A R&D Based Real Business Cycle Model

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Abstract

The New Keynesian Real Business Cycle model with staggered price adjustment is augmented with a R&D producing sector. Two sources of economic shocks are considered, namely random participation (perturbances to value of alternative investment opportunities in another sector) and financial intermediation (shocks to the cost of raising capital in the financial intermediation market). We find that, when comparing to the baseline model, both models can explain pro-cyclical R&D spending. Additionally, the investment oversensitivity problem is corrected. However, only the financial intermediation model is consistent with the observed finding that volatility of R&D is larger than that of investment and output.

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1 Introduction

There are two competing theories in the growth literature. The Solow and Swan neoclassical model, that attributes growth to the accumulation of capital and exogenous technological change. The Romer (1990), Aghion and Howitt (1997) style models, that treat technological development as a product of deliberate human effort - the accumulation of Research and Development (R&D) effort. However, ever since the seminal work of Prescott and Kydland (1982), in which the neoclassical model with exogenous technology shocks was demonstrated to be capable of exhibiting dynamics similar to those observed U.S. data\(^1\); the Solow and Swan model reigns in the business cycle theory.

This paper attempts to fill in the gap by combining the endogenous growth and business cycle theory. It focuses on ties between R&D and financing, in ways different from standard real business cycle model. The source of growth in Solow and Swan model is exogenous technological change. Cycles transpire when some external shocks hit the economy, reducing the aggregate productivity during recession, vice versa for boom. Since deliberate R&D effort drives the economy in the endogenous growth model, we introduce a new shock to the R&D production sector. Nonetheless, the proportion of R&D is at most 5% of GDP in the U.S. economy; is it possible to demonstrate that shocks in this smaller sector can generate dynamics similar to the standard shocks? If yes, how much new insight can we get from it? Does it explain some stylized fact that traditional models fail to replicate?

This paper is structured as follows. The rest of section 1 is a survey of related literature and model outline. Section 2 outlines the New Keynesian staggered price model augmented with a R&D production sector. Section 3 describes the first source of shocks to the economy- one where perturbations arise from imperfections in labor matching in the R&D sector. We will discuss the parameters chosen for the calibration and examine the impulse response functions. Section 4 deals with another variant of the model: a surprise in the financial intermediation sector that would affect capital accumulation. Section 5 compares the calibrated second moments and impulse response functions to the baseline model (New Keynesian); and section 6 concludes.

1.1 Literature Review and Motivation

Important endogenous growth models include Romer (1990), Aghion and Howitt (1997), and Grossman and Helpman (1991). The specific one considered in this paper is Aghion and Howitt (1997). We will examine the impact of interest rate, government and productivity shocks on R&D and output. For a nominal shock to have effect on real variables, certain forms of market imperfections have to be introduced. Thus, we synthesize the New Keynesian Real Business Cycle model (Clarida et al., 1999)

\(^{1}\)For a survey and comparison of real business cycle models, see Rebelo (2005).
with staggered prices, with the Aghion and Howitt (1997) R&D producing sector. This forms the baseline model. An important underlying assumption is the existence of a perfect capital market. How does a R&D firm pay for its expense before the investment turns profitable? Aghion and Howitt (1997) assume that either the intermediate firms own it and provide the funding; or the R&D firm borrows money and repays the loan by selling the patent to the intermediate firms after success. The second assumption is maintained.

The financing of innovation projects is not likely to be perfect. Instead of modelling complicated contractual arrangements between R&D firms and a third party, we propose an alternate source of economic perturbation to model financial frictions - random participation in R&D sector, which can be construed as a change in outside opportunities that drags away resources from R&D sector, similar to the Rybczynski effect in international trade. Combining endogenous growth and business cycle model might add to new insight. In fact, there has been no lack of criticisms against the Real Business Cycle Models (RBC). Bernanke and Parkinson (1991) argue that since the technology shock is roughly the same as Solow residual; if technology shocks are the source of economic fluctuations, then the covariance of Solow residuals and output should be positive. Since it is generally believed that the Great Depression cannot be attributed to a technology shock but the postwar recessions is, then Solow residuals and output should move together only in the postwar period. However, they find that the co-movement between Solow residuals and output has remained relatively intact. RBC theorists posit that an exogenous shock hits the economy, and it propagates over time due to workers' intertemporal substitution of leisure. To be more specific, as a positive technology shock increases the marginal product of labor and real wages, workers are more willing to substitute labor for leisure during economic booms, and vice versa for recessions. Taken literally, this implies that there is no involuntary unemployment during a recession. Empirically, it seems difficult, if not impossible for ideas to disappear. Hence, the idea of technological regress is impossible. Two modifications have been proposed: either assume new forms of innovations, for example government expenditure surprises (King et al., 1988), money shocks (Gali and Gertler, 1999), sunspot activity (Farmer and Guo, 1994) or departure from the neoclassical framework.

A natural alternative to the RBC model is a business cycle model with endogenous technology. Most of endogenous growth theorists recognize the importance R&D on growth. Romer (1990) contends that "technical change arises in large part because of intentional actions taken by people who respond to market incentives." Thus, technical change is endogenous, instead of exogenous as portrayed in the standard

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2 Assume that there are two industries (labor intensive and capital intensive) using two inputs (labor and capital); Rybcznski (1955) proves that endowment increases raises the output of the industry using that factor intensively and decreases the output of the other industry. The Dutch disease is one notable example of this type of phenomenon.
neoclassical model. Obviously, R&D is one of those "intentional actions", that can enhance the productivity of existing capital, improve the quality of a continuum of products (Grossman and Helpman, 1991) or create new products (Young, 1995). The strong role of R&D on growth is well-documented in the literature. Using data compiled by Scherer (1981), Griliches and Lichtenberg (1984) examine the relationship between total factor productivity (TFP) growth and R&D intensity. Griliches and Lichtenberg (1984) derive a revised version of measured TFP which "deviates from its actual TFP growth by the weighted sum of errors in the various materials deflators, using respective cost shares as weights". Using more disaggregated data than Scherer (1981), they confirm the positive relationship between TFP and R&D intensity. The authors proceed to break down R&D into own process, own product and imported product R&D (intermediate input quality improvement).

Despite the importance of R&D in the growth literature, relatively less effort has been made to synthesize the theory of short-run economic fluctuations with endogenous growth theory. Howitt (1998) is one of those few who attempt, in which final good and R&D sectors compete for the labor use. He contends that the arrival of new ‘general technological knowledge’ retards the current growth rate due to withdrawal of labor from manufacturing to the research sector. But his model has some shortcomings: the time lag between the arrival of new technological know-how and the subsequent economic boom is unreasonably long, the business cycle is deterministic, and it predicts that output is countercyclical.

Suarez and Sussman (1997) construct a model such that cycles are driven by defaults and financial distress. Others simply augment the endogenous growth models with an exogenous shocks. Schmitt-Grohe (1997) studies the business cycle fluctuations predicted by a two-sector endogenous growth model with sector-specific external increasing returns to scale. He focuses on the autocorrelations between output, hours and consumption. In essence, the model is a learning-by-doing business cycle model, in which aggregate investment is the source of increasing returns. The calibrated second moment of key variables is in sync with filtered time series. However, once the impulse response functions are examined, the impact of a shock is miniscule. Canton (2002) analyzes the impact of cyclical volatility on long-term growth. An independent and identically distributed shock is introduced into the Lucas-Uzawa model's human capital accumulation equation. Stadler (1990) shows that, with endogenous technical change, both real and money models yield very similar output processes. Learning-by-doing is again the source of externality, which is constantly perturbed by an independent and identically distributed shock. Aggregate productivity is a positive function of average labor.

Our model is different from the earlier works, in the way that the financial market is the source of economic fluctuation and the steady states exist.
1.2 Model Outline

We will develop the New Keynesian business cycle model (Clarida et al., 1999) which generates endogenous growth (Aghion and Howitt, 1997). The outline of the model is illustrated at diagram 1. The competitive final good sector combines a continuum of intermediate goods into a single final good, in the spirit of Dixit and Stiglitz (1977). The monopolistic intermediate sector uses capital and labor as inputs, taking the demand from the final goods sector as given. The production is a modified neoclassical Cobb-Douglas function. With the assumption of monopolistic competition, sticky prices, money and other nominal rigidities can easily be incorporated into the model. The intermediate firms charge a countercyclical price markup, as portrayed in Rotemberg and Woodford (1995). The consumers supply capital and labor to the intermediate firms. They own the shares of the intermediate firms. At the end of each market period, the consumers sell the used capital to a capital producing sector and repurchase them later. The capital producing firm uses last period's capital and new investment from the final goods sector to produce the current capital stock which to be used in the intermediate good's sector. Capital output is a function of investment good - capital stock ratio.

