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1 **Vibration analysis of thin-walled composite beams with I-shaped**

1 **cross-sections**

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9 6 (Dated: July 6, 2010)

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11
12 A general analytical model applicable to the vibration analysis of thin-walled composite I-beams
13 with arbitrary lay-ups is developed. Based on the classical lamination theory, this model has
14 been applied to the investigation of load-frequency interaction curves of thin-walled composite
15 beams under various loads. The governing differential equations are derived from the Hamilton's
16 principle. A finite element model with seven degrees of freedoms per node is developed to solve
17 the problem. Numerical results are obtained for thin-walled composite I-beams under uniformly
18 distributed load, combined axial force and bending loads. The effects of fiber orientation, location
19 of applied load, and types of loads on the natural frequencies and load-frequency interaction curves
20 as well as vibration mode shapes are parametrically studied.
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29 7 **Keywords:** Thin-walled composite I-beams; vibration analysis; axial force and bending loads; load-frequency
30 interaction curves
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9 NOMENCLATURE

1		
2		
3		
4	A	Cross section area
5	\bar{a}	Location of transverse load with respect to shear center
6		
7	b_1, b_3	Width and height of I-section
8		
9	E_{ij}	Stiffness coefficients of thin-walled composite beams
10		
11	E_1, E_2	Young's moduli in the 1- and 2-directions of lamina
12		
13	$(EA)_{com}$	Axial rigidity of composite beam
14		
15	$(EI_x)_{com}, (EI_y)_{com}$	Flexural rigidity with respect to x - and y -axis
16		
17	$(EI_\omega)_{com}$	Warping rigidity
18		
19	$f(z), g(z)$	Polynomial functions which depend on the loading pattern
20		
21	G_{12}	Shear moduli in the 1-2 plane of lamina
22		
23	$(GJ)_{com}$	Torsional rigidity
24		
25	I_p	Polar moment of inertia about the centroid
26		
27	$[K], [G_1], [G_2], [M]$	Stiffness, geometric and mass matrix in finite element formulation
28		
29	m_0, m_c, m_p, m_s, m_2	Inertia coefficients
30		
31	M_b	External uniform bending moment
32		
33	M_{cr_n}	Buckling moments for pure bending
34		
35	M_t	Torsional moment
36		
37	M_x, M_y	Bending moments with respect to x - and y -axis
38		
39	M_ω	Warping moment
40		
41	$\bar{M}_{cr}, \bar{M}_{x_n}$	Nondimensional bending moment
42		
43	N_0	External axial force
44		
45	N_z	Axial force
46		
47	p	Transverse load
48		
49	$P_{x_n}, P_{y_n}, P_{\theta_n}$	Flexural buckling loads in the x - and y -axis and torsional buckling load
50		
51	\bar{P}_{cr}	Nondimensional vertical concentrated load
52		
53	$\bar{P}_{x_n}, \bar{P}_{y_n}, \bar{P}_{\theta_n}, \bar{N}_{cr}$	Nondimensional axial force
54		
55	q, r	Coordinate of point on the contour in the (n, s) coordinate system
56		
57	\bar{q}_{cr}	Nondimensional uniformly distributed load
58		
59	$(\bar{Q}_{ij}^*)^k$	Transformed reduced stiffness of the k^{th} lamina
60		
61	t	Flange and web thickness of I-section
62		
63		
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1	$\mathcal{T}, \mathcal{U}, \mathcal{V}$	Kinetic energy, strain energy and potential energy
2		
3	u, v, w	Displacements of a point on the contour in the (n, s, z) coordinate system
4		
5	U, V, W	Displacement components of the pole in the (x, y, z) coordinate system
6		
7	$\bar{u}, \bar{v}, \bar{w}$	Midsurface displacements of a point on the contour in the (s, z) coordinate system
8		
9	x_p, y_p	Coordinates of pole in the (x, y) coordinate system
10		
11	α	Angle between x and tangent axis
12		
13	$\{\Delta\}$	Eigenvector of nodal displacements corresponding to an eigenvalue
14		
15	$\epsilon_z, \epsilon_z^o, \bar{\epsilon}_z$	Axial strain in the (n, s, z) coordinate system
16		
17	θ	Fiber orientation
18		
19	κ_x, κ_y	Curvatures with respect to the x - and y -axis
20		
21	$\kappa_{sz}, \kappa_\omega$	Twisting and warping curvature
22		
23	$\bar{\kappa}_{sz}, \bar{\kappa}_z$	Midsurface curvatures
24		
25	λ	Buckling parameter
26		
27	ν_{12}	Poissons ratio
28		
29	Π	Total potential energy
30		
31	ρ	Density of composite material
32		
33	σ_z, γ_{sz}	Normal and shear stresses in the (n, s, z) coordinate system
34		
35	Φ	Angle of rotation of the cross section about the pole axis
36		
37	Ψ_j, ψ_j	Interpolation function in finite element formulation
38		
39	$\omega(s)$	Warping function
40		
41	$\omega_{x_n}, \omega_{y_n}, \omega_{\theta_n}$	Flexural natural frequencies with respect to the x - and y -axis and torsional natural frequencies
42		
43	$\omega_{xx_n}, \omega_{ya_n}, \omega_{yb_n}$	Natural frequencies for simply supported composite beams under axial force and uniform bending
44		
45	$\bar{\omega}$	Nondimensional natural frequency
46		
47		

10 1. INTRODUCTION

11 Fiber-reinforced composite materials have been used over the past few decades in a variety of structures. Composites
12 have many desirable characteristics, such as high ratio of stiffness and strength to weight, corrosion resistance and
13 magnetic transparency. Thin-walled structural shapes made up of composite materials, which are usually produced

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14 by pultrusion, are being increasingly used in many civil, mechanical and aerospace engineering applications.

15 Up to the present, investigation into the vibration and stability analysis of thin-walled members has received
16 widespread attention and has been carried out extensively since the early works of Vlasov [1], Gjelsvik [2]. It is also
17 well known that the vibration behavior of these members under various loads display complex response. Barsoum
18 [3] studied the stability analysis of structural systems under non-conservative forces using Hamilton principle as
19 basis and the dynamic criterion of stability. Attard and Somerville [4] focused on free vibration analysis of straight
20 prismatic beams of general thin-walled open cross-section, under conservative and nonconservative loads. Joshi and
21 Suryanarayan [5] investigated coupled flexural-torsional vibrations of double-symmetric thin-wall beams under axial
22 loads and end moments. They found that the problem could be reduced to a beam-column problem with a zero
23 moment, so that it was possible to obtain simple algebraic expressions unifying numerical results for various boundary
24 conditions. Based on the transfer matrix method, Ohga et al. [6, 7] estimated not only the natural frequencies but
25 also vibration mode shapes of the thin-walled members under in-plane forces. Mohri et al. [8] presented a higher-
26 order non shear deformable model to investigate the dynamic behavior of thin-walled open sections in the pre- and
27 post-buckling state. In their numeric examples, they considered simply supported beams under axial and distributed
28 transverse loads. Silvestre and Camotim [9] derived of a Generalised Beam Theory (GBT) to analyse the vibration
29 behaviour of loaded cold-formed steel members. Later, they [10] continued to study local and global vibration of
30 thin-walled members under compression and non-uniform bending. The geometrically nonlinear stiffness reduction
31 caused by the presence of longitudinal stress gradients and the ensuing shear stresses was taken into account in the
32 formulation. Voros [11] analyzed the free vibration and mode shapes of straight beams where the coupling between
33 the bending and torsion was induced by steady state lateral loads. Closed form solution for the coupled frequencies
34 and mode shapes of a symmetric beam with simply supported ends under uniform bending was derived. By using
35 the power series method, Leung [12,13] developed the exact dynamic stiffness matrix including both the axial force,
36 initial torque and bending moment for the interactive axial-torsional and axial-moment buckling analysis of framed
37 structures. Recently, Leung [14] proposed a new concept of uniform torque for buckling of columns by biaxial moments
38 and uniform end torque. Second-order effects of the axial force, biaxial moments and torque were considered in the
39 analysis.

