Loschmidt Echo

Arseni Goussev[1], Rodolfo A. Jalabert[2], Horacio M. Pastawski[3], and Diego Wisniacki[4]

[1]Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, D-01187 Dresden, Germany

[2]Institut de Physique et Chimie des Matériaux de Strasbourg, UMR 7504, CNRS-UdS, 23 rue du Loess, BP 43, 67034 Strasbourg Cedex 2, France

[3]Instituto de Física Enrique Gaviola (CONICET-UNC) and Facultad de Matemática Astronomía y Física, Universidad Nacional de Córdoba, Ciudad Universitaria, Córdoba 5000, Argentina

[4]Departamento de Física, FCEyN, UBA, Ciudad Universitaria, Buenos Aires C1428EGA Argentina

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The Loschmidt echo is a measure of the revival occurring when an imperfect time-reversal procedure is applied to a complex quantum system. It allows to quantify the sensitivity of quantum evolution to perturbations. An initial quantum state $|\psi_0\rangle$ evolves during a time t under a Hamiltonian H_1 reaching the state $|\psi_t\rangle$. Aiming to recover the initial state $|\psi_0\rangle$ a new Hamiltonian $-H_2$ is applied between t and $2t$. Perfect recover of $|\psi_0\rangle$ would be achieved by choosing H_2 to be equal to H_1. This is not possible in realistic setups, and there always appears a difference between H_2 and H_1, leading to a non-perfect recovery of the initial state. The forward evolution between t and $2t$ under the Hamiltonian $-H_2$ is equivalent to a backward evolution from t to 0 under H_2, which embodies the notion of time-reversal. The Loschmidt echo studies are focused on the cases where the dynamics induced by the Hamiltonians H_1 and H_2 are non-trivial, or sufficiently complex (like that of a classically chaotic one-particle system or a many-body system).

1 Introduction

The concept of time-reversal has captured the imagination of physicists for centuries, leading to numerous vivid discussions. An emblematic example of these was the controversy around the second law of thermodynamics between Ludwig Boltzmann and Joseph Loschmidt. When Boltzmann was trying to develop the microscopic theory of the second law of thermodynamics, Loschmidt raised an objection that had profound influence on the subsequent development of the theory. He argued that, due to the time-reversal invariance of classical mechanics, evolution in which the entropy can decrease must exist. These states could be reached by reversing velocities of all molecules of the system. After such a reversal the entropy would then no longer grow but decrease, seemingly violating the second law of thermodynamics. As a response Boltzmann argued that such a time-reversal experiment would be impossible and put forward a statistical interpretation of the second law. Undoubtedly, Boltzmann's argument is a valid approach to the problem of the "arrow of time" for generic macroscopic systems. However, in quantum systems with few degrees of freedom, today's technological advances make it meaningful to address time-reversal experiments.

1.1 Pioneering experiments

Refs. [Hahn 50, Rhim 71, Brew 84, Zhan 92, Levs 98]

The first controlled experimental implementation of time-reversal was achieved in the fifties with the inversion of nuclear spins precessing around a magnetic field. Such a reversal was able to compensate the local-field inhomogeneities responsible for the decay of the free induction signal. The time-reversal was implemented through a radio-frequency pulse, leading to the formation of a revival in the induction signal, known as spin echo or Hahn echo. Early on Erwin Hahn recognized that his procedure, which he viewed as a change in the sign of the system Hamiltonian, provided a quantum implementation of the Loschmidt proposal. Interactions between spins, not reversed in Hahn's procedure, were the main cause of the decay of the echo, within a time scale known as T_2. This experiment was followed by a number of variants, both within magnetic resonance as well in other time-dependent spectroscopies. In particular, the dynamical decoupling techniques implemented in quantum registries to isolate them from their environment are variants of Hahn's original experiment.

The next level of complexity was developed with the reversal of the many-body interactions. No general recipe is available in this case, but the sign of the truncated spin-spin dipolar interaction, which is associated with the cosine of the angle between the inter-spin vector and the quantizing magnetic field, can be reversed by

rotating the spins into a quantization axis. This was the proposal of the Magic Echo procedure, implemented by Won-Kyu Rhim, Alex Pines and John Stewart Waugh in the seventies. Substantially simpler is the Polarization Echo sequence, implemented by Richard Ernst and collaborators in the nineties. In this last case the initial state has a local nature as it is labeled by the presence of a rare ^{13}C which plays the role of a local probe to inject and later on detect the polarization of a nearby ^{1}H immersed in a ^{1}H network.

This idea was further exploited by Patricia Levstein, Horacio Pastawski and Gonzalo Usaj to test the stability of many-body dynamics. They suggested that the inefficiency of the time-reversal procedure found in all the previous experiments has a connection with quantum chaos and the inherent dynamical complexity of the many-body spin system.

1.2 Definition

Refs. [Pere 84, Jala 01]

The Loschmidt echo is defined as

$$M(t) = \left| \langle \psi_0 | e^{iH_2 t/\hbar} e^{-iH_1 t/\hbar} | \psi_0 \rangle \right|^2 \tag{1}$$

where

- $|\psi_0\rangle$ is the state of the system at time 0
- H_1 is the Hamiltonian governing the forward evolution
- H_2 is the Hamiltonian governing the backward evolution
- t is the instant at which the reversal takes place

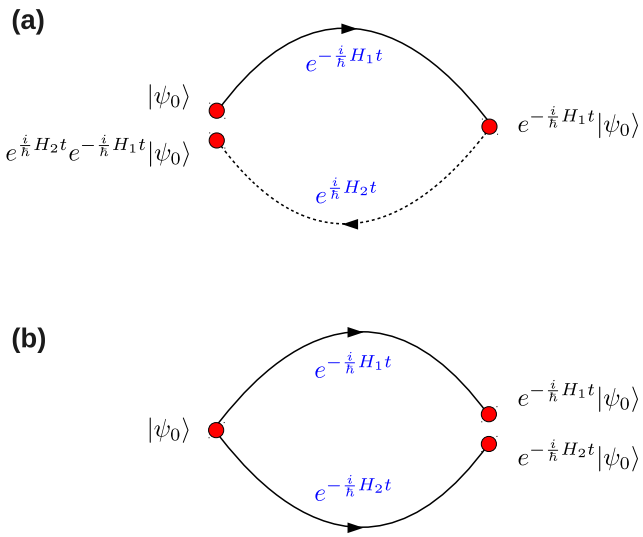


Figure 1: Schematic flow of the time-evolution for (a) the Loschmidt echo and (b) the fidelity.

The time evolution appearing in Eq. (1) is schematically represented in Fig. 1(a), where the Loschmidt echo quantifies the degree of irreversibility. Alternatively, Eq. (1) can be interpreted as the overlap at time t of two states evolved from $|\psi_0\rangle$ under the action of the Hamiltonian operators H_1 and H_2. In this case the Loschmidt echo is a measure of the sensitivity of quantum evolution to perturbations. The quantity $M(t)$ interpreted in this manner is illustrated in Fig. 1(b) and is usually referred to as fidelity. The equivalence between Loschmidt echo and fidelity is displayed in Fig. 2 for the example of an initially localized wave-packet in a Lorentz gas. The probability density of the evolved state under H_1 (H_2) is plotted in Fig. 2 (b) (c), while Fig. 2(d) presents the state resulting from the combined evolution H_1 and then $-H_2$ both for a time t. The overlap squared between the states (a) and (d) defines the Loschmidt echo, while the overlap squared between the states (b) and (c) defines the fidelity. It is important to remark that even though the probability density distributions in figures (b) and (c) are seemingly identical, the phase randomization due to the difference between the two Hamiltonians leads to a weak Loschmidt echo.

The definition given by Eq. (1) assumes that the Hamiltonian operators H_1 and H_2 are independent of time. A generalization to the case of time dependent Hamiltonians is straightforward. The case of non-Hermitian operators H_1 and H_2 can also be considered, but in such a case the equivalence between the Loschmidt echo and fidelity would not hold.

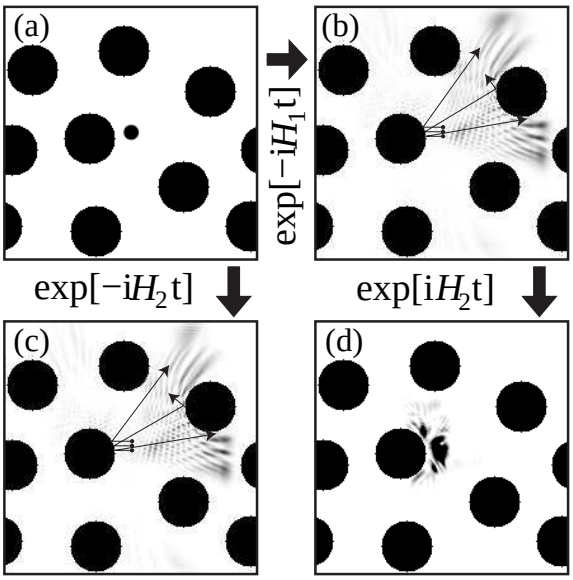


Figure 2: Wave-packet evolution in a Lorentz gas. (a) Initial state at time $t = 0$ with momentum pointing to the left. (b) State evolved with the Hamiltonian H_1 in the interval $(0, t)$. (c) State evolved with the Hamiltonian H_2 in the same time interval. (d) State evolved from that depicted in panel (b) with the $-H_2$ for the time interval $(t, 2t)$. In panels (b) and (c) the classical trajectories corresponding to three initial positions within the original wave-packet are shown for reference. The square of the overlap between the states (a) and (d), the Loschmidt Echo, is $M(t) = 0.09$, the same as that between the states of panels (b) and (c). Adapted from Ref. [Cucc 02b]. Copyright (2002), American Physical Society.

1.3 Loschmidt echo and decoherence

Refs. [Zure 01, Cucc 03, Zure 03, Cucc 04, Casa 10, Bona 11]

In an isolated quantum system the evolution is unitary; initially pure states remain pure. When the system is connected to the external world the purity of an initial state is generically lost in the course of time evolution. This ubiquitous process is referred to as decoherence. The coupling to the environment degrees of freedom, or alternatively the lack of complete knowledge of the system Hamiltonian, typically wash the specific interference properties that would be observed within a unitary evolution. Since interference is one of the most important signatures of quantum mechanics, decoherence has both, fundamental and practical interest. Concerning the former aspect, decoherence has been proposed as a road towards the classical behavior observed in macroscopic systems. On the second aspect, it is clear that decoherence represents a limitation in the implementation of quantum computers or nanoscopic devices based on quantum effects, specially when the number of qubits is scaled up, increasing the complexity of the system. The Loschmidt echo, by including a non-controlled part of the Hamiltonian, or the noisy effects of the environmental degrees of freedom, gives a way to quantify decoherence effect. By attempting the time-reversal of the controlled part of the Hamiltonian, one can quantify at which rate the neighborhood of the system (either a unitary part of larger system Hamiltonian or though the effect of infinite uncontrolled environmental degrees of freedom) acts as decoherence. Moreover, other quantifiers of decoherence, such as for instance the purity, can be shown, under certain assumptions, to be characterized by the same dependence on the underlying classical dynamics as the Loschmidt echo.

1.4 Broad interest of the Loschmidt echo

Since the beginning of this century, the Loschmidt echo has been a subject of intensive studies by researchers from different scientific communities. A list of problems, in which the concept of the Loschmidt echo appears naturally, includes

- Quantum chaos, or the quantum theory of classically chaotic systems
- Decoherence or the emergence of "classical world" (open quantum systems)
- Quantum computation and quantum information
- Spin echo in Nuclear Magnetic Resonance
- Linear waves (Elastic waves, microwaves, time-reversal mirrors, etc.)

- Nonlinear waves (Bose-Einstein condensates, Loschmidt cooling, etc.)
- Statistical mechanics of small systems
- Quantum chemistry and molecular dynamics
- Quantum phase transitions and quantum criticality
- Mathematical aspects of the Loschmidt echo

Such broad interest stems from the fact that the Loschmidt echo is a measurable quantity exhibiting, in certain regimes, a robust and universal behavior. It is by means of the forward and backward time-evolutions that the Loschmidt echo filters out some uninteresting effects, while amplifying other, more important physical processes taking place in complex quantum systems.

2 Loschmidt echo: techniques and main results

The Loschmidt echo is typically a decreasing function of t. It is therefore of foremost importance to determine the form of the decay and the associated characteristic times in various physical situations as a function of the characteristic quantities of the problem. Even though the Loschmidt echo has been addressed in a variety of scenarios, the main effort has been directed towards the simplest set-up of one-body dynamics. Well established results for the case of one-body systems, together with techniques adapted from the field of quantum chaos, are summarized in this section.

