Northumbria Research Link

Northumbria University NEWCASTLE

University Library

A review of continuum mechanics models for size-dependent analysis of beams and plates

Huu-Tai Thai [a,e], Thuc P. Vo [b,c,*], Trung-Kien Nguyen [d], Seung-Eock Kim [e,*]

[a] School of Engineering and Mathematical Sciences, La Trobe University, Bundoora, VIC 3086, Australia

[b] Duy Tan University, Da Nang, Vietnam

[c] Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne, NE1 8ST, UK

[d] Faculty of Civil Engineering, Ho Chi Minh City University of Technology and Education, 1 Vo Van Ngan Street, Thu Duc District, Ho Chi Minh City, Vietnam

[e] Department of Civil and Environmental Engineering, Sejong University, 98 Gunja Dong, Gwangjin Ku, Seoul 143-747, Republic of Korea

Abstract

This paper presents a comprehensive review on the development of higher-order continuum models for capturing size effects in small-scale structures. The review mainly focus on the size-dependent beam, plate and shell models developed based on the nonlocal elasticity theory, modified couple stress theory and strain gradient theory due to their common use in predicting the global behaviour of small-scale structures. In each higher-order continuum theory, various size-dependent models based on the classical theory, first-order shear deformation theory and higher-order shear deformation theory were reviewed and discussed. In addition, the development of finite element solutions for size-dependent analysis of beams and plates was also highlighted. Finally a summary and recommendations for future research are presented. It is hoped that this review paper will provide current knowledge on the development of higher-order continuum models and inspire further applications of these models in predicting the behaviour of micro- and nano-structures.

Keywords: Size effect; nonlocal elasticity theory; modified couple stress theory; strain gradient theory

1. Introduction

Small-scale structural elements such as beams, plates and shells are commonly used as components in micro- and nano-electromechanical systems (MEMS and NEMS), sensors, actuators and atomic force microscopes. In these applications, size effects were experimentally observed in mechanical properties [1-5]. These size effects can be captured using either molecular dynamics (MD) simulations or higher-order continuum mechanics. Although the MD method can provide accurate predictions, it is too computationally expensive. Therefore, higher-order continuum mechanics approach was widely used in the modelling of small-scale structures.

---

* Corresponding author. Tel.: +82-2-3408-3291; Fax: +82-2-3408-3332.
E-mail address: tai.thai@latrobe.edu.au (H.T. Thai); thuc.vo@northumbria.ac.uk (T.P.Vo); sekim@sejong.ac.kr (S.E. Kim).

The development of higher-order continuum theories can be traced back to the earliest work of Piola on the 19th century as demonstrated in [6-7] and the work of Cosserat and Cosserat [8] in 1909. However, until 1960s, the Cosserat brothers' idea was received considerable attention from researchers, and a large number of higher-order continuum theories have been developed. In general, these theories can be categorized into three different classes namely the strain gradient family, microcontinuum and nonlocal elasticity theories. The strain gradient family is composed of the couple stress theory, the first and second strain gradient theory, the modified couple stress theory and the modified strain gradient theory. In the strain gradient family, both strains and gradient of strains are considered in the strain energy, and thus the size effect is accounted for using material length scale parameters. In the couple stress theory initiated by Toupin [9], Mindlin and Tiersten [10] and Koiter [11], only the gradient of rotation vector is considered in the strain energy, and thus only two material length scale parameters are required. The modified couple stress theory was proposed by Yang et al. [12] based on modifying the couple stress theory. By introducing an equilibrium condition of moments of couples to enforce the couple stress tensor to be symmetric, the number of material length scale parameters of the modified couple stress theory is reduced from two to one. The first strain gradient theory initiated by Mindlin [13] considers only the first gradient of strains. One year later, Mindlin [14] derived the second strain gradient theory which is considered as the most general form of strain gradient theory accounting for both the first and second gradients of strains. Lam et al. [15] proposed the modified strain gradient theory with only three material length scale parameters by modifying Mindlin's theory by using a similar approach of Yang et al. [12]. The microcontinuum theory was developed by Eringen [16-18] consisting of micropolar, microstretch and micromorphic (3M) theories. Micropolar theory which is actually initiated by Cosserat brothers [8] is the simplest one among 3M theories, whilst micromorphic theory is the most general one among 3M theories. In 3M theories, each particle can rotate and deform independently regardless of the motion of the centroid of the particle. More details about the 3M theories as well as their applications can be found in [19-25]. The nonlocal elasticity theory was originally proposed by Kroner [26] and improved by Eringen [27-28] and Eringen and Edelen [29]. In this theory, the stress at a reference point in a continuum depends on the strains at all points of the body, and thus the size effect is captured by means of constitutive equations using a nonlocal parameter. Nonlocal elasticity theory was initially formulated in an integral form and later reformulated by Eringen [30] in a differential form by considering a specific kernel function. Compared to the integral model, the differential one is widely used for nanostructures due to its simplicity. In addition, another class of higher-order theory which is called nonlocal strain gradient theory has been recently proposed based on a combination of the nonlocal elasticity theory and the strain gradient theory. The interested reader can refer to [31-33] for more details on this theory.

Size-dependent models have been widely used for predicting the global behaviour of beam- and plate-like nanostructures such as carbon nanotubes (CNTs) and graphene sheets. CNTs were discovered by Iijima [34] by rolling graphene sheets. Based on synthesis route and reaction parameters, various types of CNTs such as single-walled carbon nanotubes (SWCNTs), double-walled carbon nanotubes (DWCNTs) and multi-walled carbon nanotubes (MWCNTs) can be obtained (see Fig. 1) by rolling single-layered graphene sheets (SLGSs), double-layered graphene sheets (DLGSs) and multi-layered graphene sheets (MLGSs) (see Fig. 2). Nanotube is a key nanostructure and has a wide range of applications in all areas of nanotechnology. Notable among them is conveying fluid [35-41] and nanofluidic devices and systems.

A large number of size-dependent models have been proposed based on various beam and plate theories. The simplest models were based on Euler-Bernoulli beam theory (EBT) and classical plate theory (CPT). These models are only appropriate for modelling of slender beams and thin plates because they ignore shear deformation effect. To overcome the limitation of the EBT and CPT, a number of shear deformation theories have been proposed. First-order shear deformation models were based on Timoshenko beam theory (TBT) and first-order shear deformation theory (FSDT). Since the in-plane displacements vary linearly through the thickness in these models, a shear correction factor is required. In order to eliminate the use of the shear correction factor and obtain a better prediction of the responses of thick beams and plates, several higher-order shear deformation theories (HSDTs) have been proposed, notable among them are Reddy beam theory (RBT) and third-order shear deformation theory (TSDT) of Reddy [42]. A comprehensive review on the plate theories can be found in the work by Thai and Kim [43].

The governing equations derived from the aforementioned size-dependent models can be solved using either analytical methods or numerical approaches. However, the application of analytical methods is limited to a particular nanostructure with simple geometry, loading and boundary conditions (BCs). For instance, Navier method is only applied for rectangular plates with simply supported BCs, whilst Levy method is only applied for rectangular plates in which two opposite edges are simply supported and the remaining two edges can have any arbitrary BCs. For the practical problems with general geometry, loading and BCs, seeking their analytical solutions is impossible because of the mathematical complexity of the size-dependent models compared to the classical ones. Therefore, numerical approaches such as finite element method, differential quadrature method, mesh-free method, Ritz method, Galerkin method, etc. become the most suitable ones for solving such problems. Among different numerical techniques, the finite element method is the most powerful tool and commonly used for the analysis of structures, and thus the development of finite element solutions for size-dependent models will be discussed in this review.

Although extensive research on small-scale beams, plates and shells has been made during the past decade,

the development of models for capturing the size effect in these structures has not been reviewed. Therefore, this paper aims to provide a comprehensive review on the development of size-dependent models for predicting the behaviour of small-scale beam- and plate-like structures. The review mainly focuses on the beam, plate and shell models which were developed based on the nonlocal elasticity theory of Eringen [30], the modified couple stress theory of Yang et al. [12] and the modified strain gradient theory of Lam et al. [15]. In addition, the development of finite element models of these theories was also highlighted and discussed in details.

2. Nonlocal elasticity theory

2.1. Review of the nonlocal elasticity theory

The nonlocal elasticity theory was initially formulated by Eringen [27-28] and Eringen and Edelen [29] by means of integral constitutive equation as

$$\sigma_{ij} = \int_{\bar{x}} k\left(\left|x-\bar{x}\right|, \kappa\right)\sigma_{ij}^{L} dx \tag{1}$$

where σ_{ij} and σ_{ij}^{L} are the components of the nonlocal and local stress tensors, respectively and k is the kernel function determined in terms of nonlocal parameter κ and neighbourhood distance $\left|x-\bar{x}\right|$ in which $\kappa = e_0 a$ and e_0 and a are the material constant and the internal characteristic length, respectively, i.e. lattice parameter, granular size or molecular diameter. The value of e_0 can be determined either from experiments or simulations. The value of e_0 was calibrated by Huang et al. [44] for the static bending analysis of SLGSs. Arash and Ansari [45] also evaluated the value of the nonlocal parameter for the free vibration of SWCNTs by comparing the predictions from the nonlocal FSDT shell model with MD simulations as shown in Fig. 3. Duan et al. [46] proposed a microstructured beam model to calibrate the value of e_0 for the free vibration analysis of nonlocal beams. Analytical expressions of e_0 were obtained based on geometrical properties and vibration modes. Zhang et al. [47-49] proposed a microstructured beam-grid model to determine the value of e_0 for the free vibration of nonlocal beams [47] and buckling and free vibration of nonlocal plates [48]. It was found that the value of e_0 varies with respect to initial stress, rotary inertia, mode shape and aspect ratio of rectangular plates. In general, a conservative estimate of the nonlocal parameter for SWCNTs is $e_0 a < 2.0$ nm [50].

By considering a specific kernel function k, Eringen [30] reformulated the nonlocal constitutive equation in a differential form as

$$\left(1 - \mu \nabla^2\right)\sigma_{ij} = \sigma_{ij}^{L} \tag{2}$$

4

where $\mu = \kappa^2$ and ∇^2 is the Laplacian operator. The explicit form of Eq. (2) can be written for three problems with isotropic materials as follows.

For one-dimensional (1D) problems:

$$\sigma_{xx} - \mu \frac{d^2 \sigma_{xx}}{dx^2} = E \varepsilon_{xx} \tag{3a}$$

$$\sigma_{xz} - \mu \frac{d^2 \sigma_{xz}}{dx^2} = \frac{E}{1+v} \varepsilon_{xz} \tag{3b}$$

For 2D problems:

$$\sigma_{xx} - \mu \left(\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{xx}}{\partial y^2} \right) = \frac{E}{1-v^2} \left(\varepsilon_{xx} + v \varepsilon_{yy} \right) \tag{4a}$$

$$\sigma_{yy} - \mu \left(\frac{\partial^2 \sigma_{yy}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2} \right) = \frac{E}{1-v^2} \left(v \varepsilon_{xx} + \varepsilon_{yy} \right) \tag{4b}$$

$$\sigma_{xy} - \mu \left(\frac{\partial^2 \sigma_{xy}}{\partial x^2} + \frac{\partial^2 \sigma_{xy}}{\partial y^2} \right) = \frac{E}{1+v} \varepsilon_{xy} \tag{4c}$$

$$\sigma_{xz} - \mu \left(\frac{\partial^2 \sigma_{xz}}{\partial x^2} + \frac{\partial^2 \sigma_{xz}}{\partial y^2} \right) = \frac{E}{1+v} \varepsilon_{xz} \tag{4d}$$

$$\sigma_{yz} - \mu \left(\frac{\partial^2 \sigma_{yz}}{\partial x^2} + \frac{\partial^2 \sigma_{yz}}{\partial y^2} \right) = \frac{E}{1+v} \varepsilon_{yz} \tag{4e}$$

For 3D problems:

$$\sigma_{xx} - \mu \left(\frac{\partial^2 \sigma_{xx}}{\partial x^2} + \frac{\partial^2 \sigma_{xx}}{\partial y^2} + \frac{\partial^2 \sigma_{xx}}{\partial z^2} \right) = \frac{E}{1-v^2} \left(\varepsilon_{xx} + v \varepsilon_{yy} + v \varepsilon_{zz} \right) \tag{5a}$$

$$\sigma_{yy} - \mu \left(\frac{\partial^2 \sigma_{yy}}{\partial x^2} + \frac{\partial^2 \sigma_{yy}}{\partial y^2} + \frac{\partial^2 \sigma_{yy}}{\partial z^2} \right) = \frac{E}{1-v^2} \left(v \varepsilon_{xx} + \varepsilon_{yy} + v \varepsilon_{zz} \right) \tag{5b}$$

$$\sigma_{zz} - \mu \left(\frac{\partial^2 \sigma_{zz}}{\partial x^2} + \frac{\partial^2 \sigma_{zz}}{\partial y^2} + \frac{\partial^2 \sigma_{zz}}{\partial z^2} \right) = \frac{E}{1-v^2} \left(v \varepsilon_{xx} + v \varepsilon_{yy} + \varepsilon_{zz} \right) \tag{5c}$$

$$\sigma_{xy} - \mu \left(\frac{\partial^2 \sigma_{xy}}{\partial x^2} + \frac{\partial^2 \sigma_{xy}}{\partial y^2} + \frac{\partial^2 \sigma_{xy}}{\partial z^2} \right) = \frac{E}{1+v} \varepsilon_{xy} \tag{5d}$$

$$\sigma_{xz} - \mu \left(\frac{\partial^2 \sigma_{xz}}{\partial x^2} + \frac{\partial^2 \sigma_{xz}}{\partial y^2} + \frac{\partial^2 \sigma_{xz}}{\partial z^2} \right) = \frac{E}{1+v} \varepsilon_{xz} \tag{5e}$$

$$\sigma_{yz} - \mu \left(\frac{\partial^2 \sigma_{yz}}{\partial x^2} + \frac{\partial^2 \sigma_{yz}}{\partial y^2} + \frac{\partial^2 \sigma_{yz}}{\partial z^2} \right) = \frac{E}{1+v} \varepsilon_{yz} \tag{5f}$$

where ε_{ij} are the components of the strain tensor; and E and ν are the Young's modulus and Poisson ratio of materials, respectively.

Compared to the integral model, the differential one is widely used for nanostructures due to its simplicity. However, the differential model may give paradoxical results for certain cases, e.g. bending and vibration problems of cantilever beams. More information about paradoxical behaviour of the differential model can be found in [51-54].

2.2. Beam models

2.2.1. Nonlocal models based on the EBT

The first nonlocal beam models based on the EBT were developed by Peddieson et al. [55] and Sudak [56]. Peddieson et al. [55] applied their model to explore the size effect on the bending behaviour of isotropic nanobeams, whilst Sudak [56] applied his model to study the buckling of MWCNTs. Since the early works by Peddieson et al. [55] and Sudak [56], there have been a large number of articles devoted to the modelling of nanobeams and CNTs using the nonlocal EBT model. For example, Zhang et al. [57] investigated the free vibration of DWCNTs. Closed-form solutions for natural frequencies of simply supported DWCNTs were obtained to study the size effect on vibration characteristics of DWCNTs. Wang et al. [58] derived a general form of closed-form solutions for buckling loads of CNTs with various BCs. Aydogdu [59] examined the size effect on the axial vibration of nanorods under different BCs and obtained explicit expressions for natural frequencies. Murmu and Pradhan [60] included thermal effects in the free vibration analysis of embedded SWCNTs using the differential quadrature (DQ) method. This approach was also used by Civalek and Demir [61] to derive bending moments and deflections of nanobeams with various BCs. The free vibration of axially loaded non-prismatic embedded SWCNTs was investigated by Mustapha and Zhong [62] using the Bubnov-Galerkin method. Li et al. [63] derived closed-form solutions for natural frequencies of axially loaded simply supported nanobeams. Ghannadpour et al. [64] employed the Ritz method solve the governing equations of the nonlocal EBT model for deflections, buckling loads and natural frequencies of nanobeams with various BCs. Based on von Karman nonlinearity, Ansari et al. [65] developed a nonlinear nonlocal EBT model for nonlinear vibrations of embedded MWCNTs in thermal environmental, whilst Fang et al. [66] developed a nonlinear nonlocal EBT model for nonlinear vibrations of embedded DWCNTs. The nonlinear free and forced vibrations of nanobeams with various BCs were examined by Simsek [67] and Bagdatli [68].

The nonlocal EBT model was also developed for nanobeams made of functionally graded (FG) materials. Simsek [69] investigated the nonlocal effect on the axial vibration of FG nanorods with variable cross-sections. The elastic modulus and mass density of nanorods were assumed to vary in the axial direction according to a power law form. Nguyen et al. [70] presented analytical solutions of the nonlocal EBT model

for the static bending analysis of FG beams with various BCs. The elastic modulus of FG nanobeams can vary in either the axial direction or transverse direction. Based on the Galerkin approach, Niknam and Aghdam [71] derived solutions of the nonlocal EBT model for natural frequencies and critical buckling loads of FG nanobeams resting on an elastic foundation. Ebrahimi and Salari [72-73] studied thermal effect on the free vibration of FG nanobeams under various BCs using a semi analytical approach. Nejad and Hadi [74] and Nejad et al. [75] examined the bending [74] and buckling [75] behaviours of FG nanobeams in which the elastic modulus can vary in both axial and transverse directions of the beam. The DQ method was used to solve the governing equations for critical buckling loads of FG nanobeams with arbitrary BCs.

Nonlinear free vibration of FG nanobeams was investigated by Nazemnezhad and Hosseini-Hashemi [76] using a nonlinear nonlocal EBT model with von Karman nonlinear theory. Analytical solutions for nonlinear natural frequencies of simply supported beams were also obtained using a method of multiple scales. El-Borgi et al. [77] developed a nonlinear nonlocal EBT model for the nonlinear free and forced vibrations of FG embedded nanobeams. The method of multiple scales was employed to solve for nonlinear frequencies of simply supported beams. Shafiei et al. [78] also developed a nonlinear nonlocal EBT model to study the nonlinear free vibration of axially FG nanobeams with variable cross-sections. Nonlinear frequencies of nanobeams under various BCs were obtained using the generalized DQ technique.

2.2.2. Nonlocal models based on the TBT

The earliest nonlocal TBT model was developed by Wang [79] to study wave propagation in CNTs. The model accounts for the shear deformation effect which becomes significant in short and stocky CNTs. Wang and Varadan [80] also developed a nonlocal TBT model, but it was applied to investigate the free vibration of both SWCNTs and DWCNTs. Closed-form solutions for natural frequencies of simply supported CNTs were also obtained. Wang et al. [81-84] derived closed-form solutions for buckling loads [81], natural frequencies [82] and deflections [83-84] of the nonlocal TBT model with four different BCs including simply supported, clamped, cantilever and propped cantilever. In these models [81-84], the transverse shear stress was based on the local theory and thus they are inconsistent. Lu et al. [85] overcame this limitation in their consistent nonlocal TBT model in which the nonlocal effect was included in both normal and transverse shear stresses. The consistent nonlocal model was also developed by Reddy and Pang [86] by reformulating both EBT and TBT using the nonlocal constitutive relations of Eringen. Closed-form solutions for deflections, buckling loads and natural frequencies were obtained for nanobeams under four different BCs. It should be noted that the closed-form solutions derived by Reddy and Pang [86] are different with those given by Wang et al. [81-84] since they were based on different TBT models.

The consistent nonlocal TBT model has been widely used to investigate the nonlocal effect in CNTs. For

example, Murmu and Pradhan [87] investigated the influences of nonlocal parameter and transverse shear deformation on the buckling of SWCNTs surrounded by an elastic medium. This work was extended by Ansari et al. [88] to include the effect of elevated temperature. Pradhan and Murmu [89] examined the nonlocal effect on the vibration of embedded SWCNTs using the consistent nonlocal TBT model and the DQ approach. Numerical solutions of the consistent TBT model were presented by Roque et al. [90] based on a meshless method with both global and local collocation techniques and radial basis functions. The vibration of embedded SWCNTs was also examined by Wu and Lai [91] using the consistent nonlocal TBT models developed based on both Reissner mixed variation theory and principle of virtual displacement. Amirian et al. [92] and Zidour et al. [93] included the thermal effect on the vibration of SWCNTs, whilst Ansari et al. [94] included the thermal effect on the dynamic stability of embedded MWCNTs. Ansari et al. [95] developed a nonlocal TBT model for the nonlinear forced vibration of magneto-electro-thermo-elastic nanobeams.

The consistent nonlocal TBT model was also developed for FG nanobeams. Simsek and Yurtcu [96] proposed both nonlocal EBT and TBT models for the bending and buckling analyses of FG nanobeams. The consistent nonlocal TBT model was extended by Rahmani and Pedram [97] to the free vibration analysis of simply supported FG nanobeams. Ebrahimi and Salari [98-99] also developed a consistent nonlocal TBT model for the buckling and free vibration analyses FG nanobeams in which thermal effects were considered.

2.2.3. Nonlocal models based on the RBT

Based on the nonlocal constitutive relations of Eringen, Reddy [100] reformulated the EBT, TBT, RBT and Levinson beam theory to include the nonlocal effect. Variational statements of four models were also derived to facilitate the development of nonlocal FE models. Closed-form solutions for deflections, buckling loads and natural frequencies were obtained for simply supported beams. Ebrahimi and Salari [101] included thermal effects in the nonlocal RBT model to examine the influences of elevated temperature and nonlocal parameter on free vibration characteristics of embedded SWCNTs. Emam [102] proposed a unified nonlinear nonlocal model for the buckling and post-buckling analyses of isotropic nanobeams. Analytical solutions for buckling load and post-buckling response were also obtained for simply supported and clamped nanobeams.

Rahmani and Jandaghian [103] extended the nonlocal RBT model to FG nanobeams. Analytical solutions for critical buckling loads were obtained for FG nanobeams under various BCs using Rayleigh-Ritz method. Ebrahimi and Barati [104] also developed a nonlocal RBT model for FG nanobeams, in which thermal effects and the interaction between the nanobeam and an elastic medium were considered.

2.2.4. Nonlocal models based on HSDTs

One of the earliest nonlocal HSDT models was developed by Aydogdu [105] for isotropic nanobeams based on the general exponential shear deformation theory of Aydogdu [106]. This theory is a general form

of the exponential shear deformation theory of Karama et al. [107] (see Table 1 for the displacement field). Thai [108] also proposed a nonlocal HSDT model for isotropic nanobeams, but it was based on the refined plate theory of Shimpi [109]. The displacement field of this theory is derived based on partitioning the displacements into shear and bending parts. Tounsi et al. [110] and Zemri et al. [111] extended the nonlocal HSDT model of Thai [108] to include thermal effects [110] and non-homogeneous behaviour of FG materials [111].

Thai and Vo [112] developed a nonlocal HSDT model for isotropic nanobeams based on the sinusoidal shear deformation theory of Touratier [113], whilst Tounsi et al. [114] proposed a nonlocal quasi-3D model for isotropic nanobeams based on the quasi-3D sinusoidal theory of Thai and Kim [115] (see Table 2). It is worth noting that unlike the HSDT model, the quasi-3D model is capable of capturing the thickness stretching effect which is significant in very thick or stocky members. The extension of the sinusoidal model of Thai and Vo [112] and quasi-3D sinusoidal model of Tounsi et al. [114] to FG nanobeams was respectively made by Ahouel et al. [116] and Chaht et al. [117]. The model of Thai and Vo [112] was also employed by Pour et al. [118] and Sadatshojaei and Sadatshojaie [119] to predict nonlinear vibration responses of SWCNTs embedded in an elastic medium.

