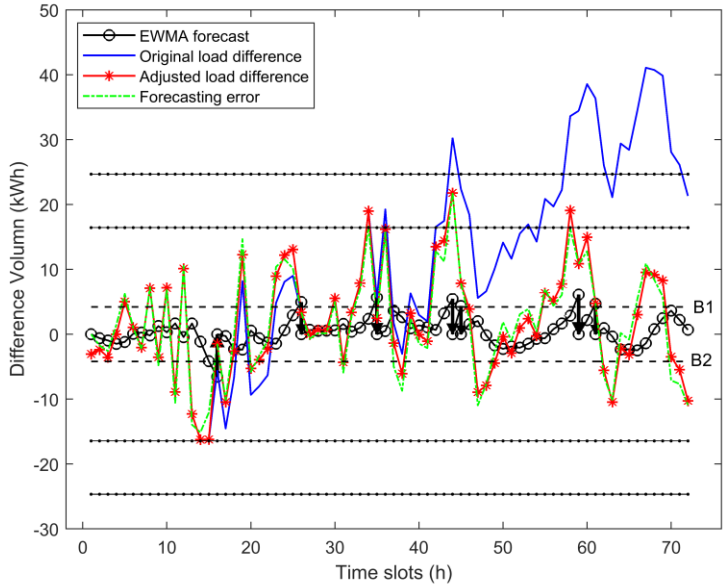

Fig. 3. Process of EPC monitoring and adjustment.

error σ (SD(σ)) of residuals is calculated by[16]

$$\sigma = \sqrt{\sum_{t=1}^{T} e_t^2 / (T-1)} \tag{31}$$

$$= \sqrt{\sum_{t=1}^{72} e_t^2 / 71} = 8.22 .$$

There are no points above the boundary 3 σ , which means adjustments will not cause abnormal phenomena.

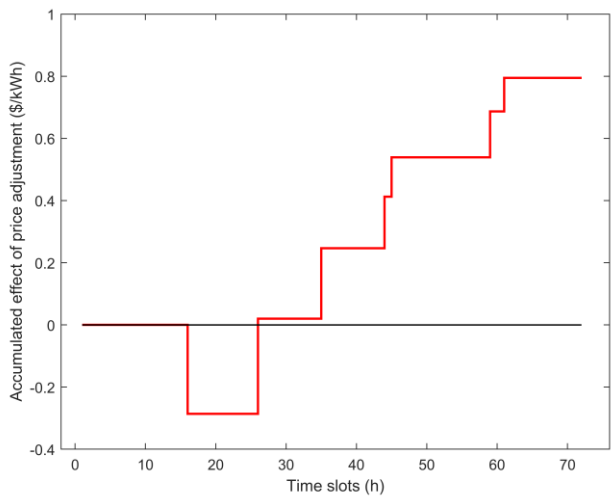

Fig. 4. Accumulated effect of price adjustment.

Fig.4 shows the accumulated effect of the price adjustment. It can be seen from Fig. 4 that the price in 16-25 (4 p.m. of Thursday to 1 a.m. of Friday) is decreasing. Meanwhile, the price has risen with the increase of reserved consumption load on Friday and Saturday, where the maximum price adjustment is $0.79/kWh according to Eqs. (27). This induces the users to buy more power after work from Thursday to Friday morning, and to store the purchased power in the battery for future when the prices rise in the coming two days.

