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# Generation Expansion Planning in Electricity Market Considering Uncertainty in Load Demand and Presence of Strategic GENCOs

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## Abstract

This paper presents a new framework to study the generation capacity expansion in a multi-stage horizon in the presence of strategic generation companies (GENCOs). The proposed three-level model is a pool-based network-constrained electricity market that is presented under uncertainty in the predicted load demand modeled by the discrete Markov model. The first level includes decisions related to investment aimed to maximize the total profit of all GENCOs in the planning horizon, while the second level entails decisions related to investment aimed at maximizing the total profit of each GENCO. The third level consists of maximizing social welfare where the power market is cleared. The three-level optimization problem is converted to a one-level problem through an auxiliary mixed integer linear programming (MILP) using primal-dual transformation and Karush-Kuhn-Tucker (KKT) conditions. The efficiency of the proposed framework is examined on MAZANDARAN regional electric company (MREC) transmission network- a part of the Iranian interconnected power system. Simulation results confirm that the proposed framework could be a useful tool for analyzing the behaviour of investment in electricity markets in the presence of strategic GENCOs.

**Keywords:** Generation expansion planning, mathematical programming of equilibrium constraints problem, dynamic planning, strategic GENCO, uncertainty, three-level model.

## Nomenclature

<b>Indices</b>			
w/w'	index for scenario	$\bar{P}_{yk}^{ES}$	capacity of existing generation unit k of strategic GENCO y (MW)
r/r'	index for year	$\bar{P}_{td}^D$	maximum load of demand d in block t (MW)
t	index for demand blocks	$C_i^S/C_k^{ES}$	marginal cost of new/existing unit of strategic GENCO (€/MWh)
y	index for GENCOs	$U_{rtd}^D$	price bid of demand d in demand block t and year r (€/MWh)
i/k	index for new/existing generation unit of strategic GENCO	$B_{mn}$	musceptance of line n-m (p.u.)
d	index for demand	$F_{nm}$	transmission capacity of line n-m (MW)
h	index for size of investment option	f	discount rate
n/m	index for bus		
<b>Parameters</b>		<b>Decision variables</b>	
$\sigma_{rt}$	weight of demand block t in year r	$X_{yri}$	capacity investment of new unit i of the strategic GENCO y in year r (MW)
$W_w$	weight of scenario w and year r	$u_{yrih}$	binary variable that is equal to 1 if the h <sup>th</sup> investment option of technology i is selected in year r, otherwise it is equal to 0
$Q_{wr}$	demand growth of scenario w in year r	$P_{yrtkw}^{Es}/P_{yrtiw}^S$	power produced by existing/new unit k/i of strategic GENCO y in year r, demand block t and scenario w (MW)
$K_{yri}$	annual investment cost of new generating unit i of strategic GENCO y in year r (€/MW)	$X_{yri}/P_{yrtiw}^{Sr}$	available Capacity/ Power produced of new unit i of strategic GENCO y in year r', in years after the installation in year r (MW)
$\bar{K}_y$	available investment budget of strategic GENCO y (M€)	$P_{rtdw}^D$	demand d, in year r, demand block t and scenario w (MW)
$X_{ih}$	option h for investment capacity of new unit i (MW)		

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$O_{yrkw}^{ES}/O_{yrtiw}^S$	price offered by existing/new unit k/i of the of strategic GENCO y in year r, demand block t and scenario w (€/MWh)
$O_{yrr'iw}^{Sr}$	price offered by new unit i of strategic GENCO y, in year r', in the years after the installation in year r, demand block t and scenario w (€/MWh)
$\lambda_{rtnw}$	location marginal price or market clearing price (€/MWh)
$C_{yrr'i}^r$	marginal cost of new unit of GENCO i, in year r', in the years after the installation in year r (€/MWh)
$\theta_{rtnw}$	voltage angle of bus n, in year r, demand block t and scenario w

## 1. Introduction

Security and quality of supply, which implies the physical availability of electric power in sufficient quantities at all times and prices that are affordable for consumers, have remained top priorities since the electricity market liberalization [1–5]. These two vital features of the electric power supply must be ensured on the short and long-term basis [6]. To achieve this goal, it is important to have a positive correlation between generation expansion and the demand growth to maintain balance between production and consumption. However, generation expansion planning has become a complex issue in the power generation industry in the competitive space [7–9]. In a limited competitive space, it is necessary that the independent system operator is equipped with the capabilities of models and computational tools to enable it to study the behaviour of the expansion of the generation sector in power systems under uncertainty [5, 7, 10–27].

Generation investment market involves competition of a set of strategic companies of which each can exact power with its decisions in the market. Given this scenario, challenges and significant difficulties will be created in strategic companies, decisions include short-term decisions which include strategic offers in the instantaneous market and long-term decisions invested in a new power plant. Also in this model, the market price is considered reliably effected by the decisions of the strategic companies. In this case, both the market price and the companies productions are variable, while investor in on-level models such as [1–3, 8, 9, 11–18] must have a prediction pattern of market price. in these models, Investment capacity for optimizing profit is affected this prediction pattern of price.

A number of models have been developed to address the generation expansion problem both before and after restructuring of the electricity market. However, many of these models are deficient in one way or another. For example, in many bi-level models such as [7, 10, 28–34] presented, the market price prediction model was used but the existence of a strategic company for the production development planning was not considered.