Berrabah et al. [120] compared the accuracy of various nonlocal HSDT models in predicting deflections, buckling loads and natural frequencies of isotropic nanobeams. The displacement fields of these nonlocal HSDT models were taken from the simple HSDT proposed by Thai and Choi [121] in which the in-plane and transverse displacements are divided into the bending and shear components as shown in Table 1. Ebrahimi and Barati [122] developed a unified nonlocal HSDT model for FG embedded nanobeams based on the simple HSDT of Thai and Choi [121]. The model was used to study the influences of both moisture and temperature on free vibration characteristics of FG embedded nanobeams. Mashat et al. [123] investigated the vibration and thermal buckling of embedded nanobeams under various BCs using a unified nonlocal HSDT model covering EBT, TBT, RBT and sinusoidal theory. Recently, Thai et al. [124] presented a simple nonlocal HSDT model for isotropic nanobeams which involves only one unknown. Closed-form solutions for deflections and natural frequencies were also obtained for nanobeams under various BCs. Numerical results indicated that the accuracy of the present theory is comparable with the nonlocal TBT model although it has only one unknown as in the case of the nonlocal EBT model.

2.3. Plate models

2.3.1. Nonlocal models based on the CPT

Zhang et al. [125] developed one of the earliest nonlocal shell model for the buckling analysis of MWCNTs under axial compression based on the classical shell theory. Closed-form solutions obtained for

buckling loads were used to examine the nonlocal effect on the axial buckling of simply supported DWCNTs. Li and Kardomateas [126-127] developed nonlocal classical shell models to examine the thermal buckling [126] and free vibration [127] of MWCNTs. The nonlocal classical shell model was also proposed by Wang and Varadan [128] and Hu et al. [129] to investigate wave propagation in CNTs. The accuracy of the nonlocal classical shell model in predicting buckling strains of axially loaded SWCNTs was also assessed by Zhang et al. [130] by comparing with the MD simulation results as shown in Fig. 4. It can be seen that for long SWCNTs with large aspect ratios, the local EBT model can give results comparable with those obtained by nonlocal EBT model and MD simulations. However, for short SWCNTs with small aspect ratios, only nonlocal shell model can give comparable predictions by the MD simulations. Rouhi and Ansari [131] also presented a nonlocal classical shell model for axial buckling of DWCNTs under various BCs. Recently, Sarvestani [132] proposed a nonlocal classical shell model for the buckling analysis of curved MWCNTs under axial compression.

One of the earliest nonlocal plate models was developed by Lu et al. [133] based on the CPT. The model was used to study the size effect on the bending and bucking behaviours of isotropic nanoplates. Duan and Wang [134] derived exact solutions of the nonlocal CPT model for the axisymmetric bending analysis of circular nanoplates under general loading. The effects of the nonlocal parameter on deflection, radial moment, circumferential moment and shear force of graphene circular sheets subjected to uniform loads with either clamped or simply supported BCs were examined. Aksencer and Aydogdu [135] derived Levy solutions of the nonlocal CPT model for buckling loads and natural frequencies of rectangular nanoplates with two opposite edges being simply supported and the remaining two edges having any arbitrary BCs. Shakouri et al. [136] employed the Galerkin approach to solve the governing equations of the nonlocal CPT model for natural frequencies of isotropic nanoplates under various BCs.

The nonlocal CPT model was also employed to capture the size-dependent behaviour of SLGSs and MLGSs. For example, Pradhan and Murmu [137] and Pradhan and Kumar [138] investigated the nonlocal effect on the bucking of SLGSs using the DQ method, while Babaei and Shahidi [139] and Farajpour et al. [140] examined the size effect on the buckling of quadrilateral SLGSs [139] and variable-thickness SLGSs [140] using Galerkin method. The free vibration behaviour of MLGS embedded in a polymer matrix was investigated by Pradhan and Phadikar [141]. It was found that the size effect increases when the number of layers increases. Shen et al. [142] extended the application of the nonlocal CPT model to examine the free vibration of a simply supported SLGS-based mass sensor. Ansari et al. [143] presented analytical expressions for natural frequencies of SLGSs with arbitrary BCs by considering interatomic potential in deriving material properties of SLGSs. Recently, Zhang et al. [144-146] employed the element-free kp-Ritz

method to solve the governing equations of the nonlocal CPT model for natural frequencies [144], nonlinear deflections [145] and bucking loads [146] of SLGSs under various BCs.

The application of the nonlocal CPT in the above-mentioned studies was limited to graphene sheets made of isotropic materials. However, the numerical results from MD simulations carried out by Ni et al. [147] indicated that the mechanical properties of graphene sheets are anisotropic because of the hexagonal structure of the unit cells of the graphene [147]. Therefore, nonlocal orthotropic CPT models were developed to account for the effect of anisotropic mechanical properties of graphene sheets. Pouresmaeeli et al. [148] developed a nonlocal CPT model for the vibration of orthotropic DLGSs embedded in an elastic medium. Mohammadi et al. [149-150] developed a nonlocal CPT model for the free vibration analysis of orthotropic embedded SLGSs in thermal environment. Both Navier and Levy solutions for natural frequencies of rectangular SLGSs were derived. Sari and Al-Kouz [151] also presented a nonlocal CPT model for the free vibration analysis of orthotropic embedded SLGSs in which the variable thickness of SLGSs was considered. Anjomshoa [152] and Anjomshoa et al. [153] proposed nonlocal CPT models to examine the buckling [152] and free vibration [153] of orthotropic circular and elliptical SLGSs embedded in an elastic medium. The nonlocal CPT model was also developed by Mohammadi et al. [154] to examine the shear buckling of orthotropic embedded SLGSs in thermal environment. Ashoori et al. [155] developed a nonlocal CPT model for thermal buckling of FG annular embedded nanoplates subjected to various types of thermal loads. Exact solutions for critical buckling temperature were also obtained for FG annular nanoplates with clamped BCs.

2.3.2. Nonlocal models based on the FSDT

The earliest nonlocal FSDT model was developed by Lu et al. [133] for isotropic nanoplates. The model was then applied to study the size effect on deflections and natural frequencies of simply supported isotropic nanoplates. Pradhan and Phadikar [156-157] presented both nonlocal CPT and FSDT models for the free vibration [156] and buckling analysis [157] of SLGSs and MLGSs. In the MLGS models, the interaction between two graphene sheets was modelled by Winkler foundation. The influences of small-scale, shear deformation, elastic modulus and stiffness of Winkler foundation on natural frequencies and critical buckling loads of simply supported graphene sheets were also investigated. Kananipour [158] also presented both nonlocal CPT and FSDT models for graphene sheets, but they were applied to the static bending analysis of DLGSs under various BCs using the DQ method. Ansari et al. [159-160] examined the vibration of SLGSs [159] and MLGSs [160] with different BCs using the nonlocal FSDT model and DQ method. The nonlocal FSDT was also employed by Samaei et al. [161] and Bedroud et al. [162] to examine the buckling of embedded SLGSs [161] and circular nanoplates [162]. Arani et al. [163] examined electro-thermal-torsional buckling of simply supported embedded double-walled boron nitride nanotubes based on the nonlocal FSDT

11

shell model. Naderi and Saidi [164] modified the nonlocal FSDT model for buckling of nanoplates by eliminating the nonlocal effect for the transverse shear stresses. The nonlinear nonlocal CPT and FSDT models were developed by Reddy [165] for the nonlinear bending analysis of isotropic nanoplates based on von Karman nonlinearity. The variational statement of these models was also presented for the development of finite element solutions.

The nonlocal FSDT models were also proposed for nanoplates made of FG and orthotropic materials. For example, Hosseini-Hashemi et al. [166] developed a nonlocal FSDT for FG circular nanoplates. Closed-form solutions for natural frequencies of circular nanoplates under various BCs were also obtained. Anjomshoa and Tahani [167] developed a nonlocal FSDT model for the free vibration analysis of orthotropic circular and elliptical SLGSs embedded in an elastic foundation. Golmakani and Rezatalab [168] presented a nonlinear nonlocal FSDT model for the nonlinear bending analysis of orthotropic embedded SLGSs using von Karman nonlinear strains. Recently, Dastjerdi et al. [169] and Dastjerdi and Jabbarzadeh [170] presented a nonlinear nonlocal FSDT model for the geometric nonlinear analysis of annular/circular orthotropic embedded SLGSs [169] and MLGSs [170] in which the effect of elevated temperature was considered.

2.3.3. Nonlocal models based on the TSDT

The nonlocal TSDT model was first presented by Aghababaei and Reddy [171] for isotropic nanoplates by reformulating the TSDT of Reddy [42] using the nonlocal constitutive relations of Eringen. Closed-form solutions for deflections and natural frequencies were also presented for simply supported nanoplates. This model was employed by Pradhan [172] and Pradhan and Sahu [173] to study the nonlocal effect on buckling loads [172] and natural frequencies [173] of simply supported SLGSs. The buckling of SLGSs was also examined by Ansari and Sahmani [174] using a unified nonlocal model representing three different theories of the CPT, FSDT and TSDT. Hosseini-Hashemi et al. [175] derived Levy solutions for critical buckling loads and natural frequencies of isotropic nanoplates. Daneshmehr et al. [176-177] extended the application of the nonlocal TSDT to the buckling [176] and free vibration analysis [177] of FG nanoplates.

2.3.4. Nonlocal models based on HSDTs

Narendar [178] proposed a nonlocal HSDT model for the buckling analysis of isotropic nanoplates based on the refined plate theory of Shimpi [109]. This model was extended by Malekzadeh and Shojaee [179] and Narendar and Gopalakrishnan [180] to the free vibration analysis of nanoplates [179] and buckling analysis of orthotropic nanoplates [180]. This model was also employed by Sobhy [181] to examine the free vibration of orthotropic DLGSs under hydrothermal conditions. Sobhy [182] presented a general HSDT model for MLGSs based on the simple HSDTs of Thai and Choi [121] (see Table 1). Analytical solutions for natural frequencies, buckling loads and buckling temperatures were also obtained for MLGSs under various BCs.

Levy solutions of the nonlocal HSDT model of Narendar [178] were derived by Sobhy [183-184] for the bending analysis of isotropic SLGSs in thermal environment [183] and orthotropic nanoplates in a hygrothermal environment [184]. Zenkour and Sobhy [185], Alzahrani et al. [186], Thai et al. [187] and Sobhy [188-189] developed nonlocal sinusoidal models for thermal buckling of embedded nanoplates [185], hydro-thermal-mechanical bending of nanoplates [186], isotropic nanoplates [187], embedded SLGSs [188] and orthotropic embedded nanoplates [189] based on the sinusoidal theory of Touratier [113]. It is noted that the nonlocal sinusoidal model developed by Sobhy [190] for FG embedded nanoplates was based on the simple sinusoidal theory of Thai and Vo [191], and thus it is simpler than the nonlocal sinusoidal models proposed in [185-189]. Belkorissat et al. [192] also developed a simple nonlocal HSDT model for FG nanoplates which is similar to the work of Sobhy [190], but it was based on the hyperbolic function of Soldatos [193]. Khorshidi and Fallah [194] reformulated the exponential theory of Karama et al. [107] for FG nanoplates. Bessaim et al. [195] developed a nonlocal quasi-3D model for the free vibration analysis of isotropic nanoplates based on the quasi-3D sinusoidal theory of Thai and Kim [115] which involves five unknowns as shown in Table 2. Recently, Sobhy and Radwan [196] also developed a nonlocal quasi-3D theory for the free vibration and buckling of FG nanoplates. The theory has five unknowns and is similar with the one proposed in [195], but it is based on a new hyperbolic function as shown in Table 2.

3. Modified couple stress theory

3.1. Review of the modified couple stress theory

The modified couple stress theory was proposed by Yang et al. [12] by modifying the classical couple stress theory of Toupin [9], Mindlin and Tiersten [10] and Koiter [11]. By introducing an additional equilibrium condition of moments of couples to enforce the couple stress tensor to be symmetric, the number of additional material length scale parameters in the modified couple stress theory is reduced from two to one. This makes the modified couple stress theory more advantageous because the determination of the material parameters is a challenging task. The strain energy U is a function of both strain and curvature as [12]

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij} \right) dV \tag{6}$$

where m_{ij} are the components of the deviatoric part of the symmetric couple stress tensor; and χ_{ij} are the components of the symmetric curvature tensor defined by

$$\chi_{xx} = \frac{\partial \theta_x}{\partial x} \tag{7a}$$

$$\chi_{yy} = \frac{\partial \theta_y}{\partial y} \tag{7b}$$

$$\chi_{zz} = \frac{\partial \theta_z}{\partial z} \tag{7c}$$

$$\chi_{xy} = \frac{1}{2}\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right) \tag{7d}$$

$$\chi_{xz} = \frac{1}{2}\left(\frac{\partial \theta_x}{\partial z} + \frac{\partial \theta_z}{\partial x} \right) \tag{7e}$$

$$\chi_{yz} = \frac{1}{2}\left(\frac{\partial \theta_y}{\partial z} + \frac{\partial \theta_z}{\partial y} \right) \tag{7f}$$

where the rotation vector θ is defined in terms of the displacement field (u_x, u_y, u_z) as

$$\theta_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z} \right) \tag{8a}$$

$$\theta_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x} \right) \tag{8b}$$

$$\theta_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) \tag{8c}$$

For a linear elastic material, m_{ij} are given as

$$m_{ij} = \frac{E}{1+v}\ell \tag{9}$$

where ℓ is the material length scale parameter. The evaluation and calibration of ℓ can be found in Refs. [197-198].

3.2. Beam models

3.2.1. Modified couple stress models based on the EBT

The earliest modified couple stress EBT model was developed by Park and Gao [199] for isotropic microbeams. They utilized their model to investigate the effect of the material length scale parameter on the deflection and bending rigidity of a cantilever epoxy beam subjected to a concentrated load at the free end. It was found that the inclusion of the material length scale parameter leads to an increase in the bending rigidity of the cantilever microbeam. This effect becomes significant when the beam thickness is small, but it is negligible with the increase of the beam thickness. This observation agrees well with the experimental data. The modified couple stress EBT model was extended by Kong et al. [200-201] to the free vibration [200] and buckling [201] problems of isotropic microbeams.

The nonlinear modified couple stress EBT model was first developed by Xia et al. [202] for the nonlinear bending, post-buckling and nonlinear free vibration analyses of isotropic microbeams based on von Karman

nonlinearity. The results indicated the importance of considering nonlinearity and size effects in the proper design of microscale devices and systems such as biosensors, atomic force microscopes and MEMS [202]. Simsek [203] also developed a nonlinear EBT model for the nonlinear bending and vibration analyses of isotropic microbeams accounting for the interaction between the beam and an elastic medium. The nonlinear EBT model was widely used to study the size effect on the nonlinear bending [204], nonlinear vibration [205-209] and post-buckling [208-209] responses of isotropic microbeams. It is worth noting that Farokhi et al. [206] considered initial geometric imperfections in the nonlinear forced vibration of beams, whilst Togun and Bagdatli [207] included the axial pretension in the nonlinear free vibration of microbeams. Meanwhile, Wang et al. [204, 208] accounted for thermal effect in the nonlinear bending [204], post-buckling and free vibration [208] of beams. Ansari et al. [209] derived closed-form solutions for the vibration and post-buckling analyses of microbeams under various BCs.

The EBT model was also applied to the FG microbeams. For example, it was employed by Asghari et al. [210] to predict the bending and free vibration behaviours of FG microbeams. Free vibration of FG tapered microbeams with material properties varying in the longitudinal direction was examined by Akgoz and Civalek [211] and Shafiei et al. [212]. It should be noted that Akgoz and Civalek [211] only considered cantilever beams, whilst Shafiei et al. [212] included the geometric nonlinearity in beams with different BCs. Simsek [213] also examined the nonlinear free vibration of axially FG microbeams. However, he used the Galerkin and He's variational method to obtain approximate solutions for the beams with simply supported and clamped BCs. Dehrouyeh-Semnani et al. [214] included initial geometric imperfections on the free vibration analysis of FG microbeams.

3.2.2. Modified couple stress models based on the TBT

Ma et al. [215] first developed the modified couple stress TBT model by extending the EBT model of Park and Gao [199] to account for the shear deformation effect. The model was employed to investigate the effects of the material length scale parameter and shear deformation on deflections and natural frequencies of simply supported isotropic microbeams. Closed-form solutions of the TBT model were derived by Asghari et al. [216] for bending response of the beams under various BCs, whilst approximate solutions of the TBT model were derived by Dos Santos and Reddy [217] for buckling loads and natural frequencies of the beams with various BCs using the Ritz method. Dehrouyeh-Semnani and Nikkhah-Bahrami [218] used both EBT and TBT models to examine the Poisson effect in isotropic microbeams. By comparing the bending rigidities and deflections of epoxy cantilever microbeams under concentrated loads predicted by the modified couple stress models and experimental tests, it was found that the inclusion of the Poisson effect in the modified couple stress models leads to underestimating the deflection of the epoxy cantilever microbeam as shown in

Fig. 5. Liu and Reddy [219] developed a modified couple stress TBT model for isotropic curved microbeams, and applied it to bending and free vibration problems of simply supported curved beams. Taati et al. [220] also developed a TBT model to investigate thermal effects in isotropic microbeams. Asghari et al. [221] presented a nonlinear TBT model for the bending and free vibration analyses of isotropic microbeams. Ghayesh et al. [222-223] also presented a nonlinear TBT model for nonlinear resonant problems of isotropic microbeams.

The TBT model was also applied to microbeams made of FG and laminated composite materials. Reddy [224] developed both EBT and TBT models for FG microbeams considering geometric nonlinearity. Closed-form expressions for deflections, buckling loads and natural frequencies of simply supported microbeams were also given. Ke et al. [225] developed a nonlinear TBT model to examine the size effect on nonlinear vibration characteristics of FG microbeams. Asghari et al. [226] developed a TBT model to investigate the size effect on the deflections and rotations of cantilever FG beams as well as on the natural frequencies of simply supported FG beams. However, the geometric nonlinearity was ignored in their model. Ke and Wang [227] utilized the TBT model to study the free vibration, static buckling and dynamic stability behaviours of FG microbeams under different BCs using the DQ method. The buckling and free vibration responses of FG microbeams at elevated temperature were investigated by Nateghi and Salamat-talab [228] using the modified couple stress TBT model. The DQ method was employed to obtain critical buckling loads and natural frequencies of FG microbeams with various BCs. Numerical results indicated that the effect of temperature becomes more significant at higher values of the ratio of the beam thickness to material length scale parameter. Simsek et al. [229] adopted the TBT model to investigate the size effect on deflections of simply supported FG microbeams subjected to uniform and concentrated loads. The application of the TBT model was extended by Chen et al. [230], Chen and Li [231], Roque et al. [232] and Mohammad-Abadi and Daneshmehr [233] to the static bending [230, 232], free vibration [231] and buckling [233] of laminated composite microbeams. Thai et al. [234] also extended the application of the TBT model to static bending, buckling and free vibration problems of FG sandwich microbeams. Recently, Krysko et al. [235] developed a TBT model for the static bending and free vibration analyses of three layer microbeams based on Grigolyuk-Chulkov theory.

3.2.3. Modified couple stress models based on the RBT

The modified couple stress RBT model was first proposed by Ma et al. [236] for isotropic microbeams. It was used to examine the size effect on the static bending and free vibration responses of simply supported microbeams. The application of the RBT model was extended by Mohammad-Abadi and Daneshmehr [237] to investigate the size effect on buckling behaviour of isotropic microbeams. Both EBT and TBT models

were also included in their works. Analytical solutions for simply supported beams were also provided for a comparison purpose.

Salamat-talab et al. [238] extended the application of the RBT model to FG microbeams, and derived closed-form solutions for deflections and natural frequencies of simply supported microbeams. Nateghi et al. [239] and Aghazadeh et al. [240] presented a unified model for buckling [239], bending and free vibration [240] of FG microbeams. The unified model covers three different beam theories of the EBT, FBT and TBT. The DQ solution method is used to solve for the buckling loads, deflections and natural frequencies of FG microbeams under different BCs. Chen et al. [241] developed a RBT model for laminated composite microbeams based on a new constitutive relation for anisotropic materials. The model was used to examine the size effect on deflections of cross-ply simply supported microbeams under uniform loads. Mohammad-Abadi and Daneshmehr [242] and Mohammad-Abadi et al. [243] extended their isotropic model in [237] to study the free vibration [242] and thermal buckling [243] of laminated composite microbeams under various BCs.

3.2.4. Modified couple stress models based on HSDTs

Darijani and Mohammadabadi [244] proposed a modified couple stress HSDT model for isotropic microbeams by separating the axial and transverse displacements into the shear and bending parts. The shape function of the shear part as shown in Table 1 was determined based on the condition that both transverse shear stress and couple stress vanish on the top and bottom surfaces of the cross-section. Recently, Noori et al. [245] presented a HSDT model for free vibration of isotropic microbeams based on a fifth-order variation of the axial displacement across the thickness. The DQ solution method was employed to solve for natural frequencies of microbeams under various BCs. Simsek and Reddy [246] developed a unified HSDT model for FG microbeams covering seven different beam theories including EBT, TBT, RBT, sinusoidal theory of Touratier [113], hyperbolic theory of Soldatos [193], exponential theory of Karama et al. [107] and general exponential theory of Aydogdu [106]. The model was applied to the bending and free vibration problems of simply supported FG microbeams. The model was also extended by Simsek and Reddy [247] and Akbarzadeh Khorshidi et al. [248] to buckling problems of FG embedded microbeams [247] and post-buckling problems of FG microbeams with general BCs [248]. Trinh et al. [249] also presented a unified modified couple stress model for FG microbeams composed of both HSDT and quasi-3D theories of beams. The displacement field of their model was based on that proposed by Thai et al. [250] in which the transverse displacements are partitioned into bending, shear and thickness stretching components as shown in Table 2.

Based on the sinusoidal theory of Touratier [113], Akgoz and Civalek [251] developed a modified couple stress sinusoidal model to investigate thermal-mechanical buckling characteristics of simply supported FG

embedded microbeams. The results indicated that the effect of elevated temperature on buckling loads of FG microbeams becomes significant when the ratio of the thickness to material length scale parameter increases [251]. Al-Basyouni et al. [252] also presented a modified couple stress sinusoidal model for FG microbeams. However, it was based on the simple sinusoidal theory proposed by Thai and Vo [191], and included the physical neutral surface of FG microbeams.

3.3. Plate models

3.3.1. Modified couple stress models based on the CPT

The modified couple stress CPT model was first proposed by Tsiatas [253] for the bending analysis of isotropic microplates with arbitrary shape. This model was extended by Yin [254] and Jomehzadeh et al. [255] for the free vibration analysis of simply support microplates [254] and Levy-type microplates [255]. Akgoz and Civalek [256] proposed a modified couple stress theory CPT model to investigate the size effect on the free vibration of simply supported SLGSs embedded in an elastic matrix. It was found that the size effect becomes remarkable for higher modes of vibration. Akgoz and Civalek [257] also included the elastic medium in CPT model for the static bending, buckling and free vibration analysis of isotropic microplates. Askari and Tahani [258] derived closed-form solutions for natural frequencies of clamped CPT microplates using extended Kantorovich method. Simsek et al. [259] adopted the CPT model to examine the size effect on the forced vibration of isotropic microplates under a moving load. The dynamic responses of microplates under various BCs were obtained using the implicit time integration method of Newmark. Zhou et al. [260] developed a modified couple stress shell model for the free vibration analysis of isotropic microshells based on the classical shell theory. It was found that the size effect becomes remarkable when the characteristic radius size is comparable to the material length scale parameter [260].