4.3. Social welfare and the profit of the electricity company

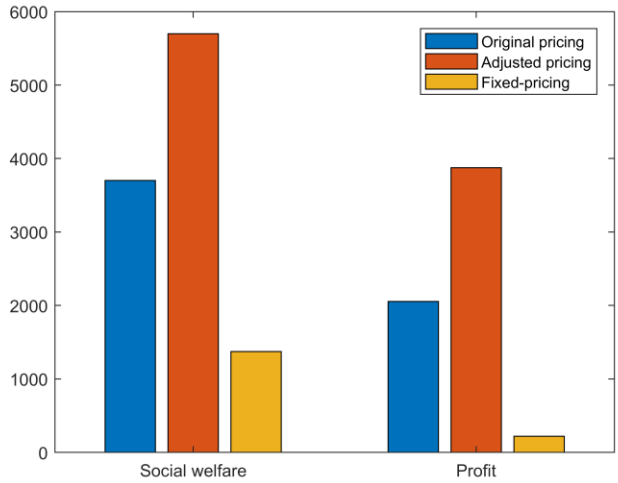

Fig. 5. Social welfare and profit in different pricing strategies.

From Fig. 5, we can see that by applying the EPC price adjustment strategy, the social welfare and the profit of the electricity company achieved the highest figures comparing to those obtained under the other two scenarios. Therefore, it can be concluded that the EPC price adjustment strategy not only helps with peak shaving and valley filling, but also contributes to improving the whole social welfare and profit of the electricity company.

5. Discussion

5.1 Performance analysis of different adjustment targets

In order to keep the stable consumption load difference during monitoring and adjusting the

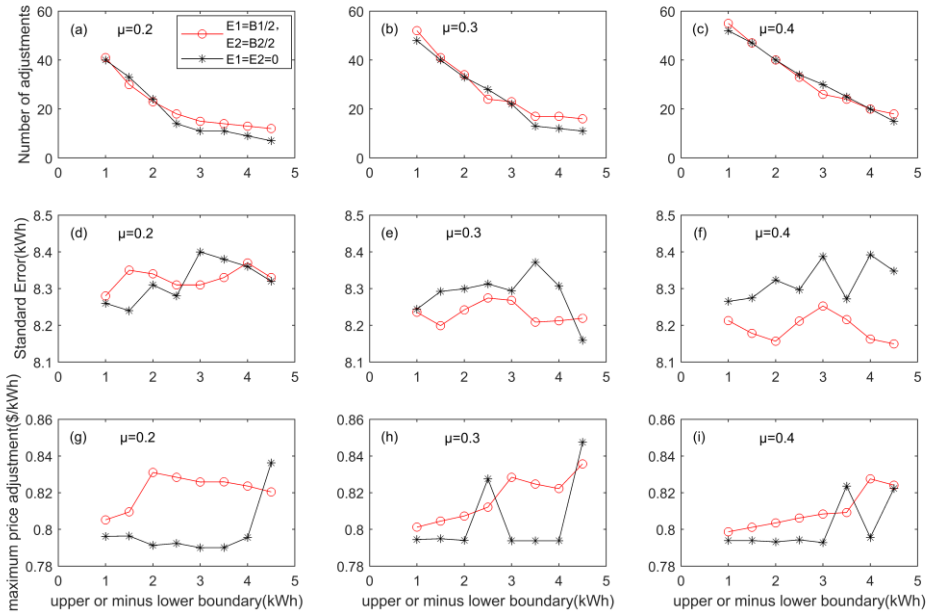

Fig. 6. Various indicators comparision with varing parameters.

system, we should consider the expected values of E_1 and E_2.

When E_1 and E_2 have different parameter values, three indicators are adopted in our simulations: number of adjustments, the standard error σ (SD (σ)) of residuals and the maximum price adjustment. Since there is no theoretical optimal formula for the parameters E_1 and E_2 up to now, we have to adjust them by experiments from Algorithm 1 instead of the optimal E_1 and E_2. If the reserved users' consumption load is consistent with the theoretical load, it would be the optimal one. At this moment, the value (EWMA value after adjustment) should be theoretically set to 0 when we apply the EPC adjustment strategy. Thus, we will discuss the situation of adjusted target values E_1 and E_2 as 0. Meanwhile, we consider to adjust E_1 and E_2 to $B_1/2$ and $B_2/2$ with different parameters $\mu \in \{0.2, 0.3, 0.4\}$ and various boundaries $B_1 = -B_2 \in \{1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5\}$ respectively. The simulation results from the implement of Algorithm 1 over 72 time slots are illustrated in Fig. 6.