40 For thin-walled composite beams, due to coupling effects from material anisotropy, these members under combined
41 axial force and bending loads simultaneously exhibit strong coupling. Therefore, their dynamic characteristics and
42 load-frequency interaction curves become very complicated. Several authors have investigated the free vibration

characteristics of axially loaded composite beams (Banerjee et al. [15,16], Li et al. [17,18], Kaya and Ozgumus [19] and Emam and Nayfeh [20]) but only a few have taken into account the effects of axial force and bending loads. By extending GBT formulation, Silvestre and Camotim [21] investigated the local and global vibration behavior of loaded thin-walled composite members, focusing on issues dealing with the variation of the fundamental frequency and vibration mode nature with the member length and applied stress level. Machado and Cortinez [22] presented free vibration of thin-walled composite beams with static initial stresses and deformations. The analysis was based on a geometrically non-linear theory based on large displacements and rotations. However, it was strictly valid for symmetric balanced laminates and especially orthotropic laminates. It is clear that the research of the vibration of thin-walled composite beams with arbitrary lay-ups under combined axial force and bending loads in a unitary manner is limited. This complicated problem has received scant attention and there is a need for further studies.

In this paper, which is an extension of the authors' previous works [23-26], vibration analysis of thin-walled composite beams with arbitrary lay-ups under combined axial force and bending loads is presented. This model is based on the classical lamination theory, and accounts for all the structural coupling coming from the material anisotropy. The governing differential equations for flexural-torsional coupled vibration are derived from the Hamilton's principle. A displacement-based one-dimensional finite element model is developed to solve the problem. Numerical results are obtained for thin-walled composite beams to investigate the effects of axial force, bending loads, fiber orientation on the natural frequencies and load-frequency interaction curves as well as vibration mode shapes.

2. KINEMATICS

The theoretical developments presented in this paper require two sets of coordinate systems which are mutually interrelated. The first coordinate system is the orthogonal Cartesian coordinate system (x, y, z) , for which the x and y axes lie in the plane of the cross section and the z axis parallel to the longitudinal axis of the beam. The second coordinate system is the local plate coordinate (n, s, z) as shown in Fig. 1, wherein the n axis is normal to the middle surface of a plate element, the s axis is tangent to the middle surface and is directed along the contour line of the cross section. The (n, s, z) and (x, y, z) coordinate systems are related through an angle of orientation α . As defined in Fig.1 a point P , called the pole, is placed at an arbitrary point x_p, y_p . A line through P parallel to the z axis is called the pole axis.

To derive the analytical model for a thin-walled composite beam, the following assumptions are made

1. The contour of the thin wall does not deform in its own plane.

2. The linear shear strain $\bar{\gamma}_{sz}$ of the middle surface is zero in each element.

3. The Kirchhoff-Love assumption in classical plate theory remains valid for laminated composite thin-walled beams.

4. Each laminate is thin and perfectly bonded.

According to assumption 1, the midsurface displacement components \bar{u}, \bar{v} at a point A in the contour coordinate system can be expressed in terms of a displacements U, V of the pole P in the x, y directions, respectively, and the rotation angle Φ about the pole axis,

$$\bar{u}(s, z) = U(z) \sin \alpha(s) - V(z) \cos \alpha(s) - \Phi(z)q(s) \quad (1a)$$

$$\bar{v}(s, z) = U(z) \cos \alpha(s) + V(z) \sin \alpha(s) + \Phi(z)r(s) \quad (1b)$$

These equations apply to the whole contour. The out-of-plane shell displacement \bar{w} can now be found from the assumption 2. For each element of middle surface, the shear strain become

$$\bar{\gamma}_{sz} = \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial s} = 0 \quad (2)$$

Eq.(2) can be integrated with respect to s from the origin to an arbitrary point on the contour,

$$\bar{w}(s, z) = W(z) - U'(z)x(s) - V'(z)y(s) - \Phi'(z)\omega(s) \quad (3)$$

where differentiation with respect to the axial coordinate z is denoted by primes (''); W represents the average axial displacement of the beam in the z direction; x and y are the coordinates of the contour in the (x, y, z) coordinate system; and ω is the so-called sectorial coordinate or warping function given by

$$\omega(s) = \int_{s_0}^s r(s)ds \quad (4a)$$

The displacement components u, v, w representing the deformation of any generic point on the profile section are given with respect to the midsurface displacements $\bar{u}, \bar{v}, \bar{w}$ by the assumption 3.

$$u(s, z, n) = \bar{u}(s, z) \quad (5a)$$

$$v(s, z, n) = \bar{v}(s, z) - n \frac{\partial \bar{u}(s, z)}{\partial s} \quad (5b)$$

$$w(s, z, n) = \bar{w}(s, z) - n \frac{\partial \bar{u}(s, z)}{\partial z} \quad (5c)$$

The strains associated with the small-displacement theory of elasticity are given by

$$\epsilon_z = \bar{\epsilon}_z + n\bar{\kappa}_z \quad (6a)$$

$$\gamma_{sz} = n\bar{\kappa}_{sz} \quad (6b)$$

87 where

$$\bar{\epsilon}_z = \frac{\partial \bar{w}}{\partial z} \quad (7a)$$

$$\bar{\kappa}_z = -\frac{\partial^2 \bar{u}}{\partial z^2} \quad (7b)$$

$$\bar{\kappa}_{sz} = -2\frac{\partial^2 \bar{u}}{\partial s \partial z} \quad (7c)$$

88 All the other strains are identically zero. In Eq.(7), $\bar{\epsilon}_z$, $\bar{\kappa}_z$ and $\bar{\kappa}_{sz}$ are midsurface axial strain and biaxial curvature of
 89 the shell, respectively. The above shell strains can be converted to beam strain components by substituting Eqs.(1),
 90 (3) and (5) into Eq.(7) as

$$\bar{\epsilon}_z = \epsilon_z^\circ + x\kappa_y + y\kappa_x + \omega\kappa_\omega \quad (8a)$$

$$\bar{\kappa}_z = \kappa_y \sin \alpha - \kappa_x \cos \alpha - \kappa_\omega q \quad (8b)$$

$$\bar{\kappa}_{sz} = \kappa_{sz} \quad (8c)$$

91 where ϵ_z° , κ_x , κ_y , κ_ω and κ_{sz} are axial strain, biaxial curvatures in the x and y direction, warping curvature with
 92 respect to the shear center, and twisting curvature in the beam, respectively defined as

$$\epsilon_z^\circ = W' \quad (9a)$$

$$\kappa_x = -V'' \quad (9b)$$

$$\kappa_y = -U'' \quad (9c)$$

$$\kappa_\omega = -\Phi'' \quad (9d)$$

$$\kappa_{sz} = 2\Phi' \quad (9e)$$

93 The resulting strains can be obtained from Eqs.(6) and (8) as

$$\epsilon_z = \epsilon_z^\circ + (x + n \sin \alpha)\kappa_y + (y - n \cos \alpha)\kappa_x + (\omega - nq)\kappa_\omega \quad (10a)$$

$$\gamma_{sz} = n\kappa_{sz} \quad (10b)$$

94 3. VARIATIONAL FORMULATION

95 The total potential energy of the system can be stated, in its buckled shape, as

$$\Pi = \mathcal{U} + \mathcal{V} \quad (11)$$

96 where \mathcal{U} is the strain energy

$$\mathcal{U} = \frac{1}{2} \int_v (\sigma_z \epsilon_z + \sigma_{sz} \gamma_{sz}) dv \quad (12)$$

97 After substituting Eq.(10) into Eq.(12), the variation of strain energy can be stated as

$$\delta\mathcal{U} = \int_0^l (N_z \delta\epsilon_z + M_y \delta\kappa_y + M_x \delta\kappa_x + M_\omega \delta\kappa_\omega + M_t \delta\kappa_{sz}) dz \quad (13)$$

98 where $N_z, M_x, M_y, M_\omega, M_t$ are axial force, bending moments in the x - and y -direction, warping moment (bimoment),
99 and torsional moment with respect to the centroid, respectively, defined by integrating over the cross-sectional area A
100 as

$$N_z = \int_A \sigma_z dsdn \quad (14a)$$

$$M_y = \int_A \sigma_z (x + n \sin \alpha) dsdn \quad (14b)$$

$$M_x = \int_A \sigma_z (y - n \cos \alpha) dsdn \quad (14c)$$

$$M_\omega = \int_A \sigma_z (\omega - nq) dsdn \quad (14d)$$

$$M_t = \int_A \sigma_{sz} n dsdn \quad (14e)$$

101 The variation of the potential of the in-plane load N_0 at the centroid and transverse load p acting on the cross
102 section at a point a distance \bar{a} above the shear center can be found in Refs. [23, 24]

$$\begin{aligned} \delta\mathcal{V} = & \int_0^l \left[N_0 [\delta U' (U' + \Phi' y_p) + \delta V' (V' - \Phi' x_p) + \delta \Phi' (\Phi' \frac{I_p}{A} + U' y_p - V' x_p)] \right. \\ & \left. - M_b (\Phi \delta U'' + U'' \delta \Phi) - \bar{a} p \Phi \delta \Phi \right] dz \end{aligned} \quad (15)$$

103 where M_b is not the actual bending moment in the beam, but the simple beam moment due to transverse load p .

