Uncertainty analysis of FRP reinforced timber beams

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ABSTRACT:
Timber has been a popular building material for centuries and offers significant sustainable credentials, high mechanical and durability properties. Availability, ease to use, convenience
and economy have made timber the most used construction material in history but, as it is a natural material, uncertainty in its mechanical characteristics is considerably higher than man-made structural materials. National codes and engineers usually employ high factor of safety to incorporate timber strength uncertainty in design of new structures and reinforcement of existing ones. This paper presents the results of 221 bending tests carried out on unreinforced and reinforced soft- and hardwood beams (fir and oakwood) and illustrates the reinforcement effect on timber capacity and strength uncertainty.

Both firwood and oakwood beams have been tested in flexure before and after the application of a composite reinforcement made of FRP (Fiber Reinforced Polymer) unidirectional sheet. The uncertainty in the strength of reinforced timber is also quantified and modelled. Test results show that the FRP reinforcement is effective for both enhancing the beam load-carrying capacity and for reducing strength uncertainties.

INTRODUCTION
The use of timber in construction is continuously increasing in Europe: information suggests that UK sawn softwood use is about 0.14 m³ per capita compared to 0.20 m³ in Germany and 0.80 m³ in Finland [1]. Timber offers significant sustainable credentials and good mechanical properties. The use of timber structural elements is also an interesting earthquake resistant solution compared to other traditional construction materials like concrete and masonry, based on its lightness, large deformation capacity and high tensile strength and strength-to-weight ratio. As a renewable and sustainable material, governments and international regulatory bodies are committed to increase the use of timber and of new wood-based products in construction, by incentivizing it by means of income-tax deduction, valuable funding.
Because wood has been used as a building material for hundreds of years [2], the upgrading of pre-existing timber structures is another important aspect: increasing the strength of timber beams when their size is incorrect over the span they need to cover or due to an increase in bending loads is often necessary in historic constructions in many parts of the planet [3]. A very large number of historic construction across Europe, representing a significant percentage of the building stock, needs to be not only preserved and protected but also maintained according to the original intended use. Conservation bodies often deal with finding new uses for redundant historic constructions without affecting their significance. As a natural material, the strength of timber is appreciably reduced by the presence of defects like knots, especially when located on the tension side, and distortion of the grain. For this reason uncertainty in the strength of timber is considerably higher compared to an artificial construction material (steel, concrete, bricks, etc.), which is produced through quality-controlled and precise manufacturing methods and processes. This uncertainty necessitates the adoption of a conservative approach in evaluating the strength of the material when designing timber beams. This aspect has not been sufficiently investigated in the past and, when an existing timber structure or component does not comply with new standards, structural engineers often opt for removal and demolition or apply strategies based upon reinforcement methods.

Remedial methods for upgrading and conservation of old timber beams include the reconstruction of deteriorated parts, the application of metal reinforcements [4-6] and, more recently, mechanical retrofitting techniques employing FRPs (Fiber Reinforced Polymers) and thermosetting resins. For example, Borri et al. [7] tested beams reinforced with carbon sheets (CFRP) applied on the tension side. The tests proved that the application of the carbon-fiber reinforcement was mainly beneficial in terms of bending capacity. Similar tests on small beams have been carried out by Plevris and Triantafillou [8], Fiorelli and Dias [9], Radford et
al. [10] and Hay et al. [11] using fiberglass sheets. The use of carbon pultruded plates has been studied by Raftery et al. [12-13], Nowak et al. [14-16], D’Ambris et al. [17], Schober and Rautenstrauch [18]. Shear or local reinforcements using FRP sheets have been studied by Triantafillou [19] and Schober et al. [20]. Glued laminated timber (glulam), made of multiple layers of dimensioned lumber bonded together with durable, moisture-resistant structural adhesives, has been also reinforced with FRPs (sheets, plates or bars) and high increases in bending capacity have been measured [21-25].

The use of composite rods or bars inserted in grooves at the tension side of timber beams has also been suggested as a means of reinforcing and repairing existing timber beams (Svecova and Eden [26], Micelli et. al. [27], Alam et al. [28]). Gentile et al. [29] tested twenty-two half scale and four full-scale timber beams strengthened using GFRP bars to failure and found a flexural strength increase up to 46%. Righetti et al. [30] studied the shear stress distribution along a groove-embedded CFRP bar.

