Performance of Digital Optical Communication Link:
Effect of In-Line EDFA Parameters

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Abstract—The performance of a base band digital optical communication link with in-line Erbium doped fiber amplifier (EDFA) is studied. A good measure for digital communication link is the Q-factor. In this paper, the Q-factor is theoretically evaluated as a function of optical signal to noise ratio (SNR_{opt}) and is compared with simulation results showing a fair agreement. This comparison helps in making an accurate prediction for the value of Q-factor when all noise sources are considered. Also, the effect of Erbium ion density and doped fiber length of EDFA on bit error probability is investigated.

I. INTRODUCTION

Most, if not all, applications of photons and lightwave signals in communications require the detection and subsequent conversion of the light to an electrical signal. In this process, the useful signal will be corrupted by noise and the ultimate sensitivity and performance of the system is limited by the noise characteristics. Optical receiver adds noise; usually thermal noise and shot noise. Optical amplifiers (OAs) can be used to improve the effective receiver sensitivity in lightwave systems. OA is a device which amplifies the input optical signal. This device works on the principle of stimulated emission [1].

There are two types of OAs which are used in optical communication systems; (i) semiconductor optical amplifiers (SOAs) and (ii) doped fiber amplifiers (DFAs). In DFAs, the optical fiber core is doped by rare earth elements such as Erbium (Er^{3+}), Holmium (Ho^{3+}), Neodymium (Nd^{3+}), Praseodymium (Pr^{3+}) and Ytterbium (Yb^{3+}). However, the most interesting element listed above is Erbium, because erbium doped fiber amplifier (EDFA) can operate in a broad range within the 1550 nm window at which the attenuation of silica fiber is minimum [2].

Because OAs add noise to the amplified signal, and at some point, this noise becomes the dominant noise source. The understanding of the noise properties of OAs and the resultant effects on systems is of crucial importance to the engineering of lightwave systems. The basic manifestation of noise in OA is in the form of amplified spontaneous emission (ASE). So, the bit error probability (BER) of an optically amplified digital communication system is affected by the ASE noise added by OA [3].

The most significant application of OAs is for amplifying the optical signals in communication links. This increases the regenerator spacing in power budget limited optical fiber links. In a point-to-point link, OA can be used as (i) post-amplifier, (ii) preamplifier and (iii) in-line amplifier. The present study is mainly concerned with in-line EDFA. Various parameters of EDFA such as Erbium ion density, doped fiber length, pumping power and doping radius can affect on ASE noise which in role affects on the value of BER.

The paper is organized as follows: section 2 gives an experimental model for point-to-point digital optical communication link with in-line EDFA. Section 3 gives a review on noise in optically amplified digital communication link including a mathematical model for BER calculation in case of in-line EDFA. Section 4 is assigned for results and discussion. This is followed by the conclusion in section 5.

II. EXPERIMENTAL MODEL

This model presents a simple base band digital optical communication system with in-line EDFA. Figure 1 shows a typical block diagram of this system.
The concerned system consists of the following components:

- Base band transmitter which consists of pseudo random bit sequence generator followed by a non-return to zero (NRZ) pulse generator.
- Optical modulator which is used to modulate the optical signal (carrier) by the output digital signal of transmitter (message). The modulator type is Mach-Zender.
- Optical source which is used to generate optical signal. In this model, a continuous wave laser of 1550 nm wavelength and power ranging from 8 to 20 dBm is used.
- Optical amplifier namely EDFA, with components shown in Fig. 2.
- Laser pump of 10 mw (1dBm) power and 980 nm wavelength. Erbium doped fiber (EDF) is of 5 m length, 2.2 µm Erbium doping radius and $1.4 \times 10^{25} / \text{m}^3$ Erbium ion density.
- Two optical fibers each of 50 km length and attenuation of 0.6 dB/km.
- PIN Photodetector.
- LPF receiver.

This system is implemented and simulated using optiwave software optisystem7. Figure 3 shows a schematic diagram of the system.

### III. MATHEMATICAL MODEL

#### A. Noise in optically amplified digital communication systems

In communication systems, where electrical, radio or optical signals are transmitted; noise can be viewed as an impairment resulting in the degradation of the information contained in the signal [4]. As mentioned previously, optical receiver adds two types of noise namely thermal noise and shot noise. Since OAs are based on the principle of stimulated emission, therefore its main contribution to noise is ASE noise.