[Insert Diagram 1]

An innovation production sector is added to the Gali and Gertler type model. As in Aghion and Howitt (1997), the invention of intermediate goods along the quality ladder, stemming from R&D is the source of endogenous economic growth.\(^3\) In Aghion and Howitt (1997) model, the R&D firms use labor and capital as inputs. We deviate from the early work and adopt a simple assumption that the final good as the sole input. Technology innovations arrive randomly with a constant Poisson arrival rate, which is a positive function of R&D input. In this way, while the stock of technology accumulates over time with a deterministic long-run trend, unlike other real business cycle model, technology regress is ruled out.

The R&D sector delivers blueprints to the intermediate firms. Their relation can be modeled in different ways. For instance, it can be portrayed in a principal-agent relation. The R&D firms enter into contractual agreement with the intermediate firms, which provided funding for innovation and promise to buy the next generation technology. Alternatively, the financing of R&D projects is supported by a perfect capital market. They repay the debt by selling the next generation patent to the intermediate firms. The former is the setting of section 3 and the latter for section 4. We intend to model two types of economic perturbation in R&D sector related to financing. Two modifications will be made to the baseline model of section 2. The impulse response functions and calibrated moments will be compared at section 5. That completes the outline of the New Keynesian with an augmented R&D sector. The next section is devoted to the delineation of the baseline model.

\(^3\)Romer (1990), in contrast, ascribes growth to increasing varieties of intermediate goods- that is horizontal growth.
2 General Equilibrium Model

2.1 The Representative Consumer

This section establishes the baseline model. Section 2.1 - 2.4 follows closely the New Keynesian business cycle model with staggered prices by Clarida et al. (1999) and Gertler (1998). To extend the model, a new R&D sector will be added in the next section. Moreover, we assume that the only form of financial asset that consumers hold is a zero coupon bond, \( B_t \). A continuum of infinitely-lived consumers, indexed from zero to one, in each market period, purchase a final good \( C_t \). They provide labor \( N_t \) to the intermediate firms, and rent physical capital \( K_t \) to them. The representative household chooses final good consumption, labor supply, capital supply and quantity of real bond holdings at each market period to maximize the following time separable utility function conditional on information available at time \( t \):

\[
E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{1-\gamma} C_{t+i}^{1-\gamma} - \frac{a_n}{1+\gamma} N_{t+i}^{1+\gamma} \right] \right\}
\]

subject to

\[
C_t = \frac{W_t}{P_t} N_t + Z_t K_{t-1} + \int_0^1 \omega(z) \pi_z(z) dz + TR_t - \frac{1}{R_{t+1}} B_t - B_{t-1} - Q_t[K_t - (1-\delta)K_{t-1}]
\]

where \( \gamma < 1, \gamma_n > 0 \) are the consumption and labor elasticity of substitution respectively. \( \beta \) is the subjective discount rate. \( a_n \) is the disutility coefficient of labor. \( \delta \) denotes the depreciation rate of capital. \( \frac{W_t}{P_t} \) is the real wage. \( Z_t \) is the rental cost of capital. The consumers own the monopolistic intermediate firms and the final good production sector. \( \pi_z(z) \) is the profit from owning intermediate firm \( z \), \( \omega(z) \) is the number of shares of the intermediate firms. The government transfers \( TR_t \) to the consumers each period. \( R_{t+1} \) is the nominal interest rate. \( \frac{1}{R_{t+1}} \) denotes the price of a zero coupon bond that pays one dollar next period. \( Q_t \) is the price of renewed capital.

Equation (2) is the consumer's budget constraint. The representative household supplies labor \( N_t \) to

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4 Staggered price is necessary for nominal shocks to have impact on real variables.

5 Ownership of final good sector is irrelevant. Constant returns to scale guarantees zero profit, so it does not enter the consumers' budget constraint.
the intermediate good sector, earning a return of \( \frac{W_t}{P_t} \) each period. He possesses the initial capital \( K_{t-1} \), rents it to the intermediate firms, making a return of \( Z_t \). They own the intermediate firms. The total profit is \( \int_{t}^{t+1} \Delta(z)Z_t(z)dz \). They invest in the bond market, making a nominal return of \( R^n_{t+1} \). At the end of each market period, the household enhances the capital by selling it to the capital producers at a price \( Z_t^{k} \)-that will be zero around balance growth path balance growth path as shown in section 2.4 - and repurchase it at a price, \( Q_t \).

Substituting the budget constraint into the objective function, define real interest rate as

\[
R_{t+1} = R^n_{t+1} E_t \left( \frac{P_t}{P_{t+1}} \right)
\]

we get the following first order conditions:

\[
\frac{W_t}{P_t} = a_n \frac{N_t^{\gamma}}{E_t^{\gamma}} \tag{3}
\]

\[
C_t = E_t (R^n_{t+1} \beta E_{t+1}^{-\gamma})^{-\gamma} \tag{4}
\]

\[
1 = E_t \left\{ R^n_{t+1} E_t \left( \frac{P_t}{P_{t+1}} \right) \beta \left( \frac{E_{t+1}}{E_t} \right)^{-\gamma} \right\} \tag{5}
\]

\[
1 = E_t \left\{ \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t} \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\} \tag{6}
\]

Equation (3) is the marginal rate of substitution between consumption and leisure. Equation (4) is the intertemporal consumption substitution condition. Equation(5) represents the Euler condition, while equation (6) denotes the capital supply condition.

### 2.2 Final Goods Output

The final good is produced by combining different varieties of differentiated intermediate goods \( z \in [0,1] \). Without loss of generality, the number of varieties is held constant, meaning all innovation is aimed at improving existing varieties\(^6\). The final good sector is perfectly competitive. The CES production is given by:

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\(^6\)For a treatment of increasing varieties, see Young (1995). Basically, there will be an additional equation that determines growth rate of varieties. The key implications of the model remain unchanged.
\[
Y_t^f = \left( \int_0^1 Y_t' (z) \frac{\varepsilon - 1}{\varepsilon} dz \right)^{\frac{\varepsilon}{\varepsilon - 1}}
\]  

(7)

\( \frac{\varepsilon}{\varepsilon - 1} \) is the constant elasticity of substitution between the intermediate goods.

A la Dixit-Stiglitz CES production exhibits diminishing marginal return, a property that will drive the firms to diversify and produce using all the intermediate goods. The final good firms choose \( Y_t^f (z) \) to minimize the total cost\(^7\).

Using the fact that elasticity of substitution between final good and intermediate good is one, the first order condition, and equating supply and demand, the lagrangian multiplier can be verified to be \( P_t \), i.e. the composite price index; while the optimal demand for intermediate good is given by:

\[
Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t
\]

(8)

The demand price elasticity is the constant \( \varepsilon \). The composite price index is given by:

\[
P_t = \left( \int_0^1 [P_t(z)]^{-\varepsilon} dz \right)^{\frac{1}{\varepsilon - 1}}
\]

(9)

The baseline model considered here is different from Gertler (1998) in the way that, the final good is used not only for consumption; the capital producers and R&D firms purchase it as input.

2.3 The Intermediate Sector

There is a continuum of monopolistically competitive firms owned by consumers, with compact support \( z \in [0,1] \). The profit is reimbursed to the consumers at the end of each period. Each firm \( z \) faces the downward sloping demand given by equation (8). Gertler (1998) assumes that the intermediate firm uses both capital and labor as inputs. The production technology takes Cobb-Douglas functional form:

\[
Y_t(z) = A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha}
\]

(10)

where \( A_t \) is a technology parameter, and its growth rate is determined by R&D sector. \( N_t(z) \) and \( K_t(z) \) are the labor and capital input demands respectively. Consumers supply labor and rent capital to intermediate firms. The relative wage is \( \frac{w_t}{P_t} \) and the rental price is \( Z_t \). The intermediate goods are sold to the final good firms at \( P_t \), which can be fixed at each market period. The price adjustment mechanisms

\(^7\)With the assumption of constant returns to scale and perfect competition, the size of each firm is indeterminate. We, therefore, derive the input demand by cost minimization.
will be discussed at next subsection.

