SEISMIC VULNERABILITY ASSESSMENT OF A MONUMENTAL MASONRY BUILDING

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Abstract. The seismic assessment of monumental buildings requires the consideration of safety and conservation objectives, both for the building and its cultural heritage assets. In order to face these issues, the paper presents the results of a diagnostic analysis carried out on a specific monumental masonry building: the Civic Museum of the small city of Sansepolcro in Tuscany. The building is one of the most important and renowned civic structures, built by a mediaeval Commune to house the town government, this building is also characterized by the presence of one of the masterpieces of late 15th-Century Renaissance art: “the Resurrection”, a large fresco painted by Piero della Francesca. Within this context, three modelling strategies of different complexity are proposed: equivalent frame model, rigid macro-block model and finite element model. In the first part, a full three-dimensional non-linear static analysis and a more simple approach directly based on the limit analysis theorems are used in order to understand the macro-scale structural behavior. Afterwards, the results of the finite element method analyses performed on a detailed 3D model of the wall panel containing the fresco are used in order to investigate the causes of the crack pattern on this important artistic asset.

Keywords: Masonry modelling; Historical buildings; Damage assessment; Finite element analysis; Non-linear static analysis; Kinematic analysis.

1 INTRODUCTION

The analysis of interpretative models that can effectively define the "structural safety" of historic constructions belonging to cultural heritage is currently a topic of great attention. Whilst for new masonry constructions it is possible to achieve important information toward the correct interpretation of their
structural behavior, and then have adequate tools to properly evaluate their response to severe earthquakes, when an analysis involves historic masonry structures such tasks becomes complex, mainly for two different reasons: i) difficulties in understanding and modeling the seismic response of historic constructions, because these were designed using an empirical approach; ii) problems in acquiring data on material mechanical properties and structural details (e.g. connections between walls, etc.), due to their dispersion and the need of avoiding expensive and destructive tests [1]. To complicate the problem, the conservation of heritage structures should last over time against degrading agents and natural decay, without losing their integrity and authenticity. This means that the need to guarantee an "acceptable level" of structural safety for building's occupants should be always related to a principle of "minimum intervention" to the building. Within this context, a common denominator among the best-known international guideline documents [2][3][4], on the assessment of cultural heritage assets, is a qualitative method. Qualitative methods are based on the analysis of historical construction stages (which can occur in different periods in history) and antique original documents, the precise survey of architectural details and the interpretation of the seismic behavior of the building, based on the damage (due to previous events, if any) or on similar structures.

Furthermore, it should be considered that any risk assessment analysis for a masonry structure should involve both architectural parts (arches, partition walls, facades, openings, etc.) and artistic assets (statues, pinnacles, frescoes, paintings, libraries). The analysis of effects of an earthquake on the artistic assets that are present in an historic construction is new and is becoming more important to the heritage bodies, mainly because extensive damage could be caused to these assets by earthquakes of small magnitudes. For their conservation and protection, the analysis of the damage mechanisms is crucial and, to evaluate their vulnerability, the development of appropriate numerical modelling methods is essential [5][6][7][8][9].

This is the background of this investigation dealing with an important historic construction, the Civic Museum of Sansepolcro, that besides being one of the most renowned civic structures built by an Italian commune during the Medieval times to house the city government, it is also characterized by the presence of one of the masterpieces of late 15th-Century Renaissance art: “the Resurrection”, a large fresco painted by Piero della Francesca.

Following the procedure introduced by Lagomarsino and Cattari [1] for the analysis of the seismic vulnerability of architectural and artistic assets, the evaluation process used to perform this task, takes into account several contributions from different research areas and involves two key stages which are iterative and continually informed each other: phase 1 (or knowledge stage) and phase 2 (or analysis stage). In such a
context, it is important to emphasize how the two subsequent levels of analysis should not be intended as a one-way process, but the results of the structural analysis has to come from the reiterative check of the outcomes of the knowledge level.