104 The variation of the kinetic energy is expressed in Ref. [25] as

$$\begin{aligned} \delta\mathcal{T} = & \int_0^l \left[m_0 \dot{W} \delta \dot{W} + [m_0 \dot{U} + (m_c + m_0 y_p) \dot{\Phi}] \delta \dot{U} + [m_0 \dot{V} + (m_s - m_0 x_p) \dot{\Phi}] \delta \dot{V} \right. \\ & \left. + [(m_c + m_0 y_p) \dot{U} + (m_s - m_0 x_p) \dot{V} + (m_p + m_2 + 2m_\omega) \dot{\Phi}] \delta \dot{\Phi} \right] dz \end{aligned} \quad (16)$$

105 where, m_0, m_c, m_p, m_s, m_2 are inertia coefficients. In order to derive the equations of motion, Hamilton's principle is
106 used

$$\delta \int_{t_1}^{t_2} (\mathcal{T} - \Pi) dt = 0 \quad (17)$$

107 Substituting Eqs.(13),(15) and (16) into Eq.(17), the following weak statement is obtained

$$\begin{aligned}
0 = & \int_{t_1}^{t_2} \int_0^l \left\{ m_0 \dot{W} \delta \dot{W} + [m_0 \dot{U} + (m_c + m_0 y_p) \dot{\Phi}] \delta \dot{U} + [m_0 \dot{V} + (m_s - m_0 x_p) \dot{\Phi}] \delta \dot{V} \right. \\
& + [(m_c + m_0 y_p) \dot{U} + (m_s - m_0 x_p) \dot{V} + (m_p + m_2 + 2m_\omega) \dot{\Phi}] \delta \dot{\Phi} \\
& - \left[N_0 [\delta U' (U' + \Phi' y_p) + \delta V' (V' - \Phi' x_p) + \delta \Phi' (\Phi' \frac{I_p}{A} + U' y_p - V' x_p)] - M_b (\Phi \delta U'' + U'' \delta \Phi) - \bar{a} p \Phi \delta \Phi \right] \\
& \left. - N_z \delta W' + M_y \delta U'' + M_x \delta V'' + M_\omega \delta \Phi'' - 2M_t \delta \Phi \right\} dz dt
\end{aligned} \tag{18}$$

108 In Eq.(18), M_b and p are the buckling moment and transverse load, and can be written for various types of loading

109 as

$$M_b = \lambda f(z) \tag{19a}$$

$$p = \lambda g(z) \tag{19b}$$

110 where λ is a buckling parameter and $f(z)$ and $g(z)$ are polynomial functions which depend on the loading pattern.

111 These functions are given as follows for various types of loading:

$$\left. \begin{cases} f(z) = 1; & g(z) = 0 & \text{for pure bending} \\ f(z) = \frac{1}{2}(\frac{l^2}{4} - z^2); & g(z) = 1 & \text{for uniformly distributed load} \\ f(z) = \frac{l}{2} - z; & g(z) = \begin{cases} 0 \\ 1 \text{ at the loading point} \end{cases} & \text{for point load at free end of a cantilever beam} \end{cases} \right\} \tag{20}$$

112 4. CONSTITUTIVE EQUATIONS

113 The constitutive equations of a k^{th} orthotropic lamina in the laminate co-ordinate system of section are given by

$$\begin{Bmatrix} \sigma_z \\ \sigma_{sz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11}^* & \bar{Q}_{16}^* \\ \bar{Q}_{16}^* & \bar{Q}_{66}^* \end{bmatrix}^k \begin{Bmatrix} \epsilon_z \\ \gamma_{sz} \end{Bmatrix} \tag{21}$$

114 where \bar{Q}_{ij}^* are transformed reduced stiffnesses. The transformed reduced stiffnesses can be calculated from the
115 transformed stiffnesses based on the plane stress ($\sigma_s = 0$) and plane strain ($\epsilon_s = 0$) assumption. More detailed
116 explanation can be found in Ref. [27].

117 The constitutive equations for bar forces and bar strains are obtained by using Eqs.(10), (14) and (21)

$$\begin{Bmatrix} N_z \\ M_y \\ M_x \\ M_\omega \\ M_t \end{Bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \\ & E_{22} & E_{23} & E_{24} & E_{25} \\ & & E_{33} & E_{34} & E_{35} \\ & & & E_{44} & E_{45} \\ \text{sym.} & & & & E_{55} \end{bmatrix} \begin{Bmatrix} \epsilon_z^\circ \\ \kappa_y \\ \kappa_x \\ \kappa_\omega \\ \kappa_{sz} \end{Bmatrix} \quad (22)$$

118 where E_{ij} are stiffnesses of thin-walled composite beams and given in Ref. [25].

15 119 5. GOVERNING EQUATIONS OF MOTION

18 120 The governing equations of motion of the present study can be derived by integrating the derivatives of the varied
19
20 121 quantities by parts and collecting the coefficients of $\delta W, \delta U, \delta V$ and $\delta \Phi$

$$N'_z = m_0 \ddot{W} \quad (23a)$$

$$M''_y + N_0(U'' + \Phi'' y_p) + (M_b \Phi)'' = m_0 \ddot{U} + (m_c + m_0 y_p) \ddot{\Phi} \quad (23b)$$

$$M''_x + N_0(V'' - \Phi'' x_p) = m_0 \ddot{V} + (m_s - m_0 x_p) \ddot{\Phi} \quad (23c)$$

$$\begin{aligned} M''_\omega + 2M'_t + N_0\left(\Phi'' \frac{I_p}{A} + U'' y_p - V'' x_p\right) + M_b U'' + \bar{a} p \Phi &= (m_c + m_0 y_p) \ddot{U} \\ &+ (m_s - m_0 x_p) \ddot{V} \\ &+ (m_p + m_2 + 2m_\omega) \ddot{\Phi} \end{aligned} \quad (23d)$$

37 122 By substituting Eqs.(9) and (22) into Eq.(23), the explicit form of governing equations of motion can be expressed

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123 with respect to the laminate stiffnesses E_{ij} as

$$E_{11}W'' - E_{12}U''' - E_{13}V''' - E_{14}\Phi''' + 2E_{15}\Phi'' = m_0\ddot{W} \quad (24a)$$

$$E_{12}W''' - E_{22}U^{iv} - E_{23}V^{iv} - E_{24}\Phi^{iv} + 2E_{25}\Phi''' + N_0(U'' + \Phi''y_p) + (M_b\Phi)'' = m_0\ddot{U} + (m_c + m_0y_p)\ddot{\Phi} \quad (24b)$$

$$E_{13}W''' - E_{23}U^{iv} - E_{33}V^{iv} - E_{34}\Phi^{iv} + 2E_{35}\Phi''' + N_0(V'' - \Phi''x_p) = m_0\ddot{V} + (m_s - m_0x_p)\ddot{\Phi} \quad (24c)$$

$$E_{14}W''' + 2E_{15}W'' - E_{24}U^{iv} - 2E_{25}U''' - E_{34}V^{iv} - 2E_{35}V''' - E_{44}\Phi^{iv} + 4E_{55}\Phi'' + N_0\left(\Phi''\frac{I_p}{A} + U''y_p - V''x_p\right) + M_bU'' + \bar{a}p\Phi = (m_c + m_0y_p)\ddot{U} + (m_s - m_0x_p)\ddot{V} + (m_p + m_2 + 2m_\omega)\ddot{\Phi} \quad (24d)$$