The input demand is derived by cost minimization. The first order conditions give:

\[
\frac{\alpha Y_i(z)}{N_i(z)} = (1 + \mu_i) \frac{W_i}{P_i} \quad (11)
\]

\[
(1 - \alpha) \frac{Y_i(z)}{K_i(z)} = (1 + \mu_i) Z_i \quad (12)
\]

where the markup \( \mu_i \) is the inverse of real marginal cost\(^8\), i.e. \( 1 + \mu_i = \frac{1}{MC_i} \).\(^9\) Equations (11) and (12) relate the demand for labor and capital to the mark-up wages and rental cost.

### 2.3.1 Optimal Pricing Setting

The staggered price component of Gertler (1998) follows closely the Calvo (1983) model. Price adjustment follows a Bernoulli distribution, with a probability \( \theta \) that price remains fixed. In each market period, only \( (1 - \theta) \) fraction of intermediate firms can readjust prices. Since the draw is independent of history and we do not need to keep track of firms changing prices. The expected time over which the price is fixed, i.e., the expected waiting time for the next price adjustment is \( \frac{1}{1 - \theta} \).\(^{10}\)

In each period, a fraction of \( \theta \) firms will be able to change price to maximize profit. The remaining \( 1 - \theta \) firms can only adjust output to meet demand. Because of constant returns to scale, the size of intermediate firms cannot be determined.

Define \( MC^n_i \) as the intermediate firm's nominal marginal cost - \( MC^n_i = P_i MC_i \), and \( \Lambda_{i,t} = \left( \frac{c^*_{t+i}}{c^*_t} \right)^\gamma \) as the ratio of the marginal utility of consumption at \( t + i \) to marginal utility at \( t \). The first order condition gives

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\(^8\) A similar scenario is minimum wage that effectively increases the marginal cost. For a general equilibrium model about how minimum wage affects growth, see Ho et al. (2007).

\(^9\) Note that the mark-up is time-variant. Equation (11) is one of the profit- maximizing condition of a monopolistic firm that marginal revenue product equal to a mark-up of marginal cost.

\(^{10}\) Alternatively, we can assume an adjustment cost: \( AC_{j,t} = \frac{\phi_y}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \hat{\mu} \right)^2 Y_t \) where \( \phi_y \) measures the degree to which firms dislike to deviate in their price setting behavior from the balance growth path inflation rate \( \hat{\mu} \). Sbordone (1998) contends that this price setting is observationally equivalent to Calvo's model.
\[ P_t^* = (1 + \mu) \sum_{i=0}^{\infty} \phi_{t,i} MC_{t+i}^n \]  

where

\[ \phi_{t,i} = \frac{E_t \left[ (\theta \beta)^i \Lambda_{t,i} Y_{t+i} \left( \frac{1}{r_{t+i}} \right)^{-\varepsilon} \right]}{E_t \sum_{i=0}^{\infty} (\theta \beta)^i \Lambda_{t,i} Y_{t+i} \left( \frac{1}{r_{t+i}} \right)^{-\varepsilon}} \]

Hence, desired price is a weighted average of the marginal cost in the future. The weight is a function of discounted income at \( t+i \), and expected life time income. The perfect price adjustment case is given by setting \( \theta = 0 \). Last, given that all firms that adjust price in time \( t \) choose the same price, the average price of firms that do not adjust is simply price of last period. The price index can be expressed as:

\[ P_t = [\theta P_{t-1}^{1-\varepsilon} + (1 + \theta) P_t^{1-\varepsilon}]^{1/\varepsilon} \]

which is a weighted average of lagged price and optimal price.

### 2.4 Capital Production

We take the initial level of capital as exogenous. In the Gertler (1998) setup, there are a large number of identical competitive capital producers, indexed by \( j \in [0,1] \). Households each period rent capital to intermediate firms, and households later sell it to the capital producers at the price \( Z_t^k \) and then repurchase it at the beginning of next period at the price of \( Q_t \). The production function for the new capital takes the form

\[ Y_t^k(j) = \phi \left[ \frac{I_t(j)}{K_t(j)} \right] K_t(j) \]

\( \phi(.) \) is the adjustment cost in the growth literature, with the properties: \( \phi'(.) > 0, \phi''(.) < 0 \), and at steady state, \( \phi(0) = 0, \phi(\frac{1}{K}) = \frac{1}{K} \), which is the balance growth path balance growth path ratio of investment to capital. The capital production satisfies the Inada condition. The marginal value of product is positive and strictly decreasing with higher investment - capital ratio. Investment is perfect substitute of consumption good. The zero profit condition with respect to \( I_t(j) \) gives

\[ Q_t \phi \left( \frac{I_t(j)}{K_t(j)} \right) = 1 \]
All capital firms are identical. Their optimal \( \frac{I_j(j)}{K_t(j)} \) is the same. The industry first order condition is given by

\[ Q_t = \Phi \left( \frac{I_t}{K_{t-1}} \right) \]  

(15)

where

\[ \Phi \left( \frac{I_t}{K_{t-1}} \right) = \frac{1}{\phi \left( \frac{I_t}{K_{t-1}} \right)} \]  

(16)

and \( K_{t-1} = \int_0^1 K_t(j) dj \). By construction \( \phi \left( \frac{1}{K} \right) = \frac{1}{K} \) at steady state. Thus, \( \phi \left( . \right) = 1 \) and the balance growth path \( Q \) is 1. The capital price \( Z_t^k \) is zero around steady state. Intuitively, one can interpret \( Q_t - Z_t^k \) as net cost of reinstalling capital, with \( Z_t^k = 0 \) as normalization. We have delineated the New Keynesian model with fixed price. A modified Aghion and Howitt (1992) R&D sector will be appended.

2.5 Research Sector

The following extension to the model is built on Aghion and Howitt (1992). There is a continuum of research firms for each intermediate good \( z \), indexed by \( z \in [0,1] \). Each of them innovates by building on existing cutting-edge technology in the economy, \( A_t^{\text{max}} \)-which is shared by all R&D firms. A perfect capital market provides financing to each R&D firm who repays the debt by selling the patent to the intermediate firms.

The R&D and intermediate firm relation is different from Aghion and Howitt (1997), in which the research sector is portrayed as in the patent-race literature that has been surveyed by Tirole (1988) and Reinganum (1989). The innovator will become the monopoly in the next market period. Competition is drastic in the sense that the existing firm will be replaced by the next innovator who can set price without restrictions from the incumbent. Section 3 will depict a principal-agency relation; and that explains the change of assumption. Unlike the Aghion and Howitt (1997) model, each R&D firm uses final good \( (\Xi_t(i)) \) as input, instead of labor alone. Define \( \xi_t(i) = \frac{\Xi_t(i)}{A_t^{\text{max}}} \), the industry Poisson arrival rate, \( \rho_t \), is given by

\[ \rho_t = \lambda p(\xi_t) \]
where $\lambda > 0$, $\rho(\cdot) > 0$, $\rho'(\cdot) < 0$, $\rho(0) = 0$, $\xi_i = \int_0^1 \xi_i(t) \, dt$.

$\lambda$ is the productivity parameter of research; $\rho(\cdot)$ measures the intensity of research, exhibiting diminishing marginal product\(^{11}\). Hence, technology is not exogenous as hypothesized in neoclassical model. Instead, technological growth is the result of deliberate use of final goods. There exists no diffusion barrier across sectors, the cutting-edge technology becomes a public good to all R&D firms. Each innovation at date $t$ by any firm $i$ permits the innovator of all firms to produce using the leading edge technology. Define $A_{z,t}$ as the technology level at intermediate sector $z$. Any innovation raises $A_{z,t}$ by a constant factor of $\gamma^g > 1$ for that firm. Each innovation raises technology up along the quality ladder, as depicted in Grossman and Helpman (1991).

