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Unknown Input Observer Based Robust Fault Estimation for Systems Corrupted by Partially-Decoupled Disturbances

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Abstract – Robust fault estimation plays an important role in real-time monitoring, diagnosis and fault-tolerance control. Accordingly, this paper aims to develop an effective fault estimation technique to simultaneously estimate the system states and the concerned faults, while minimizing the influences from process/sensor disturbances. Specifically, an augmented system is constructed by forming an augmented state vector composed of the system states and the concerned faults. Next, an unknown input observer is designed for the augmented system by decoupling the partial disturbances and attenuating the disturbances that cannot be decoupled, leading to a simultaneous estimate of the system states and the concerned faults. In order to be close to the practical engineering situations, the process disturbances in this study are assumed not to be completely decoupled. In the first part of the paper, the existence condition of such an unknown input observer is proposed to facilitate the fault estimation for linear systems subjected to process disturbances. In the second part, robust fault estimation techniques are addressed for Lipschitz nonlinear systems subjected to both process and sensor disturbances. The proposed technique is finally illustrated by the simulation studies of a three-shaft gas turbine engine and a single-link flexible joint robot.

Keywords: Fault diagnosis, fault estimation, unknown input observer, augmented system approach, linear matrix inequality.

I. INTRODUCTION

Industrial systems have become more expensive and more complex, which have a higher demand in reliability and safety. Unexpected deviation of characteristic properties or system parameters from standard condition, defined as fault, can possibly induce serious damages and even break down the system. As a result, it is paramount to detect the occurrence of

a fault at the early stage, determine the location of the fault, and identify the degree of the severity of the fault. During the last four decades, extensive results were reported on various analytical redundancy based fault diagnosis techniques [1-4] and their applications in aero systems, electro-mechanical systems, energy systems [5-8] and so forth.

The approaches of fault diagnosis can be classified into various categories from different perspectives. In the three-part review paper [9-11], fault diagnosis methods were classified into quantitative model based methods, qualitative model methods, and process history-based methods. In the recent two-part survey paper [12, 13], fault diagnosis approaches were categorized into model-based methods, signal-based methods, knowledge-based methods, and hybrid/active methods. Model-based method has been a popular tool for fault diagnosis, which can provide systematic design solutions, but the diagnosis performance highly relies on the disturbance/noise attenuation ability of the diagnosis algorithms. Observer/filter plays a key role in model-based fault diagnosis methods, which utilizes input and output data to monitor the consistency between the predicted model output and the output of the actual process, leading to a diagnosis decision. In order to attenuate the influences from the disturbances/uncertainties, one solution is to carry out various optimization calculations to make the residual sensitive to faults, but robust against the disturbances/uncertainties [14-17]. The alternative is to utilize decomposition techniques to decouple the process disturbances so that the effects from the disturbances to the residual are removed. One of the known *disturbance decomposition techniques* is differential geometric approach [18-20], which was utilized to perfectly decouple process disturbances for fault detection and isolation. Another popular disturbance decomposition technique is *unknown input observer (UIO)*, which was proposed in [21] and was then extended for nonlinear systems with Lipschitz constraints [22, 23]. The UIO methods were utilized for either *robust state estimation* [22, 23] or *robust fault diagnosis* [21, 24-30]. Specifically, an UIO based fault detection filter was proposed for linear time-invariant systems in [21]. Meanwhile, UIO techniques were developed in [24-27] for robust fault detection and isolation for a class of nonlinear systems. Moreover, UIO-based *fault/disturbance estimation and reconstruction* were addressed in [28-30]. It is noted that both

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the differential geometric decomposition techniques in [18-20] and the UIO decomposition techniques in [21-30] are constrained by some existence conditions, which would become invalid in some scenarios. As a result, there is strong motivation to develop a technique to handle the systems when the disturbances cannot be completely decoupled by using the existing *decomposition techniques* [18-30]. Recently, a few results were reported for systems subjected to partially decoupled process disturbances in [31, 32] by using the UIO techniques, however, which were proposed for state estimates only. Therefore, this motivates us to make more effort to investigate fault diagnosis issues for systems subjected to partially decoupled disturbances/uncertainties.

Fault estimation is an advanced fault diagnosis method, which not only can tell when and where the faults occur, but also can provide the sizes and shapes of the faults, which are crucial for on-line fault tolerant control and real-time decision. Fault estimation can be realized by utilizing various observer techniques such as adaptive observers [33], sliding mode observers [34], and augmented system observers (including descriptor observers) [35-37]. It is noticed that the fault estimation techniques in [33-37] attenuate the disturbances by either using optimization methods or high-gain design approaches, rather than using the disturbance decomposition techniques. It is evident that disturbance decomposition can better alleviate the adverse influences from the disturbances. Therefore, it is desirable to decouple the disturbances as much as possible. For systems subjected to the disturbances which cannot be completely decoupled, one can decouple partial disturbance components while attenuating the disturbance components that cannot be decoupled, by using optimization techniques.

In this study, an augmented system is first constructed by forming an augmented state vector composed of the system states and the concerned faults. Second, an UIO is presented for the augmented system to decouple partial process disturbances, and the existence conditions of such an observer are addressed as well. Linear matrix inequality (LMI) optimization technique is utilized to ensure the estimation error dynamics to be stable, and the disturbances that cannot be decoupled to be attenuated, leading to an effective simultaneous estimate of the states and the concerned faults, addressed in Section II. Furthermore, the UIO-based fault estimation methods are extended to Lipschitz nonlinear systems, and further extensions are then made so that the methods can be applied to the systems subjected to both process and sensor disturbances, presented in Section III. Finally, the effectiveness of the proposed methods is demonstrated by two engineering-oriented systems in Section IV: a three-shaft gas turbine engine and a single-link flexible joint robot.

The notations in the presented paper are quite standard. \mathcal{R}^n and $\mathcal{R}^{n \times m}$ stand for n -dimensional Euclidean space and the set of $n \times m$ real matrices, respectively. I_n denotes identity matrix with dimension of $n \times n$. 0 is a scalar zero or a zero matrix with appropriate zero entries. The superscript T represents the transpose of matrices or vectors. The notation $X > Y$ indicates

that the symmetric matrix $X - Y$ is positive definite. $Re(s)$ represents the real part of the complex number s . $\|\cdot\|$ denotes standard norm of vectors or matrices, while $\|d\|_{Tf} = (\int_0^{Tf} d^T(\tau) d(\tau) d\tau)^{1/2}$. Moreover, $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$.

II. UIO-BASED FAULT ESTIMATION FOR LINEAR SYSTEMS

A. System description and augmented system

Consider a dynamic system in the form of:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t) \\ y(t) = Cx(t) + Du(t) + D_f f(t) \end{cases} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ represents state vector with initial value of $x(0) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}^m$ and $y(t) \in \mathcal{R}^p$ stand for control input vector and measurement output vector, respectively, $d(t) \in \mathcal{R}^d$ is a bounded unknown input vector caused by either disturbances or modelling errors, $f \in \mathcal{R}^{l_f}$ is the fault vector involving actuator faults and sensor faults, A, B, C, D, B_d, B_f and D_f are known constant coefficient matrices with appropriate dimensions. For the simplicity of presentation, the symbol t will be omitted in the rest of the paper.

