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# Investigation of a New Effective Viscosity Model for Nanofluids

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**Abstract**

In this paper a new viscosity model is proposed in order to predict the viscosity of the nanofluids in order to bridge the gap on this issue. The effective viscosity expression proposed in this paper is based on the regression analysis of a number of carefully selected published papers which covers theoretical, experimental and numerical results. Compared with the other theoretical models that are available in literature, the presented model has a good accuracy and reliability. The proposed model has been tested in 3D horizontal pipe and the results with Nusselt number show good agreement with available data. The model was tested for a wide range of volume concentration and temperature and shows wider applicability.


Peer-review under responsibility of the organizing committee of ICCHMT2016

*Keywords:* Heat Transfer; Nanofluid; Effective viscosity

**Nomenclature**

| | |
|---|---|
| A | Area |
| $k$ | Thermal conductivity |
| $q"$ | Heat flux |
| $C_p$ | Specific heat at constant pressure |
| D | Diameter |
| μ | Viscosity |
| ρ | Density |
| ϕ | Volume fraction |

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| | |
|---|---|
| T | Temperature |
| H | Heat Transfer Coefficient |
| L | Length |
| Subscript | |
| f | base fluid |
| nf | nanofluid |
| s | solid |
| eff | effective |

## 1. Introduction

In the last two decades the nanofluids have been investigated in many practical applications due to their superior thermophysical properties compared to the conventional fluids mainly due to limited thermal conductivity. Since Choi [1] first introduced the nanofluid concept, many researchers proposed models to predict thermophysical properties for nanofluids. Many efforts have been devoted for modelling the thermal conductivity. However, only limited works were focused on the effective viscosity models. Viscosity is an important flow property of fluids. Pumping power, pressure drop in laminar flow and convective heat transfer directly affected by the viscosity of fluids. The understanding of viscosity change is vital for predicting the heat transfer performance of nanofluids.

Einstein [2] developed the first viscosity formula for micro colloids in 1906. The model was based on the assumption of viscous fluid containing spherical particles. The model was found to be valid for volume concentrations up to 2%. Brinkman [3] introduced a formula to predict the viscosity of the particles at a very low volume fraction. The model was based on assumption of spherical particles.

Batchelor [4] developed Einstein's model and proposed a new model for rigid and spherical particles with Brownian motion. The model was investigated by many researchers and found working well with low concentrations. Boungiorno [5] developed a mathematical model for colloids made of a base fluid and nanoparticles and showed that the nanofluid properties may vary significantly within the boundary layer because of the effect of the temperature gradient and thermophoresis. He attributed the convective heat transfer enhancement to the reduction of viscosity within and consequent thinning of the laminar sublayer. Tseng [6] conducted an experimental investigation for $TiO_2$, The number of data used in his experiment was only four volume concentrations. He derived a curve fit model for effective viscosity. Maiga [7] proposed a model for effective viscosity in forced convection using $Al_2O_3$ water nanofluid. Saito[8] implemented a theoretical model for very small particles with spherical rigid particles.

Nguyen [9] conducted an experimental study for $Al_2O_3$ and CuO particles nanofluid, he derived a curve fit equation for predicting the effective viscosity for nanofluids.

Grag et al [10] measured the viscosity of copper nanoparticles in Ethylene glycol. They found that the viscosity increase was almost four times of that predicted by the Einstein law of viscosity. The number of data used for introducing their model is only 5.

Wang and Xu [11] measured the viscosity for $Al_2O_3$ water, they highlighted that the enhancement of viscosity was between 20-30% for 3% volume fraction. Three volume fractions were used to measure the thermal conductivity and viscosity

From the literature it is found that there is no unified accurate model to predict the effective viscosity for nanofluids. The present work is aimed to propose a new model to predict the viscosity with a wide range for volume fraction.