The market in [30, 31] was modeled using the conjecture price approach in the lower level of the problem while

ref [29] used Cournot modeling approach their work. Although, the market price was considered a variable in the bi-level models presented for the production expansion planning problem in [9] and [13] strategic companies were not taken into account in the studies. In some other bi-level models such as [35–37] the supply function model was used to consider the strategic companies. However, these models include competition of only one strategic company with a set of non-strategic companies. Meanwhile, there is no competition among the strategic companies in the approach used in [35, 36], due to the static nature of planning. Its worth mentioning that one of the other important advantages of multi-level models is that the objectives that are in conflict with each other are observed in them. In [38], a three level model for generation expansion planning without considering the strategic companies is presented. Similarly, though competition among a set of strategic companies was considered for the generation expansion planning in [39, 40], certain important features such as dynamism, consideration for uncertainty and capacities of true investment are missing in these references.

Furthermore, the models proposed in [29, 36, 39, 41–48] did not take into account the dynamic nature of investment decisions. Similarly, uncertainty in the demand growth were not included in the models proposed in [36, 37, 44, 49] just as the investors in the models in [36, 39, 44, 50] have no choice of different technologies.

Interestingly, the present work tries to address most of the deficiencies highlighted above, to present a very accurate model. In this respect, it considers the dynamics of the investment decisions, just as it puts into account strategic GENCOs. It also considers the problem as a three-level model consisting of the transmission network constraints. The uncertainty regarding demand growth is equally accounted for in this study. In addition, the model takes into account peak and base technologies in different capacities from which the user could select. In the proposed model, small and large private GENCOs compete with one another. Fig. 1 shows the effectiveness of investment and planning.

By way of summary, the contributions of this paper are listed below:

- The proposed model for the strategic GENCOs is dynamic in nature. To the best of the authors' knowledge, none of the equilibrium programming with equilibrium constraint (EPEC) models presented in the literature considered the dynamic nature of investment decisions. Thus, the multi-period stochastic three-level model consisting of transmission network constraints is presented here.
- Inclusion of uncertainty in the load demand in the dynamic competition of strategic GENCOs: Scenarios are used to describe the uncertainty pertaining to the demand growth modelled by the discrete Markov model because no anticipativity constraints are inherent in all multi stage stochastic optimisation problems;
- Consideration of investment decision variable as discrete variables in the three-level stochastic dynamic

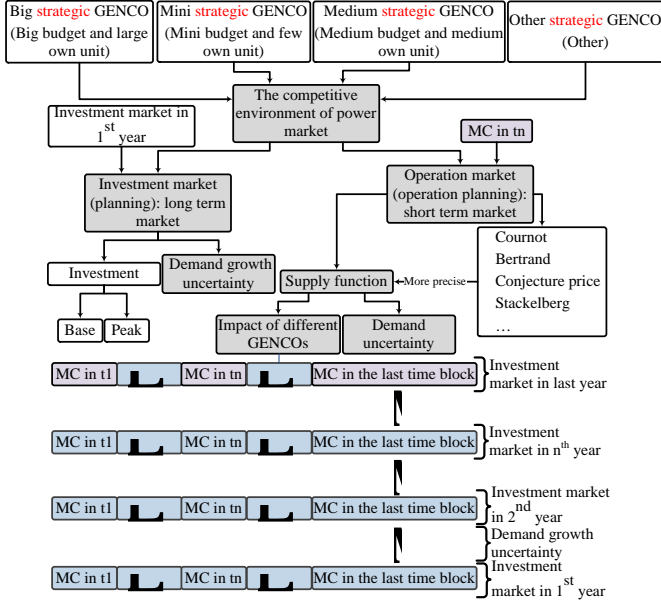


Figure 1: Effectiveness of investment and planning

model for competition among the strategic GENCOS: Two scenarios (base and peak) in which each has different option for investment capacity are considered for each candidate.

- In this model, small and large strategic GENCOS compete with one another.

## 2. The proposed algorithm

The generation planning problem in this study is solved using the algorithm presented in Algorithm 1. It is desired to have the objective function established at the scheduled time. The following procedure is repeated for each scenario. At first, for each company participating in this competition, the investment is done with the aim of maximizing profits in the market. Non-anticipativity is considered at the output of the optimization offers in the market as well as investment for each company. New peak and base units added to the existing unit in the power network is considered as the existing units in the following year. At the lowest level, the social welfare is maximized for all time blocks in the horizon year.

For simplicity, time blocks and GENCOS count blocks are not shown in the pseudo code illustrated in Algorithm 1.  $r_n$ ,  $w_n$ ,  $t_n$ , and  $y_n$  are number of stages, scenarios, time block, and GENCOS respectively.

The variables in the first level represent the invested capacity of the strategic GENCOS and their associated operation costs ( $X_{yri}$ ,  $X_{yrr_i}^r$ ,  $u_{yrih}$ ,  $C_{yrr_i}^r$ ). Therefore, their values are known in the second and third levels and they modeled as known parameters. Again, only the variables related to the second level offers of the strategic GENCOS ( $O_{yrtiw}^S$ ,  $O_{yrr_tiw}^{Sr}$ ,  $O_{yrtkw}^E$ ) in the market for the sale of electricity produced by their existing and new units are characterized with the aim