We can learn that in Fig. 6, (a,b,c) depict the number of adjustments, (d,e,f) illustrate the standard error of residuals and (g,h,i) show the maximum price adjustments. It is shown that when adjustment boundaries are set appropriately, not only a small number of adjustments can be obtained, but the standard errors of residuals are also controlled within a certain range $[8.15, 8.45]$. It is not hard to see that as the EPC adjustment boundaries ($B_1 = -B_2$) increase, the numbers of adjustments gradually decrease and tend to be stable after the boundary $B_1 = -B_2 = 4$. Furthermore, the maximum adjustments of price are mostly under the adjustment target $E_1 = B_1/2, E_2 = B_2/2 (B_1 = -B_2)$ when the adjustment targets $E_1 = E_2 = 0$ appear. For example, when the adjustment targets are set as $E_1 = E_2 = 0$, the model obtained a smaller number of adjustments (9 adjustments) and the maximum adjustment of price ($0.80/kWh) from Eqs. (27)than those of the

adjustment target $E_1 = B_1 / 2, E_2 = B_2 / 2 (B_1 = -B_2)$, which had 13 adjustments and maximum adjustment of price (\$0.82/kWh) from Eqs. (27) with μ=0.2 , B_1=4 , $B_2 = -4$. According to the observations, we conclude that we can choose proper μ, E_1, E_2 by weighing the three indicators.

5.2 Comparison with the application of the linear function adjustment strategy

To further illustrate the rationality of the proposed quadratic function adjustment strategy in Algorithm 1, we compare the operating results of the proposed strategy with that of the linear function adjustment strategy provided in [16]. Simulation assumptions are the same in Section 4 and the parameters are set as $k = 50$, $\mu = 0.2$, $B_1 = -B_2 = 4.5$, $E_1 = 0$, $E_2 = 0$.

Table 1
Comparison results with two scenarios

	Quadratic function adjustment	Linear function adjustment
Number of adjustments	7	15
Standard error	8.32	11.70
Social welfare	5626.9	3860.4
Profit	3877.3	2776.9

The simulation results over 72 time slots are shown in Table 1, Fig.7, Fig.8 and Fig.9. As indicated by Table 1 and Fig. 7 , the number of adjustments and the standard error in the scenario where the linear demand function is applied are greater than those in the scenario

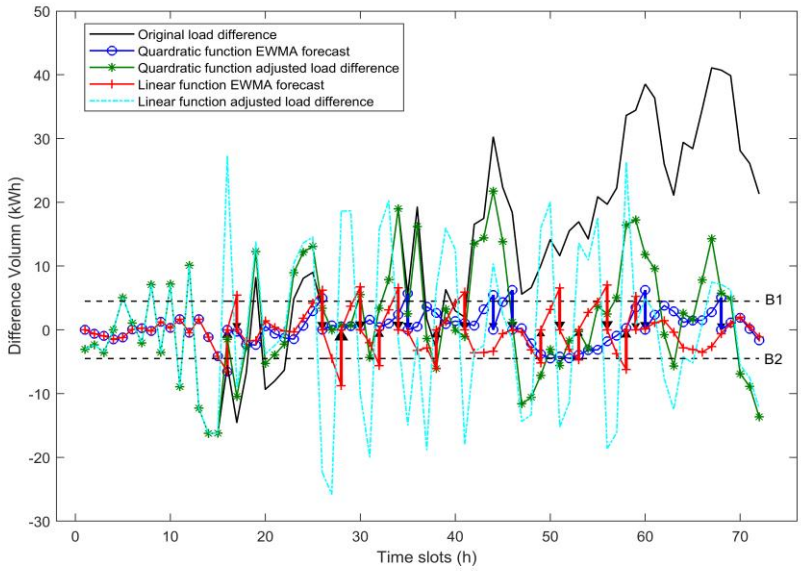

Fig. 7. Comparison of EPC monitoring and adjustment in different adjustment strategy.

where the quadratic adjustment strategy is applied.