Eq.(24) is most general form for flexural-torsional coupled vibration of thin-walled composite beams with arbitrary lay-ups under axial and bending loads and the dependent variables, W , U , V and Φ are fully coupled. For the case of thin-walled composite beams under axial force and uniform bending, if all the coupling effects and the cross section is symmetrical with respect to both x - and the y -axes, Eq.(24) can be simplified to the uncoupled differential equations as

$$(EA)_{com}W'' = \rho A\ddot{W} \quad (25a)$$

$$-(EI_y)_{com}U^{iv} + N_0U'' + M_b\Phi'' = \rho A\ddot{U} \quad (25b)$$

$$-(EI_x)_{com}V^{iv} + N_0V'' = \rho A\ddot{V} \quad (25c)$$

$$-(EI_\omega)_{com}\Phi^{iv} + \left[(GJ)_{com} + N_0\frac{I_p}{A}\right]\Phi'' + M_bU'' = \rho I_p\ddot{\Phi} \quad (25d)$$

From above equations, $(EA)_{com}$ represents axial rigidity, $(EI_x)_{com}$ and $(EI_y)_{com}$ represent flexural rigidities with respect to x - and y -axis, $(EI_\omega)_{com}$ represents warping rigidity, and $(GJ)_{com}$ represents torsional rigidity of thin-

131 walled composite beams, respectively, written as

$$(EA)_{com} = E_{11} \quad (26a)$$

$$(EI_y)_{com} = E_{22} \quad (26b)$$

$$(EI_x)_{com} = E_{33} \quad (26c)$$

$$(EI_\omega)_{com} = E_{44} \quad (26d)$$

$$(GJ)_{com} = 4E_{55} \quad (26e)$$

132 6. ANALYTICAL SOLUTIONS FOR SIMPLY SUPPORTED COMPOSITE BEAMS UNDER AXIAL FORCE AND 133 UNIFORM BENDING

134 For simply supported beams with free warping, the overall displacements modes in bending and torsion are assumed

135 as

$$U(z, t) = U_0 \sin\left(\frac{n\pi z}{L}\right) \sin(\omega t) \quad (27a)$$

$$V(z, t) = V_0 \sin\left(\frac{n\pi z}{L}\right) \sin(\omega t) \quad (27b)$$

$$\Phi(z, t) = \Phi_0 \sin\left(\frac{n\pi z}{L}\right) \sin(\omega t) \quad (27c)$$

136 Substituting Eq.(27) into Eq.(25), after integrations and some reductions, the resulting flexural and torsional equations

137 of motion are obtained in compact form as

$$\omega_{x_n}^2 (1 - \bar{P}_{x_n}) - \omega_{xx_n}^2 = 0 \quad (28a)$$

$$A \left[\omega_{y_n}^2 (1 - \bar{P}_{y_n}) - \omega^2 \right] U_0 - \bar{M}_{x_n} \sqrt{AI_p} \omega_{y_n} \omega_{\theta_n} \Phi_0 = 0 \quad (28b)$$

$$-\bar{M}_{x_n} \sqrt{AI_p} \omega_{y_n} \omega_{\theta_n} U_0 + I_p \left[\omega_{\theta_n}^2 (1 - \bar{P}_{\theta_n}) - \omega^2 \right] \Phi_0 = 0 \quad (28c)$$

138 For the above equations, it is well known that the flexural natural frequencies in the x -direction and bending

139 moments are decoupled, while, the flexural natural frequencies in the y -direction, torsional natural frequencies and

140 bending moments are coupled. They are given by the orthotropy solution for simply supported boundary condition

$$\omega_{xx_n} = \omega_{x_n} \sqrt{1 - \bar{P}_{x_n}} \quad (29a)$$

$$\omega_{ya_n} = \sqrt{\frac{\omega_{y_n}^2 (1 - \bar{P}_{y_n}) + \omega_{\theta_n}^2 (1 - \bar{P}_{\theta_n})}{2}} - \sqrt{\left[\frac{\omega_{y_n}^2 (1 - \bar{P}_{y_n}) - \omega_{\theta_n}^2 (1 - \bar{P}_{\theta_n})}{2} \right]^2 + \bar{M}_{x_n}^2 \omega_{y_n}^2 \omega_{\theta_n}^2} \quad (29b)$$

$$\omega_{yb_n} = \sqrt{\frac{\omega_{y_n}^2 (1 - \bar{P}_{y_n}) + \omega_{\theta_n}^2 (1 - \bar{P}_{\theta_n})}{2}} + \sqrt{\left[\frac{\omega_{y_n}^2 (1 - \bar{P}_{y_n}) - \omega_{\theta_n}^2 (1 - \bar{P}_{\theta_n})}{2} \right]^2 + \bar{M}_{x_n}^2 \omega_{y_n}^2 \omega_{\theta_n}^2} \quad (29c)$$

141 in which $\omega_{x_n}, \omega_{y_n}$ and ω_{θ_n} are the flexural natural frequencies in the x - and y -direction, and torsional natural fre-
 142 quencies [28]

$$\omega_{x_n} = \frac{n^2\pi^2}{l^2} \sqrt{\frac{(EI_x)_{com}}{\rho A}} \quad (30a)$$

$$\omega_{y_n} = \frac{n^2\pi^2}{l^2} \sqrt{\frac{(EI_y)_{com}}{\rho A}} \quad (30b)$$

$$\omega_{\theta_n} = \frac{n\pi}{l} \sqrt{\frac{1}{\rho I_p} \left[\frac{n^2\pi^2}{l^2} (EI_\omega)_{com} + (GJ)_{com} \right]} \quad (30c)$$

143 and $\bar{P}_{x_n}, \bar{P}_{y_n}, \bar{P}_{\theta_n}$ and \bar{M}_{x_n} are nondimensional axial force and moment.

$$\bar{P}_{x_n} = \frac{N_0}{P_{x_n}} \quad (31a)$$

$$\bar{P}_{y_n} = \frac{N_0}{P_{y_n}} \quad (31b)$$

$$\bar{P}_{\theta_n} = \frac{N_0}{P_{\theta_n}} \quad (31c)$$

$$\bar{M}_{x_n} = \frac{M_b}{M_{cr_n}} \quad (31d)$$

144 where P_{x_n}, P_{y_n} and P_{θ_n} are the flexural buckling loads in the x - and y -direction, and torsional buckling loads [29]

$$P_{x_n} = \frac{n^2\pi^2(EI_x)_{com}}{l^2} \quad (32a)$$

$$P_{y_n} = \frac{n^2\pi^2(EI_y)_{com}}{l^2} \quad (32b)$$

$$P_{\theta_n} = \frac{A}{I_p} \left[\frac{n^2\pi^2(EI_\omega)_{com}}{l^2} + (GJ)_{com} \right] \quad (32c)$$

145 and M_{cr_n} is the buckling moments for pure bending [29]

$$M_{cr_n} = \sqrt{\frac{n^2\pi^2(EI_y)_{com}}{l^2} \left[\frac{n^2\pi^2(EI_\omega)_{com}}{l^2} + (GJ)_{com} \right]} \quad (33)$$

146 7. FINITE ELEMENT FORMULATION

147 The present theory for thin-walled composite beams described in the previous section was implemented via a
 148 displacement based finite element method. The element has seven degrees of freedom at each node, three displacements
 149 W, U, V and three rotations U', V', Φ as well as one warping degree of freedom Φ' . The axial displacement W is
 150 interpolated using linear shape functions Ψ_j , whereas the lateral and vertical displacements U, V and axial rotation Φ

151 are interpolated using Hermite-cubic shape functions ψ_j associated with node j and the nodal values, respectively.

$$1 \quad W = \sum_{j=1}^2 w_j \Psi_j \quad (34a)$$

$$2 \quad U = \sum_{j=1}^4 u_j \psi_j \quad (34b)$$

$$3 \quad V = \sum_{j=1}^4 v_j \psi_j \quad (34c)$$

$$4 \quad \Phi = \sum_{j=1}^4 \phi_j \psi_j \quad (34d)$$

152 Substituting these expressions into the weak statement in Eq.(18), the finite element model of a typical element
153 can be expressed as the standard eigenvalue problem

$$18 \quad ([K] - N_0[G_1] - \lambda[G_2] - \omega^2[M])\{\Delta\} = \{0\} \quad (35)$$

154 where $[K]$, $[G_1]$, $[G_2]$ and $[M]$ are the element stiffness matrix, the element geometric stiffness matrix due to axial
155 force and bending loads as well as the element mass matrix, respectively. The explicit forms of them are given in
156 Refs. [23-26].