It can be shown that the growth rate of the economy will be\(^{12}\):

$$g_t = \frac{A_{t}^{\text{max}}}{A_{t}^{\text{max}}} = \lambda \rho(\xi_i) \ln \gamma^g$$

(17)

At any point in time, there will be a distribution of productivity parameters $A_{z,t}$ across the sectors of the economy, with support $[0, A_{t}^{\text{max}}]$, shifting rightward over time. However, the long-run distribution of relative productivity parameters $a = \frac{A_{z,t}}{A_{t}^{\text{max}}}$ will be stationary.

**Proposition 1.** The distribution of relative productivity parameters $a$, is invariant over time. The stationary distribution is given by:

$$H(a) = a^{\frac{1}{\ln \gamma}}, 0 \leq a \leq 1$$

(18)

Define $V_{t+1}$ as the value of the patent. The expected profit is the product of innovation arrival rate and $V_{t+1}$. Assuming that final good is the sole input, a typical research firm $i$ maximizes

$$\max_{\xi_i(t)} \left[ \frac{\lambda \rho(\xi_i)}{\xi_i} \right] \xi_i(t) V_{t+1} - (1 - \beta_n) \Xi_i(t)$$

(19)

where $\beta_n$ is government subsidy to research. Notice that as $\xi_i$ increases, the arrival time of next generation technology expected to be shorter; the value of the next generation patent will be lower. In contrast with the endogenous growth literature like Romer (1990), we assume diminishing returns in the R&D sector. The first order condition is given by

\(^{11}\)In the Barro and Sala-i-Martin (1994) model, the Poisson arrival rate is a negative function of task complexity. It shows diminishing returns and scale effect can be eliminated.

\(^{12}\)The proof can be found at Aghion and Howitt (1997).
where \( \nu_t = \frac{V_t}{A_{\text{max}}} \) is productivity adjusted value of the firm.

The value of \( V_{t+1} \) is determined by the following asset equation \(^{13}\):

\[
R_t^a V_{t+1} = \pi_{t+1} - \lambda \rho(\xi_{t+1}) V_{t+1}
\]

(21)

This equation has the following meaning: the net present value of the next generation technology \( R_t^a V_{t+1} \), is equal to the profit when the patent remains as the leading edge technology \( \pi_{t+1} \), less the expected capital loss that will occur when it is replaced by a new generation patent \( \lambda \rho(\xi_{t+1}) V_{t+1} \).\(^{14}\)

Note that the research curve can be lifted up in four ways: (a) decrease in interest raises the present discounted value of monopoly; (b) an increase in the magnitude of innovation \( \gamma \) also raises monopoly profit next period; (c) an increase in labor force represents scale effect; (d) an increase in \( \lambda \) increases the marginal benefit of research on the one hand, but increases the rate of creative destruction on the other hand. Aghion and Howitt (1992) showed that the former effect would dominate. Suppose the representative consumer is risk averse, though the capital curve will be upward sloping, all comparative static will remain intact.

2.6 Equilibrium

We hereby characterize the equilibrium conditions of the baseline model. All intermediate good firms are identical, the symmetric equilibria of intermediate good price, output and labor demand are characterized by the following conditions

\[
P_t(z) = P_t \quad \forall z
\]

\[
Y_t(z) = Y_t \quad \forall z
\]

\[
N_t(z) = N_t \quad \forall z
\]

At equilibrium, the consumers are indifferent between lending and borrowing, so \( B_t = 0 \). The aggregate

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\(^{13}\)If the leader innovates, as shown in Barro and Sala-i-Martin (1994), the asset equation becomes:

\[
r V_{t+1} = \pi_{t+1} - \pi_t - \lambda n_{t+1} V_{t+1} + \lambda n_{t+1} V_{t+2},
\]

\(^{14}\)Implicitly, it is assumed that the existing patent holder does not perform R&D. The heuristic reason is that all potential competitors have access to existing technology (business stealing effect); thus the patent holder and competitors have same chance of success. Once successful the outside research firm will monopolize the whole market; the marginal reward is \( V_{t+1} \). This is known as the ‘Arrow effect’ in the industrial organization literature. However, the marginal gain to the current patent holder is \( V_{t+1} - V_t \).
production can be expressed as

\[ Y_t = \left( \int_0^1 [A_t N_t(z)^\alpha K_t(z)^{-\alpha}] \frac{e^{-\lambda z}}{e^{-\lambda z}} \, dz \right)^{\frac{1}{\alpha}} \]  

(22)

It can be shown that, around the steady state, the approximate aggregate production function is

\[ Y_t = A_t N_t^\alpha K_{t-1}^{1-\alpha} \]  

(23)

This function is essentially the Cobb-Douglas aggregate production function\(^\text{15}\). The aggregate demand for the final goods is

\[ Y_t = C_t + I_t + G_t + \Xi_t \]  

(24)

where \( G_t \) is government expenditure. The government maintains a balanced budget, thus

\[ G_t = TR_t \]

The capital accumulation equation is

\[ K_t = \Phi \left( \frac{I_t}{K_{t-1}} \right) + (1-\delta)K_{t-1} \]  

(25)

Combining equation(3) and equation(11), the labor market condition is given by

\[ \alpha \frac{Y_t}{N_t} = \frac{1}{MC_t} \frac{\alpha N_t^{\alpha-1}}{C_t^{-\gamma}} \]  

(26)

Combining equations(4), (5) and (12),

\[ MC_t (1-\alpha) \frac{Y_t}{K_t} + (1-\delta)Q_{t-1} \]  

\[ E_t \left[ \frac{\frac{MC_t (1-\alpha) \frac{Y_t}{K_t}}{Q_t}}{(C_{t+1}^{\alpha+1})^{-\gamma}} \right] = E_t \left[ \frac{R_{t+1}^{\alpha} P_t}{P_{t+1}} \right] \frac{(C_{t+1}^{\alpha+1})^{-\gamma}}{(C_t^{-\gamma})} \]  

(27)

The other equilibrium conditions are equations (3) , (14), (15) and (20).

The symmetric equilibrium is characterized by \((Y_t, C_t, I_t, N_t, K_t, \Xi_t, Q_t, P_t, MC_t, R_t^{\alpha})\) – a vector that maximizes the constrained present value of the stream of utility of the representation household, the constrained present value of intermediate firms and expected profit of R&D firms, subject to an exogenous shock. That completes the description of the baseline model.

### 3 Outside Opportunities Changes

In this section, we will append a shock to the R&D production sector. The model being considered is called random participation, in the spirit of Rochet and Chone (1998) and Rochet and Stole (2002), which is an adverse selection model with random reservation utility. The randomness is change in outside...  

\(^{15}\) Labor growth rate is assumed to be constant, Ho(2007) demonstrates how illegal immigration lowers welfare.
opportunities. This shock can be interpreted as a sort of Dutch Diseases-brighter prospect at an outside sector - that drags away resources from R&D- similar to the Rybczynski effect in international trade theory.

For instance, the R&D expenditure level in Hong Kong remained low before 1998, because entrepreneurs invested heavily in the thriving real estate market. After the 1998 Asian financial crisis, the economy contracted and interest rate became higher. The real estate and stock market cooled down quickly. The government and private corporations launched many new R&D projects. The explanation is that, due to the Hong Kong linked exchange rate system, the Hong Kong interest rate has to follow U.S. interest rate change. Before 1998, the U.S. interest rate was low, while the Hong Kong inflation was high. The real exchange rate appreciated as nontraded good prices rose relative to tradable. Returns of investment in real estate market were higher than that of nontradable sector; and this pulled resources and talent out of tradable goods. After the 1998 financial crisis, all those prerequisites disappeared. The relative return of R&D became higher.

There is no lack of work purporting that asymmetric information affects innovation and adoption of new ideas (for example, James et al. (1990) and Larson and Anderson (1994)). In fact, the extension to incomplete contract framework is natural, since R&D takes place either within firms where employees-inventors are subject to contractual obligations; or between independent research units and users of their innovations or financiers.