2.1 Characteristic quantities

A number of physical quantities, describing characteristic time and energy scales of a system under consideration, prove to be especially important in the theoretical studies of the Loschmidt echo. These quantities include

- g, *mean density of states.* – By definition, $g(E) = \frac{1}{(2\pi\hbar)^d}\frac{\mathrm{d}V(E)}{\mathrm{d}E}$, where $V(E)$ is the volume of the phase-space enclosed by a surface of constant energy E. The mean level spacing Δ is given by an inverse of the density of states, $\Delta = 1/g$. Note that g is the smooth part (or average) of the full density of states, $g_{\mathrm{full}}(E) = \sum_n \delta(E - E_n)$, where $\{E_n\}$ are the eigenenergies of the Hamiltonian.
- g_{local}, *local density of states.* – Let $|\psi\rangle$ be a reference quantum state. The local density of states with respect to $|\psi\rangle$ is defined as $g_{\mathrm{local}}(E, |\psi\rangle) = \sum_n \left|\langle n|\psi\rangle\right|^2 \delta(E - E_n)$, where $\{|n\rangle\}$ and $\{E_n\}$ are respectively the eigenstates and eigenenergies of the Hamiltonian in question.
- N, *effective size of the Hilbert space for a quantum state.* – An initial quantum state $|\psi_0\rangle$ of a closed quantum system can be viewed as a linear combination of N eigenstates of the Hamiltonian. In a d-dimensional system, N is given by the volume of the phase space, accessible to $|\psi_0\rangle$ in the course of its time evolution, divided by the volume of the Planck's cell, $(2\pi\hbar)^d$.
- λ, *mean Lyapunov exponent.* – In classical systems with chaotic dynamics, a distance between two typical trajectories, initiating at infinitesimally close points in phase-space, grows exponentially with time. The rate of this growth, averaged over all trajectory pairs at a given energy, is the mean Lyapunov exponent λ. The latter characterizes dynamical instability of a classical system with respect to perturbations of its initial conditions.
- t_{E}, *Ehrenfest time.* – The center of an initially localized wave packet stays close to the phase-space trajectory of the corresponding classical particle for times shorter than the so-called Ehrenfest time, t_{E}. The latter, in chaotic systems, can be estimated as $t_{\mathrm{E}} = \frac{1}{\lambda}\ln\frac{L}{\sigma}$, where λ is the mean Lyapunov exponent of the classical system, L is the characteristic linear size of the system, and σ is the initial dispersion of the wave packet.
- t_{H}, *Heisenberg time.* – The discreteness of the energy spectrum of a closed quantum system becomes dynamically important at times comparable to (and longer than) the so-called Heisenberg time, $t_{\mathrm{H}} = \hbar/\Delta = \hbar g$, where Δ and g are the mean energy level spacing and the mean density of states, respectively. Semiclassical (short-wavelength) approximations to quantum dynamics are known to break down beyond the Heisenberg time.

2.2 Calculational techniques

2.2.1 Semiclassics

Refs. [Jala 01, Vani 03, Cucc 04, Vani 04, Vani 06, Gutk 10, Zamb 11]

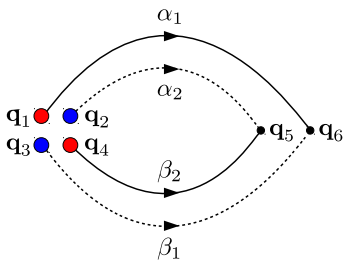
In this context, semiclassics stands for the short-wavelength approximation to the quantum evolution in terms of classical trajectories. The key ingredient is the Van Vleck-Gutzwiller approximation to the propagator (matrix element of the evolution operator in the position representation)

$$\langle \mathbf{q}'|e^{-iHt/\hbar}|\mathbf{q}\rangle = \left(\frac{1}{2\pi i\hbar}\right)^{d/2} \sum_{\gamma(\mathbf{q}\to\mathbf{q}',t)} C_\gamma^{1/2} \exp\left(\frac{i}{\hbar}R_\gamma - \frac{i\pi}{2}\nu_\gamma\right) \tag{2}$$

- γ stands for the classical trajectories going from $\mathbf{q}$ to $\mathbf{q}'$ in time t.
- $R_\gamma = \int_0^t d\tau \mathcal{L}_\gamma$ is the Hamilton's principal function for the trajectory γ. Here, $\mathcal{L}_\gamma$ denotes the Lagrangian along γ.
- $C_\gamma = |\det(-\nabla_{\mathbf{q}'}\nabla_{\mathbf{q}} R_\gamma)|$ is the stability factor of γ.
- ν_γ is the number of conjugate points along γ.
- d is the number of dimensions of the position space.

A combination of Eqs. (1) and (2) leads to the semiclassical approximation to the Loschmidt echo,

$$M(t) \;=\; \int \prod_{j=1}^{6} \mathrm{d}\mathbf{q}_j \sum_{\alpha_1,\,\alpha_2,\,\beta_1,\,\beta_2}$$



(3)

Here, the summand is a product of terms with the following pictorial representation:

- A red circles at a point $\mathbf{q}_j$ corresponds to $\langle \mathbf{q}_j|\psi\rangle$.
- A blue circles at a point $\mathbf{q}_j$ correspond to $\langle \psi|\mathbf{q}_j\rangle$. That is, every blue circle is a complex conjugate of the corresponding red circle.
- A solid curve leading from $\mathbf{q}_j$ to $\mathbf{q}_k$ along a trajectory γ corresponds to $(2\pi i\hbar)^{-d/2} C_\gamma^{1/2} \exp\big(iR_\gamma/\hbar - i\pi\nu_\gamma/2\big)$. The Hamiltonian H_1 is used if γ equals α_1, and the Hamiltonian H_2 is used if γ equals β_2.
- A dashed curve leading from $\mathbf{q}_j$ to $\mathbf{q}_k$ along a trajectory γ corresponds to $(-2\pi i\hbar)^{-d/2} C_\gamma^{1/2} \exp\big(-iR_\gamma/\hbar + i\pi\nu_\gamma/2\big)$. The Hamiltonian H_1 is used if γ equals α_2, and the Hamiltonian H_2 is used if γ equals β_1. In other words, every dashed curve is a complex conjugate of the corresponding solid curve.

In Loschmidt echo studies one is typically in interested in the case of H_2 being close to H_1. It is common to describe the perturbation by an operator defined as

$$\kappa\Sigma = H_2 - H_1\,, \tag{4}$$

where κ parametrizes its strength.

The expression in Eq. (3) is, in general, extremely complicated to calculate as it involves six spacial integrals and the sum over four classical trajectories. Semiclassical evaluations of the Loschmidt echo deal with different approximations to Eq. (3). The following steps are usually implemented in the semiclassical calculations in order to obtain meaningful analytical results:

(i) If the initial state is spatially localized around a point $\mathbf{q}_0$, the diagram of Eq. (3) can be approximated by a simpler diagram in which the points $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$, and $\mathbf{q}_4$ merge into $\mathbf{q}_0$. Within this approximation there are only two spacial integrals to be done (over $\mathbf{q}_5$, and $\mathbf{q}_6$).

(ii) Assuming that the perturbation $\kappa\Sigma$ is classically small but quantum mechanically significant (which means that the perturbation does not change the topology of the trajectories but introduces a phase difference) trajectories α_j and β_j are identified for $j = 1, 2$. This is the so-called **diagonal approximation**. The shadowing theorem ensures that the identification of classical trajectories α and β is always possible and that the resulting action difference can be obtained from the accumulated phase of the perturbation along one of the trajectories. For instance, in the case where Σ only depends on position coordinates

$$\Delta R_\gamma = -\kappa \int_0^t \mathrm{d}\tau \; \Sigma(\mathbf{q}_\gamma(\tau)) \; , \tag{5}$$

for $\gamma = \alpha_1, \alpha_2$. The diagonal approximation reduces Eq. (3) to a sum over only two trajectories, α_1 and α_2, of the unperturbed system.

(iii) A key step that allows to make further progress in the semiclassical calculation is to regroup the resulting pair of trajectories (α_1, α_2) into two different families:

- Trajectories where $\mathbf{q}_5$ is near $\mathbf{q}_6$ and α_1 evolves close to α_2.
- The rest of pairs, where the two trajectories are uncorrelated.

The semiclassical calculation usually proceeds by estimating the accumulated phases along the trajectories for the two kinds of resulting pairs, and then performing some kind of average of $M(t)$ (over the perturbation, the initial conditions or the evolution of the classical trajectories). In the case of complex classical dynamics these averages are justified and allow to obtain the mean value $\overline{M(t)}$ of the Loschmidt echo in terms of the main parameters of problem.

An alternative semiclassical formulation for fidelity amplitude $m(t)$ (defined throught $M(t) = |m(t)|^2$) which avoids the usual trajectory-search problem of the standard semiclassics, is the so called dephasing representation

$$m(t) = \int \mathrm{d}\mathbf{q}\,\mathrm{d}\mathbf{p} \; \exp\left(-\frac{i}{\hbar}\Delta R(\mathbf{q},\mathbf{p},t)\right) \; W_0(\mathbf{q},\mathbf{p}) \tag{6}$$

where

$$W_0(\mathbf{q},\mathbf{p}) = \frac{1}{(2\pi\hbar)^d} \int \mathrm{d}(\delta\mathbf{q}) \; \exp\left(-\frac{i}{\hbar}\mathbf{p}\cdot\delta\mathbf{q}\right) \; \langle \mathbf{q} + \frac{\delta\mathbf{q}}{2}|\psi_0\rangle\langle\psi_0|\mathbf{q} - \frac{\delta\mathbf{q}}{2}\rangle \tag{7}$$

is the Wigner function of the initial state $|\psi_0\rangle$ and

$$\Delta R(\mathbf{q},\mathbf{p},t) = \int_0^t \mathrm{d}\tau[\mathcal{L}_2(\bar{\mathbf{q}}(\tau),\bar{\mathbf{p}}(\tau)) - \mathcal{L}_1(\bar{\mathbf{q}}(\tau),\bar{\mathbf{p}}(\tau))] \tag{8}$$

is the action difference evaluated along the phase-space trajectory $\big(\bar{\mathbf{q}}(\tau),\bar{\mathbf{p}}(\tau)\big)$ evolved from $\big(\mathbf{q},\mathbf{p}\big)$ under the unperturbed Hamiltonian H_1. In this way the decay can be attributed to the dephasing produced by the perturbation of the actions –thus the name dephasing representation. In the case where Σ only depends on position coordinates (as in Eq. (5)) the phase difference (8) reads

$$\Delta R(\mathbf{q},\mathbf{p},t) = -\kappa \int_0^t \mathrm{d}\tau \; \Sigma(\bar{\mathbf{q}}(\tau)) \; . \tag{9}$$

For generic chaotic systems and initially localized states, the calculation of $M(t)$ using the dephasing representation follows the same lines of the standard semiclassics. The standard semiclassical approach becomes an initial-value problem once the integration over the final position is traded by the integration over the initial momentum using the stability factor C_γ as the Jacobian of the transformation. However, Eq. (6) bears the advantage that it regards $m(t)$ as the solution of an initial-value problem, as opposed to a boundary-value problem. This proves especially convenient in situations when the sums over classical trajectories are evaluated explicitly.

2.2.2 Random matrix theory

Refs. [Cucc 02a, Cerr 03, Gori 04, Stoc 04, Stoc 05, Kohl 08, Kohl 11, Kohl 12]

Random matrix theory is a powerful technique to understand the statistical behavior of quantum complex systems. Among the latter quantum systems exhibiting chaotic dynamics in the classical limit are particularly important, as their spectral properties are well described by averages taken over appropriate ensembles of matrices that satisfy certain general symmetry restrictions. The simplest Hamiltonian ensemble is that of real symmetric $N \times N$ matrices (appropriate for cases with time-reversal symmetries) usually referred to as Gaussian Orthogonal Ensemble.

The invariance of the probability distribution under orthogonal transformations and the statistical independence of the matrix elements lead to a joint probability distribution of the matrix elements of the Hamiltonian,

$$P_N(H) = K_N \exp\left(-\frac{\mathrm{tr}\,\{H^2\}}{4v}\right), \tag{10}$$

where v defines the scale of the matrix elements and K_N is a normalization constant. Each of the $N(N+1)/2$ independent matrix element H_{ij} is a zero-centered Gaussian such that

$$\begin{aligned} \overline{H_{ij}} &= \quad 0 \qquad\quad i \leq j, \\ \overline{H_{ij}^2} &= (1+\delta_{ij})v^2 \quad i \leq j. \end{aligned}$$

The averages are taken over the matrix ensemble. The Loschmidt echo in complex systems can be addressed by imposing the Random Matrix hypothesis for the forward evolution Hamiltonian H_1, for the perturbation $\kappa\Sigma$ representing the imperfection of the time-reversal, or for both. Generically, the same results are obtained, independently of the choice of the matrix considered to be random. Averages of the Loschmidt echo over matrix ensembles are justified for ergodic classical dynamics. The absence of finite classical time scales, like the inverse Lyapunov exponent or the escape rate, in the Random matrix theory hinders the description of the Loschmidt echo decays that depend on those quantities.