Incipient faults and abrupt faults are the most common faults in industrial processes. Therefore, in this study, the faults concerned are assumed to be either incipient faults or abrupt faults. As a result, the second-order derivative of the fault f should be zero piecewise. In other words, $\ddot{f} = 0$, which makes sense from the perspective of engineering practices. On the other hand, one could consider a more general case, that is, the q^{th} -order derivate of the fault is assumed to be zero as shown in [35, 36]. However, in this paper, we concentrate on the case $q = 2$ without loss of generality, and the results of the paper can easily be extended to the case when $q \geq 3$.

In addition, $B_d = [B_{d1} \ B_{d2}]$, $d = [d_1 \ d_2]^T$, $d_1 \in \mathcal{R}^{l_{d1}}$ and $d_2 \in \mathcal{R}^{l_{d2}}$, where d_1 rather than d_2 is assumed to be decoupled, and B_{d1} is of full column rank.

Define an augmented state vector as

$$\bar{x} = [x^T \ \dot{f}^T \ f^T]^T \in \mathcal{R}^{\bar{n}} \quad (2)$$

where $\bar{n} = n + 2l_f$.

As a result, we can construct an equivalent augmented system as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{B}_d d \\ y = \bar{C}\bar{x} + Du \end{cases} \quad (3)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & B_f \\ 0 & 0 & 0 \\ 0 & I_{l_f} & 0 \end{bmatrix} \in \mathcal{R}^{\bar{n} \times \bar{n}}, \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} \in \mathcal{R}^{\bar{n} \times m},$$

$$\bar{B}_d = \begin{bmatrix} B_d \\ 0 \\ 0 \end{bmatrix} \in \mathcal{R}^{\bar{n} \times l_d}, \text{ and } \bar{C} = [C \ 0 \ D_f] \in \mathcal{R}^{p \times \bar{n}}.$$

Clearly, \bar{x} contains the state vector x , the concerned fault vector f , and its first-order derivative \dot{f} . As a result, the three

components can be estimated simultaneously if an observer exists for the augmented system (3).

B. Novel unknown input observer (UIO)

Consider the following unknown input observer (UIO):

$$\begin{cases} \dot{\bar{z}} = R\bar{z} + T\bar{B}u + K(y - Du) \\ \hat{\bar{x}} = \bar{z} + H(y - Du) \end{cases} \quad (4)$$

in which $\bar{z} \in \mathcal{R}^{\bar{n}}$ is the state vector of the dynamic system (4) and $\hat{\bar{x}} \in \mathcal{R}^{\bar{n}}$ represents the estimation of $\bar{x} \in \mathcal{R}^{\bar{n}}$, while $R \in \mathcal{R}^{\bar{n} \times \bar{n}}$, $K = K_1 + K_2$, $K_1 \in \mathcal{R}^{\bar{n} \times p}$, $K_2 \in \mathcal{R}^{\bar{n} \times p}$, $T \in \mathcal{R}^{\bar{n} \times m}$ and $H \in \mathcal{R}^{\bar{n} \times p}$ are the gain matrices to be designed.

Letting estimation error $\bar{e} = \bar{x} - \hat{\bar{x}}$, and using the output equation in (4), one has

$$\begin{aligned} \bar{e} &= \bar{x} - \hat{\bar{x}} \\ &= \bar{x} - \bar{z} - H\bar{C}\bar{x} \\ &= (I_{\bar{n}} - H\bar{C})\bar{x} - \bar{z} \end{aligned} \quad (5)$$

Using (3)-(5), the derivative of \bar{e} can thus be calculated as

$$\begin{aligned} \dot{\bar{e}} &= (I_{\bar{n}} - H\bar{C})\dot{\bar{x}} - \dot{\bar{z}} \\ &= (I_{\bar{n}} - H\bar{C})(\bar{A}\bar{x} + \bar{B}u + \bar{B}_d d) - R\bar{z} - T\bar{B}u - K_1\bar{C}\bar{x} \\ &\quad - K_2(y - Du) \\ &= (\bar{A} - H\bar{C}\bar{A} - K_1\bar{C})\bar{e} + (\bar{A} - H\bar{C}\bar{A} - K_1\bar{C} - R)\bar{z} \\ &\quad + [(\bar{A} - H\bar{C}\bar{A} - K_1\bar{C})H - K_2](y - Du) \\ &\quad + [(I_{\bar{n}} - H\bar{C})\bar{B} - T\bar{B}]u + (I_{\bar{n}} - H\bar{C})\bar{B}_{d1}d_1 \\ &\quad + (I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 \end{aligned} \quad (6)$$

where $[\bar{B}_{d1} \ \bar{B}_{d2}] = \bar{B}_d$.

If one can make the following relationships hold,

$$(I_{\bar{n}} - H\bar{C})\bar{B}_{d1} = 0 \quad (7)$$

$$R = \bar{A} - H\bar{C}\bar{A} - K_1\bar{C} \quad (8)$$

$$T = I_{\bar{n}} - H\bar{C} \quad (9)$$

$$K_2 = RH \quad (10)$$

the state estimation error dynamics (6) reduces to

$$\dot{\bar{e}} = R\bar{e} + (I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2. \quad (11)$$

From (11), one can see d_1 has been decoupled under the conditions (7)-(10), but d_2 still exists. Therefore, the observer design is transformed into seeking the solution to (7), and designing an algorithm to make the observer system matrix R stable, and minimizing the influence from the unknown input d_2 .

It is ready to develop the existence condition of the UIO with original system matrices, and the following lemma is useful for the proof of Theorem 1.

Lemma 1 [2, 21]. The necessary and sufficient conditions for the existence of the UIO (4) for the system (3) are:

- (i) $\text{rank}(\bar{C}\bar{B}_{d1}) = \text{rank}(\bar{B}_{d1})$;
- (ii) (\bar{C}, \bar{A}_1) is a detectable pair, where $\bar{A}_1 = (I_{\bar{n}} - H\bar{C})\bar{A}$.

Remark 1.

(a) Condition (i) in Lemma 1 can ensure equation (7) to be solvable, and a special solution is:

$$H_* = \bar{B}_{d1}[(\bar{C}\bar{B}_{d1})^T(\bar{C}\bar{B}_{d1})]^{-1}(\bar{C}\bar{B}_{d1})^T. \quad (12)$$

(b) Condition (ii) in Lemma 1 is standard for arbitrarily assigning the unstable poles of R . Moreover, the condition (ii) is equivalent to the condition that the transmission zeros from the unknown inputs to the measurements must be stable, i.e.,

$$\begin{bmatrix} sI_{\bar{n}} - \bar{A} & \bar{B}_{d1} \\ \bar{C} & 0 \end{bmatrix} \quad (13)$$

is of full column rank for all s with $\text{Re}(s) \geq 0$.