3.1 Random Participation Model

A modification will be made to equation (20), the R&D firms' objective. For simplicity, it is assumed that the intermediate firms contract with two types of research firms such that an inefficient research agent has higher cost of delivering blueprints. The true type of R&D firms is hidden, i.e. there is a form of adverse selection. The intermediate firms offer two contracts to induce efforts (the amount of final good that the R&D firm allocates to innovation); and distort the inefficient firms' output downward to reveal the true firm type. The contractual payment provides financing to R&D firms during the innovation process; and the R&D firm has obligation to sell the patent, if there is any in the future, to the intermediate firm. Since the payment is lump-sum, it has no first order effect in the profit functions of the intermediate firm and R&D firm.

Moreover, the reservation utility is randomized to capture the idea of changes in outside opportunities to R&D entrepreneurs. For instance, oil discovery can drag away capital inputs from manufacturing - the classic Dutch Disease scenario; and abnormal returns in the real estate sector generated by the pegged

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16 The extension to continuous type is natural and can be found in Rochet and Stole (2002).
exchange rate system like Hong Kong, might drag resources and talent out of R&D.

Another interpretation is that the R&D firm may form strategic relationships with other principals in other sectors. The random reservation utility model assumes competition amongst principals. For instance, real estate industry (a non tradable) can compete with manufacturing or R&D industries. Business cycle can change the relative profitability of different industries. In each market period, the R&D firm has the option of renegotiating the contract or entering into a new contract with another principal. The random participation model is a simple way to formulate this kind of shifting strategic relationship.

Diagram 2 summarizes the timeline of principal-agent relation. The R&D firms, first, discover their true types. The intermediate firms offer them a contract. The R&D firms have the option to accept or reject the contract. The R&D firms engage in delivering blueprints. An outside opportunity shock hits the economy. The R&D firms weight in the impact, renegotiate with the intermediate firms at the next market period.

[Insert Diagram 2]

Now, we proceed to describe the random participation model. The participation constraints of the risk-neutral R&D firms are given by:

\[ u_E = \Gamma^E - A_i^{max} \xi_E^{\prime} (i) \geq \delta_E \]  
\[ u_I = \Gamma^I - A_i^{max} \kappa \xi_I^{\prime} (i) \geq \delta_I \]  

(28)  
(29)

where \( \kappa > 1 \) is the inefficiency parameter - inefficient firms require more input; \( I \) and \( E \) denote inefficient and efficient type respectively; \( \delta_i \) is the exogenous reservation participation utility; \( \Gamma^i \) is the contractual payment to research firms for delivering blueprints. The intermediate firms offer two contracts each period - \( (\Gamma^E, \xi_E^\prime) \) and \( (\Gamma^I, \xi_I^\prime) \) - to induce efforts from the R&D firms. Equation (29) indicates that the cost of an inefficient firm is higher.

We perturb the agent’s participation constraint by allowing some randomness in the decision to participate. Let the following be the probability density function of participation.

\[ M(u_i, \kappa) = P(\delta_i | \delta_i \leq u_i) \]

where \( i = I, E \), This formulation allows the possibility that \( \delta \) and \( \kappa \) to be correlated. The inverse Mill’s ratio is defined as:

\[ H(u_i, \kappa) = \frac{M(u_i, \kappa)}{M_i(u_i, \kappa)} \]

(30)

Equation (41) is non-decreasing in \( u_i \). \( M_i(u_i, \kappa) \) is the first derivative with respect to \( u_i \.

Direct revelation is implied by the incentive compatibility constraints.

\[ u_E = \Gamma^E - A_i^{max} \xi_E^{\prime} (i) \geq \Gamma^I - A_i^{max} \xi_I^{\prime} (i) \]  

(31)
\[ u_t = \Gamma^t - A_{it}^{\text{max}} \kappa^{\nu_t^t}(i) \geq \Gamma^E - A_{it}^{\text{max}} \kappa^{\nu_t^E}(i) \]  

(32)

If there is no asymmetric information, as shown at section 2, \( \lambda \frac{\rho(\xi_t)}{\xi_t} \nu_{t+1} = 1 \) and \( \lambda \frac{\rho(\xi_t)}{\xi_t} \nu_{t+1} = \kappa \) are the optimal first order conditions for efficient and inefficient firms respectively.

Intermediate firms in each sector, under adverse selection, maximize the expected payoff function,

\[
\max_{\xi_t(i)} \nu M(u_E, \kappa) \left( \lambda \frac{\rho(\xi_{t+1})}{\xi_{t+1}} \xi^E(i) V_{t+1} - (1 - \beta_n) \Xi^E(i) \right) \\
+ (1 - \nu) M(u_I, \kappa) \left( \lambda \frac{\rho(\xi_{t+1})}{\xi_{t+1}} \xi^I(i) V_{t+1} - (1 - \beta_n) \Xi^I(i) \right)
\]

\[ \text{s.t. } \delta^E_{t+1} = A_{it}^{\text{max}} \frac{\xi^I(i) (\kappa - 1) + \delta_t}{\nu} \]  

(34)

\( \nu \) is the fraction of efficient research firms, which is a known constant by assumption. Other parameters are the same as those defined at section 2. Comparing to equation (20), the intermediate firm can only maximize expected profit of different types of R&D firms, multiplied by the participation probability. Notice that \( \Xi^I_{t+1} \) equal to \( \Xi^I_t \) at the steady state. As typical in the adverse selection literature, equations (29) and (31) are binding. Combining them gives (34). The intuitive reason is that the efficient type has incentive to pretend that he is an inefficient firm; and equations (29) and (32) imply equation (28).

The first order necessary conditions are:

\[
\lambda \frac{\rho(\xi_{t+1})}{\xi_{t+1}} \nu V_{t+1} = (1 - \beta_n) A_{it}^{\text{max}}
\]

(35)

\[
\lambda \frac{\rho(\xi_{t+1})}{\xi_{t+1}} \nu V_{t+1} = (1 - \beta_n) \frac{\nu M(u_E, \kappa)(\kappa - 1) + (1 - \nu) M(u_I, \kappa)}{(1 - \nu) M(u_I, \kappa)} A_{it}^{\text{max}}
\]

(36)

Following Rochet and Stole (2002), we assume that \( M(u_E) = 1 - e^{-\delta_{t_E}} \), \( M(u_I) = 1 - e^{-\delta_{t_I}} \), i.e. exponential distribution. The participation probability does not depend on \( \kappa \). A constant has no effect on the linearized equation. Rochet and Stole (2002) assume that efficient firms participate with probability equal to one. We assume that their participation is a contant. Since it can be set arbitrarily, \( \delta_{t_E} \) is normalized to be one. The change is immaterial in the sense that a constant has no first order effect. The participation probability is randomized by assuming that \( \delta_t \) follows a stationary first order Markov process.

Implicitely, we are assuming that inefficient firm is the only group that would switch to another production sector. Each period, a change of \( \delta_t \) forces the intermediate firms to change the contract and the R&D payments. The new equilibrium conditions are (3), (14), (15), (23), (24), (25), (26), (27), (35) and (36).
3.2 Calibration

To derive the quantitative calibrated results, first, the balance growth path values are computed; second, the nonlinear system of differential equations will be log-linearized around the steady state; finally, the system will be solved by various methods to compute the second moments of simulated time series and generate impulse response functions. We adopt the Uhlig (1998) scheme using method of undetermined coefficients.

Table 1 describes the benchmark values used to compute the impulse responses of the economy to various shocks. The forth column reports the parameter values of the random participation model. For the balance growth path value of labor, we set it to 1/3, i.e., a third of total endowment of time. The scaling technology factor, \( A_t \), is normalized to 1. Following Hansen(1985) and Backus et al. (1995), the labor share of output \((1-\alpha)\) is 0.64. We set the balance growth path gross interest rate equal to 1.01 per quarter (Prescott and Kydland, 1982; Backus et al., 1995) and 1.04 annually, which implies that \( \beta \) equal to 1/1.01. The depreciation rate, is set equal to 0.035.

The coefficient of relative risk aversion, \( \gamma \), is 0.5; meaning the representative household is slightly risk averse. We find that moderate degree of risk aversion is sufficient to simulate the second moments of investment, output and R&D. Labor intertemporal substitution is 1. The random participation outside opportunity is assumed to be first order stationary Markov process, with AR(1) coefficients equal to 0.75. The Phillips relation is derived by log-linearizing equation (14):

\[
\pi_t = \lambda_x \pi_t + \beta E_t \pi_{t+1} \tag{37}
\]

Gali et al. (2001) suggests that the \( \lambda_x \) coefficient in Phillips is either 0.014 or 0.024; we choose the former.