Recently, using the Random-Matrix-Theory approach, Kohler and coworkers have established relations between the averaged fidelity decay and the so-called cross-form factor, characterizing parametric correlations of the energy spectra of the unperturbed and perturbed systems. Notably, these relations exist not only in the case of fully chaotic unperturbed and perturbed systems, but also in the case of a regular system perturbed by a chaotic perturbation.

2.2.3 Numerical simulations

Refs. [Tal 84, De R 96, Van 11]

A numerical simulation of the time-evolution of initially localized wave-packets under slightly different Hamiltonians, H_1 and H_2, is a valuable tool in understanding the behavior of the Loschmidt echo in different systems and various regimes. Even though there exist numerous approaches to solving the time-dependent Schrödinger equation on a computer, the majority of numerical studies of the Loschmidt echo have been concerned with the following two methods. Both methods provide accurate, efficient, and stable approximations to the evolution operator $e^{-iA\tau}$, where the operator A corresponds a (properly rescaled) Hamiltonian, and τ denotes a sufficiently short propagation time-step.

- *Trotter-Suzuki algorithm.* – This method involves three implementation stages. First, one decomposes A into a finite (and practically small) number of components, $A = A_1 + A_2 + \ldots + A_n$, such that the operator $e^{-iA_j t}$ can be constructed analytically for all $j = 1, 2, \ldots, n$. Second, one defines the symmetric operator $U_2(\tau) = e^{-iA_n\tau/2} \ldots e^{-iA_2\tau/2} e^{-iA_1\tau} e^{-iA_2\tau/2} \ldots e^{-iA_n\tau/2}$. Third, one constructs the operator $U_4(\tau) = U_2(p\tau)U_2(p\tau)U_2((1-4p)\tau)U_2(p\tau)U_2(p\tau)$ with $p = 1/(4-4^{1/3})$. This provides a unitary approximation to the original propagator accurate up to the 4th order in τ, i.e., $e^{-iA\tau} = U_4(\tau) + \mathcal{O}(\tau^5)$.

- *Chebyshev-polynomial expansion.* – This approach requires that the operator A is normalized in such a way that all its eigenvalues lie in the interval between -1 and 1. The method involves constructing the operator $S_N(\tau) = J_0(\tau) + 2\sum_{n=1}^{N}(-i)^n J_n(\tau) T_n(A)$, where $\{J_n\}$ and $\{T_n\}$ are, respectively, the Bessel functions of the first kind and the Chebyshev polynomials of the first kind. In actual computations, one makes use of the recurrence relation $T_{n+1}(A) = 2AT_n(A) - T_{n-1}(A)$, with $T_0(A) = 1$ and $T_1(A) = A$, to calculate $S_N(\tau)$. Then, for large values of n and fixed τ, the Bessel function $J_n(\tau)$ rapidly decays as a function of n. In fact, $|J_n(\tau)| \sim (\tau/2)^n/n!$ as $n \to \infty$. This is why $S_N(\tau)$, with a sufficiently large N, provides an extremely accurate approximation to the propagator in question, i.e., $e^{-iA\tau} = S_N(\tau) + \mathcal{O}\big((\tau/2)^N/N!\big)$.

2.3 Decay of the Loschmidt echo: different regimes

The decay of the Loschmidt echo mainly depends upon the underlying classical dynamics of the system, the initial state, and the nature and strength of the perturbation $\kappa\Sigma$.

The behavior of the Loschmidt echo is fairly well understood for single-particle quantum systems whose dynamics is fully chaotic in the classical limit. Some progress has been done towards the theory of the Loschmidt echo in systems with regular and mixed phase space.

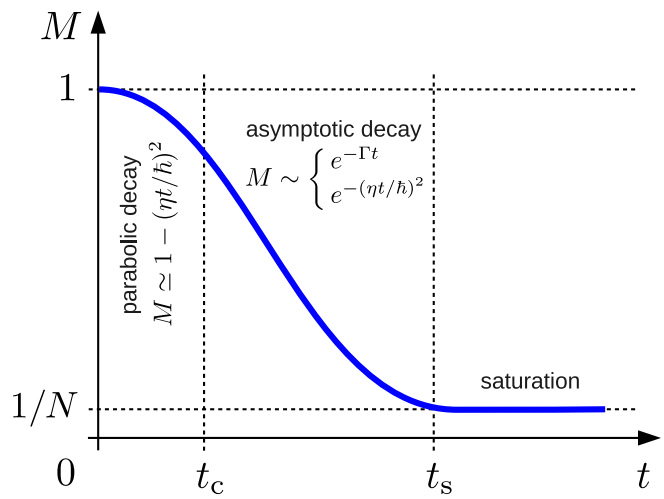


Figure 3: Decay regimes of the Loschmidt echo in quantum systems with chaotic classical limit. Characteristic crossover times are $t_c \simeq \hbar/\eta$, signaling the end of the initial parabolic decay, and t_s ($\simeq \Gamma^{-1} \ln N$ for the non-perturbative/exponential regime) indicating the onset of the saturation.

2.3.1 Chaotic dynamics

Refs. [Jala 01, Jacq 01, Cerr 02, Wisn 02, Cerr 03, Wisn 03, Cucc 04, Gori 04, Guti 09]

The time-decay of the Loschmidt echo, averaged over an ensemble of initial states, Hamilton operators, or perturbations, typically exhibits three consecutive stages (see Fig. 3):

- **Short-time parabolic decay**, $\overline{M(t)} \simeq 1 - (\eta t/\hbar)^2$. It is a short, initial stage of the Loschmidt echo decay in all quantum systems. Here, η is an average dispersion of the perturbation operator evaluated with respect to the initial state, i.e., $(\eta/\kappa)^2 = \overline{\langle\psi|\Sigma^2|\psi\rangle} - \overline{\langle\psi|\Sigma|\psi\rangle^2}$. The parabolic decay holds for times t short enough for the propagators of the unperturbed and perturbed systems, $\exp(-iH_j t/\hbar)$ with $j = 1, 2$, to be reliably approximated by their second order Taylor expansions, $1 - iH_j t/\hbar - (H_j t)^2/(2\hbar^2)$.

- **Intermediate-time asymptotic decay**. The short-time parabolic decay is typically followed by an "asymptotic" decay regime, whose functional form depends on the strength of the perturbation.

  - **Perturbative/Gaussian regime**, $\overline{M(t)} \simeq \exp(-(\eta t/\hbar)^2)$. This decay holds for "weak" Hamiltonian perturbations, such that the absolute value of a characteristic matrix element of the perturbation operator is small compared to the mean energy level-spacing of the unperturbed Hamiltonian. Since $\eta \sim \kappa$, the decay rate is quadratic in the perturbation strength.
  - **Non-perturbative/Exponential regime**, $\overline{M(t)} \simeq \exp(-\Gamma t)$. This non-perturbative decay regime is typically observed for stronger Hamiltonian perturbations, i.e., for perturbations large on the scale of the mean level spacing of the unperturbed Hamiltonian. The functional form of the dependence of the decay rate Γ on the perturbation strength κ is different for "global" and "local" Hamiltonian perturbations (see below).

- **Long-time saturation**, $\overline{M(t)} \sim N^{-1}$. At long times, the asymptotic decay of the Loschmidt echo is followed by a saturation (or freeze) at a value inversely proportional to the size N of the effective Hilbert space of the system. N is given by the volume of the phase space, that is available to the state of the system in the course of its time evolution, divided by the volume of the Planck's cell, $(2\pi\hbar)^d$. The value, at which the Loschmidt echo saturates, is independent of the perturbation strength.

  An explicit expression is available for the saturation value of the Loschmidt echo in two-dimensional quantum billiards, with strongly chaotic dynamics in the classical limit. Thus, if the initial state of the particle is given by the Gaussian wave function $\psi(\mathbf{q}) = (\pi\sigma^2)^{-1/2} \exp\left(i\mathbf{p}_0(\mathbf{q} - \mathbf{q}_0)/\hbar - (\mathbf{q} - \mathbf{q}_0)^2/(2\sigma^2)\right)$ with $\mathbf{q}_0$, $\mathbf{p}_0$, and σ denoting respectively the average position, momentum, and position uncertainty of the particle, and the area of the billiard is A, then the long-time saturation of the average Loschmidt echo is given by $\overline{M(t)} \simeq \sqrt{2\pi}\,\hbar\sigma/(|\mathbf{p}_0|A)$. This formula holds in the semiclassical regime, such that $\hbar/|\mathbf{p}_0| \ll \sigma \ll \sqrt{A}$.

The above classification is schematically illustrated in Fig. 4.

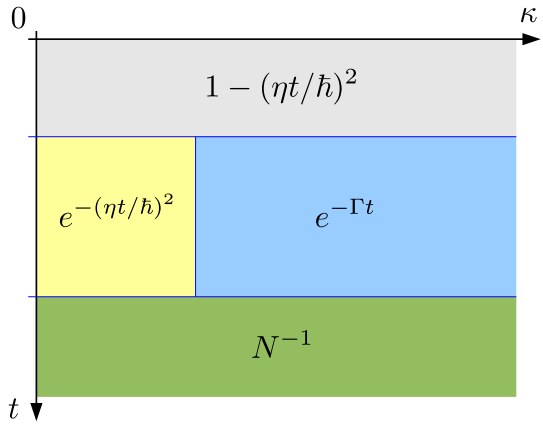


Figure 4: Typical regimes of the Loschmidt echo decay in chaotic systems.

Global perturbations

Refs. [Jala 01, Jacq 01, Cucc 02b, Cucc 04, Pros 02b]

A Hamiltonian perturbation is said to be "global" if it affects all (or a dominant part) of the phase-space that is accessible to the system in the course of its time-evolution. The accessible phase-space depends on the initial state of the system.

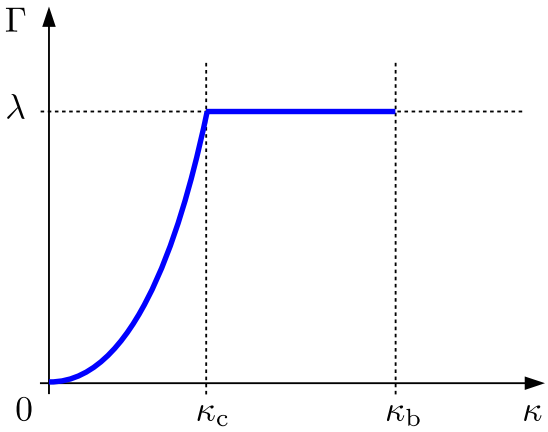


Figure 5: Exponential decay rate Γ as a function of the perturbation strength κ in the case of a global Hamiltonian perturbation.

For global perturbations, the dependence of the decay rate Γ on the perturbation strength κ can be approximated by a piecewise-continuous function, shown schematically in Fig. 5. More precisely, the semiclassical analysis of the exponential decay regime of the Loschmidt yields

$$\overline{M(t)} = e^{-c\kappa^2 t} + b(\kappa, t)\ e^{-\lambda t}\ . \tag{11}$$

The first term in Eq. (11) stems from uncorrelated pairs (α_1, α_2) of contributing trajectories, while the for second term is a contribution of correlated trajectory pairs, such that α_1 evolves close to α_2 (see Sec. 2.2.1). Unlike the constant c, the prefactor b generally exhibits an explicit dependence on both t and κ. However, this dependence is sub-exponential, and, as a consequence, the decay rate Γ of the Loschmidt echo is dominated by the minimum of $c\kappa^2$ and λ, leading to a commonly accepted (but oversimplified) interpretation of the exponential decay (see Fig. 5).

For perturbation strengths κ that are weak compared to a critical strength κ_c, the dependence is parabolic, $\Gamma = c\kappa^2$. The semiclassical calculations give the expression of c in terms of correlation functions of the perturbation, and the random matrix approach leads to the general dependence $c\kappa^2 = 2\pi\eta^2/(\hbar\Delta)$, where η is given by the mean value of the width of the local density of states with respect to the initial state $|\psi_0\rangle$. Such a behavior is commonly referred to as the **Fermi-golden-rule regime**.

For perturbations that are stronger than κ_c, but weaker than a certain bound value κ_b, the decay rate Γ is independent of κ and equals the average Lyapunov exponent λ of the underlying classical system. This decay

is known as the **perturbation-independent** or **Lyapunov regime**. This regime holds for perturbations up to the breakdown strength $\kappa_{\rm b}$, beyond which it is no longer possible to regard every trajectory of the perturbed system as a result of a continuous deformation of the corresponding unperturbed trajectory, so that bifurcations have to be taken into account. The Lyapunov regime is especially remarkable, as the decay rate is totally independent of the strength of the perturbation which is at the origin of the decay.