Theorem 1. The necessary and sufficient conditions for the existence of the UIO (4) for the system (3) are

- (i) $\text{rank}(CB_{d1}) = \text{rank}(B_{d1})$;
- (ii) $\begin{bmatrix} A & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix}$ is of full column rank;
- (iii) $\text{rank} \begin{bmatrix} sI_n - A & B_{d1} \\ C & 0 \end{bmatrix} = n + l_{d1}$ for all s with $\text{Re}(s) \geq 0$, but $s \neq 0$.

Proof. It is noted that

$$\bar{C}\bar{B}_{d1} = [C \quad 0 \quad D_f] \begin{bmatrix} B_{d1} \\ 0 \\ 0 \end{bmatrix} = CB_{d1}$$

and

$$\text{rank}(\bar{B}_{d1}) = \text{rank}(B_{d1}).$$

Therefore, one can know that condition (i) in Lemma 1, that is $\text{rank}(\bar{C}\bar{B}_{d1}) = \text{rank}(\bar{B}_{d1})$, is equivalent to the condition (i) in Theorem 1, that is, $\text{rank}(CB_{d1}) = \text{rank}(B_{d1})$.

It is noted that

$$\begin{aligned} &\text{rank} \begin{bmatrix} sI_{\bar{n}} - \bar{A} & \bar{B}_{d1} \\ \bar{C} & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} sI_n - A & 0 & -B_f & B_{d1} \\ 0 & sI_{l_f} & 0 & 0 \\ 0 & -I_{l_f} & sI_{l_f} & 0 \\ C & 0 & D_f & 0 \end{bmatrix} \\ &= \begin{cases} \text{rank} \begin{bmatrix} A & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix} + l_f, & s = 0 \\ \text{rank} \begin{bmatrix} sI_n - A & B_{d1} \\ C & 0 \end{bmatrix} + 2l_f, & s \neq 0 \end{cases} \end{aligned} \quad (14)$$

Therefore, (14) implies that the conditions (ii) and (iii) in Theorem 1 are equivalent to (13) being full of column rank for all s with $\text{Re}(s) \geq 0$, which is also equivalent to (ii) in Lemma 1. This completes the proof.

The next step for designing robust observer (4) is to make the matrix R stable and reduce the influence from the disturbance that cannot be decoupled, that is, d_2 .

Theorem 2. For system (3), there exists a robust UIO in the form of (4) such that $\|\bar{e}\|_{Tf} \leq r\|d_2\|_{Tf}$, if there exists a positive definite matrix P and matrix Q , such that

$$\begin{bmatrix} I_{\bar{n}} + \bar{A}_1^T P + P \bar{A}_1 - \bar{C}^T Q^T - Q \bar{C} & P(I_{\bar{n}} - H \bar{C}) \bar{B}_{d2} \\ * & -r^2 I_{l_{d2}} \end{bmatrix} < 0 \quad (15)$$

where $\bar{A}_1 = (I_{\bar{n}} - H \bar{C}) \bar{A}$, and $Q = P K_1$.

Proof. Take the following Lyapunov function candidate for error dynamic system (11):

$$V(\bar{e}) = \bar{e}^T P \bar{e}. \quad (16)$$

Using (11) and (16), one has

$$\begin{aligned} \dot{V}(\bar{e}) &= \bar{e}^T P \dot{\bar{e}} + \dot{\bar{e}}^T P \bar{e} \\ &= \bar{e}^T (\bar{A}_1^T P + P \bar{A}_1 - \bar{C}^T Q^T - Q \bar{C}) \bar{e} \\ &\quad + 2 \bar{e}^T P (I_{\bar{n}} - H \bar{C}) \bar{B}_{d2} d_2. \end{aligned} \quad (17)$$

Form (15), one can see

$$I_{\bar{n}} + \bar{A}_1^T P + P \bar{A}_1 - \bar{C}^T Q^T - Q \bar{C} < 0,$$

indicating $\bar{A}_1^T P + P \bar{A}_1 - \bar{C}^T Q^T - Q \bar{C} < 0$.

Apparently, when $d_2 = 0$, one can obtain $\dot{V}(\bar{e}) < 0$, implying that the error dynamics in (11) is asymptotically stable.

Let

$$\Gamma = \int_0^{Tf} (\bar{e}^T \bar{e} - r^2 d_2^T d_2) dt. \quad (18)$$

By using (17) and (18), one has:

$$\begin{aligned} \Gamma &= \int_0^{Tf} (\bar{e}^T \bar{e} - r^2 d_2^T d_2 + \dot{V}(\bar{e})) dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \\ &= \int_0^{Tf} [\bar{e}^T (I_{\bar{n}} + \bar{A}_1^T P + P \bar{A}_1 - \bar{C}^T Q^T - Q \bar{C}) \bar{e} \\ &\quad + 2 \bar{e}^T P (I_{\bar{n}} - H \bar{C}) \bar{B}_{d2} d_2 - r^2 d_2^T d_2] dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \\ &= \int_0^{Tf} [\bar{e}^T \quad d_2^T] \Pi \begin{bmatrix} \bar{e} \\ d_2 \end{bmatrix} dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \end{aligned} \quad (19)$$

where

$$\Pi = \begin{bmatrix} I_{\bar{n}} + \bar{A}_1^T P + P \bar{A}_1 - \bar{C}^T Q^T - Q \bar{C} & P(I_{\bar{n}} - H \bar{C}) \bar{B}_{d2} \\ * & -r^2 I_{l_{d2}} \end{bmatrix}.$$

Under zero initial condition $\bar{e}(0) = 0$, one has

$$\begin{aligned} \int_0^{Tf} \dot{V}(\bar{e}) dt &= \bar{e}^T(T_f) P \bar{e}(T_f) - \bar{e}^T(0) P \bar{e}(0) \\ &= V(\bar{e}(T_f)) > 0. \end{aligned} \quad (20)$$

Since $\Pi < 0$ in terms of (15), and from (19) and (20), one has $\Gamma < 0$, which indicates $\|\bar{e}\|_{Tf} \leq r\|d_2\|_{Tf}$. The proof is completed.

C. Design procedure of the UIO for fault estimation

Based on Theorems 1 and 2, we can summarize the design procedure of the UIO as follows.

Procedure1. The design of robust UIO for fault estimation

- i) Construct the augmented system in the form of (3).

- ii) Select the matrix H_* in the form of (12).

- iii) Solve the LMI (15) to obtain the matrices P and Q , and calculate the gain $K_1 = P^{-1}Q$.

- iv) Calculate the other gain matrices R , T and K_2 following the formulae (8)-(10), respectively.