The balance growth path consumption-output ratio, investment-output ratio, and R&D-output ratio are chosen to be 0.5, 0.4 and 0.1 respectively. The government plays no role in this setup. Thus, the \( G_t \) term in equation(24) will disappear; \( \beta_n \) at equations (33), (35) and equations (36) will equal to zero, i.e. no government subsidy.

For \( Y/K \), Andres et al. (2002) choose 1/8 and the balance growth path marginal cost equal to 1/1.2. They argue that a wide range choice of marginal cost can be used for calibrating their model. The labor disutility factor \( a_n \) is a free parameter in our model. We choose 1.3 to make the values of balance growth path variables consistent with our choice of balance growth path ratios. The capital production function is expressed as
\[ \phi\left( \frac{I_t}{K_{t-1}} \right) = \phi_k \left( \frac{I_t}{K_{t-1}} \right)^{\frac{1}{2}} \]  

(38)

where \( \phi_k \) can be interpreted as the adjustment cost coefficient. The Balance growth path probability of raising capital is set to be 0.15. The R&D rate of arrival is given by:

\[ \rho(\xi_t) = \phi_\rho \left( \xi_t \right)^{\frac{1}{2}} \]  

(39)

The Poisson arrival rate exhibits diminishing return, which is consistent with the idea that invention becomes more difficult as the degree of complexity increases over time. Parameters that have no second order effect, such as the research productivity coefficient \( \lambda \), on the log-linearized system can be safely ignored. Finally, the balance growth path \( \kappa \) - the inefficiency parameter of R&D firms - is equal to 1.5.

Every stationary vector autoregressive system has a unique Wold decomposition. We are going to investigate the impact of a structural shock on the path of economic variables.

The y axis of figure 2 measures the percent deviation from the steady state. A one standard deviation shock to \( \delta_t \) drives down the R&D spending of efficient firms by 0.7 percent at the initial period, and it returns to the long term path after 6 periods. Each period represents a year, because the predicted model moments will be compared to the actual data, which is observed annually. The impact on inefficient firms is smaller, approximately 0.3 after 1 period. One possible explanation is that the efficient firm's participation and incentive compatibility constraints are binding, so it is more sensitive to change in reservation utility. Another reason is the relative large portion of efficient firms in the model - \( \nu \) equal to 0.8.

[Insert Figure 2]

Demand for final goods as R&D input will decrease, which reduce the demand for capital and investment inputs. Eventually, the capital price will drop. As shown in figure 2, output drops by 0.25 percent at period 0. It settles down to the balance growth path after 4 years. Capital price change is negative 1.2 percent. The investment drop is the biggest among the variables being considered. The initial period drop is almost 2.5 percent. Both capital price and investment overshoot after 3 years.

The R&D expenditure of exhibits hump-shaped dynamic movement, since both current period and next period R&D appear in the log-linearized equation. For the inefficient type, it can be expressed as:

\[ E_r[y(t+1) - \frac{MC}{1-MC} mc_{t+1} - 0.5\xi_{t}^i - 0.5\xi_{t+1}^i] = (1-e^{-r})r + \delta_t \]  

(40)

One contribution of this paper is that, random participation in R&D sector alone, is capable of generating enough dynamic as most real business cycle models do. Boldrin and Woodford (1990) note that many of "endogenous business cycles" studies have the same basic structure as RBC - that is, it relies
on an exogenous shock which hits the aggregate production directly and then propagate through time. However, in our case, the source of shock is only confined to the R&D sector; and it is assumed that R&D makes up at most 10% of GDP. In a word, we have a shock in a small sector, but it is enough to generate sufficient deviation from balance growth path time series. We have calibrated different values of \( \nu \) (the percentage of efficient R&D firms) and, most of the results noted above remain invariant.

4. Financial Intermediation Model

In the literature, financial shocks most often take the form of monetary policy surprises (for instances King and Plosser (1984), Altig et al. (2004), and Clarida et al. (2002)) or exchange rate regime changes (for instance, Baxter and Stockman (1989)). Once nominal frictions, like fixed prices and wages, are introduced, these models can generate impulse responses similar to those of technology shocks. For instance, in the Christiano and Eichenbaum (1992) model, the household allocates a fraction of money holding to consumption goods, and the remaining lent to financial intermediaries. The household is liquidity constrained, in the sense that current consumption decision is not a function of time-t realization of monetary policy. In this way, they can show that the interest rate decreases as money supply increases—a result that the authors claim that most monetary RBC models fail to replicate- and that the liquidity effect is persistent.

Other researchers examine how financial market frictions (for example credit constraints) affect the ways that real variables respond to technology and money shocks (see Bernanke et al. (1999)). An important extension is the International Real Business Cycle (IRBC) model with incomplete financial markets. Backus et al. (1995) argue that IRBC is consistent with the international stylized fact that the cross-country consumption correlation is less than the cross-country output correlation. Williamson (1987) illustrates how financial intermediation, bankruptcy costs and credit rationing propagate the effects of a stochastic disturbance. He argues that monetary shocks generate cycles that are inconsistent with empirical evidence, lending support to RBC.

Sutherland (1996) constructs an international RBC with staggered prices and trading frictions across countries. Simulations show that volatilities of a number of variables increase with monetary shocks and increasing financial market integration. The opposite holds for real demand and supply shocks. Fuerst (1995) adds financial intermediaries into a RBC model to examine the effect of real and monetary shocks. The impulse responses are remarkably weak, so he concludes that “we have plenty of sources of business cycle shocks, but little in the way of propagation.”

Our model is different from the earlier works, in the way that the financial market is the source of economic fluctuation. King and Levin (1993) demonstrate that a higher degree of financial development, in particular lower agency cost of raising capital, has positive growth effects. The agency cost is
randomized in our model. The idea is that when positive R&D news hits the market, it lowers the cost of raising capital, thus affecting current output. As argued above, it is hard to apprehend the concept of a negative technology shock (for example, ideas cannot disappear). A negative R&D shock can however be interpreted easily. It can be due to unexpected delays of new invention, unsatisfactory research progress, termination of a project or competitors' patent, which render both internal and external financing more difficult. The financial intermediation model should work for developed countries. We will compare the calibrated moments to the observed U.S. data.

4.1 The Model

In the Aghion and Howitt (1997) model, the consumers are risk neutral. They earn their wages and dividends from owning intermediate firms, and pay the R&D firms their expected wage even though there is no invention. However, consumers in this paper are risk averse, thus financing R&D outlays at the time of no invention becomes an issue. It is assumed in section 2 that R&D projects are financed by a perfect capital market. Here, we add the assumption that raising capital involves an agency cost.

It is sometimes argued that technical know-how and scientific theories are abundant. The key problem is funding the experiments, applying the ideas for practical usage. King and Levine (1993) demonstrate how financial development bridges the gap. Letting \( f \) be the agency cost of identifying a capable researcher, and \( \psi \) be the probability of raising sufficient capital to finance the research projects, through the financial market. The new arbitrage equation (revision of equation (20) is

\[
\lambda \left[ \frac{\rho(\xi_t)}{\xi_t} \right] p_{r,i} = (1 - \beta_t) \left( 1 + \frac{f}{\psi} \right)
\]

(41)

The research arbitrage equation is essentially the same as Aghion and Howitt (1992). Profit depends on both labor and some exogenous parameters. The effective input cost is increased by a factor of \( \frac{f}{\psi} \) due to adverse selection. Higher financial development is represented by a lower \( f \). A lower \( \psi \), raises the agency cost. It is generally believed that when the stock market collapses, the financial intermediaries lose their functions of reducing moral hazard and adverse selection. The agency cost increases sharply during economic slump. For instance, corporate profits drop significantly during recession - a macroeconomic factor unrelated to firms' characteristics. However, firms' leverage increases at those times. Banks and bond holders may have reason to worry about defaults. Loan market cannot function properly, thus increasing the cost of raising capital.