157 In Eq.(35), $\{\Delta\}$ is the eigenvector of nodal displacements corresponding to an eigenvalue

$$30 \quad \{\Delta\} = \{W \ U \ V \ \Phi\}^T \quad (36)$$

34 158 8. NUMERICAL EXAMPLES

159 A thin-walled composite I-beam with length $l = 8\text{m}$ is considered to investigate the effects of axial force, bending
160 loads, fiber orientation on the natural frequencies and load-frequency interaction curves as well as vibration mode
161 shapes. The geometry of the I-section is shown in Fig. 2. Stacking sequence of this beam consists of two layers with
162 equal thickness as follows: angle-ply laminate $[\theta/-\theta]$ at the bottom flange, and unidirectional laminate $[0]_2$ at the
163 top flange and web, respectively. For this lay-up, all the coupling stiffnesses are zero, but E_{15} and E_{35} do not vanish.
164 The following engineering constants are used

$$50 \quad E_1/E_2 = 25, G_{12}/E_2 = 0.6, \nu_{12} = 0.25 \quad (37)$$

165 For convenience, the following nondimensional axial force, bending loads and natural frequency are used

$$\bar{N}_{cr} = \frac{N_{cr} l^2}{E_2 t b_3^3} \quad (38a)$$

$$\bar{M}_{cr} = \frac{M_{cr} l}{E_2 t b_3^3} \quad (38b)$$

$$\bar{P}_{cr} = \frac{P_{cr} l^2}{E_2 t b_3^3} \quad (38c)$$

$$\bar{q}_{cr} = \frac{q_{cr} l^3}{E_2 t b_3^3} \quad (38d)$$

$$\bar{\omega} = \frac{\omega l^2}{b_3} \sqrt{\frac{\rho}{E_2}} \quad (38e)$$

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14 166 As a first example, a simply-supported composite beam under under uniformly distributed load is analyzed. The
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16 167 load is applied at at the shear center, top flange and bottom flange. The first load-frequency interaction curves for
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18 168 three cases are plotted with respect to the fiber angle variation in Fig. 3. It is clear that the location of applied load
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20 169 has major effects of vibration of beams under transverse load. All three cases of groups show similar trends. That is,
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22 170 the smallest group is for the case of load at the top flange and the largest one is for the case of load at the bottom
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24 171 flange. The lowest three load-frequency interaction curves with fiber angles $\theta = 0^\circ, 30^\circ$ and 60° for three cases are
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26 172 displayed in Figs. 4, 5 and 6. At $\theta = 0^\circ$ (Fig. 4), the first and third natural frequencies decrease from $\bar{\omega}_1 = 5.05$ and
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28 173 $\bar{\omega}_3 = 20.15$ to zero, when the lateral buckling loads are reached, depending on the position of the load height, vice
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30 174 versa, the second one increases monotonically with the increase of load. As the fiber angle changes, this response is
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32 175 no longer visible. For example, at $\theta = 30^\circ$, for the case of load at shear center, with the increase of load, the first
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34 176 and third natural frequencies increase and reach local maximum values around $\bar{q} = 0.39$ and 0.51 , they decrease and
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36 177 finally vanish at $\bar{q}_{cr_1} = 0.45$ and $\bar{q}_{cr_2} = 1.64$, respectively, which are corresponding to the first and second lateral
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38 178 buckling loads (Fig. 5). The decrease becomes more quickly when uniform loads are close to lateral buckling loads.

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40 179 The next example is a simply supported composite beam under combined axial force and bending moment. The
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42 180 lowest four natural frequencies are obtained by the finite element analysis and orthotropy solution, which neglects the
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44 181 coupling effects of E_{15} and E_{35} from Eqs.(29a)-(29c), are given in Table 1. The critical flexural-torsional buckling loads
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46 182 (\bar{N}_{cr}) and critical buckling moments (\bar{M}_{cr}) for pure bending agree completely with those of previous papers [23,24].
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48 183 With the same value of bending moment, it can be seen that the natural frequencies diminish when the axial force
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50 184 changes from tensile to compressive, as expected. For unidirectional fiber direction, the lowest four natural frequencies
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52 185 by the finite element analysis exactly corresponding to the flexural-torsional coupled modes and the first flexural mode
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54 186 in x -direction by the orthotropy solution, respectively. As the fiber angle is rotated off-axis, the orthotropy solution
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56 187 and finite element analysis solution show discrepancy indicating the coupling effects become significant. It can be
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188 also explained partly by the typical normal mode shapes corresponding to the first four natural frequencies with fiber
 189 angle $\theta = 30^\circ$ for the case $(\bar{N} = 0.5\bar{N}_{cr}, \bar{M} = 0.5\bar{M}_{cr})$ in Fig. 7. It should be noticed that although a coupling
 190 stiffness E_{15} between the axial mode and the torsional mode is not null, the magnitude of induced axial displacement
 191 W is much lower than U, V and Φ and thus, is not plotted in the mode shapes. As a result, the first three natural
 192 frequency exhibits doubly coupled modes (flexural mode in y -direction and torsional mode), whereas, the fourth one
 193 displays triply coupled modes (flexural mode in the x -, y -direction and torsional mode). Therefore, the orthotropy
 194 solution is no longer valid for unsymmetrically laminated beams due to coupling effects. In order to investigate these
 195 effects further in the large bending moment region, the lowest three moment-frequency interaction curves with the
 196 fiber angle $\theta = 30^\circ$ for two cases $(\bar{N} = 0)$ and $(\bar{N} = 0.5\bar{N}_{cr})$ are displayed in Figs. 8 and 9. These figures highlight
 197 the effects of coupling on the vibration of thin-walled composite beam under axial load and bending moment. It is
 198 very interesting to note that all moment-frequency interaction curves by finite element analysis are asymmetric. This
 199 response is never seen in isotropic beams with doubly symmetric cross-section because coupling terms are not present.
 200 For the case $(\bar{N} = 0.5\bar{N}_{cr})$ in Fig. 9, due to asymmetric interaction curves, when the natural frequency vanishes,
 201 each branch always has two different buckling moments. For instance, at the lowest branch, the negative buckling
 202 moment, $\bar{M}_{cr_1} = -2.50 \times 10^{-2}$, occurs when the moment causes tension in the top flange, while the positive one,
 203 $\bar{M}_{cr_2} = 3.72 \times 10^{-2}$, corresponds to a reversal in the sense of the moment which causes compression in the top flange.
 204 As a result, this branch is disappeared when \bar{M} is slightly outside this range. As the bending moment changes, two
 205 interaction curves $(\omega_2 - M_2)$ and $(\omega_3 - M_3)$ intersect at $\bar{M} = -7.20 \times 10^{-2}$ and $\bar{M} = 12.20 \times 10^{-2}$, thus, after these
 206 values, vibration mode 2 and 3 change each other. The second branch will also be disappeared when \bar{M} is slightly
 207 outside the range of $[-12.88 \quad 14.10] \times 10^{-2}$.