## 2. Problem Description

The geometry considered in this work is shown in Fig 1, a pipe with length 1 meter and a dimeter 10 mm with constant external heat flux. The fluid enters the pipe with temperature $T_o$=293 K inlet temperature and uniform axial inlet velocity. At the pipe exit a fully developed flow condition is considered. The fluid considered in this study is $Al_2O_3$ water nanofluid. The properties of the nanoparticles are shown in Table 1.

Table 1 Nanoparticle properties

| Nanoparticle | ρ | k | $C_p$ |
|---|---|---|---|
| $Al_2O_3$ | 3950 | 35 | 765 |

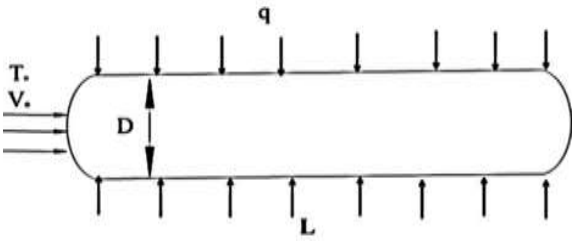


Figure 1 Problem geometry

The mesh for this problem is shown in Fig 2

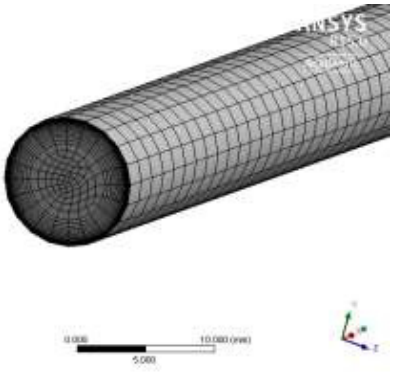

Figure 2 The domain mesh

## 3. Governing equations and thermophysical properties of nanofluid

The governing equations are continuity, momentum equation and energy equation and can be written as follows:

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla\left(\rho \vec{v}\right) = 0 \tag{1}$$

Momentum equation

$$\frac{\partial\left(\rho \vec{v}\right)}{\partial t} + \nabla\left(\rho \vec{v}\right)\vec{v} = -\nabla p + \nabla\left(\tau\right) + \rho g + F \tag{2}$$

Energy equation

$$\frac{\partial(\rho E)}{\partial t}+\nabla\left(\vec{v}\left(\rho E+P\right)\right)=\nabla\left(K_{eff}\nabla T-\sum h_j J_j+(\tau\vec{v}\right) \tag{3}$$

The density of Nanofluid is expressed as:

$$\rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s \tag{4}$$

The specific heat is expressed as:

$$C_{pnf}=\frac{(1-\varphi)\left(\rho C_p\right)_f+\varphi\left(\rho C_p\right)_s}{\rho_{nf}} \tag{5}$$

The effective thermal conductivity is written as:

$$K_{eff}=K_f\left(\frac{K_s+2K_f-2\varnothing\left(K_f-k_s\right)}{K_s+2K_f+\varnothing\left(K_f-k_s\right)}\right) \tag{6}$$

The heat transfer coefficient is calculated as:

$$h=\frac{q}{\left(T_H-T_C\right)} \tag{7}$$

Nu number is calculated from the equation:

$$Nu=\frac{h.L}{k} \tag{8}$$

**4. The Effective viscosity model:**

The proposed model was built based on a regression function from a range from available data from literature. Ten different experimental and theoretical models were used to develop the present equation for predicting the effective viscosity. The new model is expressed as:

$$\mu_{eff}=\mu_f(1+5\Phi+80\Phi^2+160\Phi^3) \tag{9}$$

The equation derived above is based on the models available from literature and are shown in Table 1. The Nanofluid is modelled as single phase, nonNewtonian and there is no slip between the particles and the base fluid.