## Algorithm 1 THE ALGORITHM FOR SOLVING THE PROPOSED IDEA

**Require:** growth demand uncertainty, network and GENCOS info

```

1: while  $r \leq r_n$  do
2:   max aim of first level
3:   if peak unit is invested then
4:     It added to the existing unit in network;
5:      $k \leftarrow k + 1$ 
6:   end if
7:   if base unit is invested then
8:     It added to existing unit in network;
9:      $k \leftarrow k + 1$ 
10:  end if
11:  enforcing dynamic decency for invested capacity
12:  Output: investment by GENCOS
13:   $y \leftarrow 1$ 
14:  while  $S \leq y_n$  do
15:    max profit of company S in second level
16:    Output: offers by GENCOS;
17:    non-anticipativity checking of primary level variables;
18:    while  $t \leq t_n$  do
19:       $W \leftarrow 1$ 
20:      while  $w \leq w_n$  do
21:        max social welfare in third level
22:        Market clearing with DC power flow
23:        Output: generation, consumption and price
24:        non-anticipativity checking of tertiary level variables;
25:         $W \leftarrow w + 1$ 
26:      end while
27:       $y \leftarrow y + 1$ 
28:    end while
29:     $r \leftarrow r + 1$ 
30:  end while
31: end while
32: return production, consumption and market clearing price

```

of maximizing the profit of both strategic GENCOS as a constraint in the first level problem. Similarly, only the variables related to the second level problem are considered as the first level variables. The invested capacity by the GENCOS and their operation cost and offers of strategic GENCOS have a certain amount in the third level which are parameterized. Only the third-level variables are included in the variables of production, consumption, buses angles, and market clearing prices ( $P_{yrtiw}^S$ ,  $P_{yrtkw}^E$ ,  $P_{rtw}^d$ ,  $P_{yrr_tiw}^{Sr}$ ,  $\theta_{rtw}$ ,  $\lambda_{rtw}$ ) which are determined with the aim of maximizing the social welfare as a constraint in the second level problem. Finally, only the variables related to the third level are accounted for as first and second level variables.

## 3. Uncertainty and non-anticipativity constraints in the dynamic problem

It should be noted that non-anticipativity constraints are inherent in all multi-stage stochastic optimisation problems [40]. If the realizations of stochastic processes are identical up to stage  $n$ , then the values of all decision variables for these scenarios need to be identical up to stage  $n$ . However, without the non-anticipativity constraints, values of decision variables for the scenarios may not be identical up to stage  $n$  [51]. Therefore, the provisions of non-anticipativity used as scenarios are not independent of each other similar to the case of a 5-stage planning.

Figure 2 shows different states of a stochastic problem as used in this study. One way to solve the problem is to include the results of each year in the studies of the following year.

This could be in form of quasi-dynamic planning [51]. It is apparent that the dynamic planning solution could be more optimal in comparison with the quasi-dynamic planning solution. If the scenarios considered in the model are independent and different from one another at all stages (node-variable formulation), the provisions should not be applied to the programming (mode 1, Figure 2).

The multistage problem as scenario-variable formulation including dependent uncertainties has two modes when the uncertainties in the scenarios considered are similar in the process of multi-stage models. The constraints related to non-anticipativity should be applied to the model (mode 3, Figure 2). If the multistage stochastic problem is solved by an optimization algorithm such as heuristic algorithm, each iteration results in different optimal solutions. It should be noted that applying the non-anticipativity constraints to the program is optional, especially if it uses GAMS solvers like MILP, LP, NLP and EXT.

Because of the structure of the optimization problem, solutions obtained for each scenario is essentially the same in mode 2 of Figure 2. Therefore, applying the non-anticipativity conditions to the program increases the complexity of the model and calculations volume which does not necessarily lead to the optimal solution. For this reason, application of the non-anticipativity constraints is considered optional in this study since the uncertainties in the defined scenarios are similar in the process of multi-stage models since they use MILP GAMS solvers. Interestingly, the dynamic stochastic approach [52] is similar to case 1 of Figure 2 while the formulated model in [53] is similar to mode 3 of Figure 2 of the work presented in this paper.

After the simulation, this subject must be checked to ensure the accuracy of the obtained optimal results. In this model, the second and third level variables must be checked to ensure the accuracy of non-anticipativity constraints. The variables of the first level which correspond to the installed generation capacity of the investing agent will not be stochastic in nature. The reason for this is that a generation company can only make one investment decision at a time as it is impossible to know which scenario is going to occur in reality. Thus, the non-anticipativity subject is not considered in the first level variables.

In this paper, the uncertainty in load demand is modelled by using the Markov chains with the demand growth is assumed to be 10% and 8% with the probabilities of 60% and 40%, respectively, in any scenario for each year. Figure 2 shows the Markov chain for the uncertainty in the demand growth. The generation expansion planning includes 16 scenarios for five years according to the scenarios taken into account for each year. Therefore, the subject of non-anticipativity conditions must be checked in these assumed scenarios as follows. The variables introduced are the same in all the scenarios for the first year. In the second year, however, only variables in scenarios 1 to 8 and that of 9 to 16 remains the same. In the third year the variables for scenarios 1 to 4, 5 to 8, 9 to 12 and 13 to 16 are similar while in the fourth year the variables are the same for scenario 1 with 2, 3

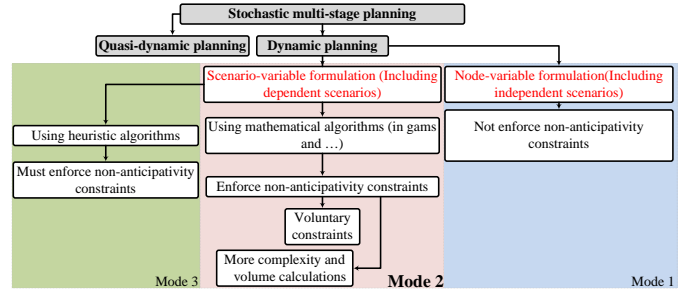


Figure 2: Different states considering non-anticipativity in the multi-stage stochastic problem