Composite sheets made of natural fibers (bamboo, flax, hemp, basalt) have been studied by Borri et al. [31] and de la Rosa García et al. [32]. More recently composite sheets made of high strength steel cords embedded into an epoxy putty have been used to reinforce timber beams [33].

Among retrofitting methods using composite materials, the subject of FRP reinforcement using pre-impregnated sheets generated considerable interest within the research community mainly because this method proved to be the most effective in terms of strength improvement. Ease of application, limited damage to the timber substrate in case of removal, low-cost and fast reinforcement procedures are the key features of the use of epoxy-bonded FRP sheets.

This paper presents the results of an experimental investigation of the behaviour of 221 unreinforced and reinforced timber beams. Reinforcement has been applied using FRP pre-
impregnated unidirectional sheets placed on the tension side of a very large number of timber beams using an epoxy gluing system. Specimens were made of common commercially-available softwood (Firwood - *Abies Alba*) and hardwood (Oakwood – *Quercus Petraea*). Enhance of the behavior of timber beams in bending by the addition of a composite reinforcement is not a new concept, but the analysis of the strength uncertainty of both commercially available unreinforced and FRP-reinforced timber beams has not been addressed before. A first attempt to address this problem is reported in [34]. Uncertainty analysis was only studied with regard to the short term static performance. No analysis was undertaken with regard to fatigue, long term and dynamic performance. The presence of FRP sheets seems to delay crack opening on the tension side, confines local rupture and bridges local defects in the timber and this has a considerable effect on the strength properties.

**UNREINFORCED TIMBER**

The bending strength of timber is governed by the modes of failure. Since the behavior of timber in compression is different from that in tension, the failure modes could be highly affected by this. Figure 1 shows different characteristic failures of beams in bending. Simple tension failure (Fig. 1a) due to a tensile stress parallel to the grain. This is common in straight-grained beams made of high quality timber, particularly when the wood is well seasoned and there is no diagonal cross grain.

The most common failure mode is the cross-grained tension, in which the fracture is caused by a tensile force acting oblique to the grain. This is a common form of failure especially where the beam has diagonal or other form of cross grain on its tension side. This failure mode, always occurring on the beam tension side, can be also activated by the presence of defect (a knot, a shake, etc.). Example of such failures are shown in Figure 2. Since the
tensile strength of wood across the grain is only a small fraction of that with the grain it is easy to see why a cross-grained timber would fail in this manner.

As stated, an interesting effect of the analysis of the failure modes is that these usually occurs for different levels of bending loads. Failure mode in Figure 1b is usually activated for low bending loads. This is also typical of low-grade timber where the high number of defects facilitates the cross-grained tension failure.

Failure on compression side is shown in Fig. 1c. This failure mode do not usually lead to the collapse of the structure as the behavior of timber in compression is plastic (Fig. 3). Failure modes in Figure 1a is usually activated for high bending loads as this occurs for straight-grained beams and tensile strength of timber is very high.

While generally tensile fracture governs bending capacity, other mode of failure is horizontal shear rupture, in which two portions of a timber beam slide along each other. This failure mode is rare for large timber beams, but it can occur in the case of large beams with openings and often require local reinforcement [35]. It is often due to shake checks, which reduce the resisting cross sectional area. The consequence of a failure in horizontal shear is to divide the beam into two or more parts the combined capacity of which is much less than that of the original beam. Figure 1d shows a large beam in which a horizontal shear failure occurred at one end.

The application of an external FRP reinforcement causes an increase in the bending capacity for different reasons. Firstly because high-strength composite material is added on the tension side increasing the resisting cross sectional area, but also because this could prevent the occurrence of a failure mode characterized by a low capacity. This is the case of a FRP-reinforcement epoxy-glued on the tension side: the initiation of the fracture mechanism produced by the grain deviation or the presence of a knot on the beam’s tension side is
postponed or stopped (Fig. 4) and the beam will fail according to a different failure mode with a higher bending capacity.