Asghari [261] proposed a nonlinear modified couple stress CPT model for the geometrically nonlinear analysis of microplates with arbitrary shapes. Wang et al. [262-263] developed a nonlinear modified couple stress CPT model to investigate the size effect on the nonlinear free vibration [262] and nonlinear bending responses [263] of circular microplates. Farokhi and Ghayesh [264] also developed a nonlinear modified couple stress CPT model for the nonlinear dynamic analysis of isotropic microplates including initial geometric imperfections.

In addition to the application to isotropic microplates, the modified couple stress CPT model was also applied to FG microplates. Ke et al. [265] studied the size effect on deflections, critical buckling loads and natural frequencies of FG annular microplates under different BCs. Asghari and Taati [266] investigated free vibration of FG microplates with arbitrary shapes. Ashoori and Sadough Vanini [267] presented a modified couple stress CPT model for the buckling analysis of FG microplates which included thermal effects and the

interaction between the plate and an elastic medium. Recently, Ashoori and Sadough Vanini [268] extended their work [267] to account for geometric nonlinearity on thermal buckling of circular FG microplates. Taati [269] derived analytical solutions of the nonlinear modified couple stress CPT model for buckling and post-buckling loads of FG microplates with various BCs subjected to in-plane shear, biaxial compression and uniformly transverse loads. Based on the classical shell theory, Beni et al. [270] developed a modified couple stress shell model to investigate the size effect on natural frequencies of simply supported FG cylindrical microshells. Tsiatas and Yiotis [271] developed a modified couple stress CPT model to investigate the size effect on the static bending, buckling and free vibration responses of skew microplates. By comparing with the nonlocal CPT mode, it was found that the effect of the material length scale parameter on critical buckling loads and natural frequencies is in contradiction with that of the nonlocal parameter of the nonlocal model.

3.3.2. Modified couple stress models based on the FSDT

One of the earliest modified couple stress FSDT models was developed by Ma et al. [272] and Ke et al. [273] for isotropic microplates. It is worth noting that the FSDT model of Ma et al. [272] considering both stretching and bending deformations, whilst Ke et al. [273] considered only bending deformation in their model. In addition, Ma et al. [272] derived closed-form solutions for bending and free vibration problems of simply supported plates, whilst Ke et al. [273] derived numerical solutions for natural frequencies of plates with simply supported and clamped BCs using the p-version Ritz method. Roque et al. [274] presented numerical solutions of the modified couple stress FSDT model for the static bending analysis of isotropic microplates using the meshless collocation method with radial basis functions. Zhou and Gao [275] developed a modified couple stress FSDT model for the axisymmetric bending analysis of isotropic circular microplates. Recently, Alinaghizadeh et al. [276] developed a modified couple stress FSDT model for static bending analysis of FG annular sector microplates. The DQ solution method was used to solve for deflection of microplates under various BCs. He et al. [277] extended the FSDT model to the static bending analysis of laminated composite skew microplates, whilst Simsek and Aydın [278] extended the FSDT model to the static bending and forced vibration analysis of FG microplates under a moving load.

Reddy and Berry [279] extended the axisymmetric FSDT model of Zhou and Gao [275] to account for the influences of geometric nonlinearity, elevated temperature and non-homogeneous behaviour of FG materials on the axisymmetric nonlinear bending analysis of circular microplates. Ke et al. [280-281] also developed an axisymmetric FSDT model for the nonlinear free vibration [280] and post-buckling analysis [281] of FG annular microplates. Thai and Choi [282] proposed nonlinear CPT and FSDT models for FG microplates. Analytical expressions for linear and nonlinear deflections, buckling loads and natural frequencies of simply

supported microplates were derived to explore the size effect on the bending, buckling and vibration responses of FG microplates. Jung et al. [283-284] included the interaction between the plate and an elastic medium in the FSDT model in investigating the size effect on the bending, vibration [283] and buckling responses [284] of simply supported FG microplates. The nonlinear FSDT models were also developed by Ansari et al. [285-286] for the nonlinear vibration [285], nonlinear bending and post-buckling analysis [286] of FG microplates. It is noted that the nonlinear FSDT model developed in [286] considered the physical neutral surface of FG plates and thus the stretching-bending coupling was eliminated. Ansari et al. [287] adopted the nonlinear FSDT model to investigate the size effect on the post-buckling path and frequency of FG microplates. Lou and He [288] also presented nonlinear CPT and FSDT models for the nonlinear bending and free vibration analysis of FG microplates. The interaction between the plate and an elastic medium, and the physical neutral surface of FG plates were taken into account in their models.

Based on the FSDT, Zeighampour and Beni [289] and Hosseini-Hashemi et al. [290] presented shell models for the free vibration analysis of isotropic cylindrical microshells [289] and spherical microshells [290]. Gholami et al. [291] also developed a FSDT shell model, but it was applied to the axial buckling and dynamic stability of FG microshells. Tadi Beni et al. [292] presented a FSDT shell model for FG cylindrical microshells, and applied to the free vibration problems. Lou et al. [293] developed a nonlinear FSDT shell model to examine the influence of the pre-buckling deformation and material length scale parameter on critical buckling loads of FG cylindrical microshells. The physical neutral surface of FG shells was considered in their model. It should be noted that the work in [293] is more advanced than that by Gholami et al. [291] since the von Karman nonlinearity and the pre-buckling deformation were taken into consideration.

3.3.3. Modified couple stress models based on the TSDT

The modified couple stress TSDT model was first developed by Gao et al. [294] for isotropic plates. The model was employed to examine the size effect on deflections and natural frequencies of simply supported microplates. This model was extended by Thai and Kim [295] and Chen et al. [296] to microplates made of FG [295] and laminated composite materials [296]. Jung and Han [297] also presented a TSDT model for FG microplates, but they used a different law to compute the equivalent mechanical properties of FG microplates. Eshraghi et al. [298] developed a TSDT model for FG microplates with annular and circular shapes. The displacement field was expressed in a unified form representing three different plate theories of CPT, FSDT and TSDT. The DQ solution method was employed to solve for static bending and free vibration problems. Eshraghi et al. [299] recently extended their previous work [298] to include thermal effects. The nonlinear TSDT model was developed by Ghayesh and Farokhi [300] to examine nonlinear vibration characteristics of isotropic microplates. Based on the TSDT, Sahmani et al. [301] developed a modified couple stress shell

model for the dynamic instability analysis of FG cylindrical microshells. Closed-form solutions were also obtained for simply supported cylindrical microshells.

3.3.4. Modified couple stress models based on HSDTs

Thai and Vo [302] proposed a modified couple stress HSDT model for FG microplates. The displacement field of the model was based on the sinusoidal theory of Touratier [113]. Closed-form solutions for deflections and natural frequencies were also derived for simply supported microplates. Darijani and Shahdadi [303] proposed a simple HSDT model for isotropic microplates by partitioning the displacements into the shear and bending components as shown in Table 1. The shape function of the shear component of the in-plane displacements was obtained based on the zero traction BCs of both transverse shear and couple stresses. He et al. [304] reformulated the refined plate theory of Shimpi [109] to account for size effects in FG microplates using the modified couple stress theory. The work carried out by Lou et al. [305] is similar to that conducted by He et al. [304]. However, Lou et al. [305] employed various shape functions of Thai and Choi [121]. Lou et al. [306] recently extended the work by He et al. [304] to include geometric nonlinearity and the interaction between the plate and an elastic medium. Recently, Trinh et al. [307] also presented a unified modified couple stress model for the buckling analysis FG microplates under mechanical and thermal loads based on a quasi-3D theory. The displacement field of their model was based on that proposed by Thai and Kim [115] in which the transverse displacements are partitioned into bending, shear and thickness stretching components as shown in Table 2.

Reddy and Kim [308] developed a nonlinear quasi-3D model for FG microplates accounting for thermal effects. It was based on the von Karman nonlinearity and a general quasi-3D theory which accounts for cubic and quadratic variations of the in-plane and transverse displacements across the thickness. The displacement field of the general quasi-3D theory shown in Table 2 contains 11 unknowns. The CPT, FSDT and TSDT can be obtained from this general theory as special cases. Closed-form solutions of this model were derived by Kim and Reddy [309] for simply supported plates. Lei et al. [310] proposed a simple quasi-3D theory for the static bending and free vibration analysis of FG microplates which involves only five unknowns. The displacement field of the model has the same form of the model proposed by Thai and Kim [115] and Thai et al. [250]. However, Lei et al. [310] utilized a cubic shape function as shown in Table 2.

4. Modified strain gradient theory

4.1. Review of the modified strain gradient theory

In this theory [15], the strain energy contains two additional gradient parts of the dilatation gradient γ and the deviatoric stretch gradient η in addition to the symmetric curvature χ_{ij}. Therefore, the strain

energy is written as [15]

$$U = \frac{1}{2}\int_V \left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}\eta_{ijk} + m_{ij}\chi_{ij}\right)dV \tag{10}$$

where the symmetric curvature tensor χ_{ij} is defined in Eq. (6). The dilatation gradient vector γ_i and the deviatoric stretch gradient tensor η_{ijk} are respectively defined in Eqs. (10) and (11) as

$$\gamma_x = \frac{\partial\varepsilon_{xx}}{\partial x} + \frac{\partial\varepsilon_{yy}}{\partial x} + \frac{\partial\varepsilon_{zz}}{\partial x} \tag{11a}$$

$$\gamma_y = \frac{\partial\varepsilon_{xx}}{\partial y} + \frac{\partial\varepsilon_{yy}}{\partial y} + \frac{\partial\varepsilon_{zz}}{\partial y} \tag{11b}$$

$$\gamma_z = \frac{\partial\varepsilon_{xx}}{\partial z} + \frac{\partial\varepsilon_{yy}}{\partial z} + \frac{\partial\varepsilon_{zz}}{\partial z} \tag{11c}$$

$$\eta_{xxx} = \frac{\partial\varepsilon_{xx}}{\partial x} - \frac{1}{5}\left[\gamma_x + 2\left(\frac{\partial\varepsilon_{xx}}{\partial x} + \frac{\partial\varepsilon_{xy}}{\partial y} + \frac{\partial\varepsilon_{xz}}{\partial z}\right)\right] \tag{12a}$$

$$\eta_{yyy} = \frac{\partial\varepsilon_{yy}}{\partial y} - \frac{1}{5}\left[\gamma_y + 2\left(\frac{\partial\varepsilon_{xy}}{\partial x} + \frac{\partial\varepsilon_{yy}}{\partial y} + \frac{\partial\varepsilon_{yz}}{\partial z}\right)\right] \tag{12b}$$

$$\eta_{zzz} = \frac{\partial\varepsilon_{zz}}{\partial z} - \frac{1}{5}\left[\gamma_z + 2\left(\frac{\partial\varepsilon_{xz}}{\partial x} + \frac{\partial\varepsilon_{yz}}{\partial y} + \frac{\partial\varepsilon_{zz}}{\partial z}\right)\right] \tag{12c}$$

$$\eta_{yyx} = \eta_{yxy} = \eta_{xyy} = \frac{1}{3}\left[\eta_{xxx} + 2\frac{\partial\varepsilon_{xy}}{\partial y} + \frac{\partial\left(\varepsilon_{yy} - \varepsilon_{xx}\right)}{\partial x}\right] \tag{12d}$$

$$\eta_{zzx} = \eta_{zxz} = \eta_{xzz} = \frac{1}{3}\left[\eta_{xxx} + 2\frac{\partial\varepsilon_{xz}}{\partial z} + \frac{\partial\left(\varepsilon_{zz} - \varepsilon_{xx}\right)}{\partial x}\right] \tag{12e}$$

$$\eta_{xxy} = \eta_{xyx} = \eta_{yxx} = \frac{1}{3}\left[\eta_{yyy} + 2\frac{\partial\varepsilon_{xy}}{\partial x} + \frac{\partial\left(\varepsilon_{xx} - \varepsilon_{yy}\right)}{\partial y}\right] \tag{12f}$$

$$\eta_{zzy} = \eta_{zyz} = \eta_{yzz} = \frac{1}{3}\left[\eta_{yyy} + 2\frac{\partial\varepsilon_{yz}}{\partial z} + \frac{\partial\left(\varepsilon_{zz} - \varepsilon_{yy}\right)}{\partial y}\right] \tag{12g}$$

$$\eta_{xxz} = \eta_{xzx} = \eta_{zxx} = \frac{1}{3}\left[\eta_{zzz} + 2\frac{\partial\varepsilon_{xz}}{\partial x} + \frac{\partial\left(\varepsilon_{xx} - \varepsilon_{zz}\right)}{\partial z}\right] \tag{12h}$$

$$\eta_{yyz} = \eta_{yzy} = \eta_{zyy} = \frac{1}{3}\left[\eta_{zzz} + 2\frac{\partial\varepsilon_{yz}}{\partial y} + \frac{\partial\left(\varepsilon_{yy} - \varepsilon_{zz}\right)}{\partial z}\right] \tag{12i}$$

$$\eta_{xyz} = \eta_{yzx} = \eta_{zxy} = \eta_{xzy} = \eta_{zyx} = \eta_{yxz} = \frac{1}{3}\left[\frac{\partial\varepsilon_{yz}}{\partial x} + \frac{\partial\varepsilon_{xz}}{\partial y} + \frac{\partial\varepsilon_{xy}}{\partial z}\right] \tag{12j}$$

For a linear elastic material, the higher-order stresses $\left(p_i, \tau_{ijk}, m_{ij} \right)$ are given as

$$p_i = \frac{E}{1+v}\ell \tag{13a}$$

$$\tau_{ijk} = \frac{E}{1+v}\ell \tag{13b}$$

$$m_{ij} = \frac{E}{1+v}\ell \tag{13c}$$

where ℓ, ℓ and ℓ are the material length scale parameters associated with dilatation gradient, deviatoric stretch gradient and symmetric curvature gradient, respectively.

4.2. Beam models

4.2.1. Strain gradient models based on the EBT

One of the earliest strain gradient EBT models was proposed by Kong et al. [311] to investigate the size effect on deflections and natural frequencies of isotropic cantilever microbeams. The accuracy of the strain gradient theory was also compared with that of the modified couple stress theory and classical theory as shown in Fig. 6. Comparison results indicated that the strain gradient model predicts the size effect better than the modified couple stress model since it considers additional dilatation gradient tensor and deviatoric stretch gradient tensor in addition to rotation gradient tensor. Akgoz and Civalek [312] extended the strain gradient EBT model to the buckling analysis of isotropic microbeams with cantilever and simply supported BCs. Akgoz and Civalek [313-316] also employed the strain gradient EBT model to study the size effects on the buckling of SWCNTs [313], static bending of SWCNTs [314], buckling of linearly tapered microbeams [315] and longitudinal vibration of microbeams [316].

Zhao et al. [317] developed a nonlinear strain gradient EBT model for the nonlinear bending, post-buckling and nonlinear free vibration analysis of isotropic microbeams. They highlighted the importance of including geometric nonlinearity and size effects in the proper design of microbeams. Rajabi and Ramezani [318] also developed a nonlinear strain gradient EBT model for isotropic microbeams, but applied it to static bending and free vibration problems. The nonlinear strain gradient EBT model was extended by Mohammadi and Mahzoon [319] to include temperature effects on the post-buckling of isotropic microbeams. Analytical solutions were also obtained for microbeams with various BCs. Vatankhah et al. [320] utilized the nonlinear strain gradient EBT model to examine the nonlinear forced vibration of isotropic microbeams.

Kahrobaiyan et al. [321] extended the application of the strain gradient EBT model to the bending and free vibration analysis of FG microbeams. The extension of this model to buckling problems FG microbeams was carried out by Akgoz and Civalek [322]. Closed-form solutions for critical buckling loads were also obtained

for FG microbeams under various BCs. Akgoz and Civalek [323] adopted the strain gradient EBT model to examine the longitudinal free vibration of FG microbeams. Rayleigh-Ritz solution technique was used to solve for natural frequencies of FG microbeams with clamped-clamped and clamped-free BCs. Rahaeifard et al. [324] developed a nonlinear strain gradient EBT model to study the influences of geometric nonlinearity and material length scale parameters on deflections and natural frequencies of FG simply supported microbeams.

4.2.2. Strain gradient models based on the TBT

Wang et al. [325] first presented a strain gradient TBT model for the static bending and free vibration analyses of isotropic simply supported microbeams. The nonlinear strain gradient TBT models were developed by Ansari et al. [326] and Asghari et al. [327] for isotropic microbeams using von Karman nonlinearity. It is worth noting that Ansari et al. [326] applied their model for nonlinear free vibration problems, whilst Asghari et al. [327] considered both nonlinear bending and nonlinear free vibration problems in their model.

Ansari et al. [328] extended the strain gradient TBT model to FG microbeams. Closed-form solutions for natural frequencies of simply supported microbeams were derived to investigate the effects of material gradient index and small-scale on the free vibration response of FG beams. Ansari et al. [329] also extended their work [328] to free vibration of curved FG microbeams. Ansari et al. [330] developed a strain gradient TBT model for thermal buckling of FG microbeams with various BCs. Recently, Ansari et al. [331] extended the strain gradient TBT model to study linear and nonlinear vibrations of fractional viscoelastic beams. It should be noted that Gholami et al. [332] did develop a strain gradient TBT model to examine the nonlinear pull-in stability and vibration of FG mircoswitches, but it was based on the most general form of the strain gradient theory of Mindlin [14] which is not covered in this review. The effect of temperature distributions on buckling characteristics of FG microbeams was also investigated. Xie et al. [333] employed the indirect radial basis function collocation approach to solve the EBT and TBT models for deflections, buckling loads and natural frequencies of FG microbeams under various BCs. It is noted that in the previous works dealing with FG microbeams, the material length scale parameters were assumed to be constant across the thickness. Therefore, Tajalli et al. [334] improved the previous strain gradient TBT model by accounting for the variation of the material length scale parameter across the beam thickness. Case studies on static bending and free vibration problems confirmed that the aforementioned assumption of constant material length scale parameters seems to be inaccurate [334].

The nonlinear strain gradient TBT model was developed by Ansari et al. [335] to investigate the influences of material length scale parameters and initial geometric imperfections on the post-buckling response of FG

microbeams. Approximate solutions for buckling loads of FG microbeams under various BCs were also presented using the DQ method. Ansari et al. [336] extended their previous work [335] to account for thermal effects.

4.2.3. Strain gradient models based on the RBT and HSDTs

Based on the strain gradient theory of Lam et al. [15], Wang et al. [337] reformulated the RBT model to account for the size effect on the static bending and free vibration responses of isotropic microbeams. Sahmani and Ansari [338] improved the strain gradient RBT model to include thermal effects and non-homogeneous behaviour of FG materials on the buckling of FG microbeams. The strain gradient RBT model was employed by Ansari et al. [339] to explore the size effect on the free vibration of simply supported FG microbeams. Zhang et al. [340] developed a RBT model for FG embedded microbeams based on the improved RBT of Shi [341]. Sahmani et al. [342] developed a nonlinear strain gradient RBT model for nonlinear free vibration of FG microbeams.

In addition to the RBT model, the HSDT models were also proposed for strain gradient microbeams based on various HSDTs of beams such as sinusoidal theory of Touratier [113], hyperbolic theory of Soldatos [193] and n-th order shear deformation theory of Xiang et al. [343] (see Table 1 for the displacement field of these theories). For example, Akgoz and Civalek [344] and Lei et al. [345] proposed strain gradient sinusoidal models for the bending and free vibration analyses of the microbeams made of isotropic materials [344] and FG materials [345] based on the sinusoidal theory of Touratier [113]. Akgoz and Civalek [346] extended their previous work [344] to buckling problems of isotropic microbeams. Akgoz and Civalek [347] also developed a strain gradient sinusoidal model for FG microbeams as in the work of Lei et al. [345]. They also proposed a new equation for calculating the shear correction factor of the TBT model. In their equation, the shear correction factor is a function of the material length scale parameters. Akgoz and Civalek [348] extended their previous work [347] to account for the interaction between the FG microbeam and an elastic medium. Based on the hyperbolic theory of Soldatos [193], Akgoz and Civalek [349] proposed a strain gradient hyperbolic model for the bending and buckling analyses of isotropic embedded microbeams. Akgoz and Civalek [350] presented a unified HSDT model for the bending analysis of simply supported embedded CNTs. The displacement field of the model was based on Simsek and Reddy [246] which covers seven beam theories including the EBT, TBT, RBT, sinusoidal theory of Touratier [113], hyperbolic theory of Soldatos [193], exponential theory of Karama et al. [107] and general exponential theory of Aydogdu [106]. Zhang et al. [351] proposed a HSDT model for the bending and free vibration analyses of FG curved microbeams based on the nth-order shear deformation theory of Xiang et al. [343].

4.3. Plate models

4.3.1. Strain gradient models based on the CPT

The earliest strain gradient CPT model was developed by Wang et al. [352] for predicting size-dependent responses of isotropic microplates. A comparison between the strain gradient model and modified couple stress model as shown in Fig. 7 indicated that the first one captures the size effect better than the second one does [352]. Bending solutions of the strain gradient CPT model was solved by Ashoori Movassagh and Mahmoodi [353] for microplates under various BCs using the extended Kantorovich method, whilst buckling solutions were analytically derived by Mohammadi and Fooladi Mahani [354] for Levy-type microplates. Mohammadi et al. [355] improved their previous work [354] using exact BCs of microplates. Wang et al. [356] derived the strain gradient CPT model for the bending analysis of microplates with various BCs. Zeighampour and Tadi Beni [357], Allahbakhshi and Allahbakhshi [358], Li et al. [359], Hosseini et al. [360] and Zhang et al. [361] extended the strain gradient CPT model to SWCNTs [357], MLGSs [358], two-layered isotropic microplates [359], multi-layered orthotropic microplates [360] and isotropic embedded microplates [361].

4.3.2. Strain gradient models based on the FSDT

One of the earliest strain gradient FSDT models was proposed by Sahmani and Ansari [362] and Ansari et al. [363] for the free vibration and thermal buckling of FG microplates. Sahmani and Ansari [362] only dealt with simply supported plates, whilst Ansari et al. [363] dealt with microplates under various BCs using the DQ method. Ansari et al. [364] developed a nonlinear strain gradient FSDT model to examine the post-buckling of FG annular microplates under thermal loading. Ansari et al. [365] extended their previous work [363] to study the effect of elevated temperature on the bending, buckling and free vibration responses of FG microplates under various BCs. It is noted that Shenas and Malekzadeh [366] also studied the influence of elevated temperature on the free vibration of FG microplates under various BCs. However, they employed the Chebyshev-Ritz method instead of the DQ approach as in the work of Ansari et al. [365]. Ansari et al. [367] developed a FSDT model for FG circular/annular microplates under various BCs using the DQ method.

Gholami et al. [368] developed a strain gradient FSDT shell model for FG cylindrical microshells. Closed-form solutions were presented for the critical buckling load of simply supported FG cylindrical microshells under axial compression. Zhang et al. [369] also developed a strain gradient FSDT shell model for FG cylindrical microshells, but it was based on the four unknown FSDT proposed by Thai and Choi [370-372]. Therefore, their model was simpler than the one proposed by Gholami et al. [368] which involves with five unknowns.