- v) Implement the robust UIO (4), and obtain the augmented estimate \hat{x} , leading to the simultaneous state and fault estimates as follows:

$$\hat{x} = [I_n \quad 0_{n \times 2l_f}] \hat{\bar{x}} \quad (21)$$

$$\hat{f} = [0_{n \times (n+l_f)} \quad I_{l_f}] \hat{\bar{x}}. \quad (22)$$

III. UIO-BASED FAULT ESTIMATION FOR LIPSCHITZ NONLINEAR SYSTEMS

A. Nonlinear systems subjected to process disturbances

In this subsection, UIO-based fault estimation approaches are to be proposed for Lipschitz nonlinear system subjected to process disturbances. The Lipschitz nonlinear system under consideration is represented as follows:

$$\begin{cases} \dot{x} = Ax + Bu + B_d d + B_f f + \Phi(t, x, u) \\ y = Cx + Du + D_f f \end{cases} \quad (23)$$

where $\Phi(t, x, u) \in \mathcal{R}^n$ is a real nonlinear vector function with Lipschitz constant θ , namely,

$$\begin{aligned} \|\Phi(t, x, u) - \Phi(t, \hat{x}, u)\| &\leq \theta \|x - \hat{x}\|, \\ \forall(t, x, u), (t, \hat{x}, u) &\in \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^m, \end{aligned} \quad (24)$$

and the other symbols are the same as defined as (1). Lipschitz nonlinear systems, locally Lipschitz nonlinear systems at least, can be found in many practical systems. All the results derived for a globally Lipschitz system can be applied to a locally Lipschitz system directly.

Defining an augmented state vector in the form of (2), one can obtain an equivalent augmented system as follows:

$$\begin{cases} \dot{\bar{x}} = \bar{A} \bar{x} + \bar{B} u + \bar{B}_d d + \bar{\Phi}(t, x, u) \\ y = \bar{C} \bar{x} + Du \end{cases} \quad (25)$$

where $\bar{\Phi}(t, x, u) = [\Phi^T(t, x, u) \quad 0 \quad 0]^T \in \mathcal{R}^{\bar{n}}$, and the other symbols are defined the same as those in (3).

A nonlinear UIO is in the form of

$$\begin{cases} \dot{\bar{z}} = R \bar{z} + T \bar{B} u + K(y - Du) + T \bar{\Phi}(t, \hat{x}, u) \\ \hat{\bar{x}} = \bar{z} + H(y - Du) \end{cases} \quad (26)$$

where $K = K_1 + K_2$, the gains H , R , T , and K_2 satisfy (7)-(10), and K_1 is to be designed.

The estimation error is defined by (5). In terms of (5), (25) and (26), the estimation error dynamics is represented as

$$\begin{aligned} \dot{\bar{e}} &= (\bar{A} - H \bar{C} \bar{A} - K_1 \bar{C}) \bar{e} + (\bar{A} - H \bar{C} \bar{A} - K_1 \bar{C} - R) \bar{z} \\ &\quad + [(\bar{A} - H \bar{C} \bar{A} - K_1 \bar{C}) H - K_2](y - Du) \\ &\quad + [(I_{\bar{n}} - H \bar{C}) \bar{B} - T \bar{B}] u + (I_{\bar{n}} - H \bar{C}) \bar{B}_{d1} d_1 \end{aligned}$$

$$+(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 + (I_{\bar{n}} - H\bar{C})\tilde{\Phi} \quad (27)$$

in which $\tilde{\Phi} = \bar{\Phi}(t, x, u) - \bar{\Phi}(t, \hat{x}, u)$.

Substitution (7)-(10) into (27) yields

$$\dot{\bar{e}} = R\bar{e} + (I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 + (I_{\bar{n}} - H\bar{C})\tilde{\Phi}. \quad (28)$$

It is time to design the observer K_1 to ensure the estimation error dynamics above to be asymptotically stable and satisfy the robust performance index. The following two lemmas are useful in deriving Theorem 3.

Lemma 2 [38]. For any matrices $X \in \mathcal{R}^{s \times t}$ and $Y \in \mathcal{R}^{t \times s}$, a time-varying matrix $F(t) \in \mathcal{R}^{t \times t}$ with $\|F(t)\| \leq 1$, and any scalar $\varepsilon > 0$, we have:

$$XF(t)Y + Y^T F^T(t)X^T \leq \varepsilon^{-1}XX^T + \varepsilon Y^T Y.$$

Lemma 3 (Schur complement) [39]. Let $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$ be

a symmetric matrix. $S < 0$ is equivalent to $S_{22} < 0$ and $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$.

Theorem 3. For system (25), there exists a robust observer in the form of (26), such that $\|\bar{e}\|_{Tf} \leq r\|d_2\|_{Tf}$, if there exists a positive definite matrix P and matrix Q , such that

$$\begin{bmatrix} \Lambda & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} & P(I_{\bar{n}} - H\bar{C}) \\ * & -r^2 I_{d2} & 0 \\ * & * & -\varepsilon I_{\bar{n}} \end{bmatrix} < 0 \quad (29)$$

where $\Lambda = (\varepsilon\theta^2 + 1)I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C}$,

$\bar{A}_1 = (I_{\bar{n}} - H\bar{C})\bar{A}$, $Q = PK_1$, ε is a given positive number, r is a performance index, standing for the magnitude of error compared with disturbances.

Proof. Choosing the Lyapunov function in the form of (16), and using (28), and noticing that $R = \bar{A}_1 - K_1\bar{C}$, one has

$$\begin{aligned} \dot{V}(\bar{e}) &= \bar{e}^T (\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C})\bar{e} \\ &\quad + 2\bar{e}^T P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 \\ &\quad + \bar{e}^T P(I_{\bar{n}} - H\bar{C})\tilde{\Phi} + \tilde{\Phi}^T (I - H\bar{C})^T P\bar{e} \end{aligned} \quad (30)$$

Applying Lemma 2 to the last two terms in (30) and using (24), one has

$$\begin{aligned} \dot{V}(\bar{e}) &\leq \bar{e}^T (\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} + \varepsilon\theta^2 I_{\bar{n}} \\ &\quad + \varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P)\bar{e} \\ &\quad + 2\bar{e}^T P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 \end{aligned} \quad (31)$$

In terms of Lemma 3, one can see that (29) implies

$$\Lambda + \varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P < 0, \quad (32)$$

which is equivalent to

$$\begin{aligned} &\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} + \varepsilon\theta^2 I_{\bar{n}} \\ &\quad + \varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P < 0. \end{aligned} \quad (33)$$

When $d_2 = 0$, from (31) and (33) one has $\dot{V}(\bar{e}) < 0$, indicating the error dynamics is asymptotically stable.