The exponential decay of the Loschmidt echo is generally followed in time by a saturation at a value on the order of N^{-1}, where N is the size of the effective Hilbert space. This implies that, in the non-perturbative regime, the saturation occurs at time $t_{\rm s} \simeq \Gamma^{-1} \ln N$. In the case of a two-dimensional chaotic billiard (introduced above), the saturation time is given by $t_{\rm s} \simeq \Gamma^{-1} \ln \big(|\mathbf{p}_0| A/(\hbar\sigma)\big) - (2\Gamma)^{-1} \ln(2\pi)$. It is interesting to note that the saturation time can, in principle, be arbitrarily long. In particular, $t_{\rm s}$ can exceed other important time scales of quantum dynamics, such as the Heisenberg time $t_{\rm H} \equiv \hbar g(E)$ with $g(E)$ denoting the density of states. Indeed, in two-dimensional billiards the saturation time $t_{\rm s} \to \infty$ as $|\mathbf{p}_0| \to \infty$, while the Heisenberg time $t_{\rm H}$ is independent of the particle's momentum.

The exponential decay, $\overline{M(t)} \simeq \exp(-\Gamma t)$, only holds for perturbations weaker than $\kappa_{\rm b}$. Beyond this threshold, the exponential regime breaks down and gives way to another regime, in which the Loschmidt echo exhibits a Gaussian dependence on time.

Some of the mentioned regimes can be obtained using alternative approaches relating the Loschmidt echo with the two-point time auto-correlation function of the generator of the perturbation.

Local perturbations

Refs. [Gous 07, Gous 08, Hohm 08, Ares 09, Kobe 11]

A Hamiltonian perturbation is said to be "local" if it is concentrated in a small region of the phase space accessible to the system. In the case of chaotic dynamics, the phase space extent of a local perturbation can be characterized by a rate γ, known as the "escape" rate, that is defined as the rate at which trajectories of the corresponding classical system visit the perturbation region. For local perturbations, the escape rate is small compared to the characteristic rate at which a typical trajectory "explores" the dynamically available phase space, and also small compared to the average Lyapunov exponent of the system.

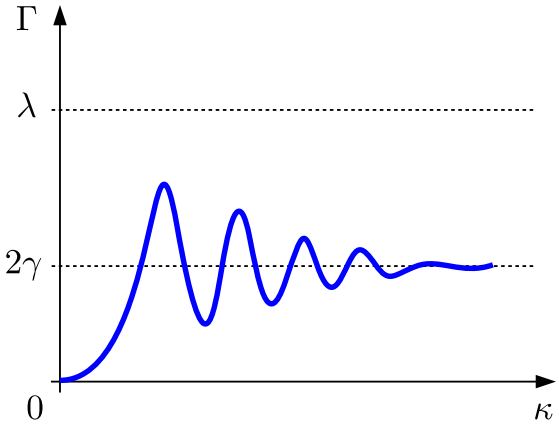


Figure 6: Exponential decay rate Γ as a function of the perturbation strength κ in the case of a local Hamiltonian perturbation.

Figure 6 illustrates the characteristic dependence of the decay rate Γ on the perturbation strength κ. The function $\Gamma(\kappa)$ exhibits a number of distinctive features. Thus, for sufficiently weak perturbations, the decay rate grows quadratically with the perturbations strength, $\Gamma \sim \kappa^2$, demonstrating the **Fermi-golden-rule regime**. In the limit of strong perturbations, $\kappa \to \infty$, the decay rate Γ saturates at a perturbation-independent value 2γ. This saturation is known as the **escape-rate regime**. The crossover from the Fermi-golden-rule to the escape rate regime is non-monotonic, and $\Gamma(\kappa)$ generally exhibits well-pronounced **oscillations**. The amplitude and frequency of these oscillations depend on the nature and physical properties of the particular Hamiltonian perturbation. An important point is that the function $\Gamma(\kappa)$ is given for all strengths κ by the width of the local density of states.

The exponential decay, $\overline{M(t)} \simeq \exp(-\Gamma t)$ with the non-monotonic function $\Gamma(\kappa)$ described above and illustrated in Fig. 6, has been obtained by semiclassical analysis. (Its existence has now been observed in numerical and laboratory experiments). The key assumption of any semiclassical reasoning is that the de Broglie wavelength corresponds to the shortest length scale of the system. This assumption is obviously violated for systems, in which the spatial linear size of the Hamiltonian perturbation is small compared to the de Broglie wavelength.

Generally, the decay of the Loschmidt echo in such systems is different from the exponential decay. For instance, two-dimensional chaotic systems in the limit of point-like Hamiltonian perturbations, for which $\gamma \to 0$, exhibit the algebraic, inverse-quadratic decay of the Loschmidt echo, $\overline{M(t)} \sim t^{-2}$.

2.3.2 Regular and mixed dynamics

Refs. [Pros 02a, Pros 03, Jacq 03, Sank 03, Wein 05, Wang 07, Gous 11]

Time dependence of the Loschmidt echo in quantum system with regular or mixed classical dynamics is typically more complex than that in fully chaotic systems. As a result of this complexity, a comprehensive classification of decay regimes is still lacking. Nevertheless, a number of important results have been established for the Loschmidt echo decay in systems whose phase space is predominantly regular in the classical limit.

In the limit of weak Hamiltonian perturbations, $\kappa \to 0$, the average Loschmidt echo generally exhibits a **Gaussian decay**. The duration $t_{\rm G}$ of this decay is inversely proportional to the perturbation strength, $t_{\rm G} \sim \kappa^{-1}$. The existence of the Gaussian decay requires the perturbation to be sufficiently weak such that the decay time $t_{\rm G}$ is long in comparison with any relaxation (averaging) time scale of the system.

If the Hamiltonian perturbation is sufficiently strong and varies rapidly (or is almost "random") along a typical classical trajectory of the unperturbed Hamiltonian, then the average Loschmidt echo is known to exhibit the **algebraic decay** $\overline{M(t)} \sim t^{-3d/2}$, with d being the dimensionality of the system. This power-law decay is faster than the decay of the overlap of the corresponding classical phase-space densities, t^{-d}.

Numerous case studies of the unaveraged (individual-realization) Loschmidt echo $M(t)$ have revealed that the functional form of the decay depends strongly upon the location of the initial quantum state with respect to the phase space of the underlying, unperturbed and perturbed, classical systems. In addition, the decay is sensitive to certain properties of the Hamiltonian perturbation. For instance, the Loschmidt echo decays differently depending on whether the perturbation is global or local in phase space, whether it vanishes under averaging over time, or whether it preserves or destroys integrability of the underlying classical system. A number of different regimes, such as the **exponential decay** and power-law decays with varies decay exponents, have been observed in numerical simulations. Interesting phenomena of **quantum revivals**, closely related to (quasi-)periodicity of the underlying classical motion, and temporary **quantum freeze** of the Loschmidt echo have been reported in various studies. However, a comprehensive classification of all decay regimes of the Loschmidt echo in regular systems still remains a challenge in the field.

2.3.3 Numerical observation of various decay regimes

Refs. [Jacq 01, Cucc 02a, Cucc 02b, Cucc 04, Gous 07, Gous 08, Ares 09, Garc 11b]

Numerical simulations have been of foremost importance for establishing the results summarized in sections 2.3.1 and 2.3.2. The main body of numerical work has been devoted to one-particle systems. Various chaotic dynamical systems, globally perturbed, have been studied numerically; among them the Lorentz gas, two-dimensional hard-wall and soft-wall billiards. As a prominent example, Fig. 7 shows the crossover from the perturbative Fermi-golden-rule regime to the Lyapunov regime of the decay rate of the Loschmidt echo as a function of the perturbation strength in the Lorentz gas. The predictions for local perturbations, described in Sec. 2.3.1, were also tested in a chaotic billiard system. In this case, a deformation of a small region of the boundary was considered as the perturbation (see Fig. 8).

Other enlightening numerical studies were carried in quantum maps. These systems possess all the essential ingredients of the chaotic dynamics and are, at the same time, extremely simple from a numerical point of view, both at the classical and at the quantum levels. The predictions for locally perturbed chaotic systems were also observed in the paradigmatic cat map. Figure 9 shows the decay rate of the Loschmidt echo as a function of the perturbation strength κ in the cat map under the action of a local perturbation. When the map was perturbed in all its phase space, three decay regimes (instead of two) were observed depending on the perturbation strength: (i) the Fermi-golden-rule regime for weak perturbations, (ii) a regime of an oscillatory dependence of the decay rate on κ at intermediate perturbation strengths, and (iii) the Lyapunov regime for strong perturbations. The complete understanding of the crossover between the last two regimes remains an open problem.

2.3.4 Beyond the "standard" picture: fluctuations, disorder, many-body systems

Refs. [Adam 03, Silv 03, Peti 05, Manf 06, Quan 06, Pozz 07, Manf 08, Zang]

Decay regimes of the average Loschmidt echo, discussed above, are the ones most commonly observed, and generally regarded as "standard", in low-dimensional quantum systems. However, when analyzing the Loschmidt echo beyond a simple averaging over the initial state or perturbation, or in a more complex setup, one often encounters departures from the standard picture.

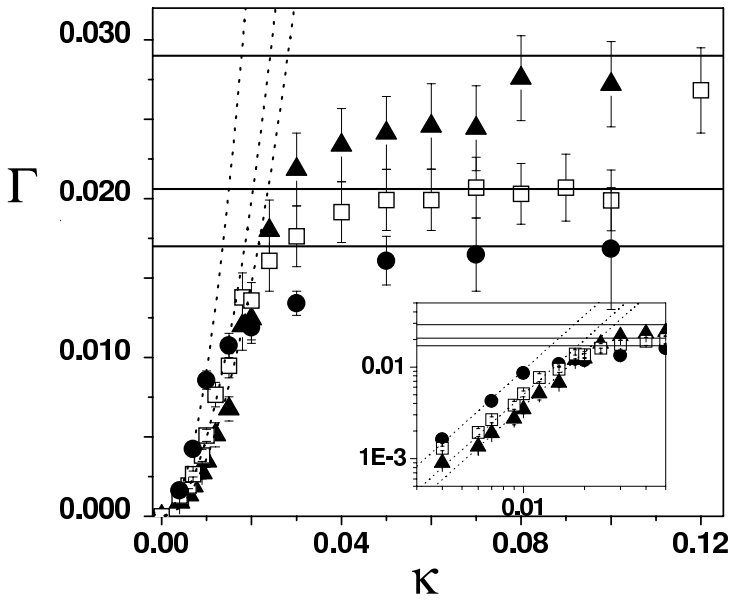


Figure 7: Decay rate of the Loschmidt echo as a function of the perturbation strength κ for the three concentrations of impurities in a Lorentz gas. The rates Γ are normalized to the group velocity of the initial wave packet. The solid lines are the corresponding classical Lyapunov exponents and the dashed lines are fits to the quadratic behavior. Inset: a log-log scale of the same data showing the quadratic increase of Γ for small perturbations. Adapted from Ref. [Cucc 04]. Copyright (2004), American Physical Society.

It was fist pointed out by Silvestrov and coworkers that, for sufficiently strong perturbations and on time scales short compared to the Ehrenfest time, statistical fluctuations generally play an important role in the problem of the Loschmidt echo decay. In particular, the average Loschmidt echo, $\overline{M(t)}$, is typically dominated by rare fluctuations, characteristic of only a small fraction of the chaotic phase space. In the main part of the phase space, however, the unaveraged Loschmidt echo, $M(t)$, decays much faster than $\overline{M(t)}$: this decay can be as fast as double-exponential, $M(t) \sim \exp(-\text{constant} \times e^{2\lambda t})$. It is only after the Ehrenfest time that $M(t)$ follows $\overline{M(t)}$ in the main part of the phase space.

The variance of the Loschmidt echo, $\overline{M^2} - \overline{M}^2$, was addressed by Petitjean and Jacquod within both a semiclassical and a Random Matrix Theory approach. The variance was shown to exhibit a rich nonmonotonous dependence on time, characterized by an algebraic growth at short times followed by an exponential decay at long times.

In disordered systems, the Loschmidt echo may exhibit even richer decay than that in low-dimensional dynamical systems, since the elastic mean-free path and the disorder correlation length enter as relevant scales. For long-range disorder (small-angle scattering) an asymptotic regime governed by the classical Lyapunov exponent emerges. A Josephson flux qubit operated at high energies leads to a chaotic dynamics, and the Lyapunov regime can be obtained when a Loschmidt setup is considered in such a system.