Table 1 and Fig. 8 also show that the social welfare and the profit of the electricity company is higher when the quadratic demand function is applied than when the linear one is applied.

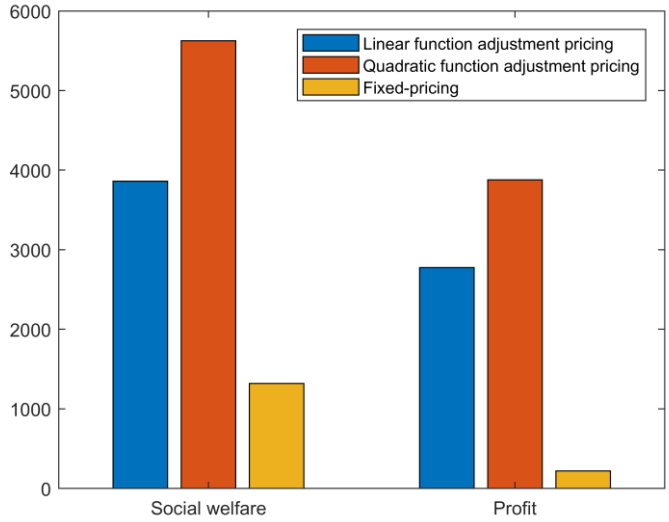

Fig. 8. Comparison of Social welfare and profit in different adjustment strategy.

Fig. 9 portrays the accumulated effect of the different price adjustment. The effect of quadratic demand function price adjustment is less than the linear one.

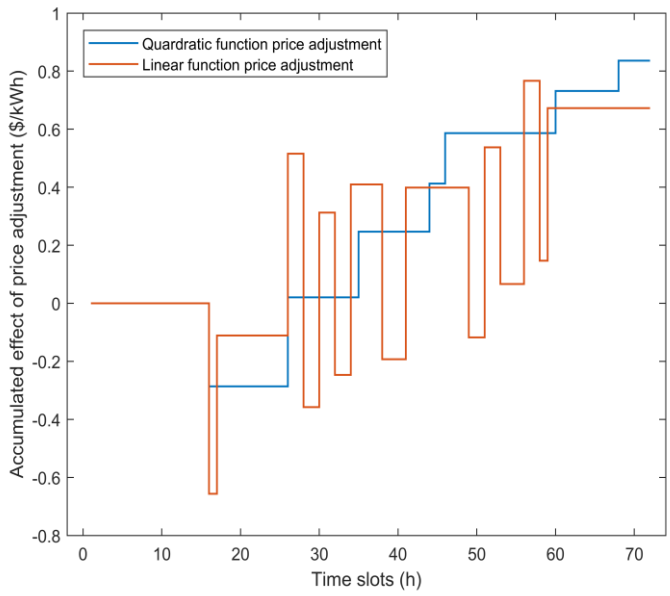

Fig. 9. Comparison of accumulated effect in different adjustment strategy.

5.3 Performance analysis on different demand function adjustment

To verify the proposed quadratic function adjustment is more reasonable to users than the linear one, the comparison between the quadratic function and the linear function adjustment method is

conducted and the results are shown in Fig. 10 and Fig. 11.