Letting

$$\Gamma_a = \int_0^{Tf} (\bar{e}^T \bar{e} - r^2 d_2^T d_2 + \dot{V}(\bar{e})) dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \quad (34)$$

and using (31), one has

$$\begin{aligned} \Gamma_a &\leq \int_0^{Tf} [\bar{e}^T (I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} \\ &\quad + \varepsilon\theta^2 I_{\bar{n}} + \varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P)\bar{e} \\ &\quad + 2\bar{e}^T P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 - r^2 d_2^T d_2] dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \\ &= \int_0^{Tf} [\bar{e}^T \quad d_2^T] \Omega \begin{bmatrix} \bar{e} \\ d_2 \end{bmatrix} dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \end{aligned} \quad (35)$$

where

$$\Omega = \begin{bmatrix} \Lambda + \varepsilon^{-1}P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} \\ * & -r^2 I_{d2} \end{bmatrix}. \quad (36)$$

In terms of Lemma 3, the inequality (29) implies $\Omega < 0$. It is also noted that $\int_0^{Tf} \dot{V}(\bar{e}) dt = V(\bar{e}(t_f)) > 0$ under zero initial condition. As a result, from (35) one has $\Gamma_a < 0$, implying

$$\|\bar{e}\|_{Tf} \leq r\|d_2\|_{Tf}. \quad (37)$$

This completes the proof.

B. Nonlinear systems with process and sensor disturbances

In this subsection, a more general case is taken into consideration, that is, a Lipschitz nonlinear system corrupted by both process and sensor disturbances, described by

$$\begin{cases} \dot{x} = Ax + Bu + B_f f + B_d d + \Phi(t, x, u) \\ y = Cx + Du + D_d d_s + D_f f \end{cases} \quad (38)$$

where D_d is constant known matrix, standing for the distribution matrix of the measurement noise $d_s \in \mathcal{R}^s$, and the other symbols are the same as defined before.

Defining an augmented state vector in the form of (2), an equivalent augmented system is given as

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u + \bar{B}_d d + \bar{\Phi}(t, x, u) \\ y = \bar{C}\bar{x} + Du + D_d d_s \end{cases} \quad (39)$$

where the symbols are the same as defined in (25) except for D_d and d_s .

The nonlinear UIO takes the same form as (26). From (26) and (39), one can see the estimation error as

$$\begin{aligned} \bar{e} &= \bar{x} - \hat{\bar{x}} \\ &= \bar{x} - \bar{z} - H(y - Du) \\ &= (I_{\bar{n}} - H\bar{C})\bar{x} - \bar{z} - HD_d d_s. \end{aligned} \quad (40)$$

Furthermore, in terms of (26), (39) and (40), one can obtain the estimation error dynamic equation as follows:

$$\begin{aligned} \dot{\bar{e}} &= R\bar{e} + (I_{\bar{n}} - H\bar{C})\bar{B}_{d2}d_2 + (I_{\bar{n}} - H\bar{C})\tilde{\Phi} \\ &\quad - K_1 D_d d_s - H D_d \dot{d}_s \end{aligned} \quad (41)$$

where $\tilde{\Phi} = \bar{\Phi}(t, x, u) - \bar{\Phi}(t, \hat{x}, u)$.

To design the parameters of the observer (26), the following theorem is addressed.

Theorem 4. For system (39), there exists a robust observer in the form of (26) such that $\|\bar{e}\|_{Tf} \leq r\|\bar{d}\|_{Tf}$, if there exists a positive definite matrix P and matrix Q , such that

$$\begin{bmatrix} \Lambda & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} & P(I_{\bar{n}} - H\bar{C}) & -QD_d & -PHD_d \\ * & -r^2I_{d2} & 0 & 0 & 0 \\ * & * & -\varepsilon I_{\bar{n}} & 0 & 0 \\ * & * & * & -r^2I_s & 0 \\ * & * & * & * & -r^2I_s \end{bmatrix} < 0 \quad (42)$$

where $\Lambda = (\varepsilon\theta^2 + 1)I_{\bar{n}} + \bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C}$,

$\bar{A}_1 = (I_{\bar{n}} - H\bar{C})\bar{A}$, $Q = PK_1$, $\bar{d} = [d_2^T \ d_s^T \ \dot{d}_s^T]^T$, ε is a given positive number, and r is a performance index.

Proof. Taking the Lyapunov function in the form of (16), using (41) and the proof methodology of (30) and (31), one has

$$\begin{aligned} \dot{V}(\bar{e}) &\leq \bar{e}^T (\bar{A}_1^T P + P\bar{A}_1 - \bar{C}^T Q^T - Q\bar{C} + \varepsilon\theta^2 I_{\bar{n}} \\ &\quad + \varepsilon^{-1} P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P) \bar{e} \\ &\quad + 2\bar{e}^T P(I - H\bar{C})\bar{B}_{d2} d_2 - 2\bar{e}^T PK_1 D_d d_s \\ &\quad - 2\bar{e}^T PHD_d \dot{d}_s. \end{aligned} \quad (43)$$

For $d_2 = 0$, and $d_s = 0$, one can see that the error estimation system (41) is asymptotically stable, similar to the proof in Theorem 3.

Letting

$$\Gamma_b = \int_0^{Tf} (\bar{e}^T \bar{e} - r^2 \bar{d}^T \bar{d} + \dot{V}(\bar{e})) dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \quad (44)$$

and using (43), one has

$$\Gamma_b = \int_0^{Tf} [\bar{e}^T \quad d_2^T \quad d_s^T \quad \dot{d}_s^T] \Psi \begin{bmatrix} \bar{e} \\ d_2 \\ d_s \\ \dot{d}_s \end{bmatrix} dt - \int_0^{Tf} \dot{V}(\bar{e}) dt \quad (45)$$

where

$$\Psi = \begin{bmatrix} \Sigma & P(I_{\bar{n}} - H\bar{C})\bar{B}_{d2} & -QD_d & -PHD_d \\ * & -r^2I_{d2} & 0 & 0 \\ * & * & -r^2I_s & 0 \\ * & * & * & -r^2I_s \end{bmatrix} \quad (46)$$

$$\Sigma = \Lambda + \varepsilon^{-1} P(I_{\bar{n}} - H\bar{C})(I_{\bar{n}} - H\bar{C})^T P \quad (47)$$

and Λ is defined in (42).

In terms of Lemma 3, the inequality (42) implies $\Psi < 0$. It is also noted that $\int_0^{Tf} \dot{V}(\bar{e}) dt = V(\bar{e}(t_f)) > 0$ under zero initial condition. Therefore, from (45), one has $\Gamma_b < 0$, indicating $\|\bar{e}\|_{Tf} \leq r\|\bar{d}\|_{Tf}$. This completes the proof.

C. Design procedure of the Nonlinear UIO for fault estimation

On the basis of Theorems 3 and 4, we can summarize the design procedure of the robust nonlinear UIO estimator as

follows.