The Loschmidt echo studies in many-body systems are comparatively less developped than those in one-body cases. The numerical calculation of the many-body Loschmidt echo is a highly demanding computational task, and the standard approximations are in difficulty for describing the small difference obtained after the forward and backward time-evolutions. For this reason, the most reliable results of Loschmidt echo in many-body systems are those of a one dimensional spin chain or in an Ising model with transverse field, where the influence of criticality in the fidelity decay has been established. Trapped Bose-Einstein condensates have been treatead as a many-body Loschmidt echo setup using mean-field approaches.

2.4 Phase-space representation and classical fidelity

Refs. [Bene 02, Bene 03, Eckh 03, Garc 03, Cucc 04, Vebl 04]

An intuitively appealing representation of the Loschmidt echo in phase space is obtained by expressing Eq. (1) as

$$M(t) = \frac{1}{(2\pi\hbar)^d} \int \mathrm{d}\mathbf{q} \int \mathrm{d}\mathbf{p}\; W_{H_1}(\mathbf{q},\mathbf{p};t)\; W_{H_2}(\mathbf{q},\mathbf{p};t)\;, \tag{12}$$

where W_{H_1} and W_{H_2} are the Wigner functions resulting from the evolution of $W_0(\mathbf{q},\mathbf{p})$, introduced in Eq. (7), under the action of the Hamiltonian operators H_1 and H_2 respectively. This approach is naturally connected

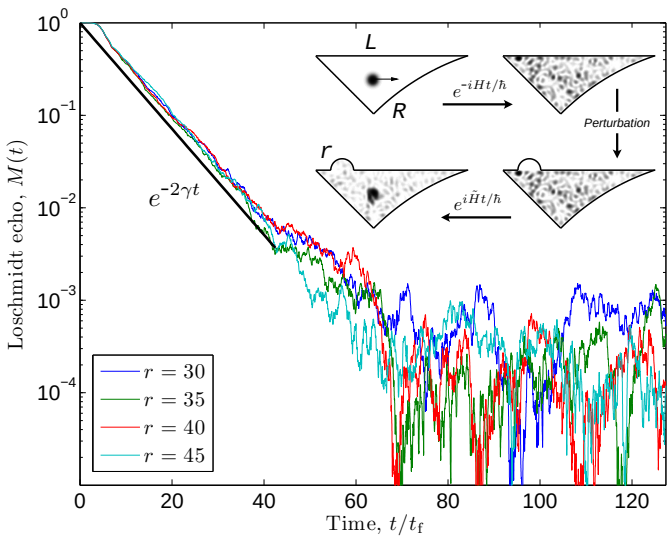


Figure 8: The Loschmidt echo decay in the desymmetrized diamond billiard for four different values of the curvature radius r of the arc boundary deformation. The solid straight line gives the trend of the $\exp(-2\gamma t)$ decay. Inset: Forward-time wave packet evolution in the unperturbed billiard, followed by the reversed-time evolution in the perturbed billiard. The arrow shows the momentum direction of the initial wave packet. The propagation time corresponds to approximately 10 collisions of the classical particle. Adapted from Ref. [Gous 07]. Copyright (2007), American Physical Society.

to the dephasing representation of Sec. 2.2.1 and provides a particularly useful framework in the study of decoherence, as the Wigner function is a privileged tool to understand the connection between quantum and classical dynamics.

An initial Gaussian wave-packet is associated with a Gaussian Wigner function that will develop in phase-space non-positive structures and be deformed under the evolution of H_1 and H_2. The sensitivity of the overlap (12) to the non-positive parts and to the deformations of the Wigner function can be related with the different decay regimes discussed in Sec. 2.3.1.

The form (12) of the quantum fidelity allows to define a classical fidelity by using in a classical problem the Liouville distributions L_{H_1} and L_{H_2} instead of W_{H_1} and W_{H_2} respectively. This definition follows from the logical representation of the Liouville distribution as the classical limit of the Wigner function. However, the definition by an overlap of distributions does not convey the sense of classical reversibility: a trajectory evolving forward in time with H_1 and backwards with H_2 would give a considerable contribution to the overlap not only if it ends up close to the initial point, but whenever the final point is in the neighborhood of those of the initial distribution. The labeling of trajectories and particles, characteristic of classical mechanics, is then not taken into account in this definition of classical fidelity.

The correspondence principle dictates that the quantum fidelity follows its classical counterpart up to the Ehrenfest time. However, such a correspondence does not imply that, in the intermediate-time asymptotic decay of globally perturbed chaotic systems, the Lyapunov regime is only present up to this characteristic time scale. In fact, the Lyapunov regime is not simply an effect of the classical-quantum correspondence. For instance, in a two-dimensional billiard, the saturation time t_S (see Sec. 2.3.1) that would signal the end of the asymptotic decay (and so the end of the Lyapunov decay if one is in the appropriate regime) can be much larger than the Ehrenfest time.

3 Experiments

3.1 Nuclear Magnetic Resonance

Ref. [Slic 90] (see particularly Sec. 8.8 and App. E)

Nuclear magnetic resonance has been the main tool to study echoes generated by different time-reversal procedures since the fifties. After the time-reversal of *individual spin* precession implemented by Erwin Hahn in the **spin echo**, one had to wait until the seventies, when Rhim and Kessemeir achieved the reversion of the spin-spin interactions. The resulting reversal of the macroscopic polarization is known as **Magic Echo** and was thoroughly studied by Rhim, Pines and Waugh. In the nineties Ernst et al. implemented a strategy to address a localized spin excitation and Levstein et al. realized that such study was optimal for a theoretical

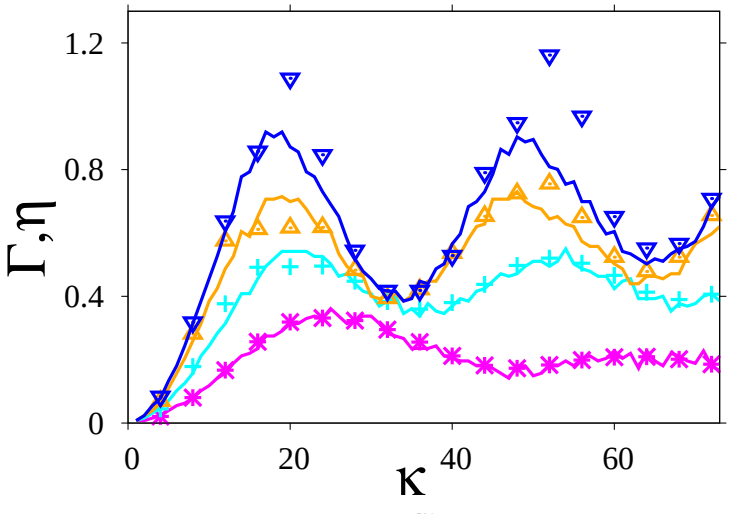


Figure 9: Decay Γ of the Loschmidt echo of a cat map perturbed locally with a shear in momentum as a function of the scaled perturbation strength κ. The symbols correspond to the following fraction of the phase space that is perturbed: $\bigtriangledown$ (40%), $\bigtriangleup$ (30%), $+$ (20%) and $*$ (10%). The width η of the local density of states is also plotted with solid lines. This width gives the spread of the states of the unperturbed system in the bases of the perturbed one It is noticeable that, the smaller is the perturbed region, the more similar to η Γ is. Adapted from Ref. [Ares 09]. Copyright (2009), American Physical Society.

description. They used crystal structures with abundant of interacting ^{1}H spins, where the dipolar coupling produces the "diffusion" of the initial polarization. The time-reversal procedure is then able to refocus the excitation generating a **Polarization Echo**. These last experiments have a direct connection with the concepts discussed in this review.

3.1.1 Basic concepts on spin dynamics

Refs. [Past 95, Madi 97, Levs 98, Dani 04, Zang]

A typical experiment starts with a sample which constitutes a network with about 10^{23} interacting spins in thermal equilibrium in presence of an external magnetic field B_0. This many-spin state is denoted as $|\Psi_{\rm eq}\rangle$. In this state every spin has an almost equal probability of being up or down since the thermal energy, k_BT, is much higher than any other relevant energy scale of the system. At time $t = 0$ a sequence of radiofrequency pulses ensures that spin at site 0th is oriented along z direction. This is represented by the action of the spin operator S_0^+ on $|\psi_{\rm eq}\rangle$. In a system with $m+1$ spins, we represent this initially excited state as

$$|\Psi_0\rangle = \frac{S_0^+|\Psi_{\rm eq}\rangle}{\left|\langle\Psi_{\rm eq}|\, S_0^- S_0^+ |\,\Psi_{\rm eq}\rangle\right|^{1/2}} = \sum_{r=1}^{2^m} \frac{e^{{\rm i}\phi_r}}{2^{m/2}} \,|\!\uparrow_0\rangle \otimes |\beta_r\rangle\,, \tag{13}$$

where the denominator $\left|\langle\Psi_{\rm eq}|\, S_0^- S_0^+ |\,\Psi_{\rm eq}\rangle\right|^{1/2}$ ensures that the initially excited state has a proper normalization and ϕ_r is a random phase that describe a mixture of states of the form

$$|\beta_r\rangle = |s_1\rangle \otimes |s_2\rangle \otimes |s_3\rangle \otimes ... \otimes |s_m\rangle \quad \text{with} \;\; |s_k\rangle \in \{|\!\uparrow\rangle\,, |\!\downarrow\rangle\}\,. \tag{14}$$

The preparation of this state involves the use of a ^{13}C nucleus as a "spy" to inject and detect polarization at the directly bonded ^{1}H spin (0th). Then, the system evolves under mutual many-body interaction described by H_1 for a period t_1. In the polarization echo experiment, this is described by effective dipolar interaction Hamiltonian H_1, truncated by the Zeeman field,

$$H_1 \underset{\substack{\text{Polarization}\\ \text{Echo Experiment}}}{\equiv} \sum_{i,j} \left[2S_i^z S_j^z - \frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) \right]. \tag{15}$$

H_1 contains flip-flop or XY operators of the form $S_i^+ S_j^- + S_i^- S_j^+$, as well as Ising terms of the form $S_i^z S_j^z$. The dipolar interaction $d_{i,j}$ constant decay with the third power of the distance between sites i and j. H_1 produces the spread of the initially localized excitation. Since total polarization is conserved under H_1, such process is commonly known as "spin diffusion". Then, a new pulse sequence rotates all the spins 90 degrees and

the irradiation with r.f. field produces a further truncation of the dipolar interaction along the rotating field. The resulting effective Hamiltonian $-H_2 = -(H_1 + \Sigma)$ acts for another period t_2. Observe that the reversal is not perfect but Σ is a small term which is due to truncations required to form the effective Hamiltonian. This constitutes the "backwards" evolution period after which the local polarization at site 0th is measured. We notice that Σ acts as a self-energy operator that may account for interactions within the same Hilbert space, and thus is just an Hermitian effective potential. However, it also may describe the interaction with a different subsystem that constitutes the "environment". In the last case $\Sigma = \Delta - \mathrm{i}\Gamma$ would also have an imaginary (non-Hermitian) component. In this last case, evolution of observables should be described by two coupled Lindblad or Keldysh equations for the density matrix requiring a self-consistent evaluation of dynamics of the system and the environment. This last situation would make the analysis less straightforward and might hinder the essential physics. Thus, we first focus on perturbations that ensure unitary evolution. An example discussed by Zangara et al. of this situation is a second frozen spin chain interacting with the first through an Ising interaction. In these cases, a polarization echo is formed when $t_2 = t_1$, i.e. after a total evolution time $2t = t_2 + t_1$. The normalized amplitude is given by the spin autocorrelation function that accounts for the observed local polarization,

$$M_{\mathrm{PE}}(t) = \frac{\langle \Psi_{\mathrm{eq}} | S_0^-(2t)\ S_0^+ | \Psi_{\mathrm{eq}} \rangle}{\langle \Psi_{\mathrm{eq}} | S_0^- S_0^+ | \Psi_{\mathrm{eq}} \rangle}\,, \tag{16}$$

where $S_0^-(2t) = e^{iH_1 t/\hbar} e^{-iH_2 t/\hbar} S_0^- e^{iH_2 t/\hbar} e^{-iH_1 t/\hbar}$ is the local spin operator, expressed in the Heisenberg representation respect to the acting Hamiltonian $H(\tau) = H_1 \theta(t-\tau) - H_2 \theta(\tau - t)$. An equivalent definition of $M_{\mathrm{PE}}(t)$ in terms of $S_0^z(2t)\ S_0^z$ was also proved useful.