Thus, since the main concern of the first stage is the achievement of an extensive knowledge of the fabric for structural analysis, in the first part of the paper, an overview of the construction stages of the Museum and in particular of the wall containing the fresco during the centuries is presented. Afterwards the geometric and damage survey, carried out to provide all the data necessary to complete the following numerical modelling stage, is described. The on-site investigation, aimed at qualitatively defining the structural composition of relevant parts of the building, is also illustrated.

Next, to study the structural behavior of the building and of its greatest artistic asset, the Piero Della Francesca’s fresco, in the second stage a performance based multi-scale approach is followed [1][10][11]. In such a context, according to the NTC [12] and IMIT [13] instructions, a global analysis of the Museum is made by using the equivalent frame approach. In particular, a full three-dimensional (3D) pushover analysis is performed to predict the macro-scale behavior of the building. Afterwards, since a comprehensive assessment would also require to analyze possible local collapse mechanisms (mainly out-of-plane ones), a macro-element approach based on the limit analysis theorems is used complementarily to the global analysis. After completing the analysis phase at macro-scale level, it has been considered crucial to have, at a more detailed level, a more precise assessment of the seismic vulnerability of the wall containing the fresco. To this end, the results of a Finite Element (FE) analyses carried out on a detailed three-dimensional model of the wall are used to provide an interpretation of the observed cracking pattern.

2 AS-BUILT INFORMATION

To study the actual structural state of both the building and the wall panel supporting the Piero Della Francesca’s fresco “the Resurrection” a standard diagnostic procedure (historical background, surveying and nondestructive testing) was conducted [14][15][16][17].

2.1 Historical Background

The Piero Della Francesca’s fresco known as “the Resurrection” is located in the Civic Museum of Sansepolcro (Italy), one of the most interesting architectural complex built by a mediaeval Commune to house the town government (Figure 1). Historically known as “Palazzo della Residenza”, since the
representatives of local power held their meetings there, this building is the result of a series of complex architectural modifications related both to the transformation of the urban site and to the change in use of the building itself. Constructed in various stages, the entire complex is, in fact, a stratified structure that consists of three main structural units (A, C, D) joined together by two 15th-century masonry buildings (E).

Figure 1: Plan layout of the Civic Museum.

The first section of the structure (A) appears to date back to the second half of the 13\textsuperscript{th} century and incorporates an older tower (B), which contained a prison in use until the 19\textsuperscript{th} century. A stonework stair connects the floors of the unit, while on its east side, a great barrel vault (G), perhaps recalling the original gate of the old town, connects the first floor of the building (the so-called “piano nobile”) with the Government building (H), headquarters of the Government Auditor.

Once it had become the seat of the Florentine Commissary along with its offices, in the middle of 15\textsuperscript{th} century, the palace was included in an urban redevelopment programme. As far back as 1444, work first began with the construction of two monumental rooms (Piero’s and Matteo di Giovanni’s rooms, C) that stood above a pre-existing underground level (a storage basement). Hence, exploiting its structural potential, new buildings (E, F), aimed at a more regular and uniform layout, were added to the structure, which took on the features of a typical 15\textsuperscript{th}-century Tuscan palace. During this period, entrusted with the job of completing the extension and modernization work of the palace’s interior, Piero Della Francesca painted the prestigious “the Resurrection” (ca. 1460) on the essential bare wall between Piero’s and Matteo di Giovanni’s rooms (Figure 2). Despite the numerous subsequent changes made over the centuries, the 15\textsuperscript{th} century layout is still
legible while visiting the present-day rooms of the Museum. The last modification of the structure dates 18th century, when a new building (D) was added, giving the structure the current L-shaped layout with an inner cloister. Converted to the Civic Museum in 1975, the complex owes much of its current appearance to work carried out between 1991 and 1997, during which the courtyard was covered with a glass structure and the original entrance, which opened onto the square, was substituted by a large glass door that maintains a continuity between the internal and the external and renders the different historical phases tangibly visible.

Figure 2: Piero Della Francesca’s fresco “the Resurrection”.