Strength grading

The main mechanical properties of timber are usually estimated using a process known as strength grading. This is usually conducted at the sawmills when the timber elements are produced. Grading is usually carried out by visual assessment or by machine by the companies selling the timber material for structural applications. Visual strength grading is made using the grader’s experience across a number of factors (dimensions and density of knots, grain deviation, annual rings characteristics, etc.) while machine strength grading is best suited to high volumes of wood where the species and the dimension of the cross section are not changed very often.

The European standard for timber [36-37] includes several strength classes. These classes are designed by a letter (D for deciduous species and C for coniferous and poplar) followed by a number. The number represents the characteristic lower 5th percentile value of the bending strength of 150 mm deep timber in MPa. Strength grading of timber beams is often done by machine to Standard EN14081 [38] to twelve classes ranging between C14 and C50 and to 5 strength classes (D30, D40, D50, D60 and D70) for softwood and hardwood, respectively.

It is recognized that some sawmills in Slovenia did not perform grading properly prior to the introduction of harmonised standards [39]. In many cases in small production sites in Europe no grading is applied or a fee is charged for this service [40-41]. In order to comply with European Standards, to avoid risks associated with unmet strength requirements and to economize on the grading process (sometimes more expensive for high quality timber), a lot of companies prefer to grade their timber production with low strength values, especially if they produce low-added value products, like timber beams for the construction industry. It is
also common that the sawmills ask the client for an additional cost for the grading service:

this often costs a fee or an additional 20% for of the price of D40 timber (and higher strength classes) and 10% for D30.

In some cases, when this is possible, both final users and producers opt not to use graded timber. Producers of engineered wood products can use material that has not been pregraded if they undertake the mechanical properties characterisation themselves. When grading is needed, a lot of sawmills grade their beams in the C16 class (for firwood beams), even if the strength quality of their products is higher, especially because the stiffness is often the controlling factor. For oakwood beams (hardwood), the typical strength class of the products on the market is D30.

The main consequence of this incorrect application of the European standard is that a very limited choice of timber is available on the market for the higher strength classes and, for the lower strength classes (C16, D30, etc), the mechanical characteristics are very scattered as this is simply used as a lower strength bound.

Experimental work

In this experimental work, a large number of oak and firwood beams were used and tested in bending before and after the application of an FRP reinforcement. For both wood species different beam dimensions were tested with cross sections varying from 20x20 mm to 200x200 mm. D30 and C16 strength classes were used for oak and firwood beams, respectively.

Mechanical properties of both wood species were partially evaluated in accordance with ASTM D143 [42]. A parallel to the grain compressive strength of 27.9 MPa (Coefficient of Variation (CoV) = 9.6%) and 31.7 MPa (CoV = 7.9%) was measured from firwood and hardwood prismatic test specimens (20x20x60 mm), respectively. The average weight
densities were 791.8 and 423.7 \( \text{kg/m}^3 \) for firwood and hardwood. Moisture contents were 12.5 and 11.9 % and were measured according to EN 13183-1 standard [43].

Unreinforced beams

Six series of bending tests were performed on unreinforced softwood (fir) and hardwood (oak) beams (Tab. 1). In total 95 unreinforced beams were subjected to four-point-bending test (Fig. 5), according to UNI EN 408 [44] standard for flexural strength estimation. The beams were new and with straight and sharp edges. All beams were found on the market and had a square cross section. The dimensions of the three series of softwood beams were 20x20x380 mm, 100x100x1950 mm and 200x200x4000 mm. For hardwood beams, dimensions were 20x20x380 mm, 67x67x1320 mm and 200x200x4000 mm.

In order to reduce the local crushing of the wood, the load was applied through two diameter steel cylinders. Displacement controlled loading ensued with a crosshead speed of 24 mm/min. The load was applied monotonically until failure by means of a hydraulic jack connected by a hydraulic circuit to a pump. The vertical displacements of the beams were recorded using inductive transducers (LVDT) in the testing region (pure bending region) to monitor the mid-span deflection and calculate the curvature.

Hardwood is usually characterized by higher mechanical properties compared to softwood. However uncertainties are usually more significant compared to softwood like fir, larch and pine woods. Grain deviation and dimensions of the knots are larger, but the density of the knots are usually smaller. For this reason it was decided to test one common type of hardwood (oak) and one of softwood (fir). The test program was divided into two series: tests on beams unreinforced and reinforced with FRP sheets. Tests results were then processed according to the indications of the reference standards and the bending strength \( f_m \) evaluated thus:
where, $F_u$ is the ultimate (maximum) load (N), $a$ is the distance between the point of application of the load and the nearest support (mm) and $W$ is the modulus of resistance of the section (mm$^3$) about the neutral axis.