#### B. Thermal noise

The thermal noise of a receiver arises from the fact that electrons in a receiver circuit have some probability of generating a current even in the absence of an optical signal. This noise, often referred to as Johnson noise, can be represented by the variance of thermal current per unit frequency [5]

$$\sigma_{th}^2 = 4kT/R,$$  \hspace{1cm} (1)

where $T$ is the absolute temperature, $k$ is Boltzmann's constant and $R$ is the detector load resistance.

#### C. Shot noise

The shot noise arises from the Poisson distribution of the electron-hole generation by the photon stream. The latter is a stochastic process having random arrival times. On average, the number of electron-hole pairs created will be proportional to the number of photons, with a given constant of proportionality.
During a given time interval, with a certain number of photons incident upon the detector, the number of electron–hole pairs generated will have fluctuations as determined by Poisson statistics [5]. A dc photocurrent of $I_{pd}$ will generate a shot noise power density of
\[ \sigma_{sh}^2 = 2eI_{pd}. \]  
(2)

D. ASE Noise

ASE is a light produced by spontaneous emission that has been optically amplified by the process of stimulated emission in gain medium. Noise associated with ASE is the limiting factor in determining the ultimate signal-to-noise ratio in any system using optical amplifiers. The output ASE power can be calculated using classical derivation in [5]

\[ P_{ASE} = n_{sp}(G - 1)h \nu B_o, \]  
(3)
where $h$ is Planck’s constant and $n_{sp}$ is the inversion parameter, given by

\[ n_{sp} = \frac{\sigma_a(\lambda)N_2}{\sigma_a(\lambda)N_2 - \sigma_e(\lambda)N_1}, \]  
(4)
where $\sigma_a(\lambda)$ and $\sigma_e(\lambda)$ are the absorption and emission cross sections, respectively, $N_1$ and $N_2$ are the population density in lower and upper states, respectively, $G$ is the overall gain of the amplifier, $h$ is Planck’s constant, $\nu$ is the optical frequency of transition and $B_o$ is the optical bandwidth. Equation (3) gives the ASE power for one polarization mode. So, for single mode fiber, the right hand side of Eq. (3) must be multiplied by a factor of 2.

When an amplified optical signal and accompanying spontaneous emission are detected in a photodetector, the noise is transformed into the electrical domain and appears along with the induced photocurrent as a noise current. Photodetection is a nonlinear square-law process. The photocurrent $I_{pd}$ is found as [5]

\[ I_{pd} \propto (E_{tot})^2 = (E_s + E_n)^2 = E_s^2 + E_n^2 + 2E_s \cdot E_n. \]  
(5)

In Eq. (5), one can identify the first term as pure signal, the second term as pure noise and it is referred to as spontaneous-spontaneous (sp-sp) beat noise, and the third term as mixing component between signal and noise and it is referred to as signal-spontaneous (s-sp) beat noise [1]. The power spectrum of current corresponding to signal-spontaneous beat noise is uniform in the frequency interval $(-B_o/2)$ to $(B_o/2)$ and has an equivalent one-sided power density of

\[ N_{sp-sp} = 2n_{sp}^2 (G - 1)^2 e^2 B_o. \]  
(7)

Figure 4 summarizes the results of Eqs. (6) and (7) and shows the electrical power spectrum of the beat noise.

![Figure 4. Electrical single sided noise power spectrum of sp-sp and s-sp beat noise per one polarization mode [5].](image)

E. BER calculation for optically amplified communication link with in-line EDFA

BER calculation can be made by treating the noise sources in terms of Gaussian noise statistics. BER for optimum setting of decision threshold for choosing whether bit is a 1 or 0 is given by the Gaussian error function [5]

\[ \text{BER} = \frac{1}{2} \text{erfc} \left( \frac{I_1 - I_0}{\sqrt{2(\sigma_1^2 + \sigma_0^2)}} \right), \]  
(8)
where $I_0$ and $I_1$ are the average photocurrent generated by a 1 bit and 0 bit, respectively. $\sigma_0^2$ and $\sigma_1^2$ are the noise variances for 0 bit and 1 bit, respectively. The BER, given by Eq. (8), can be well approximated by [5]

\[ \text{BER} = \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{Q^2}{2} \right], \]  
(9)

where $Q$ is the Q-factor, given by [5]

\[ Q = \frac{I_1 - I_0}{\sqrt{N_{tot}(1) + \sqrt{N_{tot}(0)}}}. \]  
(10)

$N_{tot}(1)$ and $N_{tot}(0)$ are the total noise for a 1 bit and 0 bit, respectively. The Q-factor can be also expressed in terms of the optical signal to noise ratio ($\text{SNR}_{opt}$) as [5]
\[ Q = \left( \frac{B_o}{B_e} \right)^{\frac{1}{2}} \frac{2SNR_{opt}}{4(SNR_{opt} + 1)^{\frac{1}{2}} + 1}, \]  

(11)

with \( B_o \) and \( B_e \) the optical and electrical bandwidths, respectively.