2.2 Surveying

Even if Sansepolcro did not experience any devastating earthquakes in the past, an evaluation of the of historical seismic activity in such area [18] shows how the Civic Museum was exposed to an earthquake of degree 9 in the Mercalli-Cancani-Sieberg (MCS) scale in the past and to seismic events stronger than magnitude 7 (MCS scale) at least three times since the Piero Della Francesca’s fresco was painted (Figure 3).
Figure 3: Historical macroseismic intensity (MCS) of the major events near Sansepolcro since 1200.

In such a context, using past earthquakes as full scale tests, the actual damage location shown by the construction is an important indication of the seismic response of the construction, and it highlights the parts of the masonry structure with higher seismic vulnerability.

The crack pattern of the construction is quite complex: since no or little coupling effect can be operated by the horizontal (flexible) floors, vertical structures (walls) tend, in fact, to behave independently. Damage seems to localize maximally in correspondence of the longitudinal façades, north and south (Figure 4), being massive structures presenting significant heights and lack of transversal walls as bracing elements. It is possible to note that on the southern façade there is a relevant out-of-plumb (of about 60 mm, based on a general estimate), as a consequence of an out-of-plane rocking displacements of the perimeter walls (with a limited detachment of the façade from the masonry structure). Under such conditions, the internal wall panel supporting the Piero Della Francesca’s fresco, being the only shear resistant wall able to restrain the overturning motion of the southern façade, manifests a severe cracking pattern, characterized by a combined failure mode with vertical and diagonal cracks (Figure 5).
Conversely, the western and eastern façades appear to be in a decent state of conservation, since almost the totality of longitudinal perimeter walls do not exhibit significant damage, with the exception of a few thin cracks in some localized areas. Finally, a loss of curvature and a high concentration of thin cracks are observable on the central part of the cloister vault in the Piero’s room (Figure 4). According to this, it would seem reasonable to assume that such crack pattern could be related to a retrofitting intervention, made in the 1940s, that, aiming to prevent the collapse of the vault, modified its static behavior by removing the filling and by demolishing and rebuilding its central portion.

2.3 Non-destructive testing

A first visual inspection of the crack pattern and of the main damage that the Piero Della Francesca’s fresco has experienced in the course of centuries, confirm the assumption that the analysis of the causes, which have produced the damage, needs a careful study of the connections between the fresco and its supporting wall. Historical sources [19] suggest that the fresco was mounted on its supporting wall at a later date, after being detached from its original support using the “stacco a massello” technique, which involves cutting the wall and removing a part of it together with both layers of plaster and the fresco painting itself. There are two possible assumptions regarding the mechanical behavior of the fresco: 1) either the wall painting was removed with its entire original substrate and, consequently, its behavior can be seen as independent from that of the supporting wall, or 2) just a thick layer of plaster is retained along with the fresco and its mechanical response is thus strictly dependent on the response of wall it is applied on.
In order to study the influence of such variables on the construction’s overall response, a thermographic investigation was conducted. This analysis was aimed to investigate the morphology and the composition of the masonry of the bearing wall, such as the presence of internal voids or masonry with different arrangement of blocks (bricks or stones) and mortar type and thickness of bed joints.

![Thermographic images of the wall containing the fresco](image)

**Figure 6:** Thermographic images of the wall containing the fresco (courtesy of Las.E.R. laboratory): a) front view (detection of the masonry texture types); b) rear view.

From the analysis carried out, it was possible to detect the presence of three different masonry texture types (Figure 6a): a) solid brick masonry entirely comprised of stretcher bricks (units running horizontally); b) stone masonry made of small and medium sized limestone elements, simply rough-hewn or boasted; c) stone masonry made of rectangular units with dressed or squared beds and mortar joints. It was also possible to detect the presence of a pre-existing chimney flue hidden under the left side of the fresco (see the dotted line in Figure 6b).

These results allow the authors to assume that the fresco was “set” into the supporting wall and consequently it is considerably influenced by it.