Table 2 models used for regression analysis

| Model | | Theo/ Exp. |
|---|---|---|
| **Enstien** | **1906** | Theoretical |
| **Brinkman** | **1952** | Theoretical |
| **Maiga** | **2005** | Numerical |
| **Buongiorno** | **2006** | Mathematical |
| **Tseng** | **2003** | Experimental |
| **Nguyen** | **2007** | Experimental |
| **Batchelor** | **1977** | Theoretical |
| **Saito** | **1950** | Theoretical |
| **Grag** | **2008** | Experimental |

## 5. Numerical procedure

The governing equations were solved numerically with the boundary conditions using ANSYS 15.0. The geometry was created in ANSYS design modeler, the computational domain was discretised in ANSYS meshing and solving the governing equations together with the boundary conditions in ANSYS FLUENT. The CFD code is based on the finite volume method and SIMPLE algorithm that solves the governing equations. Second-order upwind schemes were used for the convective fluxes. Convergence was obtained with residuals less than 10-6 for the continuity equation, the momentum equations and the energy equation. To ensure a mesh independent solution, the maximum Nu number at the outer surface was used. The solution was taken to be mesh independent when the percentage change in maximum Nu remained less than 1% to give a mesh-independent solution for all simulations in this study. Table 3 shows the mesh dependency test for maximum Nu and it is clearly seen that for the mesh with 281000 elements the maximum Nu becomes insensitive to further refinement and hence mesh was selected for the solution.

Table 3 Mesh dependence test

| Mesh | No. of Elements | $Nu_{max}$ |
|---|---|---|
| 1 | 39500 | 3258 |
| 2 | 152000 | 4761 |
| **3** | **281000** | **4821** |
| 4 | 319500 | 4823 |

## 6. Validation of Results

The present simulations were validated and the results are shown in Fig 2. The Figure illustrates the change of the surface heat transfer coefficient at the wall. As there is an external heat flux applied on the wall and the flow enters the pipe with constant temperature so it is expected the heat transfer coefficient is expected to decrease from a peak value at the inlet of the pipe and decreases until a point where becomes almost unchanged constant wall The results as seen from the graph show there is a good agreement with Bianco et al [12] results.

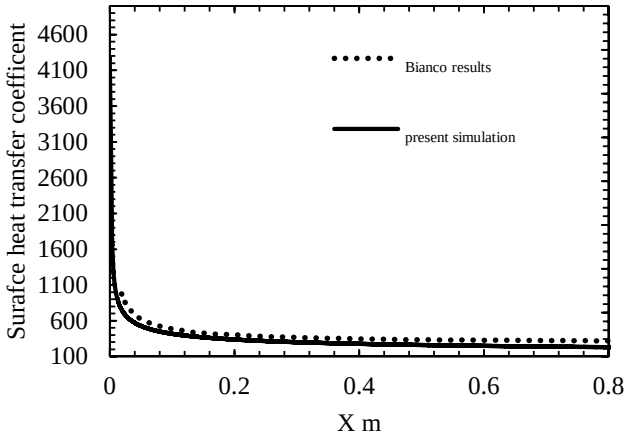


Figure 3 Results validation

## 7. Results and discussion

The results for the simulation are introduced in this section.

In Fig 4 the variation of viscosity from different models for a range of temperature is shown. It is evident that the viscosity decreases with the increase of temperature. The viscosity predicted by the present model in a good agreement with Nguyen where theoretical models underestimate the viscosity. The temperature range used in this investigation was between $0^oC$- $100^oC$. The volume fractions used between 2% and 10%.

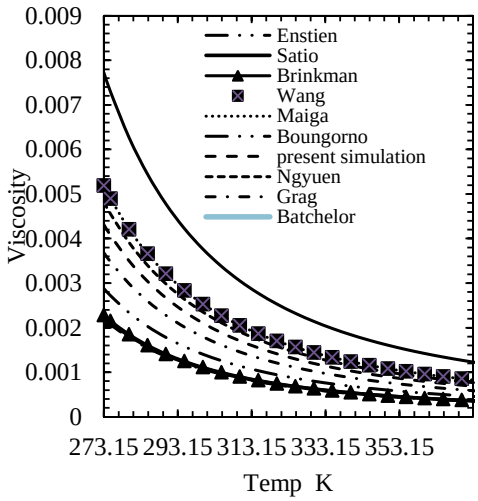


Figure 4 Effect of Temperature on the viscosity for different models

The relative viscosity prediction for different models is illustrated in Fig 5. The base nanofluid considered in this paper is water where the relative viscosity is the ratio of nanofluid to the viscosity of the base fluid. The present equation for the viscosity lies in the middle of the various models for relative viscosity, it is evident that the