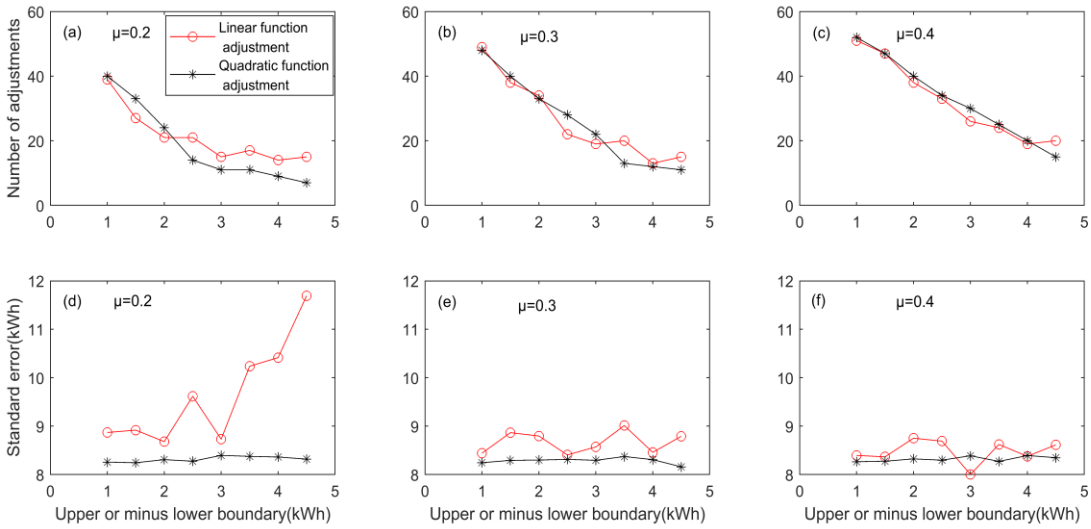

Fig.10. Indicators of different demand function adjustment with $E_1 = E_2 = 0$.

Fig. 10 shows the adjustment target $E_1 = E_2 = 0$, and Fig. 11 depicts the adjustment target the .

We observe that when EPC adjustment boundaries ($B_1 = -B_2$) are set as 4 and 4.5, the number of adjustments of the quadratic demand function adjustment, which is between 7 and 20, is less than that of the linear demand function adjustment, which is between 13 and 23 (Fig. 10. a,b, Fig. 11. a,c). What's more, the standard errors of residuals of the quadratic demand function adjustment, which is between 8.15 and 8.39, are mostly smaller than those of the linear demand function adjustments which is between 8.07 and 11.69, and the trend is more stable (Fig. 10. d, e, f, Fig. 11. d, e). It is worth mentioning that even the number of adjustments and the standard errors of residuals with the quadratic function adjustment are a little higher than the linear function adjustment (Fig. 10.c, Fig. 11. b, f) , it does

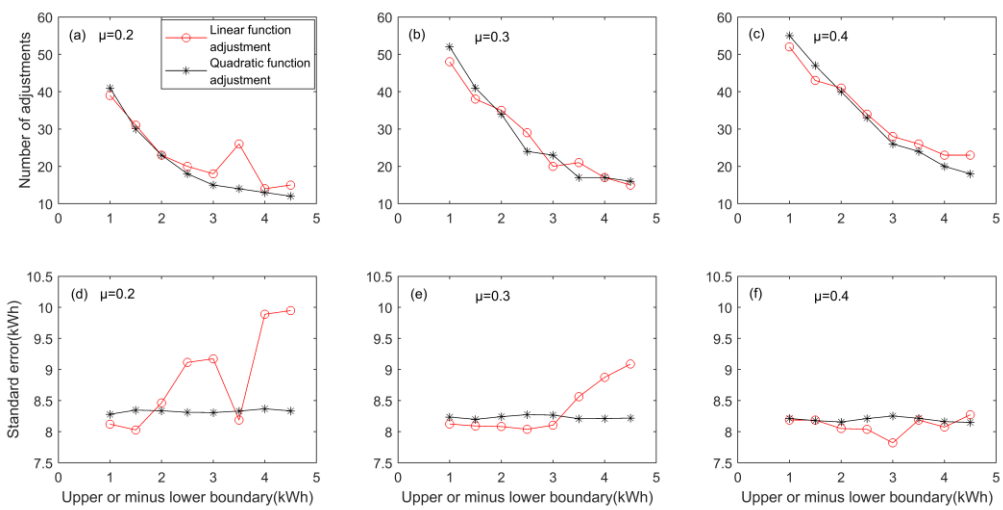

Fig.11. Indicators of different demand function adjustment with $E_1 = B_1 / 2, E_2 = B_2 / 2 (B_1 = -B_2)$.

not change the whole trend. From the observations, we can conclude that in the whole, the number of adjustments and the standard errors of residuals with the linear function adjustment are greater than those with the quadratic function adjustment.