208 The last example is the same as before except that in this case, boundary condition is clamped-free. A cantilever
 209 composite beam under combined axial force and vertical point load at shear center of free end is considered. Effect
 210 of axial force on the first load-frequency interaction curves of fiber angles 30° and 60° is investigated. Three cases
 211 of axial force is considered in Fig. 10. Both lateral buckling loads and natural frequencies increase when the axial
 212 force changes from compressive to tensile. It demonstrated again the fact that tensile forces stiffen the beam while
 213 compressive forces soften the beam. Three dimensional interaction diagram of the fundamental natural frequency,
 214 vertical load with respect to the axial compressive force change of these angles is plotted in Fig. 11. As expected,
 215 load-frequency interaction curves become smaller as the axial force increases and finally vanish at about $\bar{N} = 0.79$ and
 216 0.57 for $\theta = 30^\circ$ and 60° , respectively, which implies that at these loads, the critical flexural-torsional buckling occur as

217 a degenerated case of natural vibration and vertical load at zero value. It is from this figure that illustrate clearly the
 218 characteristic of load-load-frequency interaction curve, which explains the duality between flexural-torsional buckling
 1 load, lateral buckling load and natural frequency.
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6 220 9. CONCLUDING REMARKS

9 221 A one-dimensional finite element model was developed to study the vibration analysis of thin-walled composite
 10 222 beams with I-section. This model has been applied to the investigation of load-frequency interaction curves of beams
 11 223 under uniformly distributed load, combined axial force and bending loads. The effects of loading condition, location
 12 224 of applied load and fiber orientation on the natural frequencies, load-frequency interaction curves and mode shapes
 13 225 are investigated. Triply coupled vibration modes including the flexural mode in the x -, y -direction and torsional
 14 226 mode are included in the analysis. The present model is found to be appropriate and efficient in analyzing vibration
 15 227 problem of thin-walled composite beams under combined axial force and bending loads.
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284 CAPTIONS OF TABLES

1 285 Table 1: Effect of axial force and bending moment on the first four natural frequencies with respect to the fiber
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3 286 angle change in the bottom flange of a simply supported composite beam.
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287 CAPTIONS OF FIGURES

1 288 Figure 1: Definition of coordinates in thin-walled open sections.

2
3 289 Figure 2: Geometry of thin-walled composite I-beam.

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5 290 Figure 3: Effect of load heights on the first load-frequency interaction curves with respect to the fiber angle change
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7 291 in the bottom flange of a simply supported composite beam under uniformly distributed load.

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9 292 Figure 4: Effect of load heights on the first three load-frequency interaction curves with the fiber angle 0° in the
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11 293 bottom flange of a simply supported composite beam under uniformly distributed load.

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13 294 Figure 5: Effect of load heights on the first three load-frequency interaction curves with the fiber angle 30° in the
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15 295 bottom flange of a simply supported composite beam under uniformly distributed load.

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17 296 Figure 6: Effect of load heights on the first three load-frequency interaction curves with the fiber angle 60° in the
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19 297 bottom flange of a simply supported composite beam under uniformly distributed load.

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21 298 Figure 7: The first four normal mode shapes of the flexural and torsional components with the fiber angle 30° in
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23 299 the bottom flange of a simply supported composite beam under combined axial compressive force ($\bar{N} = 0.5\bar{N}_{cr}$) and
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25 300 bending moment ($\bar{M} = 0.5\bar{M}_{cr}$).

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27 301 Figure 8: The first three moment-frequency interaction curves with the fiber angle 30° in the bottom flange of a
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29 302 simply supported composite beam.

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31 303 Figure 9: The first three moment-frequency interaction curves with the fiber angle 30° in the bottom flange of a
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33 304 simply supported composite beam under an axial compressive force ($\bar{N} = 0.5\bar{N}_{cr}$).

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35 305 Figure 10: Effect of axial force on the first load-frequency interaction curves with fiber angles 30° and 60° in the
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37 306 bottom flange of a cantilever composite beam under point load at shear center of free end.

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39 307 Figure 11: The first load-frequency interaction curves with respect to the axial compressive force change with fiber
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41 308 angles 30° and 60° in the bottom flange of a cantilever composite beam under point load at shear center of free end.

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TABLE 1 Effect of axial force and bending moment on the first four natural frequencies with respect to the fiber angle change in the bottom flange of a simply supported composite beam.

Fiber angle	\bar{N}_{cr}	\bar{M}_{cr} ($\times 10^{-2}$)	Moment & Axial force	Present				Orthotropy			
				ω_1	ω_2	ω_3	ω_4	ω_{ya_1}	ω_{yb_1}	ω_{ya_2}	ω_{xx_1}
0	5.153	7.370	$\bar{M} = 0.5\bar{M}_{cr}$	2.914	6.204	18.657	19.830	2.914	6.204	18.657	19.830
30	2.771	4.895	$\bar{N} = 0.5\bar{N}_{cr}$	3.049	4.108	12.652	16.205	2.629	4.355	13.595	16.240
60	1.259	3.117	(compression)	2.050	3.655	5.990	12.317	1.836	3.353	6.019	14.596
90	1.112	2.905		1.895	3.674	5.450	11.146	1.695	3.360	5.453	14.523
0	5.153	7.370	$\bar{M} = 0.5\bar{M}_{cr}$	3.980	7.522	19.654	20.148	3.980	7.522	19.654	20.148
30	2.771	4.895	$\bar{N} = 0$	3.672	4.772	14.120	16.429	3.102	5.471	14.034	16.449
60	1.259	3.117	(no axial force)	2.650	3.712	7.014	13.492	2.148	4.027	6.850	14.703
90	1.112	2.905		2.448	3.721	6.422	12.270	2.021	3.945	6.268	14.617
0	5.153	7.370	$\bar{M} = 0.5\bar{M}_{cr}$	4.769	8.666	20.461	20.518	4.769	8.666	20.461	20.518
30	2.771	4.895	$\bar{N} = -0.5\bar{N}_{cr}$	4.165	5.551	15.214	16.677	3.781	6.240	14.697	16.656
60	1.259	3.117	(tension)	3.068	4.066	7.855	14.513	2.639	4.481	7.647	14.808
90	1.112	2.905		2.857	4.022	7.226	13.254	2.487	4.353	7.040	14.710

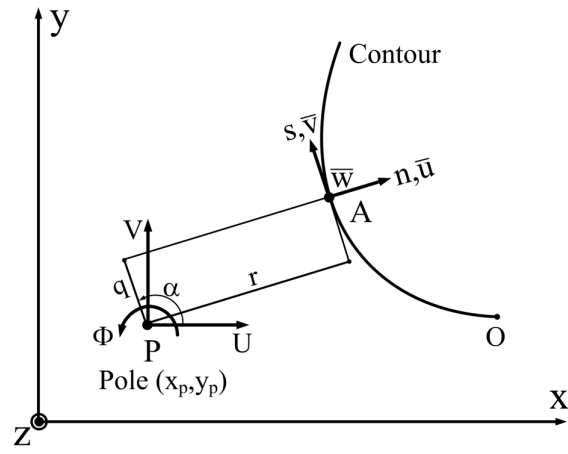


FIG. 1 Definition of coordinates in thin-walled open sections.

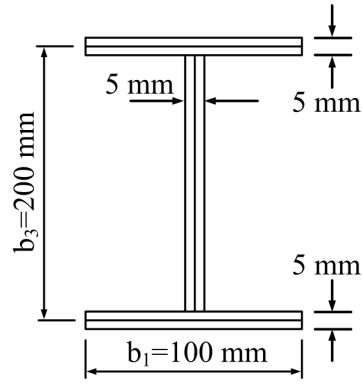


FIG. 2 Geometry of thin-walled composite I-beam.

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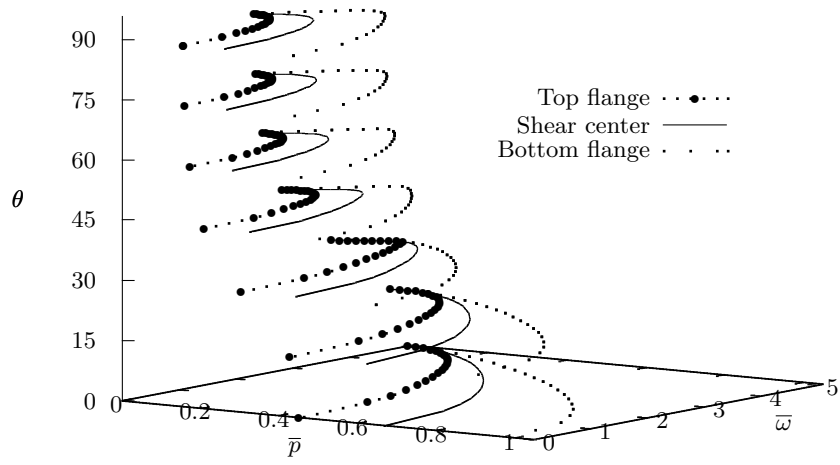


FIG. 3 Effect of load heights on the first load-frequency interaction curves with respect to the fiber angle change in the bottom flange of a simply supported composite beam under uniformly distributed load.