Procedure 2. The design of nonlinear UIO for fault estimation

- i) Construct the augmented system in the form of (25) or (39), respectively, for systems subjected to either process disturbances or both disturbances in the process and measurement.
- ii) Select the matrix H_* in the form of (12).
- iii) Solve the LMI (29) or (42) to obtain the matrices P and Q , and calculate the gain $K_1 = P^{-1}Q$.
- iv) Calculate the other gain matrices R , T and K_2 following the formulae (8)-(10), respectively.
- v) Implement the robust UIO (26), and obtain the augmented estimate \hat{x} , leading to the simultaneous state and fault estimates in the forms of (21) and (22), respectively.

IV. SIMULATION STUDY

A. Three-shaft gas turbine engine

A three-shaft gas turbine engine can be characterized by a 14-order linearized model:

$$\begin{cases} \dot{x} = Ax + Bu + B_a f_a + B_d d \\ y = Cx + Du + D_a f_a + D_s f_s \end{cases} \quad (48)$$

where the state vector, input vector and output vector are defined respectively as

$$\begin{aligned} x &= [N_L, N_I, N_H, P_{2LM}, P_{2I}, P_2, T_3, P_{4H}, P_{4I}, P_{4M}, W_H, W_C, P_5, T_6]^T, \\ u &= [W_{FE}, W_{FR}, A_f]^T, \\ y &= [W_1, W_2, P_6, T_{2LM}, T_{2I}, T_H]^T. \end{aligned}$$

The meanings of the symbols above are presented in Table 1. The coefficient matrices A, B, C , and D are provided by [37, 40], which are omitted here for brevity.

In this simulation study, three actuator faults and two sensor faults are to be considered. Therefore, $B_a = B$, $D_s = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

Denote $f = [f_a^T \ f_s^T]^T$, we can have $B_f = [B_a \ 0]$ and $D_f = [D_a \ D_s]$, correspondingly.

The unknown input disturbance vector is $d = [d_1^T \ d_2^T \ d_3^T]^T$, where $d_1 = 5 \sin(10t)$, d_2 is random number between -0.5 to 0.5 , and $d_3 = 0.5 \sin(50t)$. The control input vector is $u = [2 \ 2 \ 2]^T$.

The three actuator faults are:

$$f_{a1} = \begin{cases} 0, & t < 10 \\ t - 10, & 10 \leq t < 15 \\ 20 - t, & 15 \leq t < 20 \\ 0, & t \geq 20 \end{cases} \quad (49)$$

TABLE 1
 PARAMETER SYMBOLS OF GAS TURBINE ENGINE

A_J	Nozzle area	T_{2LM}	LP/IP inter-compressor temperature
N_H	HP shaft speed	T_3	Combustor outlet temperature
N_I	IP shaft speed	T_6	Jet pipe outlet temperature
N_L	LP shaft speed	W_1	Fan mass flow
P_{2I}	IP/P inter-compressor pressure	W_2	HP compressor mass flow
P_{2LM}	LP/IP inter-compressor pressure	W_C	Cold stream mass flow
P_{4H}	HP/IP inter-turbine pressure	W_{FE}	Engine fuel
P_{4M}	Post-turbine pressure	W_{FR}	Reheat fuel
P_5	Jet pipe pressure	W_H	Hot stream mass flow
P_6	Nozzle pressure	T_H	Thrust
P_2	Combustor pressure	T_{2I}	LP/HP inter-compressor temperature
P_{4I}	IP/LP inter-turbine pressure		

$$f_{a2} = \begin{cases} 0, & t < 25 \\ 25 - t, & 25 \leq t < 30 \\ -5, & 30 \leq t < 35 \\ t - 40, & 35 \leq t < 40 \\ 0, & t \geq 40 \end{cases} \quad (50)$$

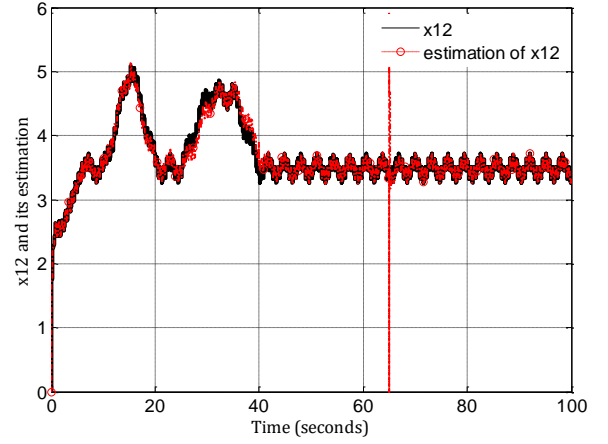
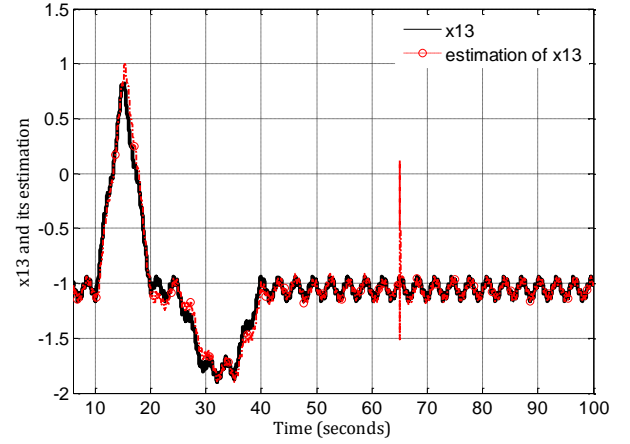
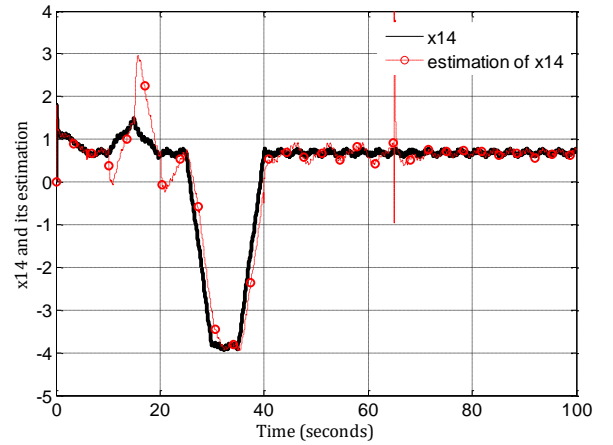
$$f_{a3} = \begin{cases} 0, & t < 2 \\ 0.2t - 0.4, & 2 \leq t < 6 \\ 0.1 \sin(2t) + 0.8, & t \geq 6 \end{cases} \quad (51)$$