Berthelemy and Varoudakis (1996) come up with the same conclusion by augmenting Aghion and Howitt (1992) model with a banking sector, which employs labor to raise loanable funds for R&D and
intermediate sector. The arbitrage condition remains unchanged, but the resources devoted to securing more savings deplete R&D. However, if only R&D sector demands for funding, then the intermediation cost will affect the arbitrage equation, and his result will be similar to King and Levine (1993).

We further assume that $\psi_t$, the probability of access to capital, follows a stationary Markov process, perturbed by an independent and identically distributed shock.

The idea is that investors are sensitive to R&D news, for instance, new formula, new design and research progress. A negative shock can be unexpected delay of new invention, unsatisfactory research progress, termination of a project or competitors patent. The stock market before 2000 is arguably such an instance. Investment was stimulated by waves of technology break-through news. A positive shock makes fund raising easier. The capital accumulation equation can be rewritten as:

$$K_t = e^{\psi_t} \phi \left( \frac{I_t}{K_{t-1}} \right) K_t + (1 - \delta) K_{t-1}$$

(42)

Where $e^{\psi_t} \phi \left( \frac{I_t}{K_{t-1}} \right) K_t$ is effective investment. The equations that characterize the equilibrium are (3), (14), (15), (23), (24), (26), (27), (41) and (42).

4.2 Parameter values and Impulse Response Functions

The parameter values are reported at the fifth column of table 1. They are the same values used for section 3, except two parameters. The coefficient of relative risk aversion, $\gamma$, is 0.9 for the financial intermediation model. The AR(1) coefficient of the shock is 0.5.

A positive shock improves the chance of raising capital (or reduction in agency cost); thus the output (R&D firms use final good as input), investment (for capital production) and the R&D spending effects are all positive as shown in figure 3. The output change after a shock is mild - 0.25% increase; it settles down to the balance growth path after 4 periods. Investment increases by 0.75%; the hump-shaped time path is attributed to lagged adjustment of capital stock (equation (42)). Similar to the random participation model, investment shock dies out at the sixth period.

[Insert Figure 3]

Capital price adjustment is almost one to one; it overshoots after 2 period. The propagation effect diminishes rather slowly. R&D expenditure, on the other hand, rises by 2 percent - the sharpest change among all variables considered. It returns to the long run path after 6 periods.

Notice the order of series volatilities: R&D, followed by investment and output. This can be explained by the fact that the random shock distorts R&D outlay directly, and capital indirectly through the capital accumulation equation. Output drops can be reconciled by the decrease in derived demand for final good. The calibrated moments of this model will be compared to the U.S. data at next section.
5. Comparing Models

Most economists agree that many economic series deviate away from a stationary trend due to some exogenous shock, and then propagates itself through time. Typical perturbations include the famous Prescott and Kyland technology shocks (Prescott and Kyland, 1982), government expenditure surprises (King et al., 1988), money shocks (Gali and Gertler, 1999) and sunspot activity (Farmer and Guo, 1994). In this section, we will examine the traditional new Keynesian model shocks - technology, interest rate and government shocks - in the Gertler-Gali model augmented with a R&D sector - the baseline model of chapter one. The impulse response functions will be compared to those of random participation and financial intermediation models. Finally, we will compare the predicted moments to the actual economic data.

5.1 Baseline Model and Parameter Values

The parameter values are reported at the third column of table 1. Readers should notice that there are 9 equations and 10 unknowns. RBC theorists assume a stochastic process for one of the variables to complete the model. The traditional RBC theorists (Prescott and Kyland, 1982) consider the impact of a productivity shock on various macroeconomic variables. In these models, monetary policy is neutral that only nominal variables will be affected. The New Keynesians argue that once some form of market imperfection (e.g. staggered price in section 2.3.1) is introduced, money is no longer neutral. Therefore, we will examine three scenarios following the framework of the standard New Keynesian RBC models, namely a surprise of government expenditure, a productivity shock and an unexpected interest rate change. The first two scenarios follow a stationary first order Autoregressive (AR) process. The AR coefficients are 0.8 and 0.95 for the government expenditure and productivity shocks respectively. The balance growth path consumption-output, investment-output, government-output and R&D-output ratios are set to 0.6, 0.15, 0.2 and 0.05 respectively.

Additionally, we can assume that the interest rate is anchored by the Taylor's rule

$$R_{t+1}^* = R^* \left( \frac{P}{P_{t-1}} \right)^{\gamma \pi} \left( \frac{Y_t}{Y_t^*} \right)^{\gamma y} e_{t}^\pi$$

(43)

$R^*$ is the real rate of interest at steady state; $Y_t^*$ is the potential output level under flexible prices. $e_{t}^\pi$ is a sequence of uncorrelated monetary policy shock, and where $\gamma_\pi > 1$ and $\gamma_y > 0$. Under the Taylor rule, the target inflation rate is zero.

The inflation coefficient of the Taylor rule $\gamma_\pi$ should be larger than 1, and the output coefficient $\gamma_y$
should be non-negative number. Based on the empirical studies of Gali et al. (2007), we set $\gamma_x$ and $\gamma_y$ equal to 1.5 and 0.5 respectively.

The impulse response functions of interest rate surprise, productivity shock and government expenditure surprise are presented in figure 1.1, 1.2 and 1.3 for the baseline model. One typical observation of this class of staggered price model is the countercyclical markup. We find the same countercyclical pattern in all models with technology and government expenditure shocks.

[Insert Figure 1.1]

In the first scenario, the monetary authority adjusts interest rate according to the Taylor rule with policy surprise. The impulse responses with respect to a one standard deviation interest rate shocks are reported in figure 1.1. The interest rate shock can explain the variation in R&D expenditure by affecting discounted profit of research project. An unexpected interest rate hike depresses output (since both capital and labor decreases) and investment. Output drops by eight percent initially and returns to the balance growth path after six periods. Capital price and investment decrease by eighteen and seventy percent respectively. Moreover, both series overshoot after four periods. With the presence of a new R&D sector, the investment overshooting problem of the New Keynesian RBC model remains.

However, R&D slightly increases, i.e. countercyclical. The reason is that, while higher interest rate (due to crowding out effect) lowers discounted franchise profit, lower output demand depresses output price, which R&D sector uses as input. In this case, the latter effect dominates.

[Insert Figure 1.2 and Figure 1.3]

Figure 1.2 shows the impact of one standard deviation positive technology (productivity) shock. The result is analogous to the traditional real business cycle model that it has positive effect on output, consumption and investment. The existence of staggered price mechanism explains for the declining marginal cost, since a positive technology shock increases marginal product of labor and capital. For a one standard deviation change in productivity, output increases by two percent, capital price by four percent. While research and development expenditure does not change at all, investment expenditure overshoots - for every one percent increase in technology shock, investment increases by sixteen percent.

As shown in figure 1.3, a one standard deviation government expenditure surprise crowds out consumption and investment, and depresses markup – another result found in the traditional New Keynesian RBC model. Output drops by half percent initially and returns to the balance growth path after four periods. Capital price and investment decrease by one and five percent respectively. Both series overshoot after four periods. The oversensitivity problem of investment is reduced. R&D expenditure, however, remains countercyclical.

Let's compare the results to the new models considered in this paper. From figure 2.1 a shock in the R&D sector – which is negative because it represents a positive shock in the alternative sector – depresses
output, investment, capital prices and R&D expenditure. For instance, capital price drops by 1.1 percent for one standard deviation random participation shock. Figure 2.2 shows the impact of a financial intermediation shock on various macroeconomic variables. Output increases by approximately 0.3 percent for one standard deviation change of financial intermediation shock. Comparing to the standard New Keynesian models (figures 1.1 - 1.3), the qualitative change of each variables after the change is the same and consistent with the stylized facts e.g. positive output effect. One virtue of the random participation and financial intermediation model is that the investment oversensitivity issue disappears. Moreover, the R&D investment is consistently procyclical. The introduction of R&D sectors and alternative shock channels improve the empirical performance of the baseline model. Note that the order of volatility is investment, output followed by R&D, which will be confronted with actual data in the next section.