The connection of this experimental observable with the concepts discussed so far are already hinted but not yet clarified by Eq. (16). Thus, we consider that the $m+1$ spins are arranged in a linear chain or in an odd sized ring where spin-spin interaction is restricted to XY terms acting on nearest-neighbors. In this case the Wigner-Jordan spin-fermion mapping allows to describe the spins system as a gas of non-interacting fermions in a 1-d lattice. In that problem, the initial excitation propagates as a single spinless fermion. In turn, the observed local polarization can be written back in terms of the evolution of an initially localized "spin wave":

$$|\psi_0\rangle = |\uparrow_0\rangle \otimes |\downarrow_1\rangle \otimes |\downarrow_2\rangle \otimes |\downarrow_3\rangle \otimes ... \otimes |\downarrow_m\rangle\ , \tag{17}$$

where $||\uparrow_n\rangle$ ($|\downarrow_n\rangle$) means that the n-th site is occupied (unoccupied). Thus, it is clear that the excitation at site 0-th can spread along the 1-d chain through the nearest neighbor flip-flop interaction. This correspondence between many-body dynamics and spin wave behavior was worked out in detail by Danieli et al. This procedure was originally employed by Pastawski et al. in the context of time-reversal experiments to predict the presence of Poincaré recurrences (Mesoscopic Echoes) which were clearly observed by Mádi et al. at the laboratory of Richard Ernst in Zurich. In this condition, the polarization detected after the time-reversal procedure results:

$$M_{\mathrm{PE}}(t) \underset{\substack{1d \text{ system} \\ \text{with } XY \text{ interaction}}}{\equiv} M(t) = \left| \langle \psi_0 | e^{iH_2 t/\hbar} e^{-iH_1 t/\hbar} | \psi_0 \rangle \right|^2 . \tag{18}$$

Notice that here $|\psi_0\rangle$ denotes a single particle wave function, there are no other operators than the propagators and the modulus square makes explicit the positivity of $M_{\mathrm{PE}}(t)$. These features that were not obvious in Eq. (16) and make it to agree with the definition of the Loschmidt echo of Eq. (1).

Many other effective many-body Halmiltonians can be experimentally reversed to obtain the revival of different observables that reduce to a detectable polarization. Thus it is now a common practice to losely call **Loschmidt Echo** to the polarization resulting from any of these possible spin dynamics reversal procedures, regardless the applicability of the single particle correspondence, with an explicit mention of the observable used.

3.1.2 Time-reversal of the dipolar many-spin interaction

Refs. [Levs 98, Usaj 98, Past 00, Zure 07]

The experiments have been carried out in different molecular crystals of the cyclopentadienyl (C_5H_5) family. There, ^{1}H nuclei are arranged in five fold rings with strong intra-ring dipolar interactions which nevertheless are not constrained to the molecule but also yield appreciable intermolecular couplings. Polarization can be injected on those ^{1}H which are close to a "spy" ^{13}C nucleus that acts as a local probe used to inject and eventually detect local polarization as represented in Fig. 10. Spreading of the initally localized spin excitation proceeds through a dipolar interaction which is truncated either in the basis of external magnetic field (Laboratory frame) or in the rotating frame of a radio frequency field. This choice is crucial in order to achieve the change of the sign and the eventual scale down of the effective Hamiltonian. Thus a spin dynamics with H_1 in the Laboratory frame is reversed by a dynamics of H_2 in the rotating frame or vice versa. The role of environment might be played by paramagnetic Co(II) or quadrupolar Mn nuclei that replace the Fe and thus they are absent for pure ferrocene

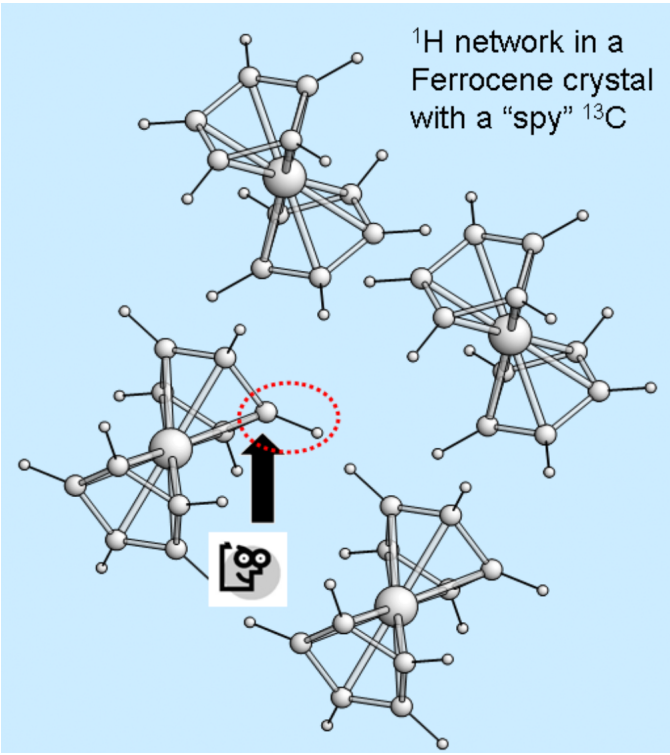


Figure 10: Ferrocene crystal showing the ^{1}H (small dots), C atoms (medium-sized dots) and Fe (large dots). A rare ^{13}C nucleus (usually called a "spy" nucleus), acting as a local probe, is shown by the arrow. Polarization evolves in the H lattice. Fe atoms at the center are not magnetically active, but when replaced by Co atoms relaxation of polarization is achieved. Adapted from Ref. [Levs 98]. Copyright (1998), American Institute of Physics.

crystals. Besides, in pure ferrocene truncation in the rotating frame give rise to non-inverted non-secular terms which, being supressed by the corresponding Zeeman energy, are at most of the order of a few percent of the matrix elements of H_2. This would yield an Hermitian Σ. The strength of these terms is inversely proportional to B_1, the strength of the r.f. pulse, and thus one has the possibility to experimentally reduce its importance by increasing the r.f. power.

The results showing the buildup of the Loschmidt echoes, under H_2 after different periods of evolution with H_1, are presented in Fig. 11. The universal features of the Loschmidt echo appear when one studies the maximum recovered polarization (Loschmidt Echo intensity) as a function of t, the evolution time before reversal. In Ferrocene crystals, once one substrates the background noise it is clear that the Loschmidt echo follows a clear Gaussian law,

$$M(t) = \exp\left[-\frac{1}{2}\left(\frac{t}{T_3}\right)^2\right], \tag{19}$$

for over two orders of magnitude and serves as a definition of the characteristic time T_3. The same decay law is observed when, using a specially tailored pulse sequence, the Hamiltonians are scaled down by a factor $n = 1, 2, 8$ and 16 respectively. i.e. the characteristic decay time is increased by the facton n. The scaling of $1/T_3$ with the Hamiltonian strength is also observed when H_1 and H_2 are scaled down by using different crystal orientations.

In contrast to Ferrocene, in Cobaltoncene crystals, one starts with a Gaussian decay, and as the dipolar Hamiltonian is scaled down, an exponential decay of the Loschmidt echo is developed and became stable with a characteristic time τ_{SE}. Thus, the overall behavior of the recovered local polarization is

$$M(t) = \exp\left[-\frac{1}{2}\left(\frac{t}{nT_3}\right)^2 - \frac{t}{\tau_{SE}}\right]. \tag{20}$$

One may assign this asymptotic exponential decay rate $1/\tau_{SE}$ to a Fermi Golden Rule decoherence rate induced by the perturbation induced by the paramagnetic nature of the Co(II) nuclei which acts as an environment. Further experiments by Pastawski et al. in crystals free of magnetic impurities confirmed the Gaussian decay with a rate $1/T_3$ that depends only weakly on the r.f. power and, when truncation terms becomes too small, saturates to an intrinsic value that scales with the strength of the reverted Hamiltonian.

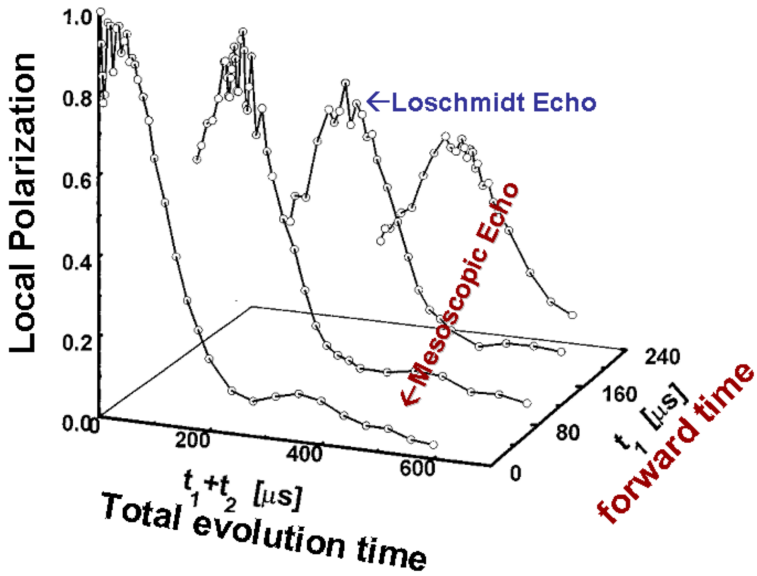


Figure 11: Formation and fading away of the Loschmidt echo with reversal time t_2 elapsed after different periods t_1 of forward evolution. Notice that the tails of the first Loschmidt Echo reflect the forward dynamics. At longer times there is a first revival asociated with the neighbors and a small revival associated with the Mesoscopic Echo resulting from the dynamics within the ring. Superimposed with the Loschmidt echo there are high frequency oscillations. These are coherent interferences resulting from an incomplete transfer between the ^{13}C, used as a local probe, and the nearby ^{1}H nuclei. Adapted from Ref. [Levs 98]. Copyright (1998), American Institute of Physics.

In summary, these experiments hinted that there is an intrinsic decoherence rate which is fixed by the inverted Hamiltonian. Thus, other than the difference between the Gaussian and exponential decay, one might say that $1/T_3$ plays a role similar to the Lyapunov exponent in the reversal of chaotic one body systems. This surprising finding suggested that, even in absence of any important perturbation, in the experimental limit of $m \to \infty$, even very small residual terms, in presence of the dynamical instability of a very complex many-body dynamics, are efficient enough to set the Loschmidt Echo decay into a perturbation independent decay regime. This plays the role of an "intrinsic decoherence" with a time scale determined by the inverted Hamiltonian.

It was precisely the hypothesis of an "intrinsic decoherence rate" what triggered the theoretical analysis of time-reversal in chaotic systems. Indeed, while dynamics of one body systems in 1D, described by Eq. (18), can not be chaotic, one deems that disorder and the more complex many-body dynamics present in higher dimensional systems, have mixing properties which makes them assimilable to chaotic systems. This observation led to G. Usaj et al. to propose that quantum chaos contains the underlying physics of a time-reversal experiment in a many-body systems. It was also argued that $M_{\rm PE}(t_R)$ constitutes an entropy measure. These ideas finally boiled down into the model proposed by Jalabert and Pastawski. While Gaussian decays appears quite naturally as a consequence of the large number statistics of many-spin states that are progressively incorporated into the dynamics, the perturbation independent decay have not found yet a straightforward explanation in this context.

3.1.3 Further time-reversal Experiments

Refs. [Levs 04, Sanc 07, Rufe 09, Sanc 09, Rama 11]

While time-reversal of different interactions is almost an unavoidable tool in experimental NMR techniques, there are not so many studies devoted to grasp at the origins of the (in)efficiency of such procedures. In particular, time-reversal was implemented using different procedures and systems that range from $3d$ and quasi-$1d$ crystals to molecules in oriented liquid crystalline phases. In these cases, also different initial states and effective Hamiltonians were studied.

Of particular interest are the double quantum (DQ) Hamiltonians,

$$H_1 \underset{\substack{\text{Loschmidt}\\ \text{Echo for } DQ}}{\equiv} H_{DQ} = \sum_{i,j} \widetilde{d}_{i,j} \left[S_i^+ S_j^+ + S_i^- S_j^- \right] . \tag{21}$$

The notation $\widetilde{d}$ for the interaction strength recalls at it is an effective Hamiltonians built up from dipole-dipole interaction by suitable pulse sequences whose effect is described with the help of the average Hamiltonian theory. Since H_{DQ} does no commute with the polarization described by $\sum_i S_i^z$, it gives a more mixing as dynamics

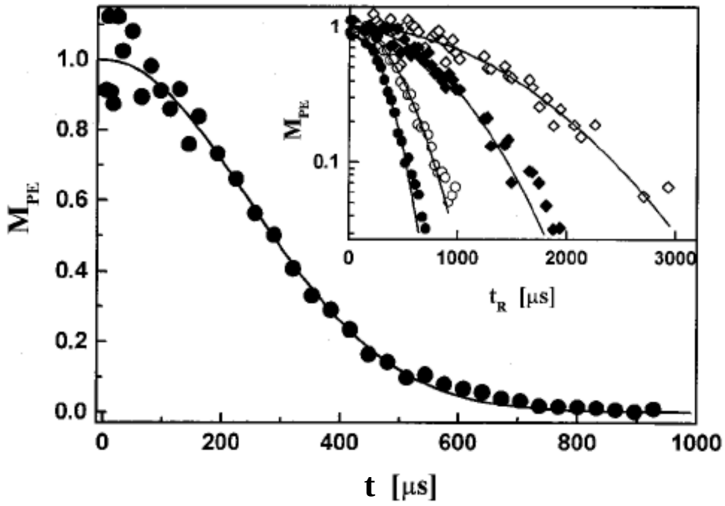


Figure 12: Attenuation of the polarization echo in the ferrocene single crystal as a function of t for $n = 1$ (REPE sequence). The line represents a Gaussian fitting. The inset shows the PE attenuation for progressively reduced dipolar dynamics, $n = 1$, 2, 8, and 16. No asymptotic regime is reached within the experimental timescale. The solid lines are Gaussian fittings. Adapted from Ref. [Usaj 98].

explores different subspaces of the Hilbert space inducing the multiple quantum coherences. Besides H_{DQ}, there are high order terms that act as an environment whose instantaneous decoherence could be described by a Fermi Golden Rule. One experimental observation is that the larger the number of subspaces correlated by the interaction (coherences of high order) the faster the decoherence.