Results for unreinforced beams are given in Table 1. In this table results are reported in terms of mean bending strength value ($f_m$) and its standard deviation. $f_{m,k}$ is the strength value at 5% of cumulative distribution function.

The relationship between bending load and mid-span displacement (Fig. 6) was initially linear. As the load increased, timber started to yield on the compression side and tensile failure occurred when the tensile strength was reached. In most cases, failure initiated by flows in the timber material (knots, grain deviation, splits or cracks). Table 1 shows that the scattering in the capacity values of un-reinforced large beams (200x200 mm and 100x100 mm cross sections), where the presence of grain deviation and knots have an influence on the failure mode, is very high. The Coefficient of Variation (CoV), also known as Relative Standard Deviation, of the bending strength was 28.26 and 34.72% for 200x200 mm cross section (oakwood) and 100x100 cross section (firwood) beams, respectively. It is worth noting that for the 95 unreinforced timber beams tested in bending, the CoV was smaller for small beams. Even if the number of tested beams was not very high, this result can be considered interesting. The explanation of this is apparent from the analysis of the dimensions of defects, mainly knots, compared to the dimensions of the timber beams: typical knot defects have a diameter varying from 3 to 10 cm and, for small beams, this may lead to early catastrophic failures when loaded, as the knot may completely interrupt the continuity of timber fibers. For this reason sawmills are forced to check small beams by discarding the defected ones or by cutting off the parts where the defects are located before commercialization. This has a positive effect on both the strength and its scattering.
When the dimensions of the beams are bigger, the effect of a single defect is limited. In this situation sawmills may pay less attention to the defects. However, large beams, when tested in bending, exhibit a large scattering in the bending strength.

Table 1 and Figures 7-8 show the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of the strength for unreinforced beams. It can be noted that the $f_{m,k}$ value was largely below 16 MPa (value given as a limit by the EN 338 standard [36] for a C16 wood) for 100x100 mm firwood beams. The difference was even bigger for 200x200 mm oakwood beams. By comparing the experimental result of $f_{m,k}$ (17.92 MPa) and the value given by the EN 338 standard (30 MPa for D30 wood) it can be noted a difference of approx. 35%. These low values of $f_{m,k}$ were clearly the consequence of the high scattering of the test results: in fact, the mean experimental value of the bending strength $f_m$ was always greater than the value given by the EN 338 standard.

It is not possible to verify how common is the fact that there are on the market timber beams that are not meeting the requirements of the EN 338 standard in terms of bending strength. However, the tests carried out in this experimental research seem to indicate that this is not very rare, especially for beams of large dimensions.

REINFORCED TIMBER

126 timber beams were reinforced using Carbon (CFRP) or Glass (GFRP) sheets. Both composite sheets had similar weight densities (0.3 and 0.288 kg/m$^2$ for carbon and glass sheet, respectively). The current market price is approx. 7.2 and 14 €/m$^2$ for carbon and glass sheet. The popularity of bonded FRP reinforcement of timber is largely due to the economy with which they may be applied with low installation times than other strengthening methods. Reinforcement can be easily made on-site (hand lay-up technique) by applying the matrix polymer (usually an epoxy resin) over the fibers (Fig. 9). The same resin is often used as
matrix polymer to form the FRP composite and as bonding adhesive with the wooden
substrate. The component materials of the FRP-strengthened beams were characterized before beams
were examined under load. Mechanical properties of glass and carbon fibers, according to the
procedure outlined in the ASTM Standard D3039 [45], are shown in Table 2. Reinforcement and resin were applied by hand lay-up (Fig. 9a, 9b). Once the composite layer
was placed over the beams (Fig. 9c), resin was applied either by pouring on by hand. The
layer was consolidated and air bubbles were removed by using squeegees and hand rollers. Beams were tested in bending according to the same test arrangement used for unreinforced
beams (Fig. 5). The failure mode was not highly influenced by the type of reinforcement
(Carbon or Glass fibers), as the failure usually occurred in the wood material, without
attaining the ultimate FRP tensile strength (Fig. 10). On the contrary, the cross sectional area
and the area fraction of the composite material had a significant influence (Tab. 3). When FRP reinforcement failure is neglected due to its high tensile strength, two different
failure mechanisms are possible. The first one involves the possibility of attaining the wood
tensile strength, while the other occurs when the compressive stress limit is reached. The two
stress limits were often attained consecutively: experimental tests have shown that the most
frequent failure mechanism was the one in which tensile failure occurred, but this was
preceded by a partial plasticization of timber material at the compression side, both for un-
reinforced and reinforced beams (Fig. 10).
The application of the composite reinforcement resulted in a downward movement of the
neutral axis position and an increase in the beam capacity, as shown in Figure 10. The
increment in the bending stiffness was usually very limited [7, 9, 20, 31]. However, some
studies reported significant increases in stiffness especially for CFRP reinforcement of lower
grade timber or high reinforcement ratios [8, 10, 12]. Analyzing the distribution of forces
over the entire section, it was possible to state that the reinforcement, applied on the tension side, was very useful in improving the ultimate resisting moment, through the contribution of an extra tensile force ($F_3$).