theoretical models predict the viscosity in the very small concentration while the viscosity underestimate the viscosity when the volume fraction is greater than 2%. Nguyen model for dimeter 47 nm is seemed to over predict the viscosity compared to other experimental models.

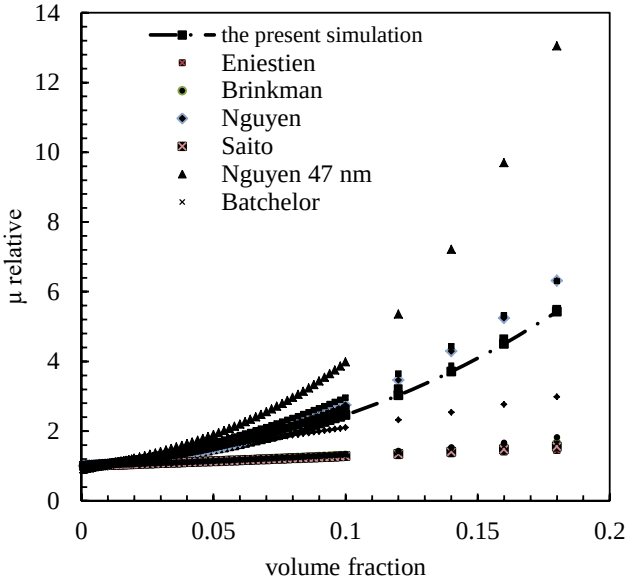


Figure 5 Effect of the Volume Fraction on the Relative Viscosity for Different Models

The results of the simulation of the viscosity in the three dimensional pipe in the present work is investigated. Fig 6 depicts the viscosity at a vertical line from the center of the pipe. The results show that the viscosity decreases with the increase of the temperature as the temperature increases from the center to the wall. The proposed model predicted the viscosity along the pipe with a reliable accuracy compared to the other four models.

While Nguyen 47 nm showed the highest value for viscosity, Brinkman model prediction value for viscosity was the lowest among the models

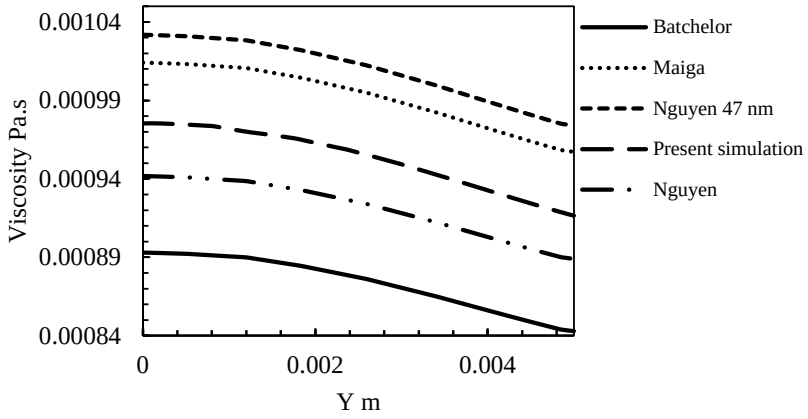


Figure 6 Viscosity from different models along a vertical line

The viscosity change along the surface was also investigated for different models. The results are shown in Fig. 7. The viscosity decreases along the surface of the pipe, this decrease is attributed to the decrease of the viscosity of the base fluid which is water in this case. The model that predicted the higher viscosity was Nguyen with 47 nm diameter, while the model predicted the lower viscosity was Brinkman model which prove that the theoretical models under predict the viscosity of the nanofluids. Moreover some of the existing models are based on a few experimental data.