5.4 Case study

Singapore's power market data [33] is used to validate the proposed EPC quadratic function adjustment strategy in this paper.

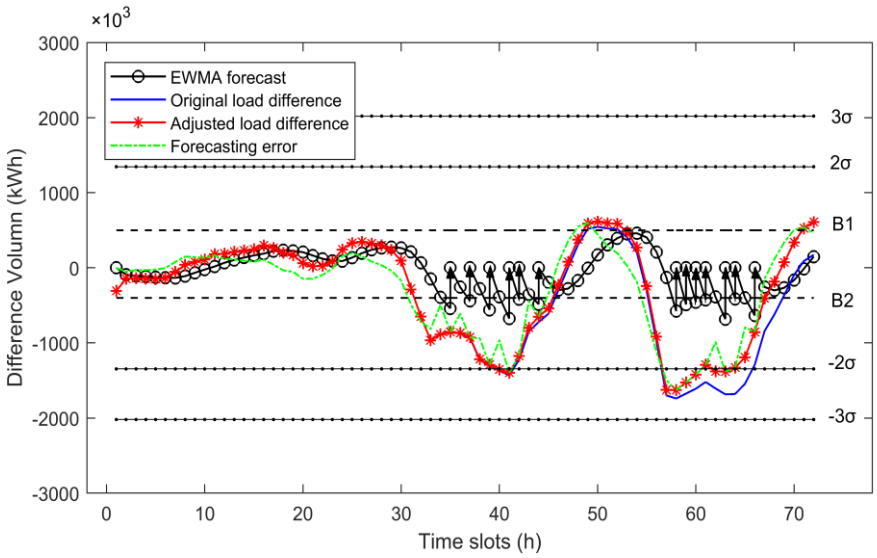

Fig. 12. Process of EPC monitoring and adjustment with realistic data

We use the real-time price data from June 2, 2017 to June 4, 2017 and power load data from May 30, 2017 to June 4, 2017 as the test data. The real-time price sequences are set as the original reserved price series $\{p_t^*\}_{t=1}^T$ in Algorithm 1, and the power load data sequences are set as the reserved consumption power load $\{x_t\}_{t=1}^T$ in Eqs. (14). The historical load sequences (power load data at the same hour of 3 days from May 30, 2017 to June 1, 2017) are set as the theoretical consumption power load $\{x_t^*\}_{t=1}^T$ in Eqs. (14). Set the initial EWMA value $\bar{p}_1 = 75$, and the other initial values are the same in Section 3.3. In Algorithm 1, the parameters are supposed as follows, $k = 50$, $\mu = 0.3$, $B_1 = 500$, $B_2 = -400$, $E_1 = 0$, $E_2 = 0$. The parameters a, b, c in cost function Eqs. (8) are respectively assumed as $0.01, 0, 0$. The test results of the monitoring and adjustment are shown in Fig. 12, Fig. 13, Fig. 14 and Fig. 15.