Furthermore, this reinforcement allowed a greater axial deformation in the compression region, as a result of the increase in the distance of the compressed wood fibers from the neutral axis. This type of intervention may be used for low grade timber due to the presence of defects, such as timber in which the ratio between ultimate tensile and compressive stresses is approx. 1. When timber yielded on the compression side, the values of forces $F_1$, $F_2$ and $F_3$ were very high. However the point of application of force $F_1$ moved downward causing a decrease of the offset of internal forces. Force $F_3$, generated by the FRP reinforcement, allowed an increase in the resisting moment.

The application of the composite reinforcement had several positive effects: 1) It caused a significant increase in the beam’s bending capacity; 2) The reinforced beams exhibited a more ductile behavior, as an higher degree of yielding was possible on the beam’s side in compression; 3) According to the results shown in Table 4, the FRP reinforcement also reduced the standard deviation in the strength value. Figures 11 and 12 show the PDF and CDF functions for reinforced beams. Several experimental tests [3] have shown that the most frequent failure is a tensile failure without the timber plasticization of the compression region, depending on the quality of the wood. This explains the need for a composite reinforcement on the tension side, especially for low-grade timber.

Figure 13 shows a comparison between the increments of reinforced firwood beams in terms of mean bending capacity and $f_{m,k}$ values. The increment, calculated using the $f_{m,k}$ values, is always bigger compared to the one based on the mean bending strengths $f_m$. The maximum ratio between the two increments ($f_{m,k}$ increment / mean capacity $f_m$ increment) was 3.24, and this occurred for 100x100 mm cross section beams reinforced with GFRPs. This increment
was usually greater for beams of large dimensions (it was approx. 1 for beams having 20x20 mm cross sections) based on the fact that larger beams contain defects of various, such as knots, slope of grain, bark pockets, etc. In this situations the application of a FRP reinforcement may produce a double positive effect as it confines local ruptures and bridges local defects in the timber.

It can be also noted that both unreinforced and reinforced timber beams were tested over a short span. This reduced the probability of the presence of a critical defect in timber, decreasing the uncertainty of timber beams, particularly when unreinforced. It is likely that with longer spans uncertainty of unreinforced beams will increase and the positive effect of the composite reinforcement should be even more noticeable. Also, it should be noted that no measures to minimize the difference in properties between the timber beams in each group or adjustment factors to the stiffness and strength values have been applied for the data reported in Tables 4 and 5.

By comparing these results with the ones reported in [46] for timber beams reinforced with unbonded composite plates, it can be noted that the increments in the bending capacity were significantly larger when the FRP reinforcement was bonded to the beam’s tension side with an epoxy adhesive. The role of the resin seems to be critical in both the stress transfer (FRP-timber) and in confining local ruptures in the timber. This had a considerable effect in reducing the uncertainties and in increasing the $f_{m,k}$ value of reinforced beams.