4.3.3. Strain gradient models based on the TSDT and HSDTs

Sahmani and Ansari [362] developed a strain gradient TSDT model for the free vibration analysis of FG

microplates. Closed-form solutions for natural frequencies were also presented for simply supported plates. Zhang et al. [373] developed a simple TSDT model for circular/annular FG microplates based on the simple TSDT proposed by Thai and Kim [374] which involves only four unknowns. The DQ solution method was used to solve for deflections, buckling loads and natural frequencies of circular/annular FG microplates with various BCs. Zhang et al. [375] developed a simple strain gradient HSDT model for FG microplates based on the simple HSDT proposed by Thai and Choi [376-379] which has only four unknowns. However, they included the interaction between the plate and elastic medium. Akgoz and Civalek [380] developed a strain gradient sinusoidal model for the bending, buckling and free vibration analysis of isotropic microplates based on the sinusoidal theory of Touratier [113].

5. Finite element models

5.1. Beam elements

5.1.1. Nonlocal elasticity elements

Based on a nonlocal EBT model, Eltaher and his colleagues [381-385] have developed nonlocal elements for nanobeams made of FG materials [381-383] and isotropic materials [384-385]. The EBT element has two nodes with six degrees of freedom (4-DOF) in which the axial and transverse displacements are respectively approximated using Lagrange and Hermite cubic interpolation functions. It is noted that Eltaher et al. [381] dealt with free vibration problems of FG nanobeams, whilst Eltaher et al. [382] dealt with bending and buckling problems of FG nanobeams. Eltaher et al. [383] also dealt with free vibration characteristics of FG nanobeams, but the physical neutral surface of FG beams was taken into account in their model. Eltaher et al. [384] examined the free vibration characteristics of isotropic nanobeams, whilst Alshorbagy et al. [385] studied the static bending of isotropic nanobeams. Marotti De Sciarra [386] presented a nonlocal element for the static bending analysis of isotropic nanobeams based on the nonlocal EBT model. The element has two nodes with 6-DOF and is based on higher-order interpolation functions. Therefore, it can accurately predict the bending behaviour of nanobeams with a coarse mesh. A case study on a cantilever nanobeam under a concentrated load indicated that the nonlocal effect does exist at both left and right sides of the concentrated load. This observation is contrary to that observed from existing finite element and analytical models indicated that the nonlocal effect only exists from the location of the point load to the free end. Nguyen et al. [387] developed a nonlocal mixed element for the static bending analysis of isotropic nanobeams. The element with two nodes is C^0 continuity and is based on Lagrange interpolation functions for both deflection and bending moment. The mixed element is also capable of capturing the nonlocal effect at both sides of the concentrated load applied on a cantilever beam.

In addition to the nonlocal EBT elements reported in the above-mentioned studies, nonlocal TBT elements

were also developed to capture the shear deformation effect in thick nanobeams. Reddy and El-Borgi [388] presented a complete theoretical development and finite element formulation of both nonlocal EBT and TBT models for the nonlinear bending analysis of isotropic nanobeams. Their models were based on the modified von Karman nonlinear theory which accounts for the nonlinear terms due to the transverse normal strain. The nonlinear EBT element used Lagrange and Hermite cubic interpolation functions to respectively approximate the axial and transverse displacements, whilst the nonlinear TBT element employed Lagrange interpolation functions for both axial and transverse displacements and rotation. Reddy et al. [389] extended their previous work in [388] to FG nanobeams. Eltaher et al. [390] developed a nonlocal TBT element for the static bending and buckling analysis of FG nanobeams. The element which accounts for the effect of the physical neutral surface has three nodes and is based on quadratic Lagrange interpolation functions.

5.1.2. Modified couple stress elements

Based on the modified couple stress theory and the von Karman nonlinear strains, Arbind and Reddy [391] and Arbind et al. [392] developed two-node EBT and TBT elements [391] and RBT element [392] for the nonlinear bending analysis of FG microbeams. The element has 3-DOF at each node. In the nonlinear EBT element, the axial and transverse displacements were approximated using Lagrange and Hermite cubic interpolation functions, respectively. Meanwhile, the nonlinear TBT and RBT elements employed Lagrange interpolation functions for both axial displacement and rotation, and Hermite cubic interpolation functions for the transverse displacement. These models were used to study the effects of material length scale parameter and geometric nonlinearity on deflections of FG microbeams. Reddy and Srinivasa [393] also developed nonlinear two-node EBT and TBT elements for microbeams which are capable of capturing moderate rotations since they were based on the modified von Karman nonlinear theory. Unlike the von Karman nonlinear theory, the modified von Karman nonlinear theory did include the nonlinear terms due to the transverse normal strain, and thus requiring 2D constitutive relations of beams. The EBT and TBT elements developed by Arbind and Reddy [391] were employed by Dehrouyeh-Semnani and Nikkhah-Bahrami [394] to examine the size effect on the bending, buckling and free vibration responses of isotropic microbeams. Kahrobaiyan et al. [395] also developed a two-node modified couple stress TBT element for the static bending analysis of isotropic microbeams. However, their element has only 2-DOF at each node and was based on the shape functions derived by directly solving the governing equations of the modified couple stress TBT model. Numerical results indicated that the load-deflection response of a cantilever microbeams predicted by their element agrees well with the experimental result as shown in Fig. 8. The accuracy and stability of the TBT elements proposed by Kahrobaiyan et al. [395] and Arbind and Reddy [391] were assessed by Dehrouyeh-Semnani and Bahrami [396]. The results indicated that both two elements give a

stable solution. However, the 6-DOF element of Arbind and Reddy [391] is more accurate than the 4-DOF element of Kahrobaiyan et al. [395] in predicting deflections of isotropic microbeams under various BCs. Recently, Karttunen et al. [397] developed an exact modified couple stress TBT element for the static analysis of FG microbeams. The element has two nodes with 3-DOF at each node. It was based on the exact shape functions derived directly from analytical solutions of the modified couple stress TBT model.

5.1.3. Strain gradient elements

Kahrobaiyan et al. [398] developed a strain gradient element for isotropic microbeams based on the EBT. The element has two nodes with 3-DOF at each node including the deflection, slope and curvature. The mass and stiffness matrices of the element were derived based on the Galerkin method with interpolation functions determined by solving directly the governing equations of the strain gradient EBT model. The element was applied to the bending analysis of a cantilever microbeam under a concentrated force at its free end. In order to account for the shear deformation effect, Zhang et al. [399] developed a two-node strain gradient TBT element for isotropic microbeams. The element has 6-DOF at each node when considering both bending and stretching deformations, and 4-DOF at each node when considering only bending deformation. The displacement field of the element is approximated using exact hyperbolic interpolation functions derived from solving directly the governing equations of the strain gradient TBT model. Numerical results indicated that the element is capable of accurately predicting the static bending, buckling and free vibration responses of isotropic microbeams. Zhang et al. [400] also presented a strain gradient TBT element for isotropic microbeams which is similar to the one developed by Zhang et al. [399]. However, it has 4-DOF per node and considers only bending deformation. Kahrobaiyan et al. [401] developed a strain gradient TBT element for isotropic microbeams. The element has two nodes with 2-DOF at each node including the deflection and rotation. The shape functions of their element were derived by directly solving the equilibrium equations of the strain gradient TBT model with the proper BCs. By comparing with experimental results, it was concluded that the present element is capable of accurately predicting the load-deflection response of a cantilever microbeams as shown in Fig. 9. The element was successfully applied to predict the deflection and natural frequency of MEMS. It should be noted that Eltaher et al. [402], Ebrahimi et al. [403] and Ansari et al. [404-406] also developed strain gradient elements for isotropic microbeams based on EBT [402-404] and TBT [405-406] models, but they were based on the nonlocal strain gradient theory and the most general form of the strain gradient theory of Mindlin [14] which are not covered in this review.

5.2. Plate elements

5.2.1. Nonlocal elasticity elements

One of the earliest nonlocal finite element models for nanoplates was developed by Phadikar and Pradhan

[407] and Ansari et al. [408] using the Galerkin method. Phadikar and Pradhan [407] developed a nonlocal CPT element for the bending, buckling and free vibration analyses of isotropic nanoplates, whilst Ansari et al. [408] proposed a nonlocal FSDT element for the free vibration analysis of MLGSs. The element developed by Phadikar and Pradhan [407] has four nodes with 3-DOF at each node and was based on Hermite cubic interpolation functions, whilst the element proposed by Ansari et al. [408] has eight nodes with 5-DOF at each node and was based on quadratic serendipity interpolation functions. Natarajan et al. [409] developed a nonlocal FSDT element for the free vibration analysis of FG nanoplates using an isogeometric analysis (IGA) in which the field variables were approximated by non-uniform rational B-splines (NURBS) basic functions as shown in Fig. 10. Nguyen et al. [410] also employed the IGA approach to develop a nonlocal element for FG nanoplates. However, their element was based on a simple quasi-3D theory with four unknowns as shown in Table 2. Ansari and Norouzzadeh [411] studied the nonlocal and surface effects on the buckling behaviour of FG nanoplates based on the FSDT and IGA approach. Sarrami-Foroushani and Azhari [412] presented a nonlocal element for the buckling and free vibration analysis of SLGSs based on the finite strip method and the refined plate theory of Shimpi [109]. Unlike the finite element method, the plate in the finite strip approach is meshed in one direction, and thus the number of DOFs is reduced.

5.2.2. Modified couple stress elements

One of the earliest modified couple stress plate elements was developed by Zhang et al. [413] for isotropic microplates based on the FSDT. The element is non-conforming and has four nodes with 15-DOF per node. Unlike the classical FSDT element, the modified couple stress FSDT element is shear locking free and thus the full integration can still be used. The element was successfully used to predict the bending, buckling and free vibration responses of isotropic microplates with various BCs. Reddy and Srinivasa [393] presented a nonlinear FSDT element for the nonlinear analysis of modified couple stress plates. Since the element was based on the modified von Karman nonlinear theory, it is capable of capturing moderate rotations. Mirsalehi et al. [414] developed a modified couple stress CPT element for FG microplates based on a spline finite strip method. The spline finite strip method is a special form of the finite strip method in which the B3-spline functions are used in the longitudinal direction and the Hermite cubic functions are used in the transverse direction of the strip [414]. The spline finite strip element was applied to predict the critical buckling loads and buckling temperatures of FG microplates under mechanical and thermal loadings. Kim and Reddy [415] developed a nonlinear modified couple stress element for FG microplates based on the general quasi-3D theory of Reddy and Kim [308] and von Karman nonlinear strains. The element is non-conforming and has four nodes with 44-DOF at each node accounting for geometric nonlinearity and requires C^1 continuity for all variables. Reddy et al. [416] presented nonlinear CPT and FSDT elements for the axisymmetric bending

analysis of circular FG microplates. The axisymmetric CPT element has two nodes with 3-DOF at each node based on Lagrange interpolation functions for the axial displacement and Hermite interpolation functions for the transverse displacement. Meanwhile, the axisymmetric FSDT element which has two nodes with 4-DOF at each node employed Lagrange interpolation functions for both axial displacement and rotation, and Hermite interpolation functions for the transverse displacement. The models were employed to study the influences of geometric nonlinearity and material length scale parameter on bending responses of FG circular plates with various BCs. Recently, Nguyen et al. [417] proposed an efficient modified couple stress element for the static bending, buckling and free vibration analyses of FG microplates based on the IGA approach. The element was based on a simple quasi-3D theory with four unknowns as shown in Table 2.

5.2.3. Strain gradient elements

For the strain gradient plate element, only two publications were found in the literature involved in the development of the finite element model for microplates based on the strain gradient theory of Lam et al. [15]. Mirsalehi et al. [418] presented a strain gradient CPT element for FG microplates using the spline finite strip method. The element was used to investigate the influences of material length scale parameters, BCs, volume fraction module and geometric dimensions on critical buckling loads and natural frequencies of FG microplates. Recently, Thai et al. [419] developed a strain gradient element for the bending, buckling and free vibration analyses of FG microplates based on the IGA approach. It should be noted that Ansari et al. [420-421] also developed strain gradient elements for isotropic microplates, but it was based on the most general form of the strain gradient theory of Mindlin [14] which is not covered in this review.

6. Concluding remarks and recommendation for future studies

The development of size-dependent models for predicting size effects on the global responses of small-scale beam, plate and shell structures was comprehensively reviewed and discussed in this paper. During the past decade, great efforts have been devoted to the development of size-dependent models based on higher-order continuum mechanics approach. This review mainly focuses on the size-dependent beam, plate and shell models developed based on the nonlocal elasticity theory, modified couple stress theory and strain gradient theory due to their common use in predicting the global behaviour of small-scale structures. Both analytical and numerical models are included in this review paper.

The review indicates that most size-dependent models have been developed in the last five years. The number of strain gradient models is small compared to the number of models developed based on the nonlocal elasticity theory and modified couple stress theory. The nonlocal beam and plate models are widely used for analysing nanostructures such as CNTs and graphene sheets, whilst the modified couple stress and strain gradient models are applied to microstructures. The review also shows that the number of relevant

papers involving in the development of finite element models is relatively small compared with the total number of papers published on analytical models.

As reviewed in this paper, most of existing size-dependent models focused on analytical solutions which are limited to beam and plate structures subjected to certain loading and boundary conditions and geometries, whereas the development of finite element solutions for size-dependent beam and plate models has not been given enough attention. Therefore, further efforts should be devoted to developing finite element models of size-dependent theories, especially the strain gradient-based models. It is noted that only one publication was found in the literature involved in the development of the finite element model for strain gradient CPT plates using the spline finite strip method.

References

[1] Stolken JS, Evans AG. A microbend test method for measuring the plasticity length scale. Acta Mater. 1998;46:5109-15.

[2] Nix WD. Mechanical properties of thin films. Metallurgical Transactions A. 1989;20:2217-45.

[3] Ma Q, Clarke DR. Size dependent hardness of silver single crystals. Journal of Materials Research. 1995;10:853-63.

[4] Fleck NA, Muller GM, Ashby MF, Hutchinson JW. Strain gradient plasticity: Theory and experiment. Acta Metall Mater. 1994;42:475-87.

[5] Chong ACM, Lam DCC. Strain gradient plasticity effect in indentation hardness of polymers. Journal of Materials Research. 1999;14:4103-10.

[6] Dell'Isola F, Andreaus U, Placidi L. At the origins and in the vanguard of peridynamics, non-local and higher-gradient continuum mechanics: An underestimated and still topical contribution of Gabrio Piola. Mathematics and Mechanics of Solids. 2015;20:887-928.

[7] Dell'Isola F, Della Corte A, Esposito R, Russo L. Some cases of unrecognized transmission of scientific knowledge: From antiquity to Gabrio Piola's peridynamics and generalized continuum theories. Generalized Continua as Models for Classical and Advanced Materials: Springer; 2016. p. 77-128.

[8] Cosserat E, Cosserat F. Théorie des corps déformables: Paris: Hermann et Fils, 1909.

[9] Toupin RA. Elastic materials with couple-stresses. Archive for Rational Mechanics and Analysis. 1962;11:385-414.

[10] Mindlin RD, Tiersten HF. Effects of couple-stresses in linear elasticity. Archive for Rational Mechanics

and Analysis. 1962;11:415-48.

[11] Koiter WT. Couple stresses in the theory of elasticity, I and II. Nederl Akad Wetensch Proc Ser B. 1964;67:17-44.

[12] Yang F, Chong ACM, Lam DCC, Tong P. Couple stress based strain gradient theory for elasticity. Int J Solids Struct. 2002;39:2731-43.

[13] Mindlin RD. Micro-structure in linear elasticity. Archive for Rational Mechanics and Analysis. 1964;16:51-78.

[14] Mindlin RD. Second gradient of strain and surface-tension in linear elasticity. Int J Solids Struct. 1965;1:417-38.

[15] Lam DCC, Yang F, Chong ACM, Wang J, Tong P. Experiments and theory in strain gradient elasticity. J Mech Phys Solids. 2003;51:1477-508.

[16] Eringen AC. Simple microfluids. Int J Eng Sci. 1964;2:205-17.

[17] Eringen AC. Linear theory of micropolar elasticity. Journal of Mathematics and Mechanics. 1966;15:909-23.

[18] Eringen AC. Micropolar fluids with stretch. Int J Eng Sci. 1969;7:115-27.

[19] Altenbach H, Eremeyev VA. Strain rate tensors and constitutive equations of inelastic micropolar materials. International Journal of Plasticity. 2014;63:3-17.

[20] Ansari R, Bazdid-Vahdati M, Shakouri A, Norouzzadeh A, Rouhi H. Micromorphic prism element. Mathematics and Mechanics of Solids. 2016;doi: 10.1177/1081286516637115.

[21] Ansari R, Bazdid-Vahdati M, Shakouri A, Norouzzadeh A, Rouhi H. Micromorphic first-order shear deformable plate element. Meccanica. 2016;51:1797-809.

[22] Ansari R, Shakouri AH, Bazdid-Vahdati M, Norouzzadeh A, Rouhi H. A nonclassical finite element approach for the nonlinear analysis of micropolar plates. J Comput Nonlinear Dyn. 2016;12:011019-12.

[23] Chen Y, Lee JD. Determining material constants in micromorphic theory through phonon dispersion relations. Int J Eng Sci. 2003;41:871-86.

[24] Eringen AC. Microcontinuum field theories: I. Foundations and solids. Springer; 1999.

[25] Eringen AC. Microcontinuum field theories: II. Fluent media: Springer, 2001.

[26] Kroner E. Elasticity theory of materials with long range cohesive forces. Int J Solids Struct. 1967;3:731-42.

[27] Eringen AC. Linear theory of nonlocal elasticity and dispersion of plane waves. Int J Eng Sci. 1972;10:425-35.

[28] Eringen AC. Nonlocal polar elastic continua. Int J Eng Sci. 1972;10:1-16.

[29] Eringen AC, Edelen DGB. On nonlocal elasticity. Int J Eng Sci. 1972;10:233-48.

[30] Eringen AC. On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves. J Appl Phys. 1983;54:4703-10.

[31] Aifantis EC. On the gradient approach – Relation to Eringen's nonlocal theory. Int J Eng Sci. 2011;49:1367-77.

[32] Lim C, Zhang G, Reddy J. A higher-order nonlocal elasticity and strain gradient theory and its

applications in wave propagation. J Mech Phys Solids. 2015;78:298-313.

[33] Narendar S, Gopalakrishnan S. Ultrasonic wave characteristics of nanorods via nonlocal strain gradient models. J Appl Phys. 2010;107:084312-8.

[34] Iijima S. Helical microtubules of graphitic carbon. Nature. 1991;354:56-8.

[35] Ansari R, Norouzzadeh A, Gholami R, Faghih Shojaei M, Darabi MA. Geometrically nonlinear free vibration and instability of fluid-conveying nanoscale pipes including surface stress effects. Microfluid Nanofluid. 2016;20:28-42.

[36] Ansari R, Gholami R, Norouzzadeh A, Darabi MA. Wave characteristics of nanotubes conveying fluid based on the non-classical Timoshenko beam model incorporating surface energies. Arab J Sci Eng. 2016;41:4359-69.

[37] Ansari R, Gholami R, Norouzzadeh A. Size-dependent thermo-mechanical vibration and instability of conveying fluid functionally graded nanoshells based on Mindlin's strain gradient theory. Thin Wall Struct. 2016;105:172-84.

[38] Ansari R, Gholami R, Norouzzadeh A, Sahmani S. Size-dependent vibration and instability of fluid-conveying functionally graded microshells based on the modified couple stress theory. Microfluid Nanofluid. 2015;19:509-22.

[39] Ansari R, Gholami R, Norouzzadeh A, Darabi M. Surface stress effect on the vibration and instability of nanoscale pipes conveying fluid based on a size-dependent Timoshenko beam model. Acta Mechanica Sinica. 2015;31:708-19.

[40] Ansari R, Norouzzadeh A, Gholami R, Shojaei MF, Hosseinzadeh M. Size-dependent nonlinear vibration and instability of embedded fluid-conveying SWBNNTs in thermal environment. Phys E. 2014;61:148-57.

[41] Wang L. Vibration analysis of fluid-conveying nanotubes with consideration of surface effects. Phys E. 2010;43:437-9.

[42] Reddy JN. A simple higher-order theory for laminated composite plates. J Appl Mech Trans ASME. 1984;51:745-52.

[43] Thai HT, Kim SE. A review of theories for the modeling and analysis of functionally graded plates and shells. Compos Struct. 2015;128:70-86.

[44] Huang LY, Han Q, Liang YJ. Calibration of nonlocal scale effect parameter for bending single-layered graphene sheet under molecular dynamics. Nano. 2012;7:125003-8.

[45] Arash B, Ansari R. Evaluation of nonlocal parameter in the vibrations of single-walled carbon nanotubes with initial strain. Phys E. 2010;42:2058-64.

[46] Duan WH, Challamel N, Wang CM, Ding Z. Development of analytical vibration solutions for microstructured beam model to calibrate length scale coefficient in nonlocal Timoshenko beams. J Appl Phys. 2013;114:104312-11.

[47] Zhang Z, Wang CM, Challamel N, Elishakoff I. Obtaining Eringen's length scale coefficient for vibrating nonlocal beams via continualization method. J Sound Vib. 2014;333:4977-90.

[48] Zhang Z, Wang CM, Challamel N. Eringen's length scale coefficient for buckling of nonlocal

rectangular plates from microstructured beam-grid model. Int J Solids Struct. 2014;51:4307-15.

[49] Zhang Z, Wang CM, Challamel N. Eringen's length-scale coefficients for vibration and buckling of nonlocal rectangular plates with simply supported edges. J Eng Mech. 2015;141:04014117-10.

[50] Wang Q, Wang CM. The constitutive relation and small scale parameter of nonlocal continuum mechanics for modelling carbon nanotubes. Nanotechnology. 2007;18:075702-4.

[51] Norouzzadeh A, Ansari R, Rouhi H. Pre-buckling responses of Timoshenko nanobeams based on the integral and differential models of nonlocal elasticity: an isogeometric approach. Applied Physics A. 2017;123:330-41.

[52] Norouzzadeh A, Ansari R. Finite element analysis of nano-scale Timoshenko beams using the integral model of nonlocal elasticity. Phys E. 2017;88:194-200.

[53] Fernandez-Saez J, Zaera R, Loya JA, Reddy JN. Bending of Euler–Bernoulli beams using Eringen's integral formulation: A paradox resolved. Int J Eng Sci. 2016;99:107-16.

[54] Challamel N, Wang CM. The small length scale effect for a non-local cantilever beam: a paradox solved. Nanotechnology. 2008;19:345703-7.

[55] Peddieson J, Buchanan GR, McNitt RP. Application of nonlocal continuum models to nanotechnology. Int J Eng Sci. 2003;41:305-12.

[56] Sudak LJ. Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics. J Appl Phys. 2003;94:7281-7.

[57] Zhang YQ, Liu GR, Xie XY. Free transverse vibrations of double-walled carbon nanotubes using a theory of nonlocal elasticity. Physical Review B. 2005;71:195404-7.

[58] Wang Q, Varadan VK, Quek ST. Small scale effect on elastic buckling of carbon nanotubes with nonlocal continuum models. Phys Lett A. 2006;357:130-5.

[59] Aydogdu M. Axial vibration of the nanorods with the nonlocal continuum rod model. Phys E. 2009;41:861-4.

[60] Murmu T, Pradhan SC. Thermo-mechanical vibration of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity theory. Comput Mater Sci. 2009;46:854-9.

[61] Civalek O, Demir C. Bending analysis of microtubules using nonlocal Euler-Bernoulli beam theory. Appl Math Model. 2011;35:2053-67.