### 3 STRUCTURAL ANALYSIS METHOD

As previously stated, the seismic assessment of monumental buildings is a difficult task for several causes: the complexity of the geometrical configuration typical of historic constructions; the limited information and data obtained from on-site experimental tests; the difficulty of the numerical modelling task due to the masonry structures’ non-linear response.
Within this context, this paper is aimed to contribute, through the application of a performance based multi-scale approach on the Civic Museum and the Piero della Francesca's fresco, to the calibration and validation of a general methodology for the damage assessment of heritage structures.

To this end, three modelling strategies of different complexity are proposed (Table 1): equivalent frame model (EFM), rigid macro-block model (RMM) and finite element model (FEM).

### Table 1: Computational models used for the modeling of structure: advantages and drawbacks.

<table>
<thead>
<tr>
<th>Modelling strategies</th>
<th>Pros</th>
<th>Cons</th>
</tr>
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</table>
| Equivalent Frame Model (EFM) | - simplicity of the interpretation of the results (also considering that professional engineers are traditionally familiar with framed structures)  
- it requires a limited number of degrees of freedom, with a reasonable computational effort  
- it is based on few mechanical parameters that may be quite simply defined and related to results of standard tests | - simplified model with a low level of accuracy in the representation of the geometric, spatial and material complexity of the structure  
- it introduces strong simplifications and unavoidable approximations of the actual behavior (e.g. related to the mechanical description of damage and dissipation mechanisms) ⇒ its accuracy depends on the consistency between the adopted hypotheses and the actual structural problem |
| Rigid Macro-block Model (RMM) | - it requires few input parameters and a limited number of degrees of freedom, allowing to simplify significantly the structural model and retaining a good accuracy in the results in the case of structures where the out of plane action in walls orthogonal to the earthquake motion is expected to prevail  
- it takes into consideration several failure mechanisms that are usually not considered by other methodologies (e.g. the out-of-plane mechanisms) | - inability to evaluate the displacements and deformations of the structure, which is fundamental for energy dissipation assessments in the current performance based design (PBD) philosophy  
- inability to take into account the dynamic interaction between adjacent buildings and the spatial heterogeneity of the load  
- difficulty to consider all possible mechanisms which can be produced by placing plastic hinges in a complex and multiple structural system |
| Finite Element Model (FEM) | - capable at giving a deep insight on the nonlinear behavior of the component materials (or their interaction) and on the local and global collapse mechanisms  
- able to reproduce, in an accurate way, the development of cracking in the structure | - time consuming during both the modelling and the results interpretation phases, so making the described approaches not suitable for non-linear analysis of real three-dimensional buildings with many degrees of freedom;  
- it requires a detailed model of the structure and a deep knowledge of the mechanical properties whose determination may often be unaffordable in practice |
At macro-scale level, a three-dimensional (3D) pushover analysis (equivalent frame model) was used to investigate the global structural performance of the building. Since, at this scale level, a comprehensive assessment would also require to analyze possible local collapse mechanisms (mainly out-of-plane ones), a macro-element approach based on the limit analysis theorems (rigid macro-block model) was used complementarily to the global analysis. Accordingly, local analyses were performed on some elementary macro-elements that, being not able to redistribute their seismic forces to the rest of the building, can be assumed as rigid and cannot be accurately analyze by the global model. Finally, a finite element analysis was conducted on a refined 3D model of the wall supporting the fresco in order to have, in this case at a meso-scale level, an accurate knowledge of the seismic vulnerability of this important artwork.

3.1 Macro-scale: pushover analysis

A pushover analysis (non-linear static analysis) with a monotonically increasing pattern of in-plane forces (representing the inertial forces that would be experienced by the masonry construction when subjected to a seismic action) was used to investigate the seismic behavior of the building [20][21][22]. Based on these assumptions, the effect of the seismic action was analyzed by considering two different load patterns (not acting simultaneously): a uniform load pattern based on lateral in-plane forces (uniform distribution) and a modal load pattern proportional to inertia masses multiplied by first-mode displacements (modal distribution).