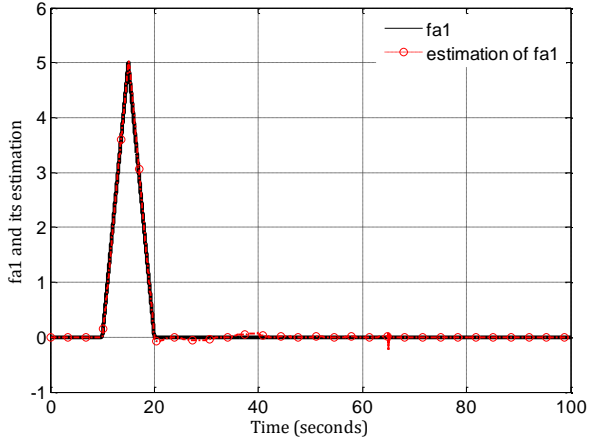
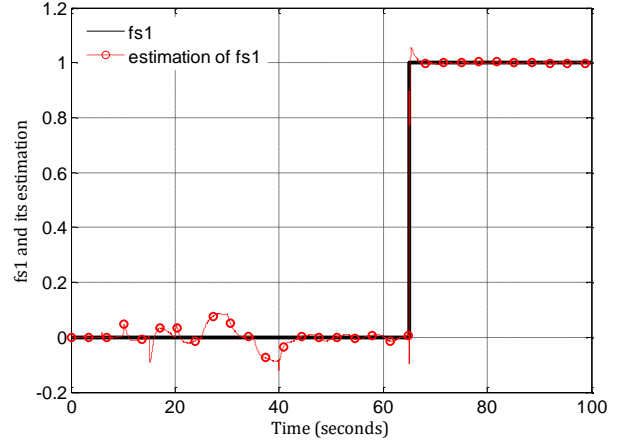
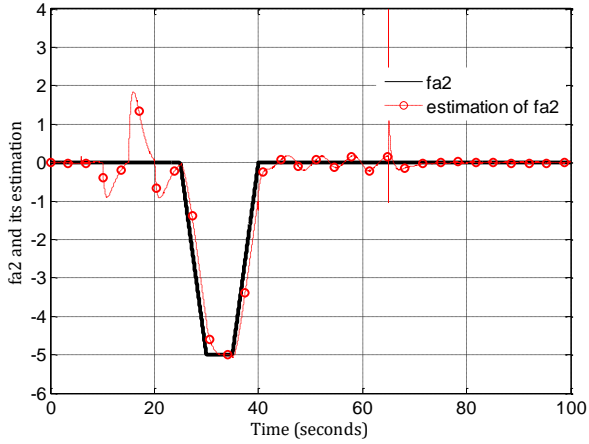
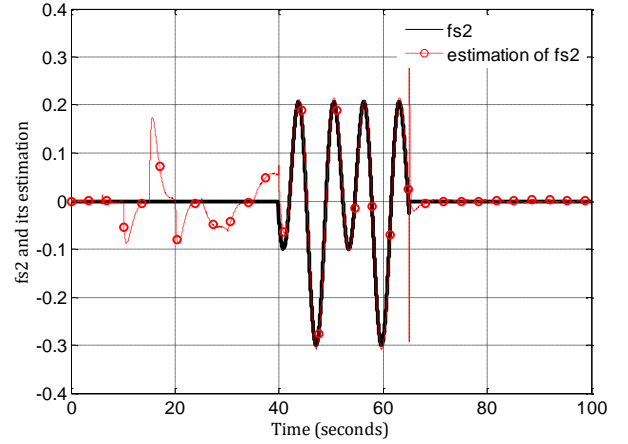
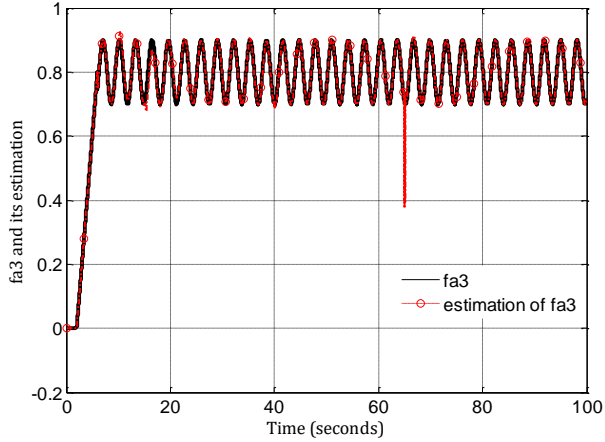
and the two sensor faults concerned are:

$$f_{s1} = \begin{cases} 0, & t < 65 \\ 1, & t \geq 65 \end{cases} \quad (52)$$

$$f_{s2} = \begin{cases} 0, & t < 40 \\ 0.1 \sin(0.5t) + 0.2 \sin(t + \frac{\pi}{2}), & 40 \leq t < 65 \\ 0, & t \geq 65 \end{cases} \quad (53)$$

By using the design procedure 1, one obtains the robust UIO in the form of (4) (the obtained observer gains are omitted here for brevity). The unknown input d_1 is decoupled whereas the influences of d_2 and d_3 are attenuated via the designed observer gains. Due to the limit of space, we only present the curves of the three dominant states (*i.e.*, states corresponding to the three dominant poles) and their estimates, shown in Figs. 1–3, showing excellent estimation performance. The estimates of the three actuator faults and two sensor faults are depicted by Figs. 4–8, respectively. It can be seen that the proposed UIO-based fault estimation techniques can successfully estimate abrupt faults, incipient faults and even sinusoidal faults.


 Fig. 1. x_{12} (the 12th state) and its estimation.

 Fig. 2. x_{13} (the 13th state) and its estimation.

 Fig. 3. x_{14} (the 14th state) and its estimation.


 Fig. 4. f_{a1} (engine fuel actuator fault) and its estimation.

 Fig. 7. f_{s1} (fan mass flow sensor fault) and its estimation.

 Fig. 5. f_{a2} (reheat fuel actuator fault) and its estimation.

 Fig. 8. f_{s2} (HP compressor mass flow sensor fault) and its estimation.

 Fig. 6. f_{a3} (nozel area actuator fault) and its estimation.

B. Single-link flexible joint robot

The single-link manipulator with revolute joints actuated by a DC motor can be described by a Lipschitz nonlinear system [41,42]:

$$\begin{cases} \dot{\theta}_m = \omega_m \\ \dot{\omega}_m = \frac{k}{J_m}(\theta_l - \theta_m) - \frac{G}{J_m}\omega_m + \frac{k_\tau}{J_m}u \\ \dot{\theta}_l = \omega_l \\ \dot{\omega}_l = -\frac{k}{J_l}(\theta_l - \theta_m) - \frac{mgh}{J_l}\sin(\theta_l) \end{cases} \quad (54)$$

where J_m represents the inertia of the DC motor, J_l is the inertia of the link, θ_m and θ_l denote the angles of the rotations of the motor and link, respectively, ω_m and ω_l are the angular velocities of the motor and link, respectively, k is torsional spring constant, k_τ is the amplifier gain, G is the viscous friction, m is the pointer mass, g is the gravity constant, and h is the distance from the rotor to the center of the gravity of the link, and u is the control input (DC voltage) to produce the motor torque. Let $x = [\theta_m \ \omega_m \ \theta_l \ 0.1\omega_l]$, the system can be written in the form of (38), where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \Phi(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.333\sin(x_3) \end{bmatrix}.$$