[62] Mustapha KB, Zhong ZW. Free transverse vibration of an axially loaded non-prismatic single-walled carbon nanotube embedded in a two-parameter elastic medium. Comput Mater Sci. 2010;50:742-51.

[63] Li C, Lim CW, Yu JL, Zeng QC. Analytical solutions for vibration of simply supported nonlocal nanobeams with an axial force. Int J Struct Stab Dyn. 2011;11:257-71.

[64] Ghannadpour SAM, Mohammadi B, Fazilati J. Bending, buckling and vibration problems of nonlocal Euler beams using Ritz method. Compos Struct. 2013;96:584-9.

[65] Ansari R, Ramezannezhad H, Gholami R. Nonlocal beam theory for nonlinear vibrations of embedded multiwalled carbon nanotubes in thermal environment. Nonlinear Dyn. 2012;67:2241-54.

[66] Fang B, Zhen YX, Zhang CP, Tang Y. Nonlinear vibration analysis of double-walled carbon nanotubes based on nonlocal elasticity theory. Appl Math Model. 2013;37:1096-107.

[67] Simsek M. Large amplitude free vibration of nanobeams with various boundary conditions based on the nonlocal elasticity theory. Compos B Eng. 2014;56:621-8.

[68] Bagdatli SM. Non-linear vibration of nanobeams with various boundary condition based on nonlocal elasticity theory. Compos B Eng. 2015;80:43-52.

[69] Simsek M. Nonlocal effects in the free longitudinal vibration of axially functionally graded tapered nanorods. Comput Mater Sci. 2012;61:257-65.

[70] Nguyen NT, Kim NI, Lee J. Analytical solutions for bending of transversely or axially FG nonlocal beams. Steel Compos Struct. 2014;17:639-63.

[71] Niknam H, Aghdam MM. A semi analytical approach for large amplitude free vibration and buckling of nonlocal FG beams resting on elastic foundation. Compos Struct. 2014;119:452-62.

[72] Ebrahimi F, Salari E. Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams with various boundary conditions. Compos B Eng. 2015;78:272-90.

[73] Ebrahimi F, Salari E. Nonlocal thermo-mechanical vibration analysis of functionally graded nanobeams in thermal environment. Acta Astronaut. 2015;113:29-50.

[74] Nejad MZ, Hadi A. Eringen's non-local elasticity theory for bending analysis of bi-directional functionally graded Euler–Bernoulli nano-beams. Int J Eng Sci. 2016;106:1-9.

[75] Nejad MZ, Hadi A, Rastgoo A. Buckling analysis of arbitrary two-directional functionally graded Euler-Bernoulli nano-beams based on nonlocal elasticity theory. Int J Eng Sci. 2016;103:1-10.

[76] Nazemnezhad R, Hosseini-Hashemi S. Nonlocal nonlinear free vibration of functionally graded nanobeams. Compos Struct. 2014;110:192-9.

[77] El-Borgi S, Fernandes R, Reddy JN. Non-local free and forced vibrations of graded nanobeams resting on a non-linear elastic foundation. Int J Non-Linear Mech. 2015;77:348-63.

[78] Shafiei N, Kazemi M, Safi M, Ghadiri M. Nonlinear vibration of axially functionally graded non-uniform nanobeams. Int J Eng Sci. 2016;106:77-94.

[79] Wang Q. Wave propagation in carbon nanotubes via nonlocal continuum mechanics. J Appl Phys. 2005;98:124301-6.

[80] Wang Q, Varadan VK. Vibration of carbon nanotubes studied using nonlocal continuum mechanics. Smart Mater Struct. 2006;15:659-66.

[81] Wang CM, Zhang YY, Ramesh SS, Kitipornchai S. Buckling analysis of micro- and nano-rods/tubes based on nonlocal Timoshenko beam theory. Journal of Physics D. 2006;39:3904-9.

[82] Wang CM, Zhang YY, He XQ. Vibration of nonlocal Timoshenko beams. Nanotechnology. 2007;18:105401-9.

[83] Wang CM, Kitipornchai S, Lim CW, Eisenberger M. Beam bending solutions based on nonlocal Timoshenko beam theory. J Eng Mech. 2008;134:475-81.

[84] Wang Q, Liew KM. Application of nonlocal continuum mechanics to static analysis of micro- and nano-structures. Phys Lett A. 2007;363:236-42.

[85] Lu P, Lee HP, Lu C, Zhang PQ. Application of nonlocal beam models for carbon nanotubes. Int J Solids Struct. 2007;44:5289-300.

[86] Reddy JN, Pang SD. Nonlocal continuum theories of beams for the analysis of carbon nanotubes. J Appl Phys. 2008;103:023511-16.

[87] Murmu T, Pradhan SC. Buckling analysis of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity and Timoshenko beam theory and using DQM. Phys E. 2009;41:1232-9.

[88] Ansari R, Gholami R, Darabi MA. Thermal buckling analysis of embedded single-walled carbon nanotubes with arbitrary boundary conditions using the nonlocal timoshenko beam theory. J Therm Stresses. 2011;34:1271-81.

[89] Pradhan SC, Murmu T. Small-scale effect on vibration analysis of single-walled carbon nanotubes embedded in an elastic medium using nonlocal elasticity theory. J Appl Phys. 2009;105:124306-9.

[90] Roque CMC, Ferreira AJM, Reddy JN. Analysis of Timoshenko nanobeams with a nonlocal formulation and meshless method. Int J Eng Sci. 2011;49:976-84.

[91] Wu CP, Lai WW. Free vibration of an embedded single-walled carbon nanotube with various boundary conditions using the RMVT-based nonlocal Timoshenko beam theory and DQ method. Phys E. 2015;68:8-21.

[92] Amirian B, Hosseini-Ara R, Moosavi H. Thermal vibration analysis of carbon nanotubes embedded in two-parameter elastic foundation based on nonlocal Timoshenko's beam theory. Arch Mech. 2012;64:581-602.

[93] Zidour M, Benrahou KH, Semmah A, Naceri M, Belhadj HA, Bakhti K, et al. The thermal effect on vibration of zigzag single walled carbon nanotubes using nonlocal Timoshenko beam theory. Comput Mater Sci. 2012;51:252-60.

[94] Ansari R, Gholami R, Sahmani S, Norouzzadeh A, Bazdid-Vahdati M. Dynamic stability analysis of embedded multi-walled carbon nanotubes in thermal environment. Acta Mech Solida Sin. 2015;28:659-67.

[95] Ansari R, Gholami R, Rouhi H. Size-dependent nonlinear forced vibration analysis of magneto-electro-thermo-elastic Timoshenko nanobeams based upon the nonlocal elasticity theory. Compos Struct. 2015;126:216-26.

[96] Simsek M, Yurtcu HH. Analytical solutions for bending and buckling of functionally graded nanobeams based on the nonlocal Timoshenko beam theory. Compos Struct. 2013;97:378-86.

[97] Rahmani O, Pedram O. Analysis and modeling the size effect on vibration of functionally graded nanobeams based on nonlocal Timoshenko beam theory. Int J Eng Sci. 2014;77:55-70.

[98] Ebrahimi F, Salari E. Thermal buckling and free vibration analysis of size dependent Timoshenko FG nanobeams in thermal environments. Compos Struct. 2015;128:363-80.

[99] Ebrahimi F, Salari E. Effect of various thermal loadings on buckling and vibrational characteristics of nonlocal temperature-dependent functionally graded nanobeams. Mech Adv Mater Struct. 2016;23:1379-97.

[100] Reddy JN. Nonlocal theories for bending, buckling and vibration of beams. Int J Eng Sci. 2007;45:288-307.

[101] Ebrahimi F, Salari E. Thermo-mechanical vibration analysis of a single-walled carbon nanotube embedded in an elastic medium based on higher-order shear deformation beam theory. J Mech Sci Technol. 2015;29:3797-803.

[102] Emam SA. A general nonlocal nonlinear model for buckling of nanobeams. Appl Math Model. 2013;37:6929-39.

[103] Rahmani O, Jandaghian AA. Buckling analysis of functionally graded nanobeams based on a nonlocal third-order shear deformation theory. Applied Physics A. 2015;119:1019-32.

[104] Ebrahimi F, Reza Barati M. Vibration analysis of nonlocal beams made of functionally graded material in thermal environment. Eur Phys J Plus. 2016;131:279-301.

[105] Aydogdu M. A general nonlocal beam theory: Its application to nanobeam bending, buckling and vibration. Phys E. 2009;41:1651-5.

[106] Aydogdu M. A new shear deformation theory for laminated composite plates. Compos Struct. 2009;89:94-101.

[107] Karama M, Afaq KS, Mistou S. Mechanical behaviour of laminated composite beam by the new multi-layered laminated composite structures model with transverse shear stress continuity. Int J Solids Struct. 2003;40:1525-46.

[108] Thai HT. A nonlocal beam theory for bending, buckling, and vibration of nanobeams. Int J Eng Sci. 2012;52:56-64.

[109] Shimpi RP. Refined plate theory and its variants. AIAA Journal. 2002;40:137-46.

[110] Tounsi A, Semmah A, Bousahla AA. Thermal buckling behavior of nanobeams using an efficient higher-order nonlocal beam theory. J Nanomech Micromech. 2013;3:37-42.

[111] Zemri A, Houari MSA, Bousahla AA, Tounsi A. A mechanical response of functionally graded nanoscale beam: An assessment of a refined nonlocal shear deformation theory beam theory. Struct Eng Mech. 2015;54:693-710.

[112] Thai HT, Vo TP. A nonlocal sinusoidal shear deformation beam theory with application to bending, buckling, and vibration of nanobeams. Int J Eng Sci. 2012;54:58-66.

[113] Touratier M. An efficient standard plate theory. Int J Eng Sci. 1991;29:901-16.

[114] Tounsi A, Benguediab S, Houari MSA, Semmah A. A new nonlocal beam theory with thickness stretching effect for nanobeams. Int J Nanosci. 2013;12:1350025-8.

[115] Thai HT, Kim SE. A simple quasi-3D sinusoidal shear deformation theory for functionally graded plates. Compos Struct. 2013;99:172-80.

[116] Ahouel M, Houari MSA, Bedia EAA, Tounsi A. Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Steel Compos Struct. 2016;20:963-81.

[117] Chaht FL, Kaci A, Houari MSA, Tounsi A, Bég OA, Mahmoud SR. Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Steel Compos Struct. 2015;18:425-42.

[118] Pour HR, Vossough H, Heydari MM, Beygipoor G, Azimzadeh A. Nonlinear vibration analysis of a

nonlocal sinusoidal shear deformation carbon nanotube using differential quadrature method. Struct Eng Mech. 2015;54:1061-73.

[119] Sadatshojaei E, Sadatshojaie A, Fakhar MH. Differential quadrature method for nonlocal nonlinear vibration analysis of a boron nitride nanotube using sinusoidal shear deformation theory. Mech Adv Mater Struct. 2016;23:1278-83.

[120] Berrabah HM, Tounsi AL, Semmah A, Adda Bedia EA. Comparison of various refined nonlocal beam theories for bending, vibration and buckling analysis of nanobeams. Struct Eng Mech. 2013;48:351-65.

[121] Thai HT, Choi DH. Efficient higher-order shear deformation theories for bending and free vibration analyses of functionally graded plates. Arch Appl Mech. 2013;83:1755-71.

[122] Ebrahimi F, Barati MR. A unified formulation for dynamic analysis of nonlocal heterogeneous nanobeams in hygro-thermal environment. Applied Physics A. 2016;122:792-806.

[123] Mashat D, Zenkour A, Sobhy M. Investigation of vibration and thermal buckling of nanobeams embedded in an elastic medium under various boundary conditions. J Mech. 2016;32:277-87.

[124] Thai S, Thai HT, Vo TP, Patel VI. A simple shear deformation theory for nonlocal beams. Compos Struct. 2017;doi: 10.1016/j.compstruct.2017.03.022.

[125] Zhang YQ, Liu GR, Wang JS. Small-scale effects on buckling of multiwalled carbon nanotubes under axial compression. Physical Review B. 2004;70:205430-6.

[126] Li R, Kardomateas GA. Thermal buckling of multi-walled carbon nanotubes by nonlocal elasticity. J Appl Mech Trans ASME. 2007;74:399-405.

[127] Li R, Kardomateas GA. Vibration characteristics of multiwalled carbon nanotubes embedded in elastic media by a nonlocal elastic shell model. J Appl Mech Trans ASME. 2007;74:1087-94.

[128] Wang Q, Varadan VK. Application of nonlocal elastic shell theory in wave propagation analysis of carbon nanotubes. Smart Mater Struct. 2007;16:178-91.

[129] Hu YG, Liew KM, Wang Q, He XQ, Yakobson BI. Nonlocal shell model for elastic wave propagation in single- and double-walled carbon nanotubes. J Mech Phys Solids. 2008;56:3475-85.

[130] Zhang YY, Wang CM, Duan WH, Xiang Y, Zong Z. Assessment of continuum mechanics models in predicting buckling strains of single-walled carbon nanotubes. Nanotechnology. 2009;20:395707-8.

[131] Rouhi H, Ansari R. Nonlocal analytical Flugge shell model for axial buckling of double-walled carbon nanotubes with different end conditions. Nano. 2012;7:1250018-10.

[132] Sarvestani HY. Buckling analysis of curved nanotube structures based on the nonlocal shell theory. Int J Multiscale Comput Eng. 2016;14:45-54.

[133] Lu P, Zhang PQ, Lee HP, Wang CM, Reddy JN. Non-local elastic plate theories. Proc R Soc A Math Phys Eng Sci. 2007;463:3225-40.

[134] Duan WH, Wang CM. Exact solutions for axisymmetric bending of micro/nanoscale circular plates based on nonlocal plate theory. Nanotechnology. 2007;18:385704-5.

[135] Aksencer T, Aydogdu M. Levy type solution method for vibration and buckling of nanoplates using nonlocal elasticity theory. Phys E. 2011;43:954-9.

[136] Shakouri A, Ng TY, Lin RM. Nonlocal plate model for the free vibration analysis of nanoplates with

different boundary conditions. J Comput Theor Nanosci. 2011;8:2118-28.

[137] Pradhan SC, Murmu T. Small scale effect on the buckling of single-layered graphene sheets under biaxial compression via nonlocal continuum mechanics. Comput Mater Sci. 2009;47:268-74.

[138] Pradhan SC, Kumar A. Buckling analysis of single layered graphene sheet under biaxial compression using nonlocal elasticity theory and DQ method. J Comput Theor Nanosci. 2011;8:1325-34.

[139] Babaei H, Shahidi AR. Small-scale effects on the buckling of quadrilateral nanoplates based on nonlocal elasticity theory using the Galerkin method. Arch Appl Mech. 2011;81:1051-62.

[140] Farajpour A, Danesh M, Mohammadi M. Buckling analysis of variable thickness nanoplates using nonlocal continuum mechanics. Phys E. 2011;44:719-27.

[141] Pradhan SC, Phadikar JK. Small scale effect on vibration of embedded multilayered graphene sheets based on nonlocal continuum models. Phys Lett A. 2009;373:1062-9.

[142] Shen ZB, Tang HL, Li DK, Tang GJ. Vibration of single-layered graphene sheet-based nanomechanical sensor via nonlocal Kirchhoff plate theory. Comput Mater Sci. 2012;61:200-5.

[143] Ansari R, Shahabodini A, Rouhi H. A nonlocal plate model incorporating interatomic potentials for vibrations of graphene with arbitrary edge conditions. Curr Appl Phys. 2015;15:1062-9.

[144] Zhang Y, Lei ZX, Zhang LW, Liew KM, Yu JL. Nonlocal continuum model for vibration of single-layered graphene sheets based on the element-free kp-Ritz method. Eng Anal Boundary Elem. 2015;56:90-7.

[145] Zhang Y, Zhang LW, Liew KM, Yu JL. Nonlocal continuum model for large deformation analysis of SLGSs using the kp-Ritz element-free method. Int J Non-Linear Mech. 2016;79:1-9.

[146] Zhang Y, Zhang LW, Liew KM, Yu JL. Buckling analysis of graphene sheets embedded in an elastic medium based on the kp-Ritz method and non-local elasticity theory. Eng Anal Boundary Elem. 2016;70:31-9.

[147] Ni Z, Bu H, Zou M, Yi H, Bi K, Chen Y. Anisotropic mechanical properties of graphene sheets from molecular dynamics. Phys B Condens Matter. 2010;405:1301-6.

[148] Pouresmaeeli S, Fazelzadeh SA, Ghavanloo E. Exact solution for nonlocal vibration of double-orthotropic nanoplates embedded in elastic medium. Compos B Eng. 2012;43:3384-90.

[149] Mohammadi M, Farajpour A, Goodarzi M, Heydarshenas R. Levy type solution for nonlocal thermo-mechanical vibration of orthotropic mono-layer graphene sheet embedded in an elastic medium. J Solid Mec. 2013;5:116-32.

[150] Mohammadi M, Moradi A, Ghayour M, Farajpour A. Exact solution for thermo-mechanical vibration of orthotropic mono-layer graphene sheet embedded in an elastic medium. Latin Am J Solids Struct. 2014;11:437-58.

[151] Sari MS, Al-Kouz WG. Vibration analysis of non-uniform orthotropic Kirchhoff plates resting on elastic foundation based on nonlocal elasticity theory. Int J Mech Sci. 2016;114:1-11.

[152] Anjomshoa A. Application of Ritz functions in buckling analysis of embedded orthotropic circular and elliptical micro/nano-plates based on nonlocal elasticity theory. Meccanica. 2013;48:1337-53.

[153] Anjomshoa A, Shahidi AR, Shahidi SH, Nahvi H. Frequency analysis of embedded orthotropic circular

and elliptical micro/nano-plates using nonlocal variational principle. J Solid Mec. 2015;7:13-27.

[154] Mohammadi M, Farajpour A, Moradi A, Ghayour M. Shear buckling of orthotropic rectangular graphene sheet embedded in an elastic medium in thermal environment. Compos B Eng. 2014;56:629-37.

[155] Ashoori AR, Salari E, Sadough Vanini SA. Size-dependent thermal stability analysis of embedded functionally graded annular nanoplates based on the nonlocal elasticity theory. Int J Mech Sci. 2016;119:396-411.

[156] Pradhan SC, Phadikar JK. Nonlocal elasticity theory for vibration of nanoplates. J Sound Vib. 2009;325:206-23.

[157] Pradhan SC, Phadikar JK. Nonlocal theory for buckling of nanoplates. Int J Struct Stab Dyn. 2011;11:411-29.

[158] Kananipour H. Static analysis of nanoplates based on the nonlocal Kirchhoff and Mindlin plate theories using DQM. Latin Am J Solids Struct. 2014;11:1709-20.

[159] Ansari R, Sahmani S, Arash B. Nonlocal plate model for free vibrations of single-layered graphene sheets. Phys Lett A. 2010;375:53-62.

[160] Ansari R, Arash B, Rouhi H. Vibration characteristics of embedded multi-layered graphene sheets with different boundary conditions via nonlocal elasticity. Compos Struct. 2011;93:2419-29.

[161] Samaei AT, Abbasion S, Mirsayar MM. Buckling analysis of a single-layer graphene sheet embedded in an elastic medium based on nonlocal Mindlin plate theory. Mech Res Commun. 2011;38:481-5.

[162] Bedroud M, Hosseini-Hashemi S, Nazemnezhad R. Buckling of circular/annular Mindlin nanoplates via nonlocal elasticity. Acta Mech. 2013;224:2663-76.

[163] Arani AG, Abdollahian M, Kolahchi R, Rahmati A. Electro-thermo-torsional buckling of an embedded armchair DWBNNT using nonlocal shear deformable shell model. Compos B Eng. 2013;51:291-9.

[164] Naderi A, Saidi AR. Modified nonlocal mindlin plate theory for buckling analysis of nanoplates. J Nanomech Micromech. 2014;4:A4013015-8.

[165] Reddy JN. Nonlocal nonlinear formulations for bending of classical and shear deformation theories of beams and plates. Int J Eng Sci. 2010;48:1507-18.

[166] Hosseini-Hashemi S, Bedroud M, Nazemnezhad R. An exact analytical solution for free vibration of functionally graded circular/annular Mindlin nanoplates via nonlocal elasticity. Compos Struct. 2013;103:108-18.

[167] Anjomshoa A, Tahani M. Vibration analysis of orthotropic circular and elliptical nano-plates embedded in elastic medium based on nonlocal Mindlin plate theory and using Galerkin method. J Mech Sci Technol. 2016;30:2463-74.

[168] Golmakani ME, Rezatalab J. Nonlinear bending analysis of orthotropic nanoscale plates in an elastic matrix based on nonlocal continuum mechanics. Compos Struct. 2014;111:85-97.

[169] Dastjerdi S, Jabbarzadeh M, Aliabadi S. Nonlinear static analysis of single layer annular/circular graphene sheets embedded in Winkler–Pasternak elastic matrix based on non-local theory of Eringen. Ain Shams Engineering Journal. 2016;7:873-84.

[170] Dastjerdi S, Jabbarzadeh M. Non-linear bending analysis of multi-layer orthotropic annular/circular graphene sheets embedded in elastic matrix in thermal environment based on non-local elasticity theory. Appl Math Model. 2017;41:83-101.

[171] Aghababaei R, Reddy JN. Nonlocal third-order shear deformation plate theory with application to bending and vibration of plates. J Sound Vib. 2009;326:277-89.

[172] Pradhan SC. Buckling of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory. Phys Lett A. 2009;373:4182-8.

[173] Pradhan SC, Sahu B. Vibration of single layer graphene sheet based on nonlocal elasticity and higher order shear deformation theory. J Comput Theor Nanosci. 2010;7:1042-50.

[174] Ansari R, Sahmani S. Prediction of biaxial buckling behavior of single-layered graphene sheets based on nonlocal plate models and molecular dynamics simulations. Appl Math Model. 2013;37:7338-51.

[175] Hosseini-Hashemi S, Kermajani M, Nazemnezhad R. An analytical study on the buckling and free vibration of rectangular nanoplates using nonlocal third-order shear deformation plate theory. Eur J Mech A Solids. 2015;51:29-43.

[176] Daneshmehr A, Rajabpoor A, Pourdavood M. Stability of size dependent functionally graded nanoplate based on nonlocal elasticity and higher order plate theories and different boundary conditions. Int J Eng Sci. 2014;82:84-100.

[177] Daneshmehr A, Rajabpoor A, Hadi A. Size dependent free vibration analysis of nanoplates made of functionally graded materials based on nonlocal elasticity theory with high order theories. Int J Eng Sci. 2015;95:23-35.

[178] Narendar S. Buckling analysis of micro-/nano-scale plates based on two-variable refined plate theory incorporating nonlocal scale effects. Compos Struct. 2011;93:3093-103.

[179] Malekzadeh P, Shojaee M. Free vibration of nanoplates based on a nonlocal two-variable refined plate theory. Compos Struct. 2013;95:443-52.

[180] Narendar S, Gopalakrishnan S. Scale effects on buckling analysis of orthotropic nanoplates based on nonlocal two-variable refined plate theory. Acta Mech. 2012;223:395-413.

[181] Sobhy M. Hygrothermal vibration of orthotropic double-layered graphene sheets embedded in an elastic medium using the two-variable plate theory. Appl Math Model. 2016;40:85-99.

[182] Sobhy M. Generalized two-variable plate theory for multi-layered graphene sheets with arbitrary boundary conditions. Acta Mech. 2014;225:2521-38.