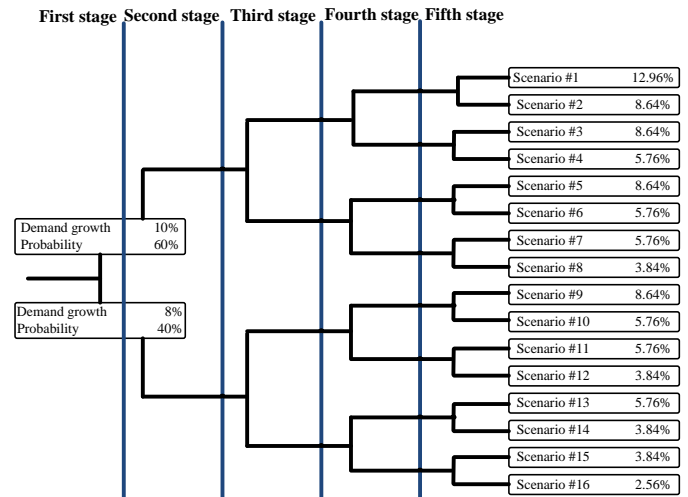


Figure 3: Scenario and non-anticipativity in the proposed model

with 4, 5 with 6, 7 with 8, 9 with 10, 11 with 12, 13 with 14 and 15 with 16. However, these variables are not the same in the last year.

#### 4. Mathematical formulation

The proposed idea presented in form of a three-level model comprises a first level, i.e. Eqs. (15)-(22), a second level, i.e. Eqs. (23)-(26), and a collection of third level problems, i.e. Eqs. (1)-(14). The first level is related to the competitive environment in which the GENCOs participate in, while the second and third levels are related to the model of each strategic company in the power market.

**Third level problem:** The third-level problem represents the market clearing. The clearing of the market for any given operating condition is represented as an optimization problem that identifies the operating decisions that maximize social welfare. The market clearing problem is constrained by DC power flow equations, transmission network limitations, upper and lower bound for production and consumption as well as non-anticipativity constraint related to the lowest level variables. The outputs of the lower level problem is nodal prices (dual variables associated to the power balance constraints), which are fed back to the medium level.

**Third-level objective function:**

$$\min \sum_y \sum_i O_{yrtiw}^S P_{yrtiw}^S + \sum_y \sum_k O_{yrtkw}^E P_{yrtkw}^E + \sum_{r'} \sum_y \sum_i O_{yrr'tiw}^S P_{yrr'tiw}^S \quad (1)$$

The market clearing problems are represented by the negative social welfare (i.e. Eq. (1)) and Eqs.(2)-(14). Note that dual variables associated with the third-problems are indicated at their corresponding constraints following a colon.

**Third-level constraints:**

$$\sum_d P_{rtdw}^D + \sum_{m \in \Phi \Phi_n} B_{nm}(\theta_{rtnw} - \theta_{rtmw}) - \sum_y \sum_i P_{yrtiw}^S - \sum_{y'} \sum_{r'} \sum_i P_{yrr'tiw}^{Sr} - \sum_y \sum_k P_{yrtkw}^E = 0 : \lambda_{rtnw} \quad \forall n, \forall t, \forall r, \forall w \quad (2)$$

Eq. (2) represents the energy balance at each bus, being the associated dual variables, LMPs ( $\lambda_{rtw}$ ) or nodal prices.

$$0 \leq P_{yrtiw}^S \leq X_{yri} : \bar{\mu}_{yrtiw}^S, \underline{\mu}_{yrtiw}^S \quad \forall y, \forall i, \forall t, \forall r, \forall w \quad (3)$$

$$0 \leq P_{yrr'tiw}^{Sr} \leq X_{yrr'i}^r : \bar{\mu}_{yrr'tiw}^{Sr}, \underline{\mu}_{yrr'tiw}^{Sr} \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\} \quad (4)$$

$$0 \leq P_{yrtkw}^E \leq \bar{P}_{yk}^E : \bar{\mu}_{yrtkw}^E, \underline{\mu}_{yrtkw}^E \quad \forall y, \forall k, \forall t, \forall r, \forall w \quad (5)$$

$$0 \leq P_{rtdw}^D \leq Q_{wr} \bar{P}_{td}^D : \bar{\mu}_{rtdw}^D, \underline{\mu}_{rtdw}^D \quad \forall d, \forall t, \forall r, \forall w \quad (6)$$

$$-\bar{F}_{nm} \leq B_{nm}(\theta_{rtnw} - \theta_{rtmw}) \leq \bar{F}_{nm} : \underline{V}_{rtnw}, \bar{V}_{rtnw} \quad \forall n, \forall m, \forall t, \forall r, \forall w \quad (7)$$

$$\pi \leq \theta_{rtnw} \leq \bar{\pi} : \underline{\zeta}_{rtnw}, \bar{\zeta}_{rtnw} \quad \forall n, \forall t, \forall r, \forall w \quad (8)$$

The third-level variable bounds are given for unit generation, demand consumption, power flow and angle bounds in Eqs. (4)-(9). Eqs. (3)-(5) are associated with limits for the generated power of new units in installed year, the power produced by new unit in the next years after installed year, and the generated power of existing units for strategic producers, respectively. In this work, demands are considered to be elastic. Eq. (6) is relevant to elastic demands. The power flow through transmission lines using a lossless DC mode limits for power and buses' voltage angles are represented in Eqs. (7)-(8), respectively.

$$\theta_{rtmw} = 0 : \zeta_t^1 \quad n = 1, \forall t, \forall r, \forall w \quad (9)$$

Eq. (9) fixes the voltage angle at the reference bus.

$$P_{yrtiw}^S = P_{yrtiw}^S : \mu_{yrtiw}^S \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall i \quad (10)$$

$$P_{yrtkw}^E = P_{yrtkw}^E : \mu_{yrtkw}^E \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall k \quad (11)$$

$$P_{yrr'tiw}^{Sr} = P_{yrr'tiw}^{Sr} : \mu_{yrr'tiw}^{Sr} \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall i, \forall r' \in \{X_{r's} > 0, r > r'\} \quad (12)$$

$$P_{rtdw}^D = P_{rtdw}^D : \mu_{rtdw}^D \quad \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall d \quad (13)$$

$$\theta_{rtmw} = P_{rtmw} : \zeta_{rtmw} \quad \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall n \quad (14)$$

The non-anticipativity equation related to the third-level variables are represented in Eqs. (10)-(14).