Among the different modelling strategies proposed in codes and literature, it was decided to adopt the equivalent frame modelling strategy (Figure 7) by considering each wall pier as a beam element in which the in-plane behavior was supposed to be elastic–plastic with limited deformation [23][24]. Similar consideration were used for spandrel walls. When the normal action is known at each iteration, i.e. in the presence of deformable floors, as was the case considered here, it is possible, in fact, to assume spandrel behavior as that of a pier rotated by 90° (the reader is referred to [5][12][13] for further details).

At this scale, the model was built considering the masonry as a macroscopically isotropic material (i.e. uncut stone masonry), whose mechanical properties, evaluated in accordance with current Italian Code [12], were conservative estimations of the main values for existing buildings. In such a context, the Young (E) and shear (G) moduli were assumed equal to 1230 N/mm² and 410 N/mm², respectively, whereas the compressive and shear strength were assumed equal to 2.0 N/mm² and 0.035 N/mm², respectively. Finally, in agreement with the Italian Code [12] three different collapse mechanisms were studied for both the wall piers
and spandrels: “rocking” or flexural failure, shear sliding failure and diagonal shear cracking (the reader is referred to [13] for further details).

![Figure 7](image)

**Figure 7**: a) Geometrical model; b) Equivalent frame model.

The horizontal displacement of the structure's control node (placed in center of mass of the roof level) versus the corresponding total shear is reported in Figure 8. This diagram can be seen as the *capacity curve* of the structure in the transversal direction as it gives, among other information, the maximum seismic force that is bearable by the construction, i.e. the level of the Peak Ground Acceleration (PGA) needed for inducing the first failure mechanism.
Figure 8: Capacity curve (maximum displacement vs total shear at the base) and failure progression sequence for the equivalent frame model in the y-direction.

Considering all the combinations and all the seismic directions requested by the Italian Code [12], the most severe loading condition was in the y-direction (Y seismic action, Figure 9) for modal distribution and results (obtained by using the commercial software Aedes Pcm [25]) with respect to this case were studied in detail. In this direction, the three-dimensional strength of the structure is low and insufficient (40% lower than that related to the X seismic action), as a result of the progression of the out-of-plane bending failure of the orthogonal bearing walls (see point A, B and C of the capacity curve, Figure 8). This can be motivated on one hand by the presence of few bearing walls in the transversal direction (y-direction), and on the other hand by the fact that dead loads on floors were mainly distributed to walls parallel to the longitudinal direction (x-direction).
This comes in agreement with the damage observed in these wall panels. In y-direction, the failure is, in fact, mainly associated with the presence of in-plane mechanisms (shear failure). Figure 9b shows the cracking pattern assessed at the end of the analysis on the wall panel supporting the fresco. As it can be observed, the in-plane behavior of the wall is governed by the shear failure of all the spandrel walls of the second and third floor accompanied by the shear failure and damage of the piers close to the chimney flue at the second floor.

The intensity measure (IM) used for the assessment of the seismic response was the PGA. More specifically, the seismic vulnerability was assessed through a comparison of the maximum value of the PGA (obtained by using the procedure proposed by Lagomarsino and Cattari [1], $IM_{kn} = 0.165g$) compatible with the fulfillment of the assumed performance level (Life Safety) and the reference target value ($IM_{kn} = a_g \cdot S = 0.345g$, where $a_g = 0.260g$ is the value of PGA on rock and $S$ is a factor depending on the ground type, which in this case - deep deposits of dense sand and stiff clay - is assumed equal to 1.330) of the seismic demand (in terms of PGA) given by the Italian code [12]. Since the value (0.478) of the safety factor ratio $IM_{kn} / IM_{kn}$ is considerably lower than the minimum level of safety (0.780) proposed by code requirements [4], the structure is clearly not verified.

3.2 Macro-scale: limit analysis

In the previous results of the pushover analysis, the assessment of the historic construction was developed by considering the entire building, using the total seismic structural mass. Hence, in this step of the
assessment it is necessary to identify the possibility of suffering local mechanisms (mainly out-of-plane ones) that involve only a fraction of the total mass. By mapping the damage produced by past seismic events [26][27][28], it is evident, in fact, that the most damage suffered by existing masonry buildings presents local characteristics, indicating the high seismic vulnerability of single structural elements that, being not able to redistribute their seismic forces to the rest of the building [29][30][31], cannot be accurately analyzed through a global analysis.