One of the interesting properties of $1d$ systems, is that the dynamics induced by a H_{DQ} has a precise correspondence with an XY chain. Thus, the Loschmidt Echo is given by Eq. (18). In real systems, such as Hydroxyapatite (HAp), inter-chain interactions are significative, quantum Zeno effect induced by a strong dynamics along the chain prevents the development of coherences beyond second order and Eq. (18) remains valid. This explains why the Loschmidt echo observed by Rufeil-Fiori et al. for the double quantum Hamiltonian shows a Loschmidt echo with an exponential decay described by the Fermi golden rule.

This decay contrasts with results for an Adamantane crystal, a highly connected $3d$ system. There, a Fermi-function like decay is indicative of the two time scales. Each molecule has 16 spins which do not have direct interactions, thus remain independent for short times until they interact through neighbor molecules. In this case decoherence remains weak while intermolecular correlation builds up. Once neighboring molecules become fully coupled the density of directly connected states becomes very high and the Fermi-golden-rule controlled exponential decay takes over.

Spin dynamics in various liquid crystals was studied in series of papers whose aim was to reduce the number of interacting spins to those within each molecule. However, the experiences have shown that a number of residual interaction remain significative and this strongly compromises the effectiveness of the Loschmidt echo sequences.

3.2 Microwave billiards

Refs. [Scha 05a, Scha 05b, Hohm 08, Body 09, Kobe 10, Kobe 11]

Microwave-frequency electromagnetic waves in quasi-two-dimensional cavities are effectively governed by the Helmholtz equation, which is equivalent to the stationary Schrödinger equation. This equivalence allows one to address the Loschmidt echo in laboratory experiments with microwave billiards.

Quantities directly measured in microwave experiments are frequency-dependent scattering matrix elements, $S_{ab}(\nu)$ and $S'_{ab}(\nu)$, corresponding to the unperturbed and perturbed chaotic billiard systems, respectively. Here, the indices a and b refer to the antennae (or scattering channels) involved in the experiment. The change from the frequency domain to the time domain is achieved by the Fourier transforms, $\hat{S}_{ab}(t) = \int d\nu\, e^{-2\pi i\nu t} S_{ab}(\nu)$ and $\hat{S}'_{ab}(t) = \int d\nu\, e^{-2\pi i\nu t} S'_{ab}(\nu)$, performed over an appropriate frequency window. The sensitivity of a scattering process to system perturbations are then quantified by the *scattering fidelity amplitude*

$$f_{ab}(t) = \frac{\langle \hat{S}^*_{ab}(t)\hat{S}'_{ab}(t)\rangle}{\sqrt{\langle|\hat{S}_{ab}(t)|^2\rangle\langle|\hat{S}_{ab}(t)|^2\rangle}}\,, \tag{22}$$

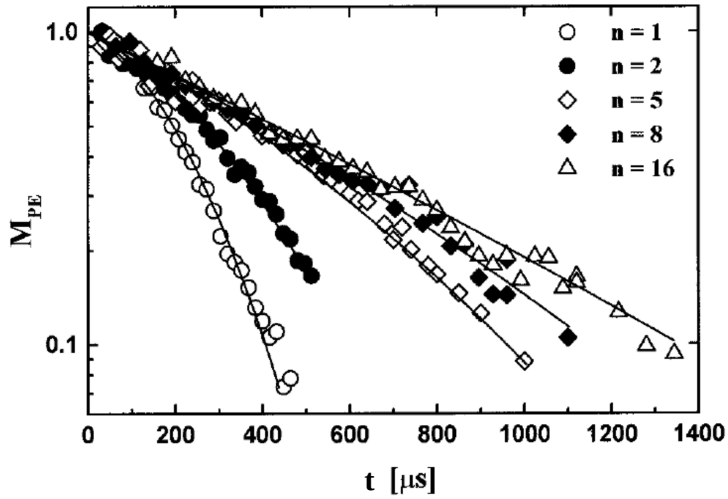


Figure 13: Attenuation of the polarization echo in the cobaltocene sample as a function of t for $n = 1$, 2, 5, 8, 16. The experimental data show a clear crossover between a dominant Gaussian attenuation to an exponential one. The solid lines represent fittings of the whole set of data to Eq. (20). Adapted from Ref. [Usaj 98].

where the asterisk stands for the complex conjugation, and the angular brackets denote an ensemble average over different realizations of the experiment (e.g., different positions of scatterers, antennae, etc.). The *scattering fidelity* itself is a real-valued quantity defined as $F_{ab}(t) = |f_{ab}(t)|^2$. The numerator in the right-hand-side of Eq. (22) quantifies correlations between scattering matrix elements of the unperturbed and perturbed system. The decay of the numerator however is dominated by the decay of the autocorrelations, so that the denominator is introduced to compensate for the autocorrelation contributions to the correlation decay.

In chaotic systems and in the case of a weak coupling of the measuring antennae to the system, the scattering fidelity is known to approach the Loschmidt echo for a random initial state. This makes microwave billiards well suited for experimental studies in the field. In particular, microwave studies provided compelling experimental evidence for the validity of some of the known decay regimes of the Loschmidt echo for global and local Hamiltonian perturbations.

3.3 Elastic waves

Refs. [Lobk 03, Gori 06]

Sound waves that travel through a elastic medium are multiple scattered by its inhomogeneities generating a slowly decaying wave diffusion. If no change occurs in the medium over time, the usually called coda waves are highly repeatable, that is, for identical excitations the waveforms are indistinguishable. While if the medium is perturbed, the change in the multiple scattered waves will result in an observable change in the coda waves. Lobkis and Weaver have measured the sensitivity to temperature changes of elastic coda waves in aluminum alloy blocks. They used the cross correlation function between two signals obtained at different temperatures T_1 and T_2

$$X(\varepsilon) = \frac{\int \mathrm{d}t\ S_{T_1}(t)\ S_{T_2}(t(1+\varepsilon))}{\sqrt{\int \mathrm{d}t\ S^2_{T_1}(t)\ \int \mathrm{d}t\ S^2_{T_2}(t(1+\varepsilon))}}\ . \tag{23}$$

to evaluate the distortion that is defined as $D(t) = -\ln(X_{\max})$, where the time dependence is given by the "age" of the signal. If formulated as a scattering process, it is shown that $D(t) = -\ln[f(t)]$, where $f(t)$ is the scattering fidelity that was introduced to analyze the fidelity decay in microwave billiards. For sufficiently chaotic dynamics in systems weakly coupled to decay channels, the scattering fidelity approaches the standard fidelity amplitude (the Loschmidt echo is the absolute square of the fidelity amplitude).

The results obtained in acoustic signals traveling in an aluminum blocks were understood using the random matrix predictions for the standard fidelity (see Fig. 17). A surprising and unexpected finding is that the scattering fidelity decay for blocks with chaotic and regular regular classical dynamics are explained by the same random matrix expressions.

3.4 Cold atoms

Refs. [Ande 03, Ande 04, Ande 06, Wimb 06, Mart 08, Wu 09, Kriv 11, Ulla 11, Dube 12]

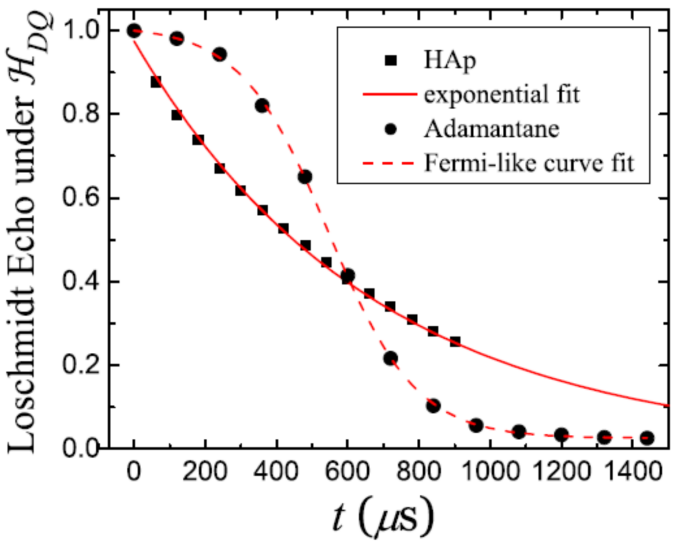


Figure 14: Loschmidt Echo experiment based on H_{DQ} and -H_{DQ}, for hydroxyapatite (HAp) (quasi-1d crystal) and Adamantane, a fully 3-d crystal. Here, the source of Σ are very small high order terms in the effective Hamiltonian.

The Loschmidt echo, or rather a quantity closely related to the Loschmidt echo, has been carefully studied in laboratory experiments with cold atoms trapped inside optical cavities. These experiments offer an atom-optics realization of the echo in two-dimensional quantum billiards with underlying chaotic or mixed classical dynamics. The basic idea underlying the experiments with atom-optics billiards is as follows. An effectively two-level atom, with the internal states denoted by $|1\rangle$ and $|2\rangle$, is initially prepared in a state $|1\rangle \otimes |\psi\rangle$, where $|\psi\rangle$ stands for the spatial component of the initial state. A laboratory observation of the echo involves first exposing the atom to a sequence of microwave-frequency electromagnetic pulses and then measuring the probability P_2 of finding the atom in the internal state $|2\rangle$.

In the protocol targeting the Loschmidt echo, $\left|\langle\psi|e^{iH_2t/\hbar}e^{-iH_1t/\hbar}|\psi\rangle\right|^2$, an atom is first irradiated with a so-called $\pi/2$ pulse, which changes the atomic state into an equiprobable superposition of $|1\rangle \otimes |\psi\rangle$ and $|2\rangle \otimes |\psi\rangle$. The pulse is practically instantaneous and introduces almost no change to the spatial component of the state. Then, the atom is left to evolve in an optical trap for a time t, during which components $|1\rangle$ and $|2\rangle$ of the atomic state propagate under Hamiltonians H_1 and H_2 respectively. The difference between H_1 and H_2 originates from a difference in dipole interaction potentials exerted upon the states $|1\rangle$ and $|2\rangle$ by the optical trap. Finally, another $\pi/2$ pulse is applied to the atom and the probability P_2 of finding the atom in the internal state $|2\rangle$ is measured; P_2 turns out to be a quantity closely related to the Loschmidt echo.

Realistic experiments however deal with thermal incoherent mixtures of initial states, rather than with pure states. As a result, the echo amplitude, due to an individual state of this thermal mixture, contributes to P_2 with an effectively random phase, smearing out the echo signal. In order to overcome this difficulty, Andersen, Davidson, Grünzweig, and Kaplan addressed a new echo measure, $\left|\langle\psi|e^{iH_2t/2\hbar}e^{iH_1t/2\hbar}e^{-iH_2t/2\hbar}e^{-iH_1t/2\hbar}|\psi\rangle\right|^2$, closely related to the Loschmidt echo. In their experiments, "two-level" atoms, initially prepared in the state $|1\rangle \otimes |\psi\rangle$, were successively exposed to (i) a $\pi/2$ pulse, (ii) evolution for a time $t/2$, (iii) a π pulse swapping the population of the internal states $|1\rangle$ and $|2\rangle$, (iv) another evolution for a time $t/2$, (v) a $\pi/2$ pulse, and finally (vi) a measurement of P_2. Under this protocol, each state of the thermal mixture contributed to the probability P_2 with the same phase. This allowed for a reliable measurement of the echo even for ensembles of more than a million of thermally populated states.