For further illustration purpose, we combine the impulse responses of investment, R&D expenditure, output, capital price and marginal cost to New Keynesian technology and financial intermediation shocks in separate graphs (figures 3.1-3.5).\(^\text{17}\) Investment increases by 16 and 0.75 percent subject to New Keynesian productivity and financial intermediation shock, respectively (figure 3.1). The output deviation under financial intermediation shock is relatively moderate – 0.3 percent only - comparing to 2 percent of New Keynesian productivity shock (figure 3.2). Capital price responses is four times higher under New Keynesian productivity shock (figure 3.3). R&D expenditure actually does not change when there is New Keynesian productivity shock; but it increases 2 percent subject to a financial intermediation shock (figure 3.4). Finally, marginal cost drops by 4 percent under New Keynesian productivity shock comparing to 0.5 percent of financial intermediation shock (figure 3.5). The section demonstrates that only the financial intermediation model is consistent with actual U.S data.\(^\text{18}\)

5.2 Comparing to the US Data

It is worth noting that the government, technology, interest rate shocks in baseline model, as well as the random participation, all imply that the variance of investment is largest, followed by R&D and output. The financial intermediation model, on the other hand, predicts that R&D volatility is highest, followed by investment and output.

[Insert Figure.3 ]

Column two of table 2 reports the U.S. volatilities of aggregate output, investment and R&D expenditure, using 1953-2010 Chained-type real GDP and Gross Domestic Investment data from Bureau

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\(^{17}\) We are virtually comparing figures 1.2 with 2.2. No comparison is made with government expenditure surprises (figure 1.3) because they are the reverse images of figure 1.2 and their order of variance is similar. Since the impulse responses of macroeconomic variables to interest rate shock (figure 1.1) are clearly too large; no comparison is made.

\(^{18}\) For this reason, we do not compare figure 1.2 to 2.1.
National Science Foundation collects data for industry, government, university R&D expenditure. We choose to report the result for industry R&D, since it makes up 70% of total expenditure and the computed volatility with other R&D categories remain the same. All data are denominated in 2000 constant dollars. Following the real business cycle literature, we compute the logged series, demeaning them by Hodrick-Prescott filter, and finally compute variances and covariance by the residual series. The graph of HP-filtered GDP and expenditure on R&D were plotted and reported in Figure 4. (The cyclical components is reported in the left panel of Figure 3 while the growth component is reported in the right panel of Figure 3 ) The correlation coefficient between log real GDP and log R&D is 0.37. The calibrated results are reported in table 2.

We found that, R&D expenditure has highest variance, followed by investment and then output. Hence, only the financial intermediation model is consistent with this finding. The second moments of U.S. filtered series is compared to the calibrated moments of the financial intermediation model. Following Hansen and Wright (1997), we compute the relative standard deviation of investment and R&D expenditure relative to GDP. As seen in the third column, the calibrated output and R&D volatilities are relatively larger than observed U.S. data; however, the calibrated relative investment volatility is very close to actual U.S. data equal to 3.5. The predicted R&D- output standard deviation is 9.4, twice as high as the observed series. If, however, the assumed R&D share is increased from 0.1 to 0.15, the model now predicts a volatility of R&D of only 5.1, closer to the actual data. The table also examines the autocorrelations. As a whole, the signs of calibrated correlations are correct, though it over predicts them.

6 Conclusion

There is relatively limited work on endogenous growth and business cycle models. This paper attempts to bridge the gap in the literature. Since R&D is the core component in most of the endogenous models, we try to introduce shocks in the R&D sector and then to examine the impact on the rest of the economy. In this paper, a well-studied New Keynesian Model is augmented with a R&D production sector depicted in Aghion and Howitt (1992). Two sources of exogenous shocks to the economy are proposed, namely random participation (shocks to outside opportunities to R&D resources) and financial intermediation (financial market imperfection).

We find that the new Keynesian model, when augmented with an R&D sector; the traditional shocks like interest rate, government expenditure and productivity shocks do not perform well. Investment is very sensitive to an exogenous shock. For instance, as shown in figure 1.1, a one standard deviation shock in interest rate reduces investment by 70 percent. Moreover, R&D is either counter-cyclical or acyclical.
In each of these new models considered in this paper, R&D is procyclical, consistent with the U.S. data. Also, the financial intermediation and random participation models do not suffer from the strong investment overshooting problem of the baseline model. Finally, we find that only the financial intermediation model is consistent with the observed fact that the volatility of R&D is larger than that of investment and output.

References


Economic Studies 69, 277-311.


Diagram 1
Model Outline

Representative Consumer

N_t(z)
K_t(z)
π_t(z)
C_t

Intermediate Sector

A_t^{max}

R&D

Final Good

Y_t(z)

Capital Producing sector

K_{t-1}(z_t)

I_t

ϕ(I_t/K_{t-1}(z_t)) (Ω)

Appendix
Diagram 2

Intermediate firm offers a contract
R&D firm accepts or declines an offer
The contract is executed
An outside opportunity changes R&D firm’s reservation utility
The contract is renegotiated
## Table 1 Parameter values of various models.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Gertler-Gali</th>
<th>Random Participation</th>
<th>Financial Intermediation</th>
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<tr>
<td>$\gamma$</td>
<td>Relative risk aversion of consumption</td>
<td>0.5</td>
<td>0.5</td>
<td>0.9</td>
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<td>Autoregressive Coefficients</td>
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<td></td>
<td>Interest rate shock</td>
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<td></td>
<td>Government expenditure shock</td>
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<tr>
<td></td>
<td>Random Participation shock</td>
<td>0.75</td>
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<tr>
<td></td>
<td>Financial intermediation shock</td>
<td>0.5</td>
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<td></td>
<td>Standard deviation of shock</td>
<td>0.712</td>
<td>0.612</td>
<td>0.612</td>
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<td>$\xi$</td>
<td>Balance growth path Consumption Output ratio</td>
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<td>0.5</td>
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<td>$\eta$</td>
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<td>$\zeta$</td>
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<td>Nil</td>
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<td>$\frac{R&amp;D}{r}$</td>
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<td>0.1</td>
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<td>L</td>
<td>Balance growth path of employment</td>
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<td>1/3</td>
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<td>A</td>
<td>Technology parameter</td>
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<td>R</td>
<td>Gross interest rate per quarter</td>
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<td>$\xi$</td>
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<td>MC</td>
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<td>$\kappa$</td>
<td>Inefficiency Parameter of R&amp;D firms</td>
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<td>$1 - \alpha$</td>
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<td>Consumer’s subjective discount rate</td>
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<td>$a_n$</td>
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<td>Symbol</td>
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<td>( \delta )</td>
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<td>( \gamma_\pi )</td>
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<td>( \gamma_y )</td>
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<td>( \lambda_\pi )</td>
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<td>( \psi )</td>
<td>Balance growth path probability of raising sufficient capital</td>
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Figure 1.1 Impulse Responses to a Shock in Interest Rate Policy

Figure 1.2 Impulse Responses to a Shock in Technology
Figure 1.3 Impulse Responses to a Shock in Government Expenditure
Figure 2.1 Impulse Responses to Random Participation Shock

![Impulse Responses to Random Participation Shock](image1)

Figure 2.2 Impulse Responses to a Financial Intermediation Shock

![Impulse Responses to a Financial Intermediation Shock](image2)
<table>
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<th>U.S</th>
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<tr>
<td></td>
<td>( \sigma_y = 0.5; )</td>
<td>( \frac{\sigma_I}{\sigma_y} = 0.4; )</td>
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<tr>
<td>( \sigma_y )</td>
<td>0.026</td>
<td>0.062</td>
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<td>( \frac{\sigma_I}{\sigma_y} )</td>
<td>3.5</td>
<td>3.7</td>
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<td>( \frac{\sigma_{rd}}{\sigma_y} )</td>
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<td>Corr(( y_t, RD_{t-1} ))</td>
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<td>0.37</td>
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<td>Corr(( y_t, RD_{t+1} ))</td>
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<td>Corr(( y_t, I_t ))</td>
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<td>0.95</td>
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<td>Corr(( y_t, I_{t+1} ))</td>
<td>0.69</td>
<td>0.86</td>
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Figure 4. HP-filtered GDP and on R&D expenditure

Note: HPTRENDLGDP is the growth component of log GDP from hp filter; HPTRENDLRD is the growth component of log RD expenditure from the hp filter. HPLGDP is the cyclical component of log GDP from hp filter, HPLRD is the cyclical component of log GDP hp filter.