According to this, a macro-element approach based on the limit analysis theorems (rigid macro-block model) was used complementarily to the global analysis. The structure was idealized here as an assemblage of elementary macro-elements, modelled as a set of rigid elements, selected on the basis of the observed damage pattern, where applicable, and the numerical damage obtained through the pushover analysis.

Following the selection of the elementary macro-elements, a number of failure mechanisms were evaluated and the corresponding value of the collapse load multiplier ($a_0$) calculated using the principle of the virtual works [13]:

$$\alpha_0 \left( \sum_{k=1}^{n_e} P_k \delta_{x,k} + \sum_{i=1}^{n_w} P_i \delta_{x,i} \right) - \sum_{k=1}^{n_e} P_k \delta_{y,k} - \sum_{j=1}^{n_f} F_j \delta_j = W$$

where $P_k$ represents the self-weight of each macro-element; $P_i$ denotes the weight transmitted to the macro-element by adjacent structures; $F_j$ is the generic external force applied to a macro-element part; $\delta_{x,k}$ and $\delta_{x,i}$ represent the horizontal virtual displacements of each macro-element centroid point; $\delta_{y,k}$ and $\delta_j$ represent the vertical virtual displacements of the points of application of $P_k$ and $F_j$, respectively; $W$ represents the total work done by the internal forces.

Once the value of the collapse load multiplier ($a_0$) is identified, the seismic spectral acceleration, activating the mechanism ($a_0^*$), can be defined as follows:

$$a_0^* = \frac{\alpha_0 \sum_{i=1}^{n_e} P_i}{M^* CF}$$

where $M^*$ represents the effective mass and $CF$ denotes the confidence factor (equal to 1.12 according to [4]). Fulfilling of the analyzed limit state can be verified with the following inequality:

$$a_q \geq a_0^* S$$

(3)
where $a_S$ is the spectral acceleration for the elastic design (seismic demand, $IM_{el} = 0.345g$) and $q$ represents the behavior factor (equal to 2.0 as suggested by [12]).

Figure 10 and Figure 11 show the failure mechanism for the lateral bearing walls and for the main façade. The evaluation of the safety factor ratio upon Eqs. (1)-(3) was made by using the commercial software Aedes Pcm (2014).

Figure 10: Lateral wall overturning: a) free-standing wall (Mechanism 01); b) wall restrained at the top (Mechanism 02).
Figure 11: Main façade overturning: free-standing wall (Mechanism 03).

Results from the limit analysis (Table 2) agree with those obtained using the pushover analysis via equivalent frame approach. The critical behavior of the building is activated when the seismic load is acting on the transversal direction (y-direction, Figure 10). The absence of buttresses is the main cause of this poor seismic behavior. The effectiveness and quality of the connection (e.g. tie rods) between the perimeter wall panels and bearing walls in the transversal direction is not able to improve this poor behavior. Other local mechanisms could be activated in some parts of the building (e.g., main façade overturning, Figure 11). However, looking at the crack pattern, it can be noted that these further failure mechanisms did not occur, most likely for an effective connection of the façade with the transversal walls as well as to the anticipation of other mechanisms.

<table>
<thead>
<tr>
<th>Title 1</th>
<th>Collapse multiplier ($a_0$)</th>
<th>Seismic spectral acceleration ($a_0^*$)</th>
<th>Seismic demand</th>
<th>Safety factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism 01 (lateral wall)</td>
<td>0.080</td>
<td>0.102g</td>
<td>0.345g</td>
<td>0.591</td>
</tr>
<tr>
<td>Mechanism 02 (lateral wall)</td>
<td>0.086</td>
<td>0.075g</td>
<td>0.345g</td>
<td>0.434</td>
</tr>
<tr>
<td>Mechanism 03 (main façade)</td>
<td>0.052</td>
<td>0.062g</td>
<td>0.345g</td>
<td>0.359</td>
</tr>
</tbody>
</table>