Equation (11) seems nonlinear and can be used to derive the optical SNR needed to obtain a given BER, for an ideal system with only amplifier noise and without nonlinearities or inter-symbol interference [5].

In case of in-line amplifier operating with moderately large optical input signals, the signal to noise ratio at the amplifier output is dominated by signal spontaneous beat noise. In this limit, the equivalent electrical SNR at the amplifier output is given by [5]

\[ SNR = \frac{GP_{to}}{4h \nu n_{sp} (G - 1) B_o}. \]  

(12)

Provided that \( G \) is reasonably high, the SNR is determined only by the input power and the inversion parameter \( n_{sp} \). More specifically, the SNR is independent of the gain. This is an important result that governs the system performance of in-line amplifiers [5].

IV. RESULTS AND DISCUSSION

A. Q-factor as a function of optical SNR

Figure 5. illustrates the relation between Q-factor and \( SNR_{opt} \), as stated in Eq. (11) and as obtained from simulation results in case of optical link with amplifier noise only. A fair agreement is noticed between theory and simulation, irrespective of the small deviation.

To investigate the effect of shot noise and thermal noise on the value of Q-factor, the optical link is simulated at three different values of Erbium ion density; typically \( 1.4 \times 10^{23}/m^3 \), \( 1.4 \times 10^{24}/m^3 \) and \( 1.4 \times 10^{25}/m^3 \). Simulation results are presented in Fig. 7, where one can deduce that the value of BER decreases with the Erbium ion density.

This result may help in generalizing the expression of Q-factor given by Eq. (11). Therefore, one can write

\[ Q_{all} = Q_{ASE} - 0.15, \]  

(13)

where \( Q_{ASE} \) and \( Q_{all} \) are the values of Q-factor in case of considering amplifier noise only and when all noise sources are considered.

B. Effect of Erbium ion density on BER

To investigate the effect of Erbium ion density in EDFA on BER, the optical link is simulated at three different values of Erbium ion density; typically \( 1.4 \times 10^{23}/m^3 \), \( 1.4 \times 10^{24}/m^3 \) and \( 1.4 \times 10^{25}/m^3 \). Simulation results are presented in Fig. 7, where one can deduce that the value of BER decreases with the Erbium ion density.
To investigate the effect of Erbium ion density more clearly, BER is plotted versus Erbium ion density as shown in Fig. 8. It is clear that, BER decreases rapidly with Erbium ion density. This is because the EDFA gain increases with the Erbium ion density and consequently, the received signal power and O-factor will increase. But, the value of ASE noise power will also increase as Erbium ion density increases. So, beyond a certain value of Erbium ion density, BER tends to be constant.

![Fig. 8. BER Vs Erbium ion density.](image)

C. Effect of doped fiber length on BER

The effect of the doped fiber length is investigated at three different values; typically 5, 10 and 15m. Simulation results are presented in Fig. 9. From simulation results, it easily noticed that, the value of BER decreases with the doped fiber length. Similar to the ion density, the effect of the doped fiber length is studied in Fig. 10. Here, it is clear that, the value of BER decreases with the doped fiber length (log BER decreases linearly). This behavior can be explained in the same manner of previous section.

![Fig. 9. BER Vs transmitted signal power for three values of doped fiber length.](image)

![Fig. 10. BER Vs doped fiber length.](image)

V. CONCLUSION

In this work, a base band digital optical communication system with in-line EDFA was studied by evaluating Q-factor as function of SNRopt considering amplifier noise only. It has been shown that a general expression of Q-factor can be predicted when shot noise and thermal noise are also considered. Also, the effect of two EDFA parameters; namely, Erbium ion density and doped fiber length on system performance were investigated by evaluating BER as function of Erbium ion density and doped fiber length. The obtained results showed that, BER decreases rapidly with Erbium ion density, and beyond a certain value of Erbium ion density, BER tends to be constant. Also, the obtained results showed that BER decreases linearly with the doped fiber length.

VI. REFERENCES