More recently, atom interferometry has also been used to investigate several aspects of time-reversal in quantum kicked rotor systems. Wu, Tonyushkin, and Prentiss studied the dynamics of laser-cooled rubidium atoms subjected to periodically pulsed optical standing waves, as an atom-optics realization of the kicked rotor. Their experiments have demonstrated that quantum fidelity of a system, that is chaotic in the classical limit, can survive strong perturbations over long time without decay. In a similar setup, Ullah and Hoogerland have performed an experiment that demonstrated the possibility of using time-reversal evolution for cooling atomic matter waves. The problem of fidelity decay in kicked atom-optics systems has recently drawn considerable interest among theoreticians.

3.5 Time-reversal mirrors

Refs. [Fink 99, Calv 08, Tadd 09]

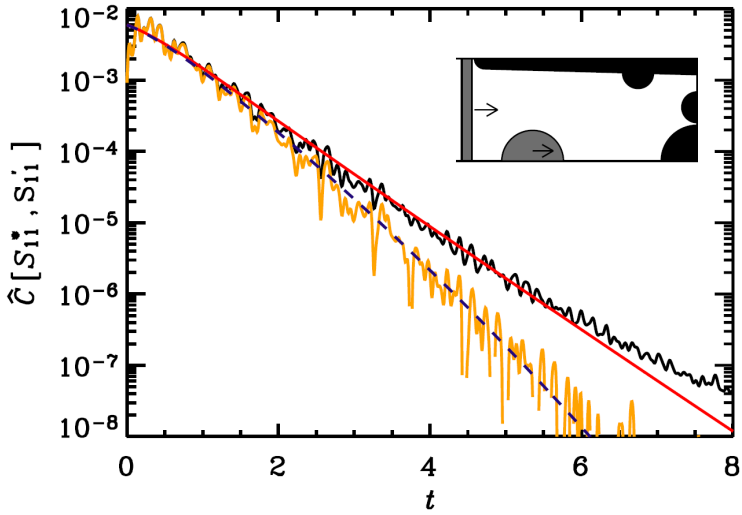


Figure 15: Correlation function $\hat{C}[S^*_{11}, S'_{11}] \propto \langle \hat{S}^*_{11}(t)\hat{S}'_{11}(t)\rangle$. The experimental results for the autocorrelation are shown in black, while the correlation of perturbed and unperturbed system are shown in gray/orange. The smooth solid curve corresponds to the theoretical autocorrelation function, and the dashed curve to the product of autocorrelation function and fidelity amplitude. The inset shows the billiard geometry used. Movable parts are marked with an arrow. Adapted from Ref. [Scha 05b]. Copyright (2005), American Physical Society.

Another time-reversal attempt conceptually close to the Loschmidt echo is that of time-reversal mirrors, developed in the last twenty years. Such a procedure has been successfully implemented in various setups where classical waves propagate through a complex media (going from acoustic to electromagnetic waves). In the time-reversal mirror protocol an initially localized pulse is recorded by a collection of receiver-emitter transducers during a time interval where the wave suffers multiple scattering. The later re-emission of the time-reversed signal by each transducer during an interval of time equal to the recording one leads to the refocusing of the signal in the region of the original excitation.

Two features of this protocol have lead to considerable surprise among the practitioners:

(i) even one transducer is enough to obtain a good reproduction of the original signal

(ii) the quality of the refocusing was improved by the complexity of the media yielding the multiple scattering of the waves.

A semiclassical theory of time-reversal focusing can be built up in terms of propagators and classical trajectories. The two previous features can be acknowledged by the semiclassical theory.

Time-reversal mirrors and Loschmidt echos differ since the former aims the refocalization of a wave which is localized in space and time, while the latter attempt to time-reverse a quantum state. A common aspect between both protocols is the fact that the Hamiltonian for the forward and backward evolutions can differ due to modifications of the environment during the process.

Time-reversal mirrors are not only conceptually important, but also have very important technological applications as for example brain therapy, lithotripsy, nondestructive testing and telecommunications. Recently a sensor of perturbations was proposed and demonstrated combining the ideas of the Loschmidt echo and time reversal mirror in classical waves.

4 Past, present, and future of the Loschmidt echo studies

Since the early discussions between Boltzmann and Loschmidt on irreversibility and time-reversal, it has been clearly established that chaos is the source of the irreversibility in statistical mechanics. However, when considering the sources of irreversibility in a quantum system with a few degrees of freedom Quantum Mechanics, which is the fundamental theory of the microscopic world, does not allow for chaotic behavior in the sense in which the latter appears in Classical Mechanics. That is, there is no exponential separation between states with nearby initial conditions since quantum evolution is unitary. Trying to understand the origin of irreversibility in quantum mechanics, Asher Peres proposed in 1984 as an alternative, to study the stability of quantum motion owing to perturbations in the Hamiltonian. In his seminal paper, Peres considered $M(t)$, later called Loschmidt echo, as a measure of sensibility and reversibility of quantum evolutions. He aimed to distinguish classically

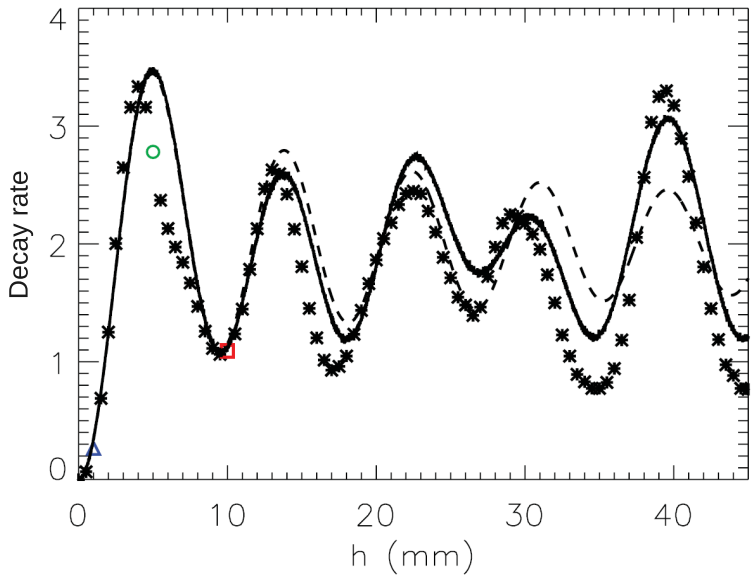


Figure 16: Decay rate of the Loschmidt echo in a microwave billiard as a function of the displacement h of a small flat part of the billiard boundary. The solid (dashed) curve corresponds to the decay rate predicted by the semiclassical theory taking (not taking) into account classical trajectories multiply scattered within the boundary deformation region. Adapted from Ref. [Kobe 11]. Copyright (2011), American Physical Society.

chaotic or integrable dynamics according to the speed at which the fidelity decays. Peres reached the conclusion that the long-time behavior of the fidelity (or saturation, following the terminology of Sec. 2.3.1) in classically chaotic systems is characterized by smaller values and smaller fluctuations as compared to the case of regular dynamics. In view of numerous possibilities of different behaviors that are allowed in regular systems, such a conclusion does not always hold.

It is interesting to notice that Peres' article appeared almost at the same time than two other seminal works studying the relevance of the underlying classical dynamics for quantum properties. These are the random-matrix description of the statistical properties of classically chaotic systems by Oriol Bohigas and collaborators; and the understanding of the spectral rigidity from a semiclassical analysis of periodic orbits proposed by Michael Berry. The field of Quantum Chaos developed building upon these founding ideas and most of the subsequent works were concerned with the spectral properties of classically chaotic systems. Comparatively very little work was done along the Peres' proposal in the decade following it.

Motivated by the puzzles posed by echo experiments in extended dipolar coupled nuclear spin systems, Rodolfo Jalabert and Horacio Pastawski studied in 2001 the behavior of the Loschmidt echo for classically chaotic systems. They found that, depending on the perturbation strength, the decay of the Loschmidt echo exhibits mainly two different behaviors. For weak perturbations, the decay is exponential with the rate that depends on the perturbation strength and that is given by the width of the local density of states (usually called the Fermi-golden-rule regime). For stronger perturbations, however, there is a crossover to a perturbation-independent regime, characterized by an exponential decay with the rate equal to the average Lyapunov exponent of the underlying classical system.

The connection of the quantum Loschmidt echo with the classical chaos generated a great activity in the field. Researches from different fields, such as quantum chaos, solid state physics, acoustics, and cold atom physics, have made important contributions towards understanding various aspects of the Loschmidt echo. During the first years of the last decade the studies were mainly focused on one-body aspects of the problem and on the connection between the Loschmidt echo and decoherence; many interesting experiments were also performed.

In the last years, the interest has primarily been focused on various many-body aspects of the Loschmidt echo. For example, there are studies that consider the decay of the Loschmidt echo as a signature of a quantum phase transition, or that concentrate on the relation between the Loschmidt echo and the statistics of the work done by a quantum critical system when a control parameter is quenched. In view of recent improvements in experimental many-body system techniques, the issue of the Loschmidt echo decay has become more concrete.

Some of the more important open questions in the field are:

- The Lyapunov (perturbation-independent) decay regime has been one of the most influential breakthroughs in the theory of the Loschmidt echo. However, an experimental observation of the regime in simple well-controlled systems is still lacking.

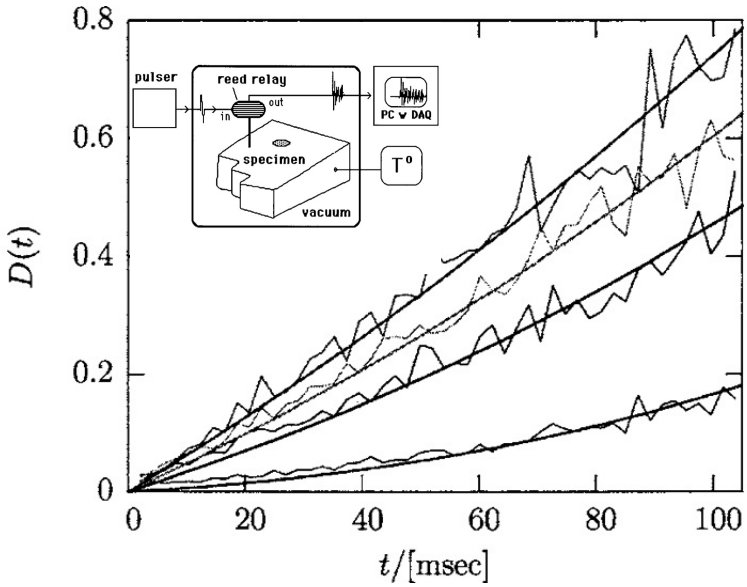


Figure 17: The distortion as a function of time for coda waves that travel in an aluminum alloy block at different temperatures. The thin lines correspond to measurements in different frequency ranges. The thick lines show the corresponding theoretical curves obtained using Random matrix theory. Inset: Laboratory configuration. An aluminum specimen, typically with nonparallel faces and a defocusing cylindrical hole, is cooled in a vacuum as temperature and ultrasonic response are monitored. A reed relay isolates the response from the pulser. Adapted from Refs. [Lobk 03] and [Gori 06]. Copyright (2003, 2006), American Physical Society.

- The first experimental measure of the Loschmidt echo was done in a many-body spin system using NMR techniques. However, theoretically little is know about the behavior of the Loschmidt echo in many-body systems. Very interesting experiments are now being carried on using NMR.
- The semiclassical theory of the Loschmidt echo has proved to be a powerful tool to understand many aspects of its behavior. But this approach has some well-recognized difficulties, such as the root search problem or the exponentially growing number of classical orbits needed for the semiclassical expansion. Recently, a simple semiclasical dephasing representation of the Loschmidt echo was proposed that does not suffer of the usual problems of the semiclassical theories. The range of validity is however unknown.
- There is a vast amount of work characterizing the decay regimes of the Loschmidt echo as universal. For chaotic systems perturbed with global or local perturbations the behavior of the decay rate of the Loschmidt echo is well established (see Sec. 2.3). However, recent works in quantum maps that are perturbed in all phase space, have reported non-universal oscillatory behavior in the decay rate of the Loschmidt echo as a function of the perturbation strength. Deviations from the perturbation independence are found usually in the form of oscillations around the Lyapunov exponent. Moreover, there are cases where deviations are considerably large rendering the Lyapunov regime non-existing. The fully understanding of this non-universal behavior, that was also observed in a Josephson flux quit, is still lacking [Garc 11b, Pozz 07].
- The semiclassical theory of the Lyapunov decay of the Loschmidt echo is based on highly localized initial states and the diagonal approximation of Eq. (3) for $M(t)$. But recent results using the semiclassical dephasing representation have shown a Lyapunov regime in the mean value of the fidelity amplitude $\langle m(t)\rangle$ [Garc 11a, Zamb 11].
- Systems with regular or fully chaotic dynamics are rather exceptional in nature. A generic system has mixed phase space consisting of integrable islands immersed in a chaotic sea. Few general results are known about the Loschmidt echo in this scenario.

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7 External links

- The Loschmidt Echo homepage