3.3 Meso-scale: FEM based analysis

Finally, since according to the guidelines developed by the PERPETUATE research project [32] - the seismic assessment of the cultural heritage is directly related to the behavior of the architectural asset, a relationship may be found between the structural damage and the consequences to the cultural heritage assets. Under such conditions, after completing the analysis phase at macro-scale level, it was considered crucial to have, at a more detailed level, a more accurate analysis of the seismic response of the wall panel supporting
the fresco. With this aim, the results of the 3D model FEM analyses were used in order to provide an interpretation of the observed damage pattern.

![Figure 12: FEM model: mesh discretization and material map.](image)

A complete three-dimensional FE analysis was thus carried out using the commercial computer software ANSYS [33][34]. An accurate numerical model was created, using available photographs, drawings and the thermographic survey. The numerical model was used to reproduce as accurately as possible the structural geometry, focusing on the wall connections, the geometrical and structural irregularities and the variations in the wall thickness. To this end, the geometry of the wall was firstly defined with a CAD drawing, next the volumes were imported and discretized using eight-node hexahedron elements (Solid 45). Figure 12 shows the complete FE model: it consists of 112,128 elements and 163,952 nodes, with 394,994 degrees of freedom.

To represent the masonry behavior, a physically non-linear model was considered by using damage mechanics. For the purpose of simplicity, both elastic and perfectly plastic behaviors were used: (a) with isotropic behavior and (b) with infinite ductility assumptions for the material. A noteworthy point is that these hypotheses do not include two distinctive features of masonry at failure, the first related to the lower value of the tensile vertical strength with respect to the horizontal in-plane one (essentially due to brick staggering), the latter related to the behavior of joints when damage occurs. Nevertheless, these aspects cannot be easily studied. On the other hand, the hypothesis of negligible tensile strength with frictional behavior is widely accepted for the analysis of historical buildings [5]. Within this approach, a tension cut-off and Mohr Coulomb type failure criteria were assumed for masonry. This elastic-plastic model, initially used for concrete, accounts for both crushing and cracking failure modes by means of a smeared model. More
specifically, the brittle behavior of masonry was here defined by means of only two material parameters: $f_c$ (uniaxial compressive strength) and $f_t$ (uniaxial tensile strength). Also, to reach the highest level of reliability of the proposed model, the simulation of the connections between elements belonging to different historical periods was performed using unilateral contact interfaces. The joint modeling requires the use of flexible-flexible contact elements (contact pair type Target 170/Contact 174 in ANSYS). In detail, in this application, a unilateral contact law was applied in the tangential direction, assuming that sliding may occur – or not – by using the Coulomb friction law and a friction coefficient of 0.4 [35]. As it regards the behavior in the normal direction, friction conditions are described by the same Coulomb friction model, indicating that masonry has negligible tensile strength in such direction and a gap may appear when the compressive stresses become negligible.

Due to the historical and artistic value of the fresco and to reduce the impact on conservation, in-situ tests could not be used for estimating the mechanical properties of the different types of brickwork forming the structure. Therefore, data on the masonry material properties were taken from normative reference [13] and existing literature - i.e. constructions with similar masonry arrangement [36] - using mechanical characteristics found in these investigations. Table 3 summarizes the values used for the evaluation of the non-linear model parameters with respect to the wall’s masonry typologies.

| Table 3: Constitutive parameters of the materials used for non-linear FEM analyses. |
|-----------------------------------------------|-----------------|-----------------|-----------------|
| Title 1                                       | Solid brick masonry | Uncut stone masonry | Dressed rectangular stone masonry |
| Young modulus (MPa)                           | 1500             | 1230             | 2800             |
| Poisson’s ratio                               | 0.25             | 0.25             | 0.25             |
| Specific weight (kg/m3)                       | 1800             | 2000             | 2200             |
| Uniaxial compressive strength (MPa)           | 2.40             | 2.00             | 7.00             |
| Uniaxial tensile strength (MPa)               | 0.09             | 0.05             | 0.16             |