[183] Sobhy M. Levy-type solution for bending of single-layered graphene sheets in thermal environment using the two-variable plate theory. Int J Mech Sci. 2015;90:171-8.

[184] Sobhy M. Hygrothermal deformation of orthotropic nanoplates based on the state-space concept. Compos B Eng. 2015;79:224-35.

[185] Zenkour AM, Sobhy M. Nonlocal elasticity theory for thermal buckling of nanoplates lying on Winkler–Pasternak elastic substrate medium. Phys E. 2013;53:251-9.

[186] Alzahrani EO, Zenkour AM, Sobhy M. Small scale effect on hygro-thermo-mechanical bending of nanoplates embedded in an elastic medium. Compos Struct. 2013;105:163-72.

[187] Thai HT, Vo TP, Nguyen TK, Lee J. A nonlocal sinusoidal plate model for micro/nanoscale plates. Proc Inst Mech Eng, Part C: J Mech Eng Sci. 2014;228:2652-60.

[188] Sobhy M. Thermomechanical bending and free vibration of single-layered graphene sheets embedded in an elastic medium. Phys E. 2014;56:400-9.

[189] Sobhy M. Natural frequency and buckling of orthotropic nanoplates resting on two-parameter elastic foundations with various boundary conditions. J Mech. 2014;30:443-53.

[190] Sobhy M. A comprehensive study on FGM nanoplates embedded in an elastic medium. Compos Struct. 2015;134:966-80.

[191] Thai HT, Vo TP. A new sinusoidal shear deformation theory for bending, buckling, and vibration of functionally graded plates. Appl Math Model. 2013;37:3269-81.

[192] Belkorissat I, Houari MSA, Tounsi A, Bedia EAA, Mahmoud SR. On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model. Steel Compos Struct. 2015;18:1063-81.

[193] Soldatos KP. A transverse shear deformation theory for homogeneous monoclinic plates. Acta Mech. 1992;94:195-220.

[194] Khorshidi K, Fallah A. Buckling analysis of functionally graded rectangular nano-plate based on nonlocal exponential shear deformation theory. Int J Mech Sci. 2016;113:94-104.

[195] Bessaim A, Houari MSA, Bernard F, Tounsi A. A nonlocal quasi-3D trigonometric plate model for free vibration behaviour of micro/nanoscale plates. Struct Eng Mech. 2015;56:223-40.

[196] Sobhy M, Radwan AF. A new quasi 3D nonlocal plate theory for vibration and buckling of FGM nanoplates. Int J Appl Mech. 2017;9:1750008-29.

[197] Dehrouyeh-Semnani AM, Nikkhah-Bahrami M. A discussion on evaluation of material length scale parameter based on micro-cantilever test. Compos Struct. 2015;122:425-9.

[198] Khajueenejad F, Ghanbari J. Internal length parameter and buckling analysis of carbon nanotubes using modified couple stress theory and Timoshenko beam model. Materials Research Express. 2015;2:105009-10.

[199] Park SK, Gao XL. Bernoulli-Euler beam model based on a modified couple stress theory. J Micromech Microengineering. 2006;16:2355-9.

[200] Kong S, Zhou S, Nie Z, Wang K. The size-dependent natural frequency of Bernoulli–Euler micro-beams. Int J Eng Sci. 2008;46:427-37.

[201] Kong S, Zhou S, Nie Z, Wang K. Size effect on the buckling loads of slender columns based on a modified couple stress theory. Journal of Mechanical Strength. 2009;31:136-9.

[202] Xia W, Wang L, Yin L. Nonlinear non-classical microscale beams: Static bending, postbuckling and free vibration. Int J Eng Sci. 2010;48:2044-53.

[203] Simsek M. Nonlinear static and free vibration analysis of microbeams based on the nonlinear elastic foundation using modified couple stress theory and He's variational method. Compos Struct. 2014;112:264-72.

[204] Wang YG, Lin WH, Liu N. Nonlinear bending and post-buckling of extensible microscale beams based

on modified couple stress theory. Appl Math Model. 2015;39:117-27.

[205] Wang YG, Lin WH, Liu N. Nonlinear free vibration of a microscale beam based on modified couple stress theory. Phys E. 2013;47:80-5.

[206] Farokhi H, Ghayesh MH, Amabili M. Nonlinear dynamics of a geometrically imperfect microbeam based on the modified couple stress theory. Int J Eng Sci. 2013;68:11-23.

[207] Togun N, Bagdatli SM. Size dependent nonlinear vibration of the tensioned nanobeam based on the modified couple stress theory. Compos B Eng. 2016;97:255-62.

[208] Wang YG, Lin WH, Zhou CL, Liu RX. Thermal postbuckling and free vibration of extensible microscale beams based on modified couple stress theory. J Mech. 2014;31:37-46.

[209] Ansari R, Ashrafi MA, Arjangpay A. An exact solution for vibrations of postbuckled microscale beams based on the modified couple stress theory. Appl Math Model. 2015;39:3050-62.

[210] Asghari M, Ahmadian MT, Kahrobaiyan MH, Rahaeifard M. On the size-dependent behavior of functionally graded micro-beams. Mater Des. 2010;31:2324-9.

[211] Akgoz B, Civalek O. Free vibration analysis of axially functionally graded tapered Bernoulli-Euler microbeams based on the modified couple stress theory. Compos Struct. 2013;98:314-22.

[212] Shafiei N, Kazemi M, Ghadiri M. Nonlinear vibration of axially functionally graded tapered microbeams. Int J Eng Sci. 2016;102:12-26.

[213] Simsek M. Size dependent nonlinear free vibration of an axially functionally graded (AFG) microbeam using He's variational method. Compos Struct. 2015;131:207-14.

[214] Dehrouyeh-Semnani AM, Mostafaei H, Nikkhah-Bahrami M. Free flexural vibration of geometrically imperfect functionally graded microbeams. Int J Eng Sci. 2016;105:56-79.

[215] Ma HM, Gao XL, Reddy JN. A microstructure-dependent Timoshenko beam model based on a modified couple stress theory. J Mech Phys Solids. 2008;56:3379-91.

[216] Asghari M, Kahrobaiyan MH, Rahaeifard M, Ahmadian MT. Investigation of the size effects in Timoshenko beams based on the couple stress theory. Arch Appl Mech. 2011;81:863-74.

[217] Dos Santos JVA, Reddy JN. Free vibration and buckling analysis of beams with a modified couple-stress theory. Int J Appl Mech. 2012;4:1250026-28.

[218] Dehrouyeh-Semnani AM, Nikkhah-Bahrami M. A discussion on incorporating the Poisson effect in microbeam models based on modified couple stress theory. Int J Eng Sci. 2015;86:20-5.

[219] Liu YP, Reddy JN. A nonlocal curved beam model based on a modified couple stress theory. Int J Struct Stab Dyn. 2011;11:495-512.

[220] Taati E, Molaei Najafabadi M, Basirat Tabrizi H. Size-dependent generalized thermoelasticity model for Timoshenko microbeams. Acta Mech. 2014;225:1823-42.

[221] Asghari M, Kahrobaiyan MH, Ahmadian MT. A nonlinear Timoshenko beam formulation based on the modified couple stress theory. Int J Eng Sci. 2010;48:1749-61.

[222] Ghayesh MH, Farokhi H, Amabili M. Nonlinear dynamics of a microscale beam based on the modified couple stress theory. Compos B Eng. 2013;50:318-24.

[223] Ghayesh MH, Amabili M, Farokhi H. Three-dimensional nonlinear size-dependent behaviour of

Timoshenko microbeams. Int J Eng Sci. 2013;71:1-14.

[224] Reddy JN. Microstructure-dependent couple stress theories of functionally graded beams. J Mech Phys Solids. 2011;59:2382-99.

[225] Ke LL, Wang YS, Yang J, Kitipornchai S. Nonlinear free vibration of size-dependent functionally graded microbeams. Int J Eng Sci. 2012;50:256-67.

[226] Asghari M, Rahaeifard M, Kahrobaiyan MH, Ahmadian MT. The modified couple stress functionally graded Timoshenko beam formulation. Mater Des. 2011;32:1435-43.

[227] Ke LL, Wang YS. Size effect on dynamic stability of functionally graded microbeams based on a modified couple stress theory. Compos Struct. 2011;93:342-50.

[228] Nateghi A, Salamat-talab M. Thermal effect on size dependent behavior of functionally graded microbeams based on modified couple stress theory. Compos Struct. 2013;96:97-110.

[229] Simsek M, Kocaturk T, Akbas SD. Static bending of a functionally graded microscale Timoshenko beam based on the modified couple stress theory. Compos Struct. 2013;95:740-7.

[230] Chen W, Li L, Xu M. A modified couple stress model for bending analysis of composite laminated beams with first order shear deformation. Compos Struct. 2011;93:2723-32.

[231] Chen WJ, Li XP. Size-dependent free vibration analysis of composite laminated Timoshenko beam based on new modified couple stress theory. Arch Appl Mech. 2013;83:431-44.

[232] Roque CMC, Fidalgo DS, Ferreira AJM, Reddy JN. A study of a microstructure-dependent composite laminated Timoshenko beam using a modified couple stress theory and a meshless method. Compos Struct. 2013;96:532-7.

[233] Mohammad-Abadi M, Daneshmehr AR. An investigation of modified couple stress theory in buckling analysis of micro composite laminated Euler–Bernoulli and Timoshenko beams. Int J Eng Sci. 2014;75:40-53.

[234] Thai HT, Vo TP, Nguyen TK, Lee J. Size-dependent behavior of functionally graded sandwich microbeams based on the modified couple stress theory. Compos Struct. 2015;123:337-49.

[235] Krysko AV, Awrejcewicz J, Pavlov SP, Zhigalov MV, Krysko VA. Mathematical model of a three-layer micro- and nano-beams based on the hypotheses of the Grigolyuk-Chulkov and the modified couple stress theory. Int J Solids Struct. 2017;117:39-50.

[236] Ma HM, Gao XL, Reddy JN. A nonclassical Reddy-Levinson beam model based on a modified couple stress theory. Int J Multiscale Comput Eng. 2010;8:167-80.

[237] Mohammad-Abadi M, Daneshmehr AR. Size dependent buckling analysis of microbeams based on modified couple stress theory with high order theories and general boundary conditions. Int J Eng Sci. 2014;74:1-14.

[238] Salamat-talab M, Nateghi A, Torabi J. Static and dynamic analysis of third-order shear deformation FG micro beam based on modified couple stress theory. Int J Mech Sci. 2012;57:63-73.

[239] Nateghi A, Salamat-talab M, Rezapour J, Daneshian B. Size dependent buckling analysis of functionally graded micro beams based on modified couple stress theory. Appl Math Model. 2012;36:4971-87.

[240] Aghazadeh R, Cigeroglu E, Dag S. Static and free vibration analyses of small-scale functionally graded beams possessing a variable length scale parameter using different beam theories. Eur J Mech A Solids. 2014;46:1-11.

[241] Chen W, Chen W, Sze KY. A model of composite laminated Reddy beam based on a modified couple-stress theory. Compos Struct. 2012;94:2599-609.

[242] Mohammad-Abadi M, Daneshmehr AR. Modified couple stress theory applied to dynamic analysis of composite laminated beams by considering different beam theories. Int J Eng Sci. 2015;87:83-102.

[243] Mohammad-Abadi M, Daneshmehr AR, Homayounfard M. Size-dependent thermal buckling analysis of micro composite laminated beams using modified couple stress theory. Int J Eng Sci. 2015;92:47-62.

[244] Darijani H, Mohammadabadi H. A new deformation beam theory for static and dynamic analysis of microbeams. Int J Mech Sci. 2014;89:31-9.

[245] Noori J, Fariborz SJ, Vafa JP. A higher-order micro-beam model with application to free vibration. Mech Adv Mater Struct. 2016;23:443-50.

[246] Simsek M, Reddy JN. Bending and vibration of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory. Int J Eng Sci. 2013;64:37-53.

[247] Simsek M, Reddy JN. A unified higher order beam theory for buckling of a functionally graded microbeam embedded in elastic medium using modified couple stress theory. Compos Struct. 2013;101:47-58.

[248] Akbarzadeh Khorshidi M, Shariati M, Emam SA. Postbuckling of functionally graded nanobeams based on modified couple stress theory under general beam theory. Int J Mech Sci. 2016;110:160-9.

[249] Trinh LC, Nguyen HX, Vo TP, Nguyen T-K. Size-dependent behaviour of functionally graded microbeams using various shear deformation theories based on the modified couple stress theory. Compos Struct. 2016;154:556-72.

[250] Thai HT, Vo TP, Bui TQ, Nguyen TK. A quasi-3D hyperbolic shear deformation theory for functionally graded plates. Acta Mech. 2014;225:951-64.

[251] Akgoz B, Civalek O. Thermo-mechanical buckling behavior of functionally graded microbeams embedded in elastic medium. Int J Eng Sci. 2014;85:90-104.

[252] Al-Basyouni KS, Tounsi A, Mahmoud SR. Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Compos Struct. 2015;125:621-30.

[253] Tsiatas GC. A new Kirchhoff plate model based on a modified couple stress theory. Int J Solids Struct. 2009;46:2757-64.

[254] Yin L, Qian Q, Wang L, Xia W. Vibration analysis of microscale plates based on modified couple stress theory. Acta Mech Solida Sin. 2010;23:386-93.

[255] Jomehzadeh E, Noori HR, Saidi AR. The size-dependent vibration analysis of micro-plates based on a modified couple stress theory. Phys E. 2011;43:877-83.

[256] Akgoz B, Civalek O. Free vibration analysis for single-layered graphene sheets in an elastic matrix via modified couple stress theory. Mater Des. 2012;42:164-71.

[257] Akgoz B, Civalek O. Modeling and analysis of micro-sized plates resting on elastic medium using the modified couple stress theory. Meccanica. 2013;48:863-73.

[258] Askari AR, Tahani M. Analytical determination of size-dependent natural frequencies of fully clamped rectangular microplates based on the modified couple stress theory. J Mech Sci Technol. 2015;29:2135-45.

[259] Simsek M, Aydın M, Yurtcu HH, Reddy JN. Size-dependent vibration of a microplate under the action of a moving load based on the modified couple stress theory. Acta Mech. 2015;226:3807-22.

[260] Zhou X, Wang L, Qin P. Free vibration of micro- and nano-shells based on modified couple stress theory. J Comput Theor Nanosci. 2012;9:814-8.

[261] Asghari M. Geometrically nonlinear micro-plate formulation based on the modified couple stress theory. Int J Eng Sci. 2012;51:292-309.

[262] Wang YG, Lin WH, Liu N. Large amplitude free vibration of size-dependent circular microplates based on the modified couple stress theory. Int J Mech Sci. 2013;71:51-7.

[263] Wang YG, Lin WH, Zhou CL. Nonlinear bending of size-dependent circular microplates based on the modified couple stress theory. Arch Appl Mech. 2014;84:391-400.

[264] Farokhi H, Ghayesh MH. Nonlinear dynamical behaviour of geometrically imperfect microplates based on modified couple stress theory. Int J Mech Sci. 2015;90:133-44.

[265] Ke LL, Yang J, Kitipornchai S, Bradford MA. Bending, buckling and vibration of size-dependent functionally graded annular microplates. Compos Struct. 2012;94:3250-7.

[266] Asghari M, Taati E. A size-dependent model for functionally graded micro-plates for mechanical analyses. J Vib Control. 2013;19:1614-32.

[267] Ashoori AR, Sadough Vanini SA. Thermal buckling of annular microstructure-dependent functionally graded material plates resting on an elastic medium. Compos B Eng. 2016;87:245-53.

[268] Ashoori AR, Sadough Vanini SA. Nonlinear thermal stability and snap-through behavior of circular microstructure-dependent FGM plates. Eur J Mech A Solids. 2016;59:323-32.

[269] Taati E. Analytical solutions for the size dependent buckling and postbuckling behavior of functionally graded micro-plates. Int J Eng Sci. 2016;100:45-60.

[270] Beni YT, Mehralian F, Zeighampour H. The modified couple stress functionally graded cylindrical thin shell formulation. Mech Adv Mater Struct. 2016;23:791-801.

[271] Tsiatas GC, Yiotis AJ. Size effect on the static, dynamic and buckling analysis of orthotropic Kirchhoff-type skew micro-plates based on a modified couple stress theory: comparison with the nonlocal elasticity theory. Acta Mech. 2015;226:1267-81.

[272] Ma HM, Gao XL, Reddy JN. A non-classical Mindlin plate model based on a modified couple stress theory. Acta Mech. 2011;220:217-35.

[273] Ke LL, Wang YS, Yang J, Kitipornchai S. Free vibration of size-dependent Mindlin microplates based on the modified couple stress theory. J Sound Vib. 2012;331:94-106.

[274] Roque CMC, Ferreira AJM, Reddy JN. Analysis of Mindlin micro plates with a modified couple stress theory and a meshless method. Appl Math Model. 2013;37:4626-33.

[275] Zhou SS, Gao XL. A nonclassical model for circular mindlin plates based on a modified couple stress theory. J Appl Mech. 2014;81:051014 -8.

[276] Alinaghizadeh F, Shariati M, Fish J. Bending analysis of size-dependent functionally graded annular sector microplates based on the modified couple stress theory. Appl Math Model. 2017;44:540-56.

[277] He D, Yang W, Chen W. A size-dependent composite laminated skew plate model based on a new modified couple stress theory. Acta Mech Solida Sin. 2017;30:75-86.

[278] Simsek M, Aydın M. Size-dependent forced vibration of an imperfect functionally graded (FG) microplate with porosities subjected to a moving load using the modified couple stress theory. Compos Struct. 2017;160:408-21.

[279] Reddy JN, Berry J. Nonlinear theories of axisymmetric bending of functionally graded circular plates with modified couple stress. Compos Struct. 2012;94:3664-8.

[280] Ke LL, Yang J, Kitipornchai S, Bradford MA, Wang YS. Axisymmetric nonlinear free vibration of size-dependent functionally graded annular microplates. Compos B Eng. 2013;53:207-17.

[281] Ke LL, Yang J, Kitipornchai S, Wang YS. Axisymmetric postbuckling analysis of size-dependent functionally graded annular microplates using the physical neutral plane. Int J Eng Sci. 2014;81:66-81.

[282] Thai HT, Choi DH. Size-dependent functionally graded Kirchhoff and Mindlin plate models based on a modified couple stress theory. Compos Struct. 2013;95:142-53.

[283] Jung WY, Park WT, Han SC. Bending and vibration analysis of S-FGM microplates embedded in Pasternak elastic medium using the modified couple stress theory. Int J Mech Sci. 2014;87:150-62.

[284] Jung WY, Han SC, Park WT. A modified couple stress theory for buckling analysis of S-FGM nanoplates embedded in Pasternak elastic medium. Compos B Eng. 2014;60:746-56.

[285] Ansari R, Faghih Shojaei M, Mohammadi V, Gholami R, Darabi MA. Nonlinear vibrations of functionally graded Mindlin microplates based on the modified couple stress theory. Compos Struct. 2014;114:124-34.

[286] Ansari R, Gholami R, Faghih Shojaei M, Mohammadi V, Darabi MA. Size-dependent nonlinear bending and postbuckling of functionally graded Mindlin rectangular microplates considering the physical neutral plane position. Compos Struct. 2015;127:87-98.

[287] Ansari R, Faghihshojaei M, Mohammadi V, Gholami R, Darabi MA. Size-dependent vibrations of post-buckled functionally graded mindlin rectangular microplates. Latin Am J Solids Struct. 2014;11:2351-78.

[288] Lou J, He L. Closed-form solutions for nonlinear bending and free vibration of functionally graded microplates based on the modified couple stress theory. Compos Struct. 2015;131:810-20.

[289] Zeighampour H, Beni YT. A shear deformable cylindrical shell model based on couple stress theory. Arch Appl Mech. 2015;85:539-53.

[290] Hosseini-Hashemi S, Sharifpour F, Ilkhani MR. On the free vibrations of size-dependent closed micro/nano-spherical shell based on the modified couple stress theory. Int J Mech Sci. 2016;115–116:501-15.

[291] Gholami R, Ansari R, Darvizeh A, Sahmani S. Axial buckling and dynamic stability of functionally

graded microshells based on the modified couple stress theory. Int J Struct Stab Dyn. 2015;15:1450070-24.

[292] Tadi Beni Y, Mehralian F, Razavi H. Free vibration analysis of size-dependent shear deformable functionally graded cylindrical shell on the basis of modified couple stress theory. Compos Struct. 2015;120:65-78.

[293] Lou J, He L, Wu H, Du J. Pre-buckling and buckling analyses of functionally graded microshells under axial and radial loads based on the modified couple stress theory. Compos Struct. 2016;142:226-37.

[294] Gao XL, Huang JX, Reddy JN. A non-classical third-order shear deformation plate model based on a modified couple stress theory. Acta Mech. 2013;224:2699-718.

[295] Thai HT, Kim SE. A size-dependent functionally graded Reddy plate model based on a modified couple stress theory. Compos B Eng. 2013;45:1636-45.

[296] Chen W, Xu M, Li L. A model of composite laminated Reddy plate based on new modified couple stress theory. Compos Struct. 2012;94:2143-56.

[297] Jung WY, Han SC. Static and eigenvalue problems of sigmoid functionally graded materials (S-FGM) micro-scale plates using the modified couple stress theory. Appl Math Model. 2015;39:3506-24.

[298] Eshraghi I, Dag S, Soltani N. Consideration of spatial variation of the length scale parameter in static and dynamic analyses of functionally graded annular and circular micro-plates. Compos B Eng. 2015;78:338-48.

[299] Eshraghi I, Dag S, Soltani N. Bending and free vibrations of functionally graded annular and circular micro-plates under thermal loading. Compos Struct. 2016;137:196-207.

[300] Ghayesh MH, Farokhi H. Coupled size-dependent behavior of shear deformable microplates. Acta Mech. 2016;227:757-75.

[301] Sahmani S, Ansari R, Gholami R, Darvizeh A. Dynamic stability analysis of functionally graded higher-order shear deformable microshells based on the modified couple stress elasticity theory. Compos B Eng. 2013;51:44-53.

[302] Thai HT, Vo TP. A size-dependent functionally graded sinusoidal plate model based on a modified couple stress theory. Compos Struct. 2013;96:376-83.

[303] Darijani H, Shahdadi AH. A new shear deformation model with modified couple stress theory for microplates. Acta Mech. 2015;226:2773-88.

[304] He L, Lou J, Zhang E, Wang Y, Bai Y. A size-dependent four variable refined plate model for functionally graded microplates based on modified couple stress theory. Compos Struct. 2015;130:107-15.

[305] Lou J, He L, Du J. A unified higher order plate theory for functionally graded microplates based on the modified couple stress theory. Compos Struct. 2015;133:1036-47.

[306] Lou J, He L, Du J, Wu H. Nonlinear analyses of functionally graded microplates based on a general four-variable refined plate model and the modified couple stress theory. Compos Struct. 2016;152:516-27.

[307] Trinh LC, Vo TP, Thai HT, Mantari JL. Size-dependent behaviour of functionally graded sandwich

microplates under mechanical and thermal loads. Compos B Eng. 2017;124:218-41.

[308] Reddy JN, Kim J. A nonlinear modified couple stress-based third-order theory of functionally graded plates. Compos Struct. 2012;94:1128-43.

[309] Kim J, Reddy JN. Analytical solutions for bending, vibration, and buckling of FGM plates using a couple stress-based third-order theory. Compos Struct. 2013;103:86-98.