**Second-level problem:** The problem solved by each strategic GENCO to determine its best offering decisions is formulated as a bi-level model that is organized as a third and second levels in the proposed three-level model. The

Second-Level Represents the investment problem of a dominant GENCO S who is seeking to maximize the present value of the total investment profit.

The dynamic dependency constraints exist in the second level as a result of the dynamic nature of the planning of the problem. The second-level constraints include investment budget limit and non-anticipativity related to the company's offer which must be positive.

**Second-level objective function**

$$\min \sum_r \left( \frac{1}{1+f} \right)^r \sum_i K_{yri} X_{yri} - \sum_w \sum_r W_{wr} \left( \frac{1}{1+f} \right)^r \sum_t \sigma_{rt} \left[ \sum_i P_{yrtiw}^S (\lambda_{rtn(i \in n)_w} - C_{yrr'i}^r) + \sum_{r'} \sum_i P_{yrr'tiw}^{Sr} (\lambda_{rtn(i \in n)_w} - C_{yrr'i}^r) + \sum_k P_{yrtkw}^E (\lambda_{rtn(k \in n)_w} - C_k^E) \right] \quad (15)$$

Eq. (15) is the negative profit (investment cost minus expected profit) in the planning horizon, which comprises two terms. The first term of Eq. (15) is associated to the investment cost of new thermal units including peak and base technologies, whereas the second term is the expected profit obtained by selling energy in the spot market.

**Second-level constraints:**

$$\sum_r \left( \frac{1}{1+f} \right)^r \sum_i K_{yri} X_{yri} \leq \bar{K}_y : \Delta_y \quad \forall y \quad (16)$$

The available budget limitation is represented by Eq. (16) for investment in the new base or peak unit which reflects the limited financial resources available to the market for each GENCO.

$$O_{yrtiw}^S \geq 0 : \underline{\zeta}_{yrtiw}^S \quad \forall y, \forall r, \forall t, \forall i, \forall w \quad (17)$$

$$O_{yrtkw}^E \geq 0 : \underline{\zeta}_{yrtkw}^E \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (18)$$

$$O_{yrr'tiw}^{Sr} \geq 0 : \underline{\mu}_{yrr'tiw}^{Sr} \quad (19)$$

Eqs. (17)-(19) mean that the GENCO's offer is positive so that Eq. (19) is related to the strategic offer of the new units in the following years.

$$O_{yrtiw}^S = O_{yrtiw}^S : \mu_{yrtiw}^S \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall i \quad (20)$$

$$O_{yrtkw}^E = O_{yrtkw}^E : \mu_{yrtkw}^E \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall k \quad (21)$$

$$O_{yrr'tiw}^{Sr} = O_{yrr'tiw}^{Sr} : \mu_{yrr'tiw}^{Sr} \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall i \quad (22)$$

The non-anticipativity equations related to GENCO offer are represented in Eq. (20)-(22).

**First-level problem:** The objective function in the first level is minimization of the present value minus the expected profit (investment cost minus expected profit) of all strategic GENCOs in the planning horizon. The first level constraints include bounds on investment, dynamic dependency constraints related to the company investment and dynamic dependency constraints related to the marginal cost of newly added units.

**First-level objective function:**

$$\min \sum_y \sum_r \left( \frac{1}{1+f} \right)^r \sum_i K_{yri} X_{yri} - \sum_w \sum_r W_{wr} \left( \frac{1}{1+f} \right)^r \sum_t \sigma_{rt} \left[ \sum_i P_{yrtiw}^S (\lambda_{rtn(i \in n)_w} - C_{yrr'i}^r) + \sum_{r'} \sum_i P_{yrr'tiw}^{Sr} (\lambda_{rtn(i \in n)_w} - C_{yrr'i}^r) + \sum_k P_{yrtkw}^E (\lambda_{rtn(k \in n)_w} - C_k^E) \right] \quad (23)$$

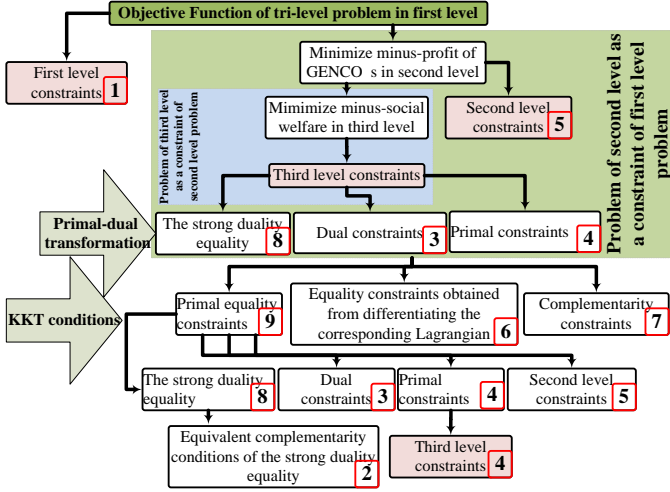


Figure 4: Converting the three-level problem into a one-level problem considering uncertainty

Eq. (23) is the present value of the expected profit of all GENCOs in the planning horizon. The profit of all producers is the summation of profit for all producers.