Fig. 12 shows that after 13 minor changes, the load differences series are more stable than that of the original load differences. The empirical average adjustment interval (AAI) is AAI=71/13=5.5, the standard error of residuals calculated by Eqs.(13) is $s = 672.9$. No points outside the $3s$ limits show that there is also no evidence for special reasons. From Fig. 13, we can learn about the fact that the adjusted power load is more close to the optimal one than the original load, which can reach the desired effect. It is shown in Fig. 14 that the electricity price has decreased with the decreasing power load in 13 time slots. The maximum adjustment of price is - \$2.953´ 10^3/kWh. The users are encouraged to buy more power load and store. The whole social

welfare calculated by Eqs.(5) is 7.945×10^{8} and the profit calculated by Eqs.(9) is 6.159×10^{7}. Fig. 15 shows that by running the EPC adjustment strategy, the smart grid system can get higher social welfare and profit than what can be obtained in the compared one.

It is easy to see that our EPC adjustment strategy not only improves the profit of the electricity company and the whole social welfare, but also contributes to balancing energy provision and preventing of power system blackouts.

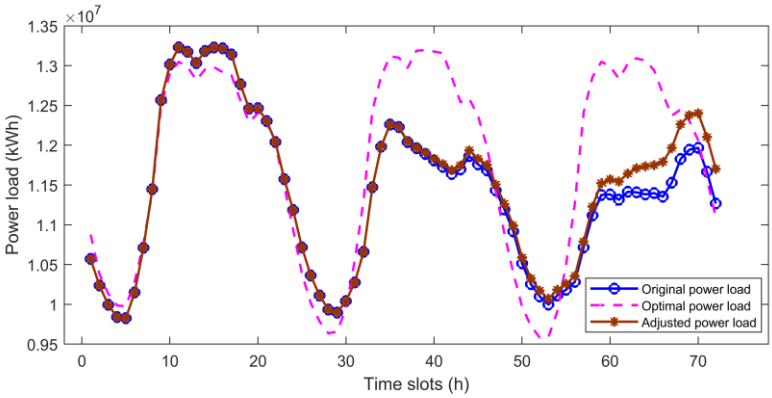

Fig. 13. Comparison of Power loads with realistic

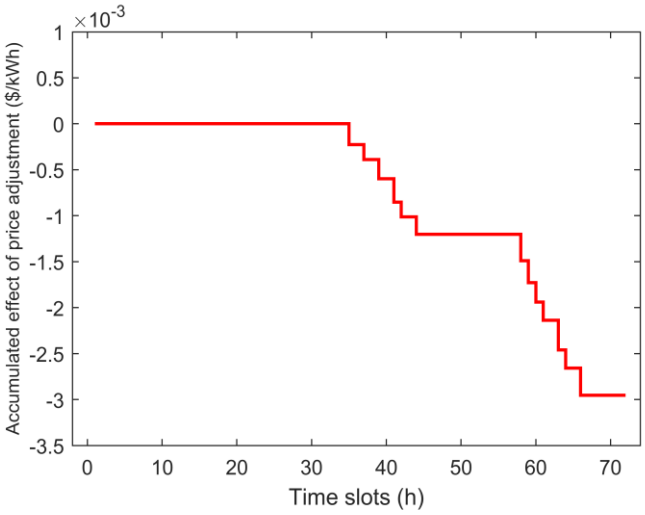

Fig. 14. Accumulated effect of price adjustment with realistic data.

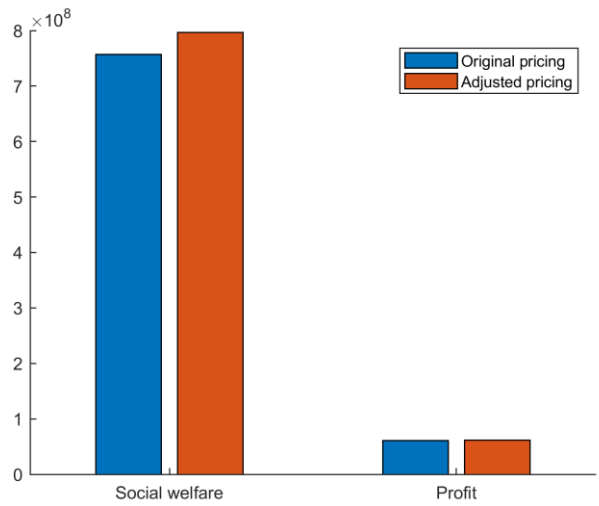

Fig. 15. Comparison of social welfare and profit with realistic data.