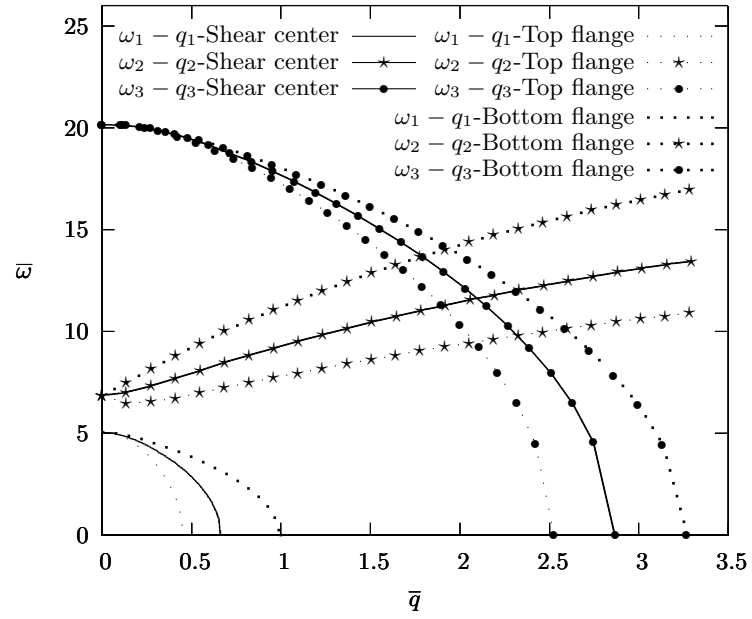


FIG. 4 Effect of load heights on the first three load-frequency interaction curves with the fiber angle 0° in the bottom flange of a simply supported composite beam under uniformly distributed load.

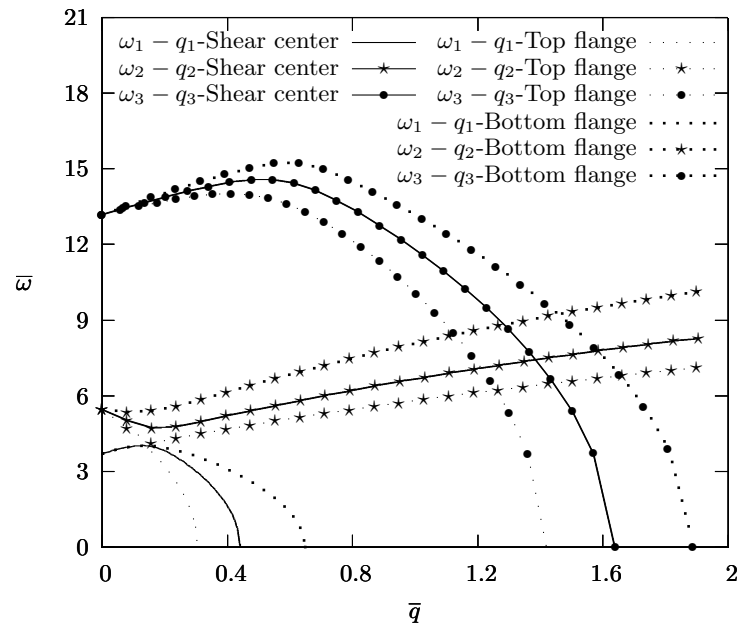


FIG. 5 Effect of load heights on the first three load-frequency interaction curves with the fiber angle 30° in the bottom flange of a simply supported composite beam under uniformly distributed load.

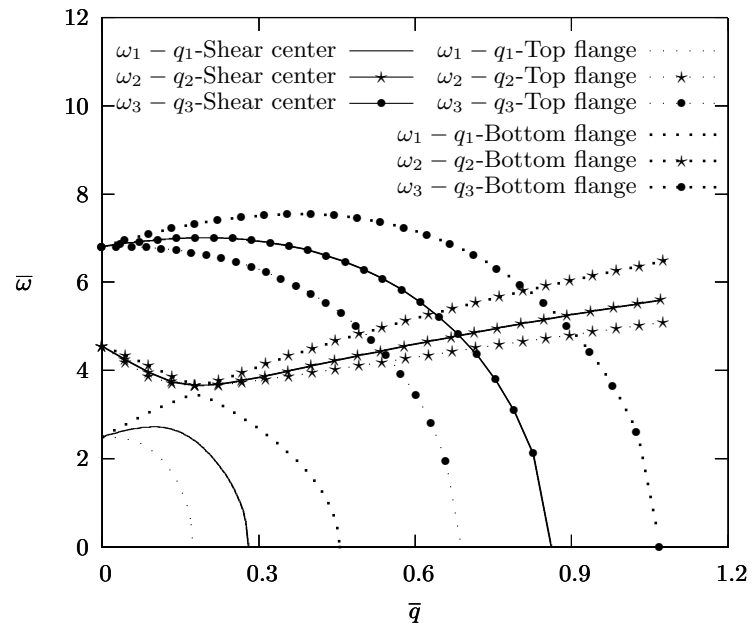


FIG. 6 Effect of load heights on the first three load-frequency interaction curves with the fiber angle 60° in the bottom flange of a simply supported composite beam under uniformly distributed load.