It has to be noted that  $i \in \mathbb{N}$  means the new power plant  $i$  is installed at bus  $n$ .

**First-level constraints:**

$$X_{yri} = \sum_h u_{yrih} \cdot X_{ih} \quad \forall i, \forall r, \forall y \quad \sum_h u_{yrih} = 1, u_{yrih} \in \{0, 1\}, \quad \forall i, \forall r, \forall y \quad (24)$$

It states that investment options for new thermal units are only available in discrete blocks.

$$X_{yrr'i} = X_{yri} \quad \forall r, r' \subset \{X_{ri} > 0, r > r'\}, \quad \forall i, \forall y \quad (25)$$

The dynamic dependency constraints on the base and peak investment decision variables are represented in Eq. (25). The new units are operated in the coming years as existing units in the network.

$$C_{yrr'i} = C_i \quad \forall r, r' \subset \{X_{ri} > 0, r > r'\}, \quad \forall i, \forall y \quad (26)$$

Similarly, the dynamic dependency constraints on decision variables of the marginal cost of new units are presented in Eq. (26).

## 5. Converting the three-level to one-level problem

The market and the proposed algorithm are expressed using a multi-level model to implement the proposed idea in this article. GAMS solvers are used to solve the model proposed in the paper. For this purpose, the multi-level model is converted to a one-level problem and then the optimal solution is obtained using the available solvers as shown in Figure 4.

The constraints of the first level of this three-level model include the candidate capacities for investment and the model for the presence of each strategic company in the market. Each strategic company is considered as a bi-level model including second and third level whose aims at second and third levels are to maximize the company's profit as well as

maximize social welfare. At first, the bi-level model of each company is converted to a one-level model to solve. Then, the bi-level model created in the next step (the objective function of the three-level model with candidate capacities constraints as a part of the first level constraint and the model of one level of each strategic company) must be converted to a one level model. In the next step, the mathematical model becomes one MILP problem after linearisation. This final problem can be solved by with the available solver.

To convert the bi-level model of each company to a one-level model, both the KKT conditions and the Primal-dual transformation could be used. The bi-level model of each strategic company becomes a one level model mixed-integer LP due to the complementary nature of the constraints. When KKT conditions are used, lower level problem of the achieved bi-level model is non-convex because generating of the binary variables from enforcing the KKT conditions on the third level. Therefore the final bi-level model cannot be converted to a one-level. Thus, in this stage, the Primal-dual transformation is used to convert the bi-level model of each strategic company i.e. second and third levels to one level-model from which blocks 8, 3 and 4 are obtained (refer to part 1 in the appendix).

In the next step, we are again faced with the problem of bi-level that must be converted to the one-level problem using one of two mentioned methods such that in this stage, block 1 is considered as first level constraint and blocks 4, 3, 8 and 5 considered the second-level constraints (refer to part 2 in the appendix). In this study, the KKT condition is enforced, so the second-level problem includes blocks 8, 3 and 4 as the second-level problem constraints obtained. The reason for this is that the converted problem comes in the form of MILP resulting in blocks 6, 7 and 9 (refer to part 3 in the appendix). Block 9 itself includes blocks 3, 4, 5 and 8. Note that the strong duality equality of the primal-dual transformation is a non-linear expression.

The obtained one-level model has non-linear phrases. These non-linear phrases must be linearised and replaced with equivalent linear relationship. Therefore, in the last step, the strong duality equality of Primal-dual transformation achieved in the previous steps is replaced with its equivalent KKT condition resulting in the transformation of block 10 to 2 (refer to part 4 in the appendix). These equivalent KKT conditions are in form of MILP

Note that in the previous section, the candidate capacities for investment was modelled as a continuous variable in each strategic company. On the other hand, investors select various generation technologies with a certain capacity. Therefore, it is logical that these variables are considered discrete. It should also be noted that the one-level model of each company must be convex to be used in the next step. The one level model of each company is not convex if the model has the integer variables.

Therefore, the variables of company investment cannot be assumed as a discrete variable in the model of each strategic company. Thus, in this paper, the variables are considered to be continuous in the model of each strategic company.



Therefore, the capacity limitation is applied in higher level constraints. This means that the selected investment capacity should be from among the specified candidates. Finally, the problem with the first level objective function and the equivalent constraints of three level problem is transformed to one level, which includes blocks 1 to 8. Explaining in detail about this subject is addressed in [37].

## 6. CASE STUDY: MREC NETWORK

In this section, the efficiency of the proposed framework is examined through a case study. To validate the proposed model, it is implemented on a 2-bus network from [39]. In this case, planning included in the competition of three strategic GENCOs with one another at stage one (static planning) without uncertainty. The network data of the case study system is extracted from [37]. The data of the existing generation units for all strategic GENCOs are considered in Table 1. The second column provides the capacity of each unit, which comprises two generation blocks (columns 3-4), with their corresponding production costs (columns 5-6).

In a three-level competition, it is assumed that the strategic GENCOs 1,2 and 3 participate in the market. Table 1 shows the results of generation expansion planning for the three-level competition for both static and stochastic dynamic as well as for 5-level competition for different cases. The second column of Table 3 is related to the planning stage while the third column illustrates the total installed capacity having the base technology in the parenthesis. Note that both nodes N1 and N2 are candidates to locate new units. The next two columns provide the Total profit (M€) and Social welfare (M€) during the planning period. The CPU time for each case is shown in the last column.