At this scale level, an equivalent static analysis was performed, in which the building was considered to be loaded by both seismic horizontal actions (applied at the floor levels) and vertical actions, i.e. live loads and self-weight. More in details, the seismic loads were modelled by a set of horizontal forces (acting along the y direction) which were applied with the distribution shown in Figure 13, where $F_i$ represents the ultimate shear at the $i$th floor (with $i = 1$ to 3, being 3 the total number of floors) obtained according the redistribution of seismic actions produced by pushover analysis at global scale. As it regards the boundary conditions, the
model was perfectly constrained at the base and no kinematic constraints were applied between the wall and the surrounding structure.

![Figure 13: Application of the seismic action at every level.](image)

Figure 14 reports the cracking pattern observed at the end of the finite element analysis on the bearing wall. This provided further comparison with the observed damage when compared to the failure progression produced by pushover analysis examined earlier in this paper (Figure 9b). Cracking is not present on the whole construction, but mainly on the southern (with diagonal and vertical cracks) and upper part of the wall close to the chimney flue (with vertical cracks between the inner and the outer leaf of the wall). This comes in agreement with the damage assessed at the end of the pushover analysis, where also shear failure of the spandrel walls and the middle pier of the second floor was calculated as the most unfavorable. As a general remark, it is worth pointing out how the calculated cracking pattern matches the real crack distribution recorded in the wall relatively well (see Figure 4b). Cracks are located in the neighborhood of the observed crack pattern, indicating how the numerical analysis, mainly from a qualitative point of view, appears to be able to give an explanation of the damage.
As in the previous analyses, the seismic vulnerability was evaluated comparing the reference target value ($IM_{kn} = 0.345g$) of the seismic demand with the maximum value of the PGA compatible with the fulfillment of the assumed performance level, the difference being the type of target performance level taken into account: artistic assets conservation instead of life safety. Assuming as seismic capacity, for the fresco, the value of the PGA at the formation of the first crack in the bearing wall ($IM_{kn} = 0.196g$), the safety factor ratio $IM_{kn} / IM_{kn}$ (equal to 0.568) is again less than the minimum level of safety (0.780) proposed by code requirements and the structure is clearly not verified.

4 CONCLUSIONS

This paper reports result of a diagnostic analysis carried out on an important historic construction, the Civic Museum of Sansepolcro, characterized by the presence of one of the masterpieces of late Renaissance art: the Piero Della Francesca’s “the Resurrection”.

Accepting that preservation of architectural heritage requires an accurate assessment of the seismic vulnerability in order to reduce the interventions, for the purpose of conservation, a performance based multi-scale approach was used in the evaluation procedure. In particular, the use of a 3D pushover analysis (equivalent frame approach) allowed, on the one hand, to refine the results of the limit analysis (macro-element strategy) and, on the other one, to calibrate the use of a refined 3D finite element model (with non-linear constitutive laws for masonry) for catching the response of the wall containing the fresco.
The results highlight some problems related to the ability of the construction to fulfill the performance levels for both the safety of people who daily use the Museum and the conservation of the fresco. As it is possible to notice, the simplified scheme of limit analysis (rigid macro-block model), in agreement with the results obtained with the non-linear static analysis (equivalent frame model), shows that the structural behavior in the transversal direction is poor and insufficient due to the out-of-plane mechanism of orthogonal bearing walls. This vulnerability is probably due to the fact that dead loads on floors were mainly distributed to longitudinal walls as well as to the presence of few bearing walls in the transversal direction. The combined application of pushover and FE analyses confirmed some structural deficiencies also in the in-plane behavior of the wall panel supporting the fresco, showing how the observed damage is mainly associated with the presence of shear failure mechanisms.

Finally, the FE analysis, providing a screening of the most vulnerable parts of the wall supporting the fresco, suggests that measures should be taken sooner or later to meet a satisfactory level of safety and protection. Without interventions aimed at eliminating the causes, the surveyed cracking pattern will became “denser” and more serious in case of a seismic event.

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