6. Conclusion

An Engineering Process Control monitoring and quadratic function adjustment strategy is introduced to monitor the users' consumption load and adjust the price to encourage the users to consume electricity appropriately. The electricity company monitors the exponential weighted moving average predict values of the difference between the theoretical consumption load given by the real-time pricing model and the users' real-time reservation consumption load by presetting the upper or lower boundaries for them. The price adjustment is used to induce the users to change their consumption requirement loads under the demand response mechanism for the power price. The price will be adjusted only if the predicted exponential weighted moving average value exceeds the boundaries

The numerical results show that the proposed strategy is superior to the linear function adjustment in the performance indices: Number of adjustments, Standard error, Social welfare and Profit. It is more reasonable and realistic than the linear one. In general, the proposed Engineering Process Control adjustment strategy not only has good performance in obtaining balanced power provision and consumption requirement load, but also can improve supplier's profit and social welfare.

Acknowledgements

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Appendix A. Lagrange Dual Method

Regarding the problem (10)-(13) as the primal problems, the Lagrangian is defined as[8]

$$L(x_i^t, L_t, \lambda_t) = \sum_{i=1}^{N} U(x_i^t, \omega_i^t) - C\left(L_t\right) - \lambda_t\left(\sum_{i=1}^{N} x_i^t - L_t\right) \tag{A1}$$

where λ_t is a Lagrange multiplier for a fixed $t \in \mathrm{T}$.

(A1) is rewritten as

$$L(x_i^t, L_t, \lambda_t) = \sum_{i=1}^{N} \left(U(x_i^t, \omega_i^t) - \lambda_t x_i^t \right) + \lambda_t L_t - C\left(L_t\right) \tag{A2}$$

Then, the maximum Lagrange function is expressed as

$$g(\lambda_t) = \max_{x_i^t, L_t} L(x_i^t, L_t, \lambda_t) \tag{A3}$$

Thus, the Lagrange dual problem is as follows,

$$D(\lambda_t) = \min_{\lambda_t} g(\lambda_t) \tag{A4}$$

Because of the strong duality property, the primal problem (10-13) is not different from dual problem (A4). Meantime, we can also get the optimal Lagrange multiplier by solving the dual problem (A4). Due to the independence of the users, we reformulated (A4) as

$$g(\lambda_t) = \sum_{i=1}^{N} \phi_i\left(\lambda_t\right) + \varphi\left(\lambda_t\right) \tag{A5}$$

where

$$\phi_i\left(\lambda_t\right) = \max_{m_i^t \le x_i^t \le M_i^t} \left(U(x_i^t, \omega_i^t) - \lambda_t x_i^t \right) \tag{A6}$$

$$\varphi(\lambda_t) = \max_{L_t^{\min} \le L_t \le L_t^{\max}} \left(\lambda_t L_t - C\left(L_t\right) \right) \tag{A7}$$

The first term of (A5) is about the users, which can be resolved into N independent problems shown in (A6). The second term is about the electricity company with (A7). Obviously, if we set the price value p_t equal to the optimal Lagrange multiplier λ_t^* in time slot t, (A6) is the maximal welfare function (5) of the users, and (A7) is profit function (9) of electricity company. By solving the dual problem (A4), we can get the optimal Lagrange multiplier λ_t^*, which is the theoretically optimal price p_t^* in time slot $t \in \mathrm{T}$, and the optimal consumption load is $x_t^* = \sum_{i=1}^{N} x_i^t$, the sum of N users' consumption loads. The optimal price p_t^* is often called Shadow Price, the pricing basis of electricity. At the same time, it also reflects the scarcity degree of consumption loads. That is, the price is higher with large consumption loads, and lower with small consumption loads.

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