The case study is the MAZANDARAN regional electric company (MREC) transmission network- a part of the IRAN interconnected power system. The single-line diagram of MREC transmission network is shown in Figure 5 while the load demand information could be found in [37]. The annual growth of the demand for the first year is assumed 6.2% while subsequent years are treated based on the explanation related to Figure 3. The price bids of the demands are 35.75, 28.721, and 27.357(€/MWh) for peak, shoulder, and off-peak blocks, respectively.

To ease computational burden, those transmission lines that operate at 230kV with respect to their capacities are not explicitly modeled. Thus, the 230kV network is considered as a bus. In this case study, two strategic GENCOs (i.e., Producers 1 and 2) are considered, meaning that, the competition between the players is duopoly. It should be noted that the generation units NEKA and ALIABAD are owned by two different Producers, 1 and 2, respectively. The candidate buses for construction of available investment budget is assumed 75M€ over the planning period for each GENCO.

The total capacity of the existing units is 3155MW, of which 69.57% of it (i.e., 2195MW) belongs to Producer 1, connected to buses NEKA4 and NEKA2, as indicated in Figure 5, and the remaining capacity to Producer 2. The operation

Table 3: Information relating to the candidate units for investment

Candidate unit	Annualized capital ( $K_{yri}$ ) cost (€/MW)	$X_{ih}$ (MW)	$C_i^S / C_k^{ES}$ Production cost (€/MWh)
Base technology	30000	0-500-750-1000	10
Peak technology	6000	0-200-300-400-500 600-700-800-900-1000	14

costs of the existing units are presented in Figure 5. Types and data for investment options are given in Table 3. Here, only one generation block is assumed for the new units. The results of the simulations are illustrated in Figure 6.

Shown in the figure are the results of generation expansion planning for the duopoly competition. It can be seen that the investment share of GENCO 1 is 300MW, which takes five years to build up at NEKA. On the other hand, GENCO 2's investment share totals 1800MW, which means there is a build-up of 1100MW on the 230kV network while the rest power is contributed by ALIABAD. The price for all years is the same and equal to 33.55€/MWh. The social welfare is 2157M€ and it is planned to increase on yearly basis above the previous year. While the profit made by strategic GENCO 1 decreases over time, the one of strategic GENCO 2 increases. The share of the two GENCOs in the total profit is 44.04% and 55.98% for strategic GENCO 1 and 2, respectively from a total profit of 2054M€.

Table 1: Units data for the two-bus test network

Existing unit	Capacity (MW)	Capacity of block 1 (MW)	Capacity of block 2 (MW)	Production cost of block 1 (€/MWh)	Production cost of block 2 (€/MWh)
GENCO 1	60	30	30	12	14
GENCO 2	60	30	30	12	14
GENCO 3	120	60	60	13	15
GENCO 4	50	25	25	18.6	20.03
GENCO 5	76	30	46	11.46	11.96

Table 2: The results of generation expansion planning for a three-level competition

Case	Stage	Total capacity added (base) MW-Location [by]	Total profit (M€)	Social welfare (M€)	CPU time (S)
Static: Tri-level competition [34]	Horizon stage	200(180)-N1 [GENCO 1:180-GENCO 3:20]	12.34	19.61	33.821
Stochastic dynamic : Tri-level competition	1	300(0)-N1[GENCO 1]	9.43	22.22	451.051
	2	0	10.64	28.29	
	3	0	10.71	28.8	
	4	0	10.69	29.1	
	5 (horizon stage)	200(0)-N2[GENCO 2]	10.68	29.4	
Stochastic dynamic : 5-level competition	1	300(0)-N1[GENCO 3]	17.93	16.85	634.128
	2	0	19.54	39.98	
	3	500(500)-N1[GENCO 5]	15.45	27.64	
	4	0	30.24	54.8	
	5 (horizon stage)	0	30.22	54.6	



Total peak unit*10 MW	70	20	40	30	50
Average price (/MWh)	33,551				
New peak unit by GENCO 1*10MW	0				30 (neka)
Net profit of GENCO 1 *10M	21.132	20.227	17.580	16.332	15.214
New peak unit by GENCO 2*10MW	70(Sari)	20:Sari	40 (Aliabad)	30 (Aliabad)	20 (sari)
Net profit of GENCO 2 *10M	18.551	20.452	23.589	25.367	26.960
Total profit* 10M	39.683	40.679	41.169	41.688	42.174
Social welfare* 10M	33.847	41.638	43.783	47.068	49.398
	1	2	3	4	5
	Stage of planning				

Figure 6: Generation expansion planning in a duopoly competition

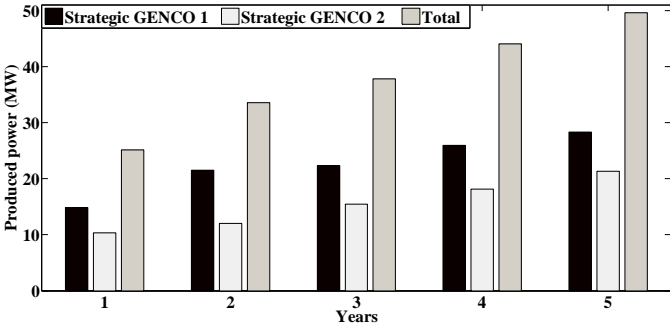


Figure 7: Energy production of the strategic GENCOs in the planning period

Figure 7 shows the production of strategic GENCOs 1 and 2 in the planning period. It can be seen that the production share of strategic GENCO 1 is 59.39% of the total energy while the production share of the strategic GENCO 2 is 40.61%.