On the contrary, the difference in terms of capacity increments between GFRP- and CFRP reinforced beams was smaller. For high reinforcement area fractions (Fig. 13) the ratio between these increments decreased. By comparing the test results of GFRP- and CFRP-reinforced beams for the same cross section (Tab. 5), it can be noted a limited difference in terms of capacity increments for beams reinforced with the two FRP types. CFRP had a much higher tensile strength (3388 MPa, Tab. 2) compared to GFRP (1568 MPa) but this did
not cause a significant increase in the beam bending capacity. Because failure always occurred on the beam’s tension side, the composite tensile strength could not be completely exploited during the tests and this reduced the importance of using a carbon sheet, more expensive and with higher mechanical properties.

With regard to the flexural stiffness $r$, the application of a FRP reinforcement did cause a significant increase in the mean value of this mechanical property. Flexural stiffness was calculated from the bending load $F$ – midspan deflection ($\delta$) graph by considering the slope of the secant line between $F_1 = 0.1x F_{\text{max}}$ and $F_2 = 0.5x F_{\text{max}}$:

$$r = \frac{F_2 - F_1}{\delta_{F_2} - \delta_{F_1}}$$

(2)

where $\delta_{F_2}$ and $\delta_{F_1}$ are the corresponding values of the midspan deflection.

For both unreinforced and reinforced beams, the CoV of the flexural stiffness $r$ was always smaller compared to the CoV of the strength. Defects in timber affect more the strength than the stiffness, causing a smaller scattering of the $r$ values.

FRP reinforcement also produced a limited increase of the flexural stiffness ($r$ increment = approx. 5-15%) based on the fact that the reinforcement area fractions (Tab. 3) were very small. Furthermore, the orientation of the FRP sheet (parallel to the neutral axis of the beam’s section) (Fig. 10) produced a very small increase in the cross section’s total second moment.

By comparing the increments of $r$ and $K_r$ values (flexural stiffness at 5% of cumulative distribution function), it can be noted that these increments were similar (Tab. 5) highlighting the fact that the application of the reinforcement was not able to reduce stiffness uncertainty.

CONCLUSIONS

Epoxy-bonded FRP sheets appear to have good potential to strengthen existing deficient timber beams. In this experimental investigation 221 fir and oakwood beams were tested and
it was demonstrated that the application of small quantities of composite reinforcement, besides being an effective method of increasing timber beam’s capacity, also reduced the uncertainties in the strength. Tests results showed that the typical failure modes for unreinforced and reinforced beams were gross-grained tension and knot initiated. Ductile compression did not produce the beam failure and the rupture always occurred on the tension side. The application of an epoxy-bonded FRP sheet confined local rupture and bridged local defects in the timber and this had a considerable effect on the beam capacity and on the scattering of the results. The negative defects effect on the tension side was effectively reduced by the application of the FRP reinforcing. Increments in the mean strength up to 122% and decrements in the CoV values up to 62.5% were experimentally found. All tested timber beams (made of firwood and oakwood) met, after reinforcement, the requirement of the EN 338 standard for the strength class for which they were commercialized and sold. Finally it is worth noting that a limited difference in terms of capacity increments was recorded for beams reinforced with the two FRP types (GFRP and GFRP). Because failure always occurred on the timber beam’s tension side, the FRP tensile strength could not be completely exploited during the tests and this reduced the importance of using a composite material with higher mechanical properties (CFRP).

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<table>
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<tr>
<th>Wood species</th>
<th>Cross section (mm)</th>
<th>Sample size</th>
<th>Weight density (kg/m$^3$)</th>
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<th>Bending Strength (MPa)</th>
<th>CoV (%)</th>
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<td>46.2</td>
</tr>
<tr>
<td>Oak</td>
<td>67x67</td>
<td>20</td>
<td>755.8</td>
<td>14.4</td>
<td>60.94</td>
<td>16.90</td>
<td>10.3</td>
<td>44.1</td>
</tr>
<tr>
<td>Oak</td>
<td>200x200</td>
<td>5</td>
<td>796.0</td>
<td>11.5</td>
<td>33.83</td>
<td>28.26</td>
<td>9.6</td>
<td>17.9</td>
</tr>
</tbody>
</table>

Table 2: Results of mechanical characterization of FRP-materials.