The fault and disturbance distribution matrices are respectively $B_{fa} = B$, and

$$B_d = \begin{bmatrix} -0.2 & 0.01 & -0.02 \\ -0.1 & 0.02 & -0.04 \\ 0.1 & -0.02 & 0.04 \\ 0.2 & 0.02 & -0.04 \end{bmatrix}, D_d = \begin{bmatrix} 0.1 \\ -0.02 \end{bmatrix}.$$

The actuator fault is:

$$f_a = \begin{cases} 1 + 0.1\sin(4t) & t \geq 4 \\ 0.5(t-2) & 2 \leq t < 4, \\ 0 & t < 2 \end{cases}$$

The unknown input disturbances are as follows: $d_1 = 5\sin(10t)$, corrupted by a uniform-random-number signal, $d_2 = 2\sin(10t)$, $d_3 = \sin(20t)$ and $d_s = 0.1\sin(10t)$. The control input is added as $u = 2\sin(2\pi t)$ and the initial state value is given as $x(0) = [0.01 \ -5 \ 0.01 \ 5]^T$.

Choose $r = 0.58$, $\varepsilon = 50$, and using the procedure 2, we can obtain the observer gains as follows:

$$H = \begin{bmatrix} 0.8000 & 0.4000 \\ 0.4000 & 0.2000 \\ -0.4000 & -0.2000 \\ -0.8000 & -0.4000 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.2000 & -0.4000 & 0 & 0 & 0 & 0 \\ -0.4000 & 0.8000 & 0 & 0 & 0 & 0 \\ 0.4000 & 0.2000 & 1 & 0 & 0 & 0 \\ 0.8000 & 0.4000 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$K = K_1 + K_2 = \begin{bmatrix} -2851.2 & 5761.3 \\ 396.33 & -910.70 \\ 3172.4 & -6393.8 \\ -1638.4 & 3224.6 \\ -6308.5 & 12617 \\ -14930 & 29859 \end{bmatrix},$$

$$R = \begin{bmatrix} -1595.9 & -7993.9 & -19.440 & 0 & 0 & -8.6400 \\ 210.82 & 1232.3 & 38.880 & 0 & 0 & 17.280 \\ 1781.0 & 8875.5 & 9.7200 & 10 & 0 & 4.3200 \\ -924.22 & -4496.9 & 17.490 & 0 & 0 & 8.6400 \\ -3540.0 & -17541 & 0 & 0 & 0 & 0 \\ -8378.3 & -41513 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

By choosing the above parameters, d_1 is decoupled and the influences of d_2 , d_3 and d_s are minimized. The curves displayed in Figs. 9–11 exhibit the estimation performance for

angular velocities of the motor and link, and actuator fault respectively.

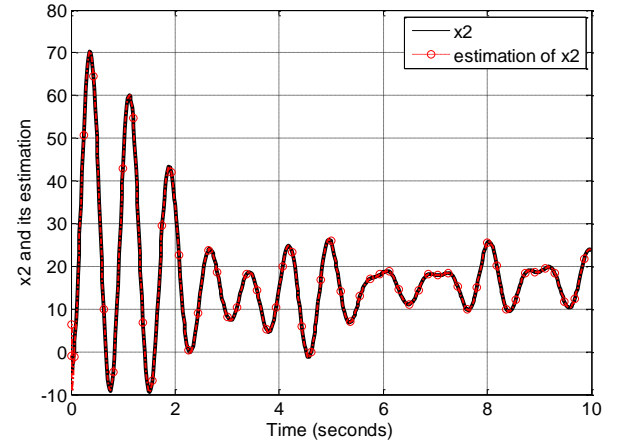


Fig. 9: x_2 (motor angular velocity) and its estimation.

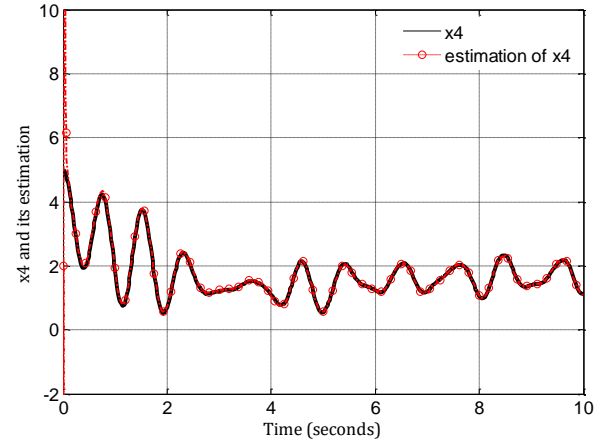


Fig. 10: x_4 (10% link angular velocity) and its estimation.

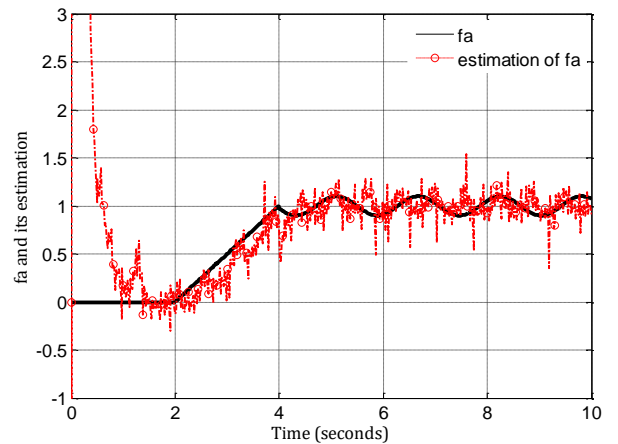


Fig. 11: f_a (DC input voltage actuator fault) and its estimation.

V. CONCLUSION

In this paper, a novel UIO-based simultaneous state and fault estimation technique has been proposed, which can be utilized to handle systems subjected to partially decoupled process disturbances and even sensor disturbances. The design procedures of the estimators for both linear and nonlinear systems are presented. The robustness is ensured by decoupling partial process disturbances with the UIO approach, and attenuating the process disturbances and sensor disturbances that cannot be decoupled, with LMI optimisation technique. The simultaneous estimation is realized with the integration of the system augmentation and the estimator design for the augmented system. The proposed techniques have been illustrated by using two engineering-oriented systems: the three-shaft gas turbine engine and the single link robot. The proposed techniques have great potentials to apply to various engineering systems. It is encouraged to extend the proposed techniques to more complex systems such as nonlinear systems, delay systems, and distributed systems. In addition, from the viewpoint of digital monitoring and real-time implementation, it is of interest to investigate discrete-time systems and delta operator systems [43-45] by taking into account stability, robustness, reliability and optimal operation performance.

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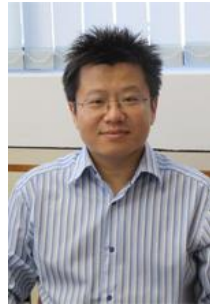
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