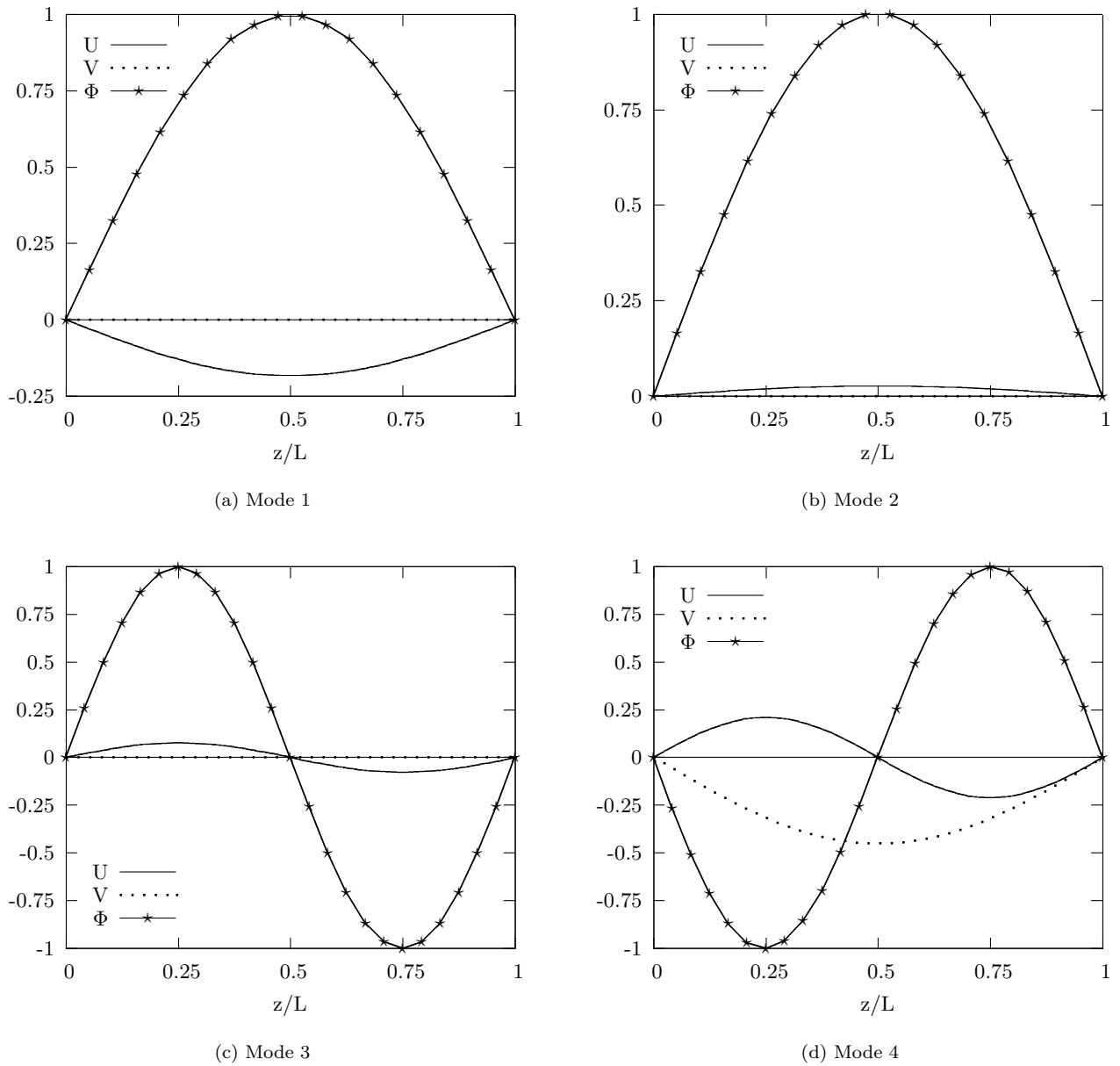


FIG. 7 The first four normal mode shapes of the flexural and torsional components with the fiber angle 30° in the bottom flange of a simply supported composite beam under combined axial compressive force ($\bar{N} = 0.5\bar{N}_{cr}$) and bending moment ($\bar{M} = 0.5\bar{M}_{cr}$).

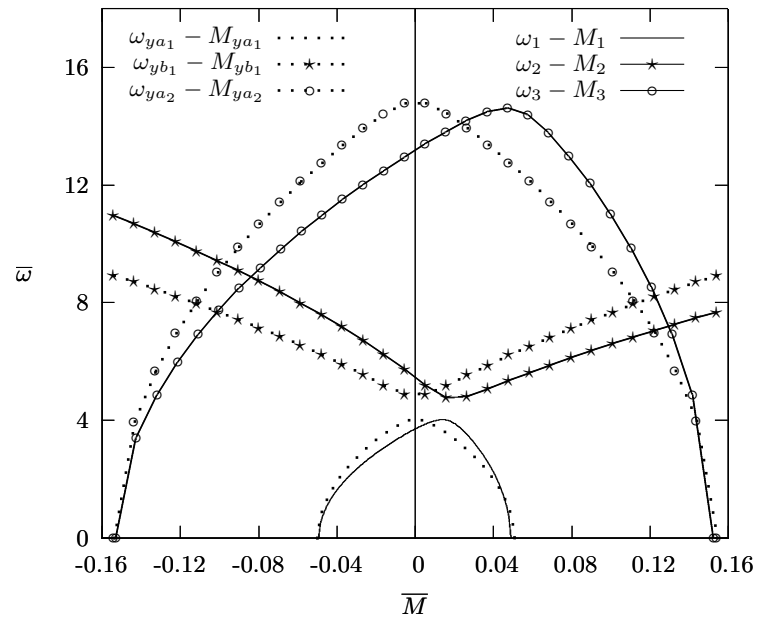


FIG. 8 The first three moment-frequency interaction curves with the fiber angle 30° in the bottom flange of a simply supported composite beam.

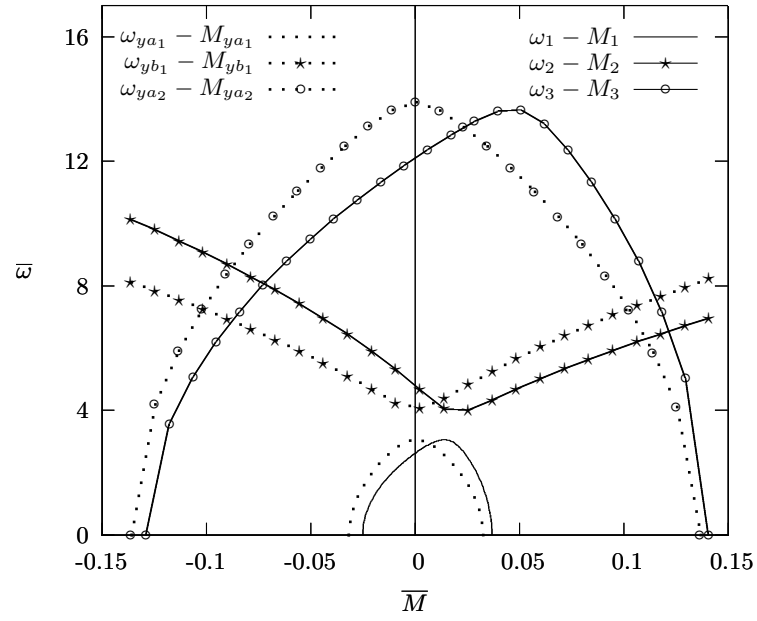


FIG. 9 The first three moment-frequency interaction curves with the fiber angle 30° in the bottom flange of a simply supported composite beam under an axial compressive force ($\bar{N} = 0.5\bar{N}_{cr}$).

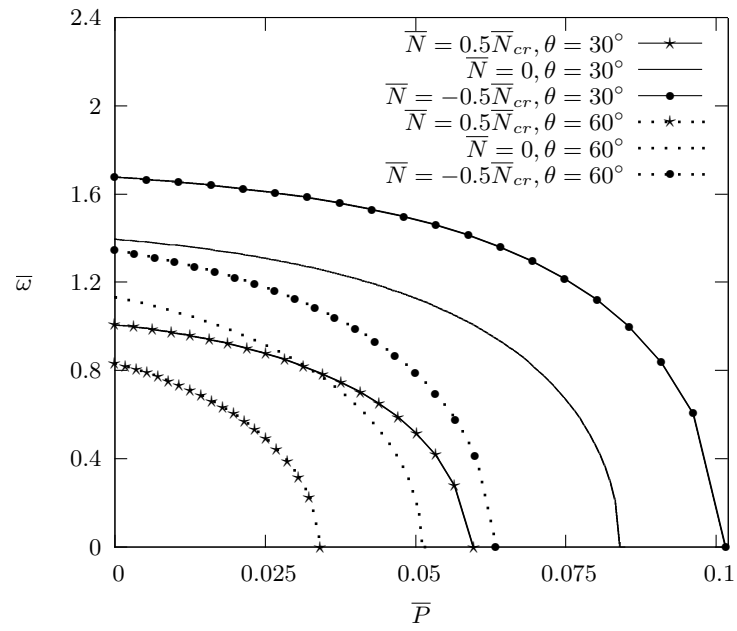


FIG. 10 Effect of axial force on the first load-frequency interaction curves with fiber angles 30° and 60° in the bottom flange of a cantilever composite beam under point load at shear center of free end.

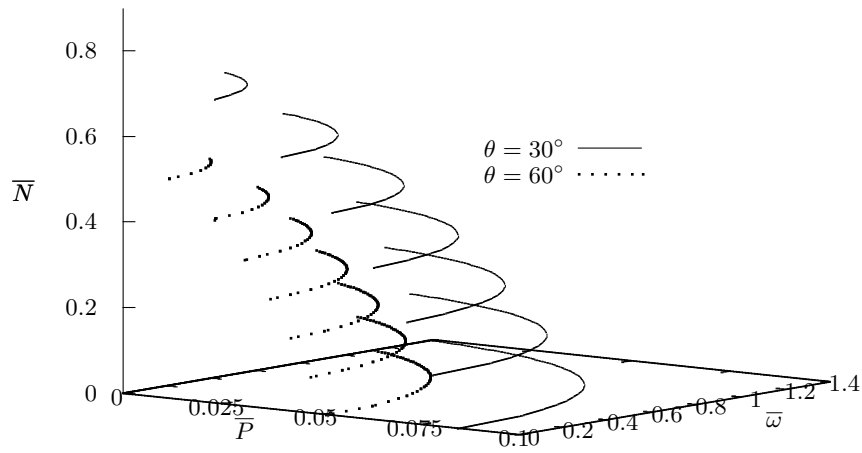


FIG. 11 The first load-frequency interaction curves with respect to the axial compressive force change with fiber angles 30° and 60° in the bottom flange of a cantilever composite beam under point load at shear center of free end.