[310] Lei J, He Y, Zhang B, Liu D, Shen L, Guo S. A size-dependent FG micro-plate model incorporating higher-order shear and normal deformation effects based on a modified couple stress theory. Int J Mech Sci. 2015;104:8-23.

[311] Kong S, Zhou S, Nie Z, Wang K. Static and dynamic analysis of micro beams based on strain gradient elasticity theory. Int J Eng Sci. 2009;47:487-98.

[312] Akgoz B, Civalek O. Strain gradient elasticity and modified couple stress models for buckling analysis of axially loaded micro-scaled beams. Int J Eng Sci. 2011;49:1268-80.

[313] Akgoz B, Civalek O. Buckling analysis of cantilever carbon nanotubes using the strain gradient elasticity and modified couple stress theories. J Comput Theor Nanosci. 2011;8:1821-7.

[314] Akgoz B, Civalek O. Investigation of size effects on static response of single-walled carbon nanotubes based on strain gradient elasticity. Int J Comput Methods. 2012;9:1240032-19

[315] Akgoz B, Civalek O. Buckling analysis of linearly tapered micro-Columns based on strain gradient elasticity. Struct Eng Mech. 2013;48:195-205.

[316] Akgoz B, Civalek O. Longitudinal vibration analysis for microbars based on strain gradient elasticity theory. J Vib Control. 2014;20:606-16.

[317] Zhao J, Zhou S, Wang B, Wang X. Nonlinear microbeam model based on strain gradient theory. Appl Math Model. 2012;36:2674-86.

[318] Rajabi F, Ramezani S. A nonlinear microbeam model based on strain gradient elasticity theory. Acta Mech Solida Sin. 2013;26:21-34.

[319] Mohammadi H, Mahzoon M. Thermal effects on postbuckling of nonlinear microbeams based on the modified strain gradient theory. Compos Struct. 2013;106:764-76.

[320] Vatankhah R, Kahrobaiyan MH, Alasty A, Ahmadian MT. Nonlinear forced vibration of strain gradient microbeams. Appl Math Model. 2013;37:8363-82.

[321] Kahrobaiyan MH, Rahaeifard M, Tajalli SA, Ahmadian MT. A strain gradient functionally graded Euler–Bernoulli beam formulation. Int J Eng Sci. 2012;52:65-76.

[322] Akgoz B, Civalek O. Buckling analysis of functionally graded microbeams based on the strain gradient theory. Acta Mech. 2013;224:2185-201.

[323] Akgoz B, Civalek O. Longitudinal vibration analysis of strain gradient bars made of functionally graded materials (FGM). Compos B Eng. 2013;55:263-8.

[324] Rahaeifard M, Kahrobaiyan MH, Ahmadian MT, Firoozbakhsh K. Strain gradient formulation of functionally graded nonlinear beams. Int J Eng Sci. 2013;65:49-63.

[325] Wang B, Zhao J, Zhou S. A micro scale Timoshenko beam model based on strain gradient elasticity theory. Eur J Mech A Solids. 2010;29:591-9.

[326] Ansari R, Gholami R, Darabi MA. A nonlinear Timoshenko beam formulation based on strain gradient theory. J Mech Mater Struct. 2012;7:195-211.

[327] Asghari M, Kahrobaiyan MH, Nikfar M, Ahmadian MT. A size-dependent nonlinear Timoshenko microbeam model based on the strain gradient theory. Acta Mech. 2012;223:1233-49.

[328] Ansari R, Gholami R, Sahmani S. Free vibration analysis of size-dependent functionally graded microbeams based on the strain gradient Timoshenko beam theory. Compos Struct. 2011;94:221-8.

[329] Ansari R, Gholami R, Sahmani S. Size-dependent vibration of functionally graded curved microbeams based on the modified strain gradient elasticity theory. Arch Appl Mech. 2013;83:1439-49.

[330] Ansari R, Gholami R, Faghih Shojaei M, Mohammadi V, Sahmani S. Buckling of FGM Timoshenko microbeams under in-plane thermal loading based on the modified strain gradient theory. Int J Multiscale Comput Eng. 2013;11:389-405.

[331] Ansari R, Faraji Oskouie M, Rouhi H. Studying linear and nonlinear vibrations of fractional viscoelastic Timoshenko micro-/nano-beams using the strain gradient theory. Nonlinear Dyn. 2017;87:695-711.

[332] Gholami R, Ansari R, Rouhi H. Studying the effects of small scale and Casimir force on the non-linear pull-in instability and vibrations of FGM microswitches under electrostatic actuation. Int J Non-Linear Mech. 2015;77:193-207.

[333] Xie X, Zheng H, Yang H. Indirect radial basis function approach for bending, free vibration and buckling analyses of functionally graded microbeams. Compos Struct. 2015;131:606-15.

[334] Tajalli SA, Rahaeifard M, Kahrobaiyan MH, Movahhedy MR, Akbari J, Ahmadian MT. Mechanical behavior analysis of size-dependent micro-scaled functionally graded Timoshenko beams by strain gradient elasticity theory. Compos Struct. 2013;102:72-80.

[335] Ansari R, Shojaei MF, Mohammadi V, Gholami R, Darabi MA. Buckling and postbuckling behavior of functionally graded Timoshenko microbeams based on the strain gradient theory. J Mech Mater Struct. 2012;7:931-49.

[336] Ansari R, Faghih Shojaei M, Gholami R, Mohammadi V, Darabi MA. Thermal postbuckling behavior of size-dependent functionally graded Timoshenko microbeams. Int J Non-Linear Mech. 2013;50:127-35.

[337] Wang B, Liu M, Zhao J, Zhou S. A size-dependent Reddy-Levinson beam model based on a strain gradient elasticity theory. Meccanica. 2014;49:1427-41.

[338] Sahmani S, Ansari R. Size-dependent buckling analysis of functionally graded third-order shear deformable microbeams including thermal environment effect. Appl Math Model. 2013;37:9499-515.

[339] Ansari R, Gholami R, Sahmani S. Free vibration of size-dependent functionally graded microbeams based on the strain gradient reddy beam theory. Int J Comput Methods Eng Sci Mech. 2014;15:401-12.

[340] Zhang B, He Y, Liu D, Gan Z, Shen L. Size-dependent functionally graded beam model based on an improved third-order shear deformation theory. Eur J Mech A Solids. 2014;47:211-30.

[341] Shi G. A new simple third-order shear deformation theory of plates. Int J Solids Struct. 2007;44:4399-417.

[342] Sahmani S, Bahrami M, Ansari R. Nonlinear free vibration analysis of functionally graded third-order shear deformable microbeams based on the modified strain gradient elasticity theory. Compos Struct. 2014;110:219-30.

[343] Xiang S, Jin Y-x, Bi Z-y, Jiang S-x, Yang M-s. A n-order shear deformation theory for free vibration of functionally graded and composite sandwich plates. Compos Struct. 2011;93:2826-32.

[344] Akgoz B, Civalek O. A size-dependent shear deformation beam model based on the strain gradient elasticity theory. Int J Eng Sci. 2013;70:1-14.

[345] Lei J, He Y, Zhang B, Gan Z, Zeng P. Bending and vibration of functionally graded sinusoidal microbeams based on the strain gradient elasticity theory. Int J Eng Sci. 2013;72:36-52.

[346] Akgoz B, Civalek O. A new trigonometric beam model for buckling of strain gradient microbeams. Int J Mech Sci. 2014;81:88-94.

[347] Akgoz B, Civalek O. Shear deformation beam models for functionally graded microbeams with new shear correction factors. Compos Struct. 2014;112:214-25.

[348] Akgoz B, Civalek O. Bending analysis of FG microbeams resting on Winkler elastic foundation via strain gradient elasticity. Compos Struct. 2015;134:294-301.

[349] Akgoz B, Civalek O. A novel microstructure-dependent shear deformable beam model. Int J Mech Sci. 2015;99:10-20.

[350] Akgoz B, Civalek O. Bending analysis of embedded carbon nanotubes resting on an elastic foundation using strain gradient theory. Acta Astronaut. 2016;119:1-12.

[351] Zhang B, He Y, Liu D, Gan Z, Shen L. A novel size-dependent functionally graded curved mircobeam model based on the strain gradient elasticity theory. Compos Struct. 2013;106:374-92.

[352] Wang B, Zhou S, Zhao J, Chen X. A size-dependent Kirchhoff micro-plate model based on strain gradient elasticity theory. Eur J Mech A Solids. 2011;30:517-24.

[353] Ashoori Movassagh A, Mahmoodi MJ. A micro-scale modeling of Kirchhoff plate based on modified strain-gradient elasticity theory. Eur J Mech A Solids. 2013;40:50-9.

[354] Mohammadi M, Fooladi Mahani M. An analytical solution for buckling analysis of size-dependent rectangular micro-plates according to the modified strain gradient and couple stress theories. Acta Mech. 2015;226:3477-93.

[355] Mohammadi M, Fooladi M, Darijani H. Exact boundary conditions for buckling analysis of rectangular micro-plates based on the modified strain gradient theory. Int J Multiscale Comput Eng. 2015;13:265-80.

[356] Wang B, Huang S, Zhao J, Zhou S. Reconsiderations on boundary conditions of Kirchhoff micro-plate model based on a strain gradient elasticity theory. Appl Math Model. 2016;40:7303-17.

[357] Zeighampour H, Tadi Beni Y. Cylindrical thin-shell model based on modified strain gradient theory. Int J Eng Sci. 2014;78:27-47.

[358] Allahbakhshi A, Allahbakhshi M. Vibration analysis of nano-structure multilayered graphene sheets using modified strain gradient theory. Front Mech Eng. 2015;10:187-97.

[359] Li A, Zhou S, Wang B. A size-dependent model for bi-layered Kirchhoff micro-plate based on strain gradient elasticity theory. Compos Struct. 2014;113:272-80.

[360] Hosseini M, Bahreman M, Jamalpoor A. Using the modified strain gradient theory to investigate the size-dependent biaxial buckling analysis of an orthotropic multi-microplate system. Acta Mech. 2016;227:1621-43.

[361] Zhang L, Liang B, Zhou S, Wang B, Xue Y. An application of a size-dependent model on microplate with elastic medium based on strain gradient elasticity theory. Meccanica. 2016;52:251-62.

[362] Sahmani S, Ansari R. On the free vibration response of functionally graded higher-order shear deformable microplates based on the strain gradient elasticity theory. Compos Struct. 2013;95:430-42.

[363] Ansari R, Gholami R, Faghih Shojaei M, Mohammadi V, Darabi MA. Thermal buckling analysis of a mindlin rectangular FGM microplate based on the strain gradient theory. J Therm Stresses. 2013;36:446-65.

[364] Ansari R, Shojaei MF, Mohammadi V, Gholami R, Rouhi H. Size-dependent thermal buckling and postbuckling of functionally graded annular microplates based on the modified strain gradient theory. J Therm Stresses. 2014;37:174-201.

[365] Ansari R, Hasrati E, Faghih Shojaei M, Gholami R, Mohammadi V, Shahabodini A. Size-dependent bending, buckling and free vibration analyses of microscale functionally graded mindlin plates based on the strain gradient elasticity theory. Latin Am J Solids Struct. 2016;13:632-64.

[366] Shenas AG, Malekzadeh P. Free vibration of functionally graded quadrilateral microplates in thermal environment. Thin Wall Struct. 2016;106:294-315.

[367] Ansari R, Gholami R, Faghih Shojaei M, Mohammadi V, Sahmani S. Bending, buckling and free vibration analysis of size-dependent functionally graded circular/annular microplates based on the modified strain gradient elasticity theory. Eur J Mech A Solids. 2015;49:251-67.

[368] Gholami R, Darvizeh A, Ansari R, Hosseinzadeh M. Size-dependent axial buckling analysis of functionally graded circular cylindrical microshells based on the modified strain gradient elasticity theory. Meccanica. 2014;49:1679-95.

[369] Zhang B, He Y, Liu D, Shen L, Lei J. Free vibration analysis of four-unknown shear deformable functionally graded cylindrical microshells based on the strain gradient elasticity theory. Compos Struct. 2015;119:578-97.

[370] Thai HT, Choi DH. A simple first-order shear deformation theory for laminated composite plates. Compos Struct. 2013;106:754-63.

[371] Thai HT, Choi DH. A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates. Compos Struct. 2013;101:332-40.

[372] Thai HT, Nguyen TK, Vo TP, Lee J. Analysis of functionally graded sandwich plates using a new first-order shear deformation theory. Eur J Mech A Solids. 2014;45:211-25.

[373] Zhang B, He Y, Liu D, Lei J, Shen L, Wang L. A size-dependent third-order shear deformable plate model incorporating strain gradient effects for mechanical analysis of functionally graded circular/annular microplates. Compos B Eng. 2015;79:553-80.

[374] Thai HT, Kim SE. A simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded plates. Compos Struct. 2013;96:165-73.

[375] Zhang B, He Y, Liu D, Shen L, Lei J. An efficient size-dependent plate theory for bending, buckling and free vibration analyses of functionally graded microplates resting on elastic foundation. Appl Math Model. 2015;39:3814-45.

[376] Thai HT, Choi DH. A refined plate theory for functionally graded plates resting on elastic foundation. Composites Science and Technology. 2011;71:1850-8.

[377] Thai HT, Choi DH. A refined shear deformation theory for free vibration of functionally graded plates on elastic foundation. Compos B Eng. 2012;43:2335-47.

[378] Thai HT, Choi DH. An efficient and simple refined theory for buckling analysis of functionally graded plates. Appl Math Model. 2012;36:1008-22.

[379] Thai HT, Choi DH. Analytical solutions of refined plate theory for bending, buckling and vibration analyses of thick plates. Appl Math Model. 2013;37:8310-23.

[380] Akgoz B, Civalek O. A microstructure-dependent sinusoidal plate model based on the strain gradient elasticity theory. Acta Mech. 2015;226:2277-94.

[381] Eltaher MA, Emam SA, Mahmoud FF. Free vibration analysis of functionally graded size-dependent nanobeams. Appl Math Comput. 2012;218:7406-20.

[382] Eltaher MA, Emam SA, Mahmoud FF. Static and stability analysis of nonlocal functionally graded nanobeams. Compos Struct. 2013;96:82-8.

[383] Eltaher MA, Alshorbagy AE, Mahmoud FF. Determination of neutral axis position and its effect on natural frequencies of functionally graded macro/nanobeams. Compos Struct. 2013;99:193-201.

[384] Eltaher MA, Alshorbagy AE, Mahmoud FF. Vibration analysis of Euler-Bernoulli nanobeams by using finite element method. Appl Math Model. 2013;37:4787-97.

[385] Alshorbagy AE, Eltaher MA, Mahmoud FF. Static analysis of nanobeams using nonlocal FEM. J Mech Sci Technol. 2013;27:2035-41.

[386] Marotti De Sciarra F. Finite element modelling of nonlocal beams. Phys E. 2014;59:144-9.

[387] Nguyen NT, Kim NI, Lee J. Mixed finite element analysis of nonlocal Euler-Bernoulli nanobeams. Finite Elem Anal Des. 2015;106:65-72.

[388] Reddy JN, El-Borgi S. Eringen's nonlocal theories of beams accounting for moderate rotations. Int J Eng Sci. 2014;82:159-77.

[389] Reddy JN, El-Borgi S, Romanoff J. Non-linear analysis of functionally graded microbeams using Eringen's non-local differential model. Int J Non-Linear Mech. 2014;67:308-18.

[390] Eltaher MA, Khairy A, Sadoun AM, Omar FA. Static and buckling analysis of functionally graded Timoshenko nanobeams. Appl Math Comput. 2014;229:283-95.

[391] Arbind A, Reddy JN. Nonlinear analysis of functionally graded microstructure-dependent beams. Compos Struct. 2013;98:272-81.

[392] Arbind A, Reddy JN, Srinivasa AR. Modified couple stress-based third-order theory for nonlinear analysis of functionally graded beams. Latin Am J Solids Struct. 2014;11:459-87.

[393] Reddy JN, Srinivasa AR. Non-linear theories of beams and plates accounting for moderate rotations and material length scales. Int J Non-Linear Mech. 2014;66:43-53.

[394] Dehrouyeh-Semnani AM, Nikkhah-Bahrami M. The influence of size-dependent shear deformation on mechanical behavior of microstructures-dependent beam based on modified couple stress theory. Compos Struct. 2015;123:325-36.

[395] Kahrobaiyan MH, Asghari M, Ahmadian MT. A Timoshenko beam element based on the modified couple stress theory. Int J Mech Sci. 2014;79:75-83.

[396] Dehrouyeh-Semnani AM, Bahrami A. On size-dependent Timoshenko beam element based on modified couple stress theory. Int J Eng Sci. 2016;107:134-48.

[397] Karttunen AT, Romanoff J, Reddy JN. Exact microstructure-dependent Timoshenko beam element. Int J Mech Sci. 2016;111-112:35-42.

[398] Kahrobaiyan MH, Asghari M, Ahmadian MT. Strain gradient beam element. Finite Elem Anal Des. 2013;68:63-75.

[399] Zhang B, He Y, Liu D, Gan Z, Shen L. Non-classical Timoshenko beam element based on the strain gradient elasticity theory. Finite Elem Anal Des. 2014;79:22-39.

[400] Zhang L, Wang B, Liang B, Zhou S, Xue Y. A size-dependent finite-element model for a micro/nanoscale timoshenko beam. Int J Multiscale Comput Eng. 2015;13:491-506.

[401] Kahrobaiyan MH, Asghari M, Ahmadian MT. A strain gradient Timoshenko beam element: application to MEMS. Acta Mech. 2015;226:505-25.

[402] Eltaher MA, Hamed MA, Sadoun AM, Mansour A. Mechanical analysis of higher order gradient nanobeams. Appl Math Comput. 2014;229:260-72.

[403] Ebrahimi F, Ansari R, Shojaei MF, Rouhi H. Postbuckling analysis of microscale beams based on a strain gradient finite element approach. Meccanica. 2016;51:2493-507.

[404] Ansari R, Shojaei MF, Ebrahimi F, Rouhi H, Bazdid-Vahdati M. A novel size-dependent microbeam element based on Mindlin's strain gradient theory. Eng Comput. 2016;32:99-108.

[405] Ansari R, Shojaei MF, Rouhi H. Small-scale Timoshenko beam element. European Journal of Mechanics-A/Solids. 2015;53:19-33.

[406] Ansari R, Faghih Shojaei M, Ebrahimi F, Rouhi H. A non-classical Timoshenko beam element for the postbuckling analysis of microbeams based on Mindlin's strain gradient theory. Arch Appl Mech. 2015;85:937-53.

[407] Phadikar JK, Pradhan SC. Variational formulation and finite element analysis for nonlocal elastic nanobeams and nanoplates. Comput Mater Sci. 2010;49:492-9.

[408] Ansari R, Rajabiehfard R, Arash B. Nonlocal finite element model for vibrations of embedded multi-layered graphene sheets. Comput Mater Sci. 2010;49:831-8.

[409] Natarajan S, Chakraborty S, Thangavel M, Bordas S, Rabczuk T. Size-dependent free flexural vibration behavior of functionally graded nanoplates. Comput Mater Sci. 2012;65:74-80.

[410] Nguyen NT, Hui D, Lee J, Nguyen-Xuan H. An efficient computational approach for size-dependent analysis of functionally graded nanoplates. Comput Methods Appl Mech Eng. 2015;297:191-218.

[411] Ansari R, Norouzzadeh A. Nonlocal and surface effects on the buckling behavior of functionally graded nanoplates: an isogeometric analysis. Phys E. 2016;84:84-97.

[412] Sarrami-Foroushani S, Azhari M. Nonlocal buckling and vibration analysis of thick rectangular nanoplates using finite strip method based on refined plate theory. Acta Mech. 2016;227:721-42.

[413] Zhang B, He Y, Liu D, Gan Z, Shen L. A non-classical Mindlin plate finite element based on a modified couple stress theory. Eur J Mech A Solids. 2013;42:63-80.

[414] Mirsalehi M, Azhari M, Amoushahi H. Stability of thin FGM microplate subjected to mechanical and thermal loading based on the modified couple stress theory and spline finite strip method. Aerosp Sci Technol. 2015;47:356-66.

[415] Kim J, Reddy JN. A general third-order theory of functionally graded plates with modified couple stress effect and the von Kármán nonlinearity: theory and finite element analysis. Acta Mech. 2015;226:2973-98.

[416] Reddy JN, Romanoff J, Loya JA. Nonlinear finite element analysis of functionally graded circular plates with modified couple stress theory. Eur J Mech A Solids. 2016;56:92-104.

[417] Nguyen HX, Nguyen TN, Abdel-Wahab M, Bordas SPA, Nguyen-Xuan H, Vo TP. A refined quasi-3D isogeometric analysis for functionally graded microplates based on the modified couple stress theory. Comput Methods Appl Mech Eng. 2017;313:904-40.

[418] Mirsalehi M, Azhari M, Amoushahi H. Buckling and free vibration of the FGM thin micro-plate based on the modified strain gradient theory and the spline finite strip method. Eur J Mech A Solids. 2017;61:1-13.

[419] Thai S, Thai HT, Vo TP, Patel VI. Size-dependent behaviour of functionally graded microplates based on the modified strain gradient theory and isogeometric analysis. Comput Struct. 2017;doi: 10.1016/j.compstruc.2017.05.014.

[420] Ansari R, Faghih Shojaei M, Mohammadi V, Bazdid-Vahdati M, Rouhi H. Triangular Mindlin microplate element. Comput Methods Appl Mech Eng. 2015;295:56-76.

[421] Ansari R, Faghih Shojaei M, Shakouri AH, Rouhi H. Nonlinear bending analysis of first-order shear deformable microscale plates using a strain gradient quadrilateral element. J Comput Nonlinear Dyn. 2016;11:051014-18.

[422] Mukhopadhyay P, Gupta RK. Graphite, Graphene, and their polymer nanocomposites: CRC Press, 2012.

[423] Pumera M. Graphene-based nanomaterials and their electrochemistry. Chemical Society Reviews. 2010;39:4146-57.