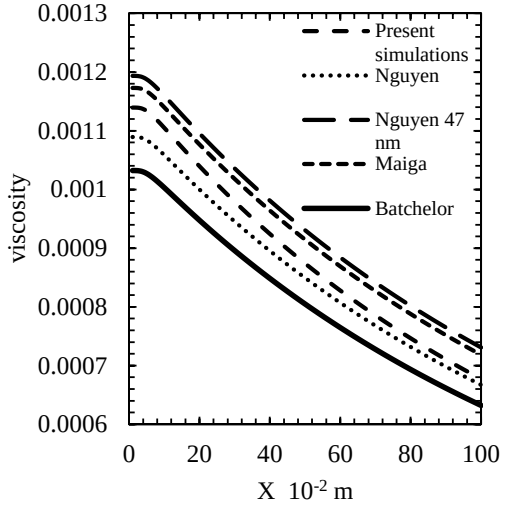


Figure 7 Viscosity change on the wall line for different models

The effect of volume fraction of the heat transfer performance is also investigated. The Fig 8 shows the surface heat transfer coefficient on the wall for different volume fractions. It was observed that increasing the volume fraction leads to an increase of the heat transfer coefficient along the wall, water was found to have the lowest heat transfer coefficient

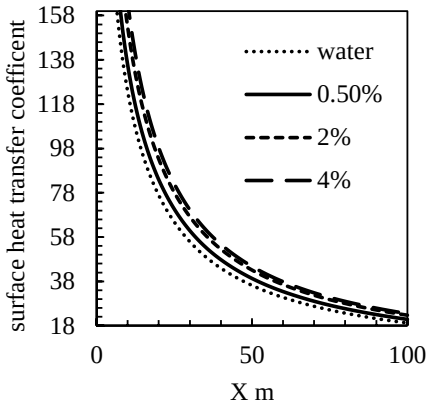


Figure 8 Effect of volume fraction on the heat transfer coefficient

The simulations were also extended for a range of Re numbers to study the effect of any change on the heat transfer rate. In Fig 9 the effect of Re number on Nu is plotted on the surface of the cylinder. It was found that increase of Re number promotes the Nu number. The simulation was carried out for different Re 100, 200, 300 and 400.

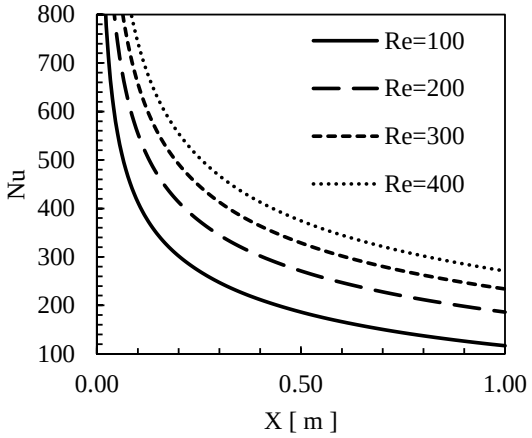


Figure 9 Effect of Re on Nu

## 8. Conclusion

A new equation for predicting the effective viscosity is proposed in this work. The proposed model demonstrated a reliable accuracy for a range of temperature and volume fractions compared to the theoretical models where these models underestimate the viscosity at higher volume fractions. A numerical simulation is carried out for the effective viscosity for nanofluids. The simulations also show that the effective viscosity model increases with the increase of the volume fraction.

The effect of Re number on the heat transfer performance was also studied and the results showed that the increase in Re lead to an increase in Nu.

The volume fraction effect on the heat transfer coefficient at the surface was also investigated. The heat transfer was promoted with the increase in volume fraction. Water was found to have the poorest heat transfer performance.

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