<table>
<thead>
<tr>
<th>Composite type</th>
<th>CFRP</th>
<th>GFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layout</td>
<td>Textile</td>
<td>Textile</td>
</tr>
<tr>
<td>No. of samples tested</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Fiber orientation</td>
<td>Unidirectional</td>
<td>Unidirectional</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>417.6**</td>
<td>78.65**</td>
</tr>
<tr>
<td>Weight density (kg/m$^3$)</td>
<td>0.3</td>
<td>0.288</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>3388**</td>
<td>1568**</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>0.165*</td>
<td>0.118*</td>
</tr>
<tr>
<td>Elongation at failure (%)</td>
<td>1.0</td>
<td>2.1</td>
</tr>
</tbody>
</table>

* nominal ply thickness ** using nominal thickness for calculation
Table 3: Reinforcement of FRP-materials.

<table>
<thead>
<tr>
<th>Beam cross section (mm)</th>
<th>20x20</th>
<th>67x67</th>
<th>100x100</th>
<th>200x200</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of beams tested</td>
<td>50</td>
<td>35</td>
<td>24</td>
<td>17</td>
</tr>
<tr>
<td>No. of composite layers</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>GFRP area fraction (%)</td>
<td>0.590</td>
<td>0.176</td>
<td>0.118</td>
<td>0.059</td>
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<tr>
<td>CFRP area fraction (%)</td>
<td>0.825</td>
<td>0.246</td>
<td>0.165</td>
<td>0.082</td>
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<tr>
<td>Sheet width (mm)</td>
<td>20</td>
<td>67</td>
<td>100</td>
<td>100</td>
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</table>

Table 4: Test results for reinforced wood beams.

<table>
<thead>
<tr>
<th>Wood species</th>
<th>Cross section (mm)</th>
<th>Sample size</th>
<th>Weight density (kg/m$^3$)</th>
<th>Moisture content (%)</th>
<th>Bending Strength (MPa)</th>
<th>Reinforcement</th>
<th>CoV (%)</th>
<th>Standard deviation (MPa)</th>
<th>$f_{mk}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fir</td>
<td>20x20</td>
<td>20</td>
<td>423.3</td>
<td>10.2</td>
<td>70.1</td>
<td>GFRP</td>
<td>13.1</td>
<td>9.11</td>
<td>55.1</td>
</tr>
<tr>
<td>Fir</td>
<td>20x20</td>
<td>20</td>
<td>423.3</td>
<td>10.2</td>
<td>94.0</td>
<td>CFRP</td>
<td>16.0</td>
<td>15.0</td>
<td>69.2</td>
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<tr>
<td>Fir</td>
<td>100x100</td>
<td>14</td>
<td>417.0</td>
<td>14.3</td>
<td>32.8</td>
<td>GFRP</td>
<td>18.7</td>
<td>6.11</td>
<td>22.7</td>
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<tr>
<td>Fir</td>
<td>100x100</td>
<td>10</td>
<td>417.0</td>
<td>14.3</td>
<td>39.3</td>
<td>CFRP</td>
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<td>25.9</td>
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<td>6</td>
<td>430.8</td>
<td>11.3</td>
<td>45.8</td>
<td>GFRP</td>
<td>10.7</td>
<td>4.91</td>
<td>37.7</td>
</tr>
<tr>
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<td>CFRP</td>
<td>8.84</td>
<td>4.32</td>
<td>41.1</td>
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<td>10</td>
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<td>7.35</td>
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<td>20</td>
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<td>14.4</td>
<td>89.60</td>
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<td>18.6</td>
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<td>83.10</td>
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<td>8.80</td>
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<td>10.6</td>
<td>5.14</td>
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</table>
Table 5: Effects of reinforcement.

<table>
<thead>
<tr>
<th>Cross section (mm)</th>
<th>Wood species</th>
<th>Reinforcement</th>
<th>Mean strength increment</th>
<th>CoV decrement (%)</th>
<th>$f_{m,k}$ increment (%)</th>
<th>Stiffness $r$ increment (%)</th>
<th>$K_r$ increment (%)</th>
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<tbody>
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<td>GFRP</td>
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<td>GFRP</td>
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<td>9.20</td>
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<td>CFRP</td>
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<td>114</td>
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<td>Oak</td>
<td>GFRP</td>
<td>47.0</td>
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