(a) SWCNT   (b) DWCNT   (c) MWCNT
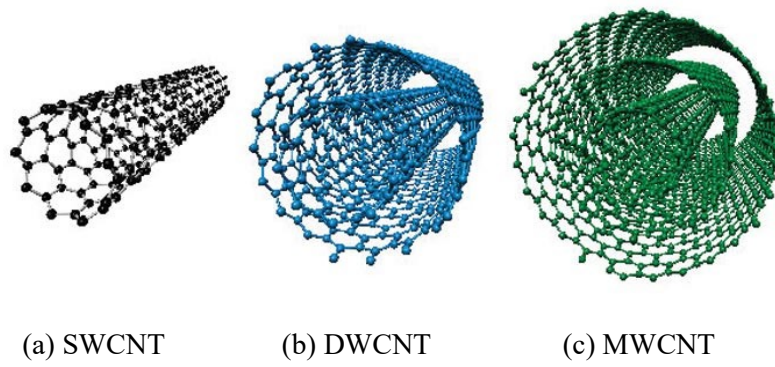
Fig. 1 Schematic illustration of different form of CNTs [422]
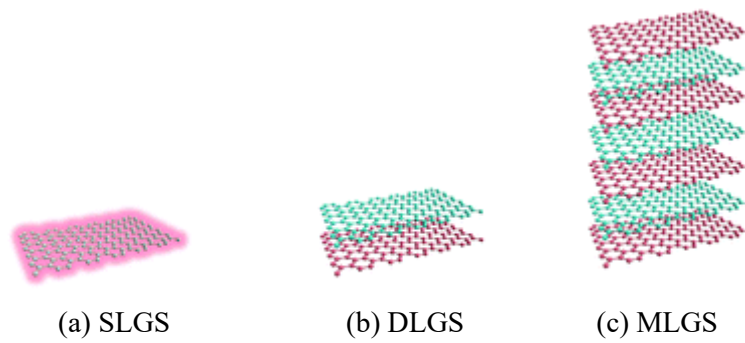

(a) SLGS   (b) DLGS   (c) MLGS
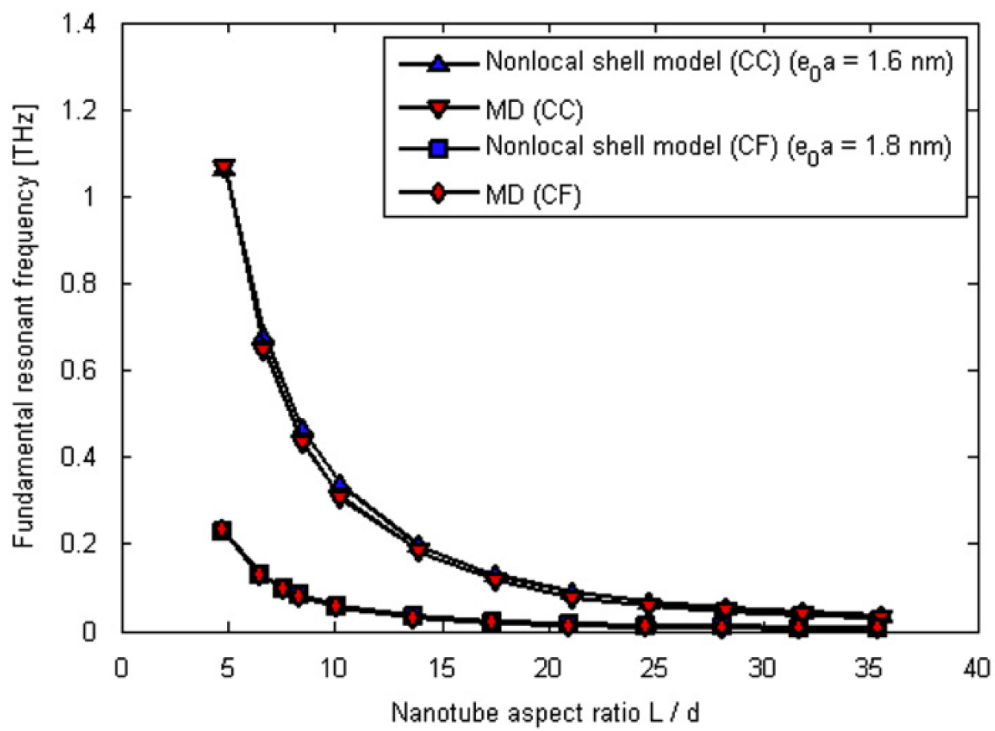
Fig. 2 Graphene-based nanomaterials [423]

Fig. 3 Fundamental frequencies of clamped (CC) and cantilever (CF) beams [45]
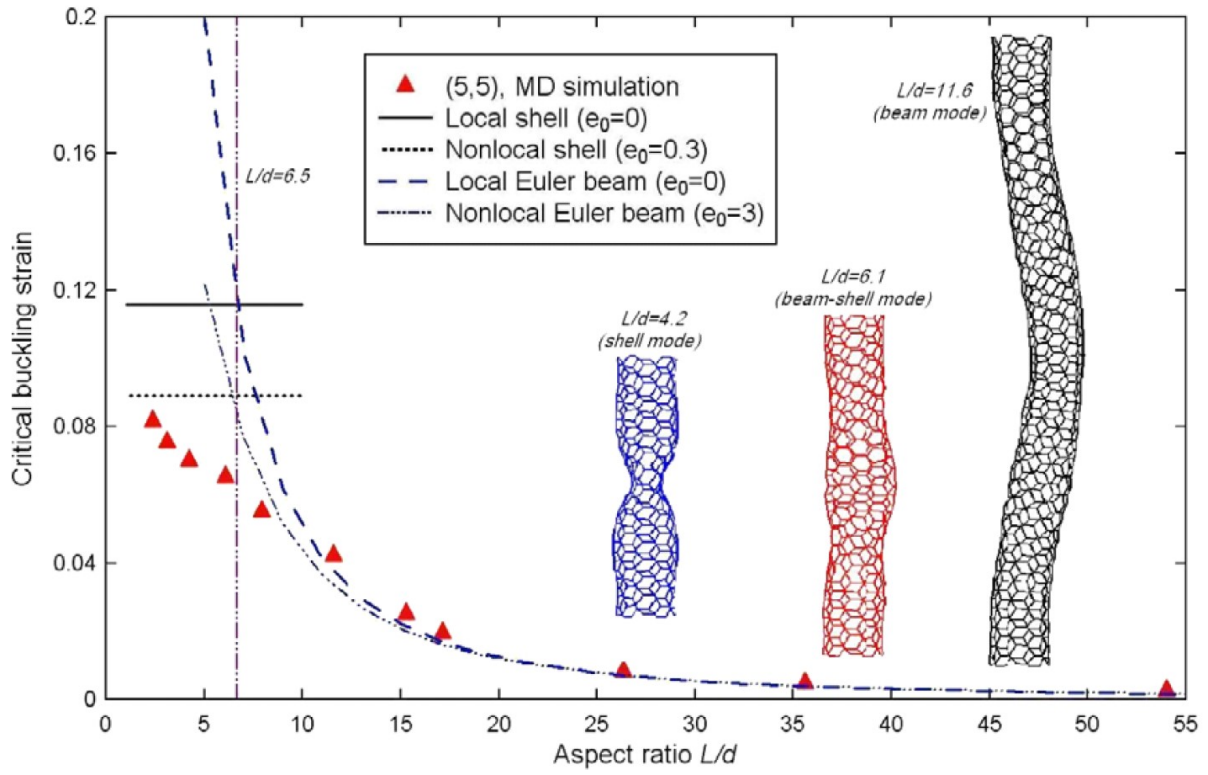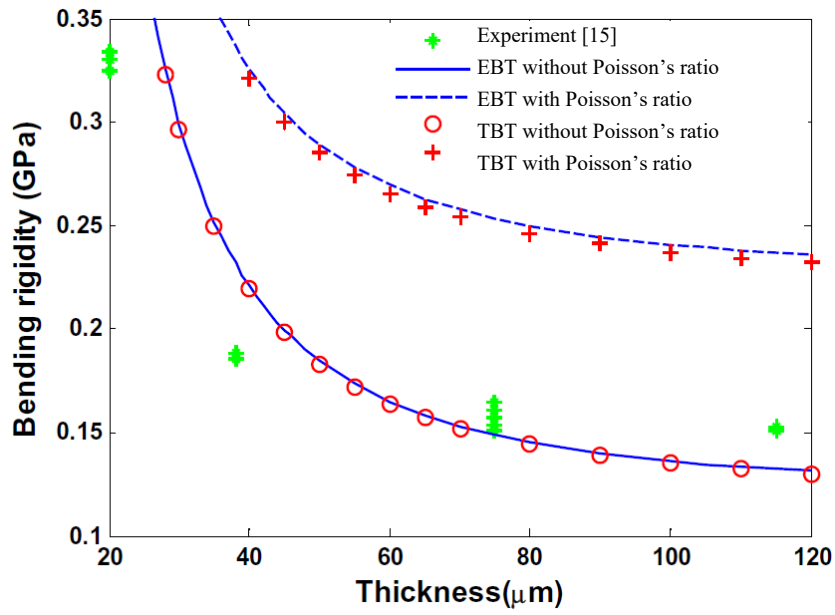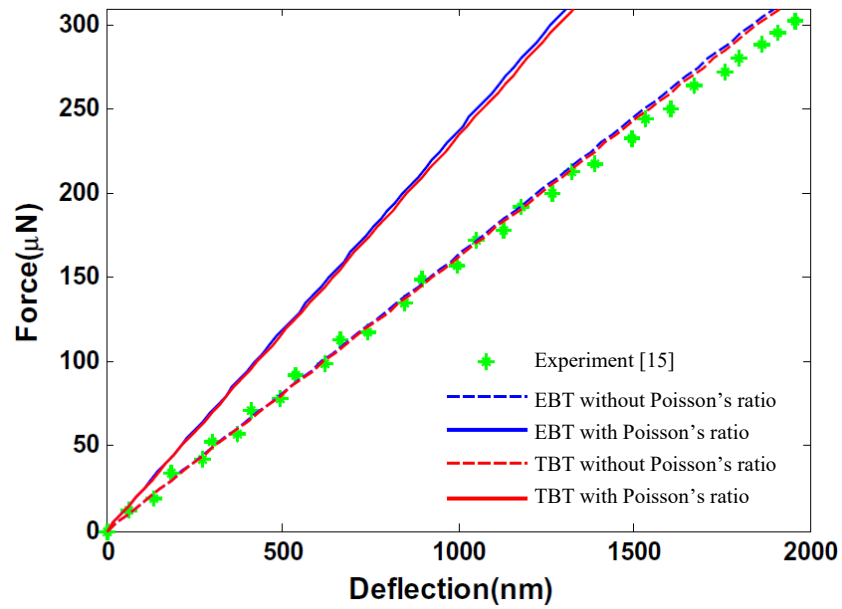
57

Fig. 4 Comparison of various continuum mechanics models with MD simulations for (5,5) SWCNTs [130]

(a) Bending stiffness versus thickness

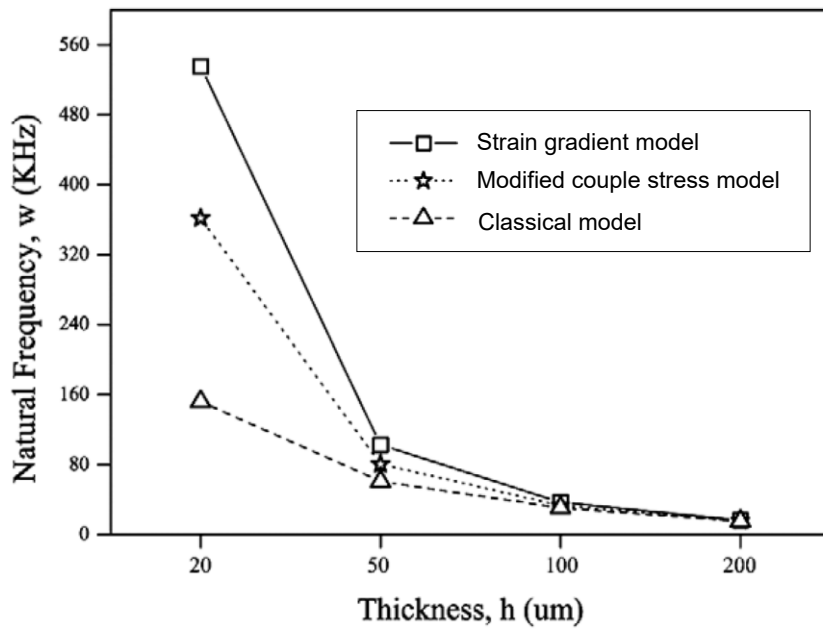(b) Load-deflection response ($h = 38$ μm)

Fig. 5 Effect of Poisson's ratio in a cantilever microbeam [218]
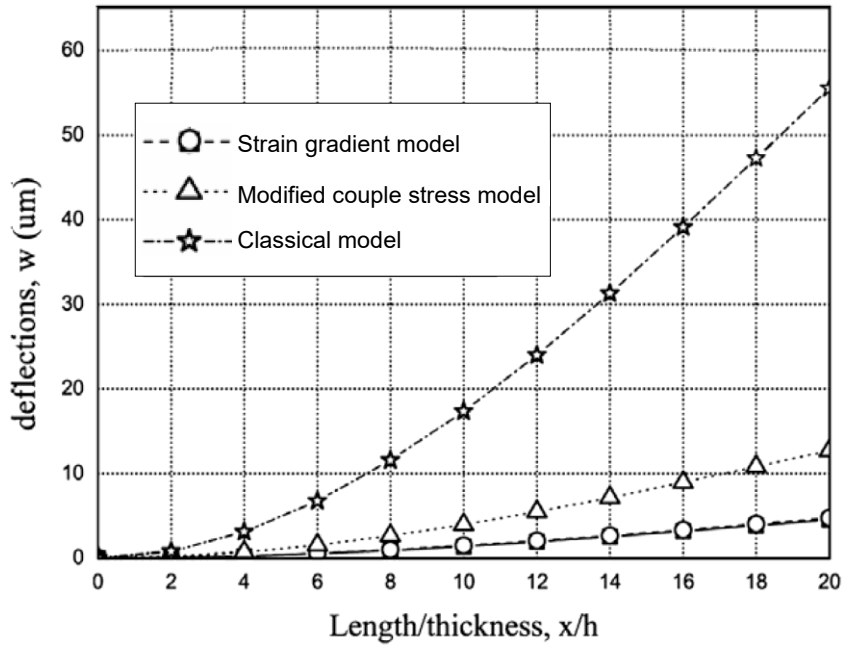
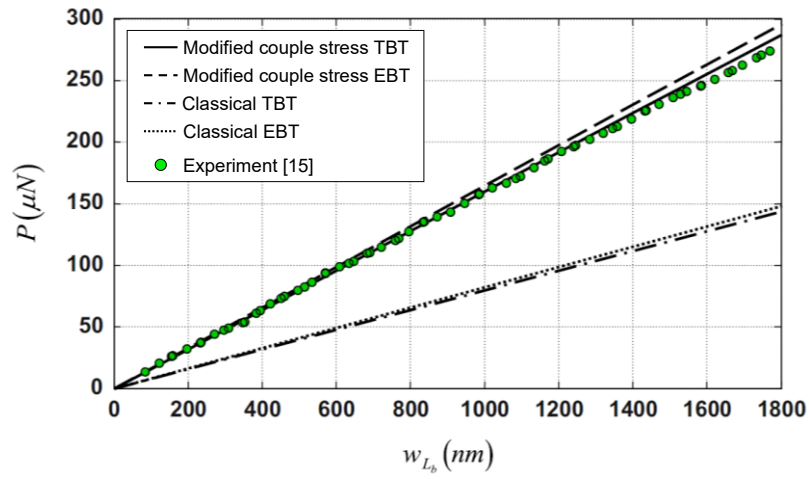Fig. 6 Comparison between strain gradient model and couple stress model for microbeams [311]

61

Fig. 8 Comparison of couple stress model with experimental result for cantilever microbeams [395]
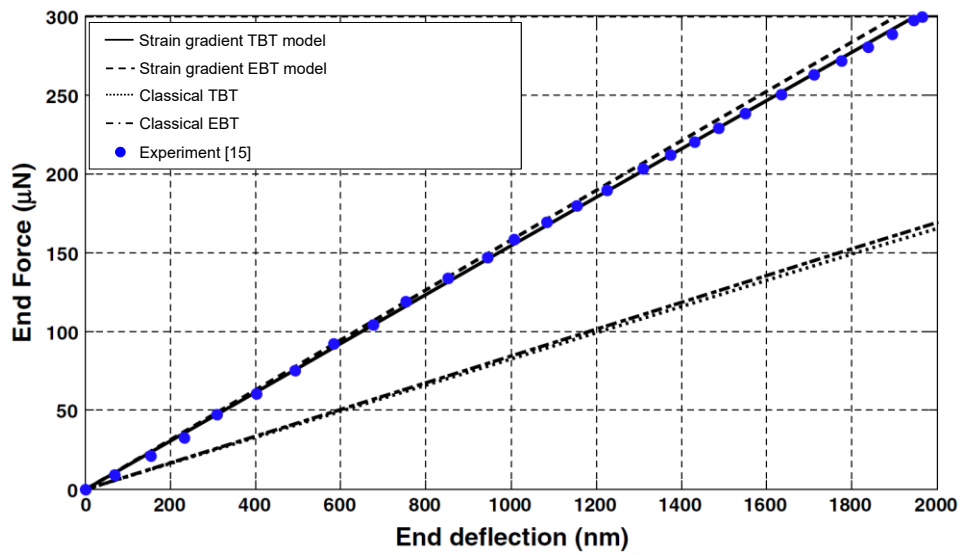

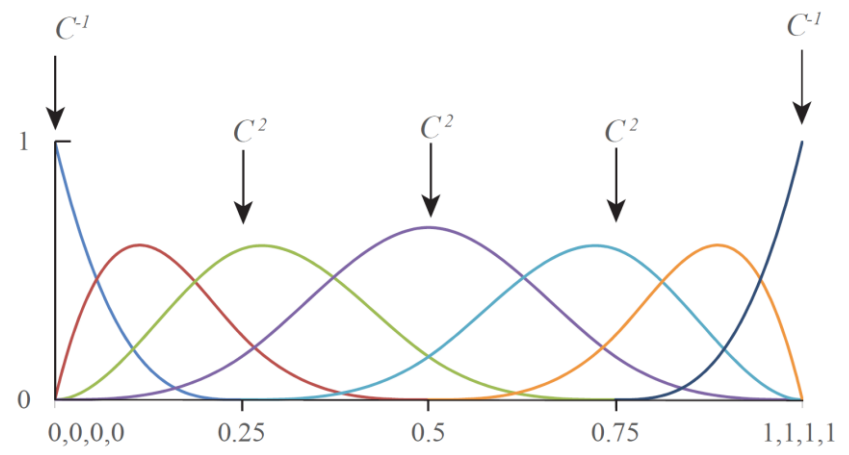Fig. 9 Comparison of strain gradient model with experimental result for cantilever microbeams [401]

Fig. 10 NURBS basic functions

Table 1. Displacement field of HSDTs

Reference	Displacement field	Shape function	Unknown
Aydogdu [106] Karama et al. [107] Touratier [113] Soldatos [193]	$u_x(x,y,z,t)=u(x,y,t)-z\dfrac{\partial w}{\partial x}+f(z)\varphi_x(x,y,t)$ $u_y(x,y,z,t)=v(x,y,t)-z\dfrac{\partial w}{\partial y}+f(z)\varphi_y(x,y,t)$ $u_z(x,y,z,t)=w(x,y,t)$	$f(z)=z\alpha^{\frac{-2(z/h)^2}{\ln\alpha}}$, with $\alpha>0$ [106] $f(z)=ze^{-2(z/h)^2}$ [107] $f(z)=\dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$ [113] $f(z)=h\sinh\left(\dfrac{z}{h}\right)-z\cosh\dfrac{1}{2}$ [193]	$u,v,w,\varphi_x,\varphi_y$
Shimpi [109] Thai and Choi [121] Thai and Vo [191] Darijani and Mohammadabadi [244] Darijani and Shahdadi [303]	$u_x(x,y,z,t)=u(x,y,t)-z\dfrac{\partial w_b}{\partial x}-f(z)\dfrac{\partial w_s}{\partial x}$ $u_y(x,y,z,t)=v(x,y,t)-z\dfrac{\partial w_b}{\partial y}-f(z)\dfrac{\partial w_s}{\partial y}$ $u_z(x,y,z,t)=w_b(x,y,t)+w_s(x,y,t)$	$f(z)=\left[\dfrac{5}{3}\left(\dfrac{z}{h}\right)^2-\dfrac{1}{4}\right]z$ [109] $f(z)=\dfrac{4z^3}{3h^2}$ [121] $f(z)=z-\dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$ [121, 191] $f(z)=z-h\sinh\left(\dfrac{z}{h}\right)+z\cosh\dfrac{1}{2}$ [121] $f(z)=z-ze^{-2(z/h)^2}$ [121] $f(z)=\dfrac{z^5}{20h^4}-\dfrac{z^3}{24h^2}-\dfrac{63z}{64}+\alpha$ [244] $f(z)=z\left[-\dfrac{121}{16}+\dfrac{455}{4}\left(\dfrac{z}{h}\right)^2-483\left(\dfrac{z}{h}\right)^4+660\left(\dfrac{z}{h}\right)^6\right]$ [303]	u,v,w_b,w_s
Xiang et al. [343]	$u_x(x,y,z,t)=u(x,y,t)+z\varphi_x(x,y,t)-f(z)\left(\varphi_x+\dfrac{\partial w}{\partial x}\right)$ $u_y(x,y,z,t)=v(x,y,t)+z\varphi_y(x,y,t)-f(z)\left(\varphi_y+\dfrac{\partial w}{\partial y}\right)$ $u_z(x,y,z,t)=w(x,y,t)$	$f(z)=\dfrac{1}{n}\left(\dfrac{2}{h}\right)^{n-1}z^n$, with $n=3,5,7,9...$	$u,v,w,\varphi_x,\varphi_y$

Note: h is the thickness. For the beam model, the displacement u_y is equal to zero, and all non-zero generalised displacements are independent of the y coordinate.

Table 2. Displacement field of quasi-3D theories

References	Displacement field	Shape function	Unknown
Thai and Kim [115] Sobhy and Radwan [196] Thai et al. [250] Lei et al. [310] Trinh et al. [307]	$u_x(x,y,z,t) = u(x,y,t) - z\dfrac{\partial w_b}{\partial x} - f(z)\dfrac{\partial w_s}{\partial x}$ $u_y(x,y,z,t) = v(x,y,t) - z\dfrac{\partial w_b}{\partial y} - f(z)\dfrac{\partial w_s}{\partial y}$ $u_z(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) + g(z)w_z(x,y,t)$	$f(z) = z - \dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)$ [115, 307] $f(z) = z - \dfrac{z\cosh\left(\dfrac{\pi}{2}\right) - \dfrac{h}{\pi}\sin\left(\dfrac{\pi z}{h}\right)}{\cosh\left(\dfrac{\pi}{2}\right) - 1}$ [196] $f(z) = z - h\sinh\left(\dfrac{z}{h}\right) + z\cosh\dfrac{1}{2}$ [250] $f(z) = \dfrac{4z^3}{3h^2}$ [307, 310] $g(z) = 1 - f'(z)$	u, v, w_b, w_s, w_z
Reddy and Kim [308]	$u_x(x,y,z,t) = u(x,y,t) + z\theta_x + z^2\phi_x + z^3\psi_x$ $u_y(x,y,z,t) = u(x,y,t) + z\theta_y + z^2\phi_y + z^3\psi_y$ $u_z(x,y,z,t) = w(x,y,t) + z\theta_z + z^2\phi_z$		$u, v, w, \theta_x, \theta_y, \theta_z,$ $\phi_x, \phi_y, \phi_z, \psi_x, \psi_y$
Nguyen et al. [410, 417]	$u_x(x,y,z,t) = u(x,y,t) - z\dfrac{\partial w_b}{\partial x} - f(z)\dfrac{\partial w_s}{\partial x}$ $u_y(x,y,z,t) = v(x,y,t) - z\dfrac{\partial w_b}{\partial y} - f(z)\dfrac{\partial w_s}{\partial y}$ $u_z(x,y,z,t) = w_b(x,y,t) + g(z)w_s(x,y,t)$	$f(z) = \pi z\left(-1 + \dfrac{9z^2}{5h^2} - \dfrac{28z^4}{25h^4}\right), g(z) = \dfrac{1}{8}f'(z)$ [410] $f(z) = z\left(8 - \dfrac{10z^2}{3h^2} - \dfrac{6z^4}{5h^4} - \dfrac{8z^6}{7h^6}\right), g(z) = \dfrac{3}{20}f'(z)$ [417]	u, v, w_b, w_s

Note: h is the thickness. For the beam model, the displacement u_y is equal to zero, and all non-zero generalised displacements are independent of the y coordinate.