## 7. CONCLUSION

A new three-level dynamic framework programming has been presented to study the generation capacity expansion in a restructured power system under uncertainty. This paper provided a methodology to characterize the interactions among strategic GENCOs and to find generation investment equilibria in a network-constrained electricity pool.

To numerically validate the proposed methodology, the MAZANDARAN regional electric company's (MREC) transmission network- a subset of the IRAN interconnected power system is adopted. The results obtained showed that while strategic GENCO 1 had its profit decreasing, strategic GENCO 2 experienced an increase in profit over time. However, the impact of investment incentives could be included in the proposed model as future work. This could be expanded to consider the impacts of transmission expansion plans, availability of gas transmission network and tax policy.

## 8. Acknowledgements

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## Appendix:

### 9. Part 1: Enforcing primal-dual transformation to third level

TransactiveAs stated so far the model of each strategic company includes Eqs. (1) to (22). The third level problem (Eqs. (1)-(14)) is convex because of linearity. As a result of transforming the third optimization problem to equivalent constraints we can use both primal-dual transformation and KKT conditions. Because the complimentary constraints obtained from KKT conditions are mixed integer, the obtained constraints, in this stage primal-dual transformation is used. The constraints equivalent to the third level optimization level problem includes primal constraints, dual constraints and the strong duality equality.

#### 9.1. TransactiveDual constraints (block 3 in Figure 4)

TransactiveL<sub>y</sub> is the Lagrangian function of third level problem.

TransactiveSome of the equations in using KKT method are obtained from the derivative of Lagrange expression relative to decision variables of which the equations are equivalent to the set of dual constraints of primal-dual transformation.

$$\frac{\partial l_y}{\partial P_{yritw}^S} = O_{yritw}^S - \lambda_{yritw} + \bar{\mu}_{yritw}^S - \underline{\mu}_{yritw}^S - \mu_{yritw}^{Sr} + \sum_{w'(w,w' \in \{Q_{w'r} = Q_{w'r'}\})} \mu_{yritw}^{S'} \quad (27)$$

$$\frac{\partial l_y}{\partial P_{yrtkw}^E} = O_{yrtkw}^E - \lambda_{yrtkw} + \bar{\mu}_{yrtkw}^E - \underline{\mu}_{yrtkw}^E - \mu_{yrtkw}^E + \sum_{w'(w,w' \in \{Q_{w'r} = Q_{w'r'}\})} \mu_{yrtkw}^{E'} \quad (28)$$

$$\frac{\partial l_y}{\partial P_{yrtw}^D} = -U_{yrtw}^D + \lambda_{yrtw} + \bar{\mu}_{yrtw}^D - \underline{\mu}_{yrtw}^D - \mu_{yrtw}^D + \sum_{w'(w,w' \in \{Q_{w'r} = Q_{w'r'}\})} \mu_{yrtw}^{D'} \quad (29)$$

$$\begin{aligned} \frac{\partial l_y}{\partial P_{yritw}^{Sr}} &= O_{yritw}^{Sr} - \lambda_{yritw} + \bar{\mu}_{yritw}^{Sr} - \underline{\mu}_{yritw}^{Sr} - \mu_{yritw}^{Sr} \\ &+ \sum_{w'(w,w' \in \{Q_{w'r} = Q_{w'r'}\})} \mu_{yritw}^{Sr'} = 0 : \rho_{yritw}^{Sr} \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, 1\} \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{\partial l_y}{\partial \theta_{rtmw}} &= \sum_m B_{nm} (\lambda_{rtmw} - \lambda_{rtmw}) + \sum_m B_{nm} (\bar{V}_{rtmw} - \bar{V}_{rtmw}) \\ &+ \sum_m B_{nm} (V_{rtmw} - V_{rtmw}) + \bar{\zeta}_{rtmw} - \underline{\zeta}_{rtmw} - \zeta_{rtmw} \\ &\sum_{w'(w,w' \in \{Q_{w'r} = Q_{w'r'}\})} \zeta_{rtmw} + (\zeta_{rtmw}^1)|_{n=1} = 0 : \rho_{rtmw}^\theta \quad \forall r, \forall t, \forall n, \forall w \end{aligned} \quad (31)$$

#### 9.2. TransactiveThe strong duality equality (block 8 in Figure 4)

TransactiveStrong dual theory relation is as follows:

$$\begin{aligned}
& \sum_y \sum_i O_{yrtiw}^S P_{yrtiw}^S + \sum_y \sum_k O_{yrtkw}^E P_{yrtkw}^E + \sum_{r'} \sum_y \sum_i O_{yrr'tiw}^S P_{yrr'tiw}^S + \sum_d \sum_{wr} Q_{wr} P_{rtw}^D + \sum_{n,m} \bar{F}_{nm} (V_{rtnmw} + \bar{V}_{rtnmw}) + \sum_n \pi (\zeta_{rtnw} + \bar{\zeta}_{rtnw}) = 0 : \phi_{rtw}^{SD} \\
& + \sum_y \sum_i X_{yrti} \bar{\mu}_{yrtiw}^S + \sum_y \sum_i \sum_{r'} X_{yrr'i} \bar{\mu}_{yrtiw}^{Sr} + \sum_y \sum_k P_{yk}^E \bar{\mu}_{yrtkw}^E + \sum_d Q_{wr} P_{rtw}^D + \sum_{n,m} \bar{F}_{nm} (V_{rtnmw} + \bar{V}_{rtnmw}) + \sum_n \pi (\zeta_{rtnw} + \bar{\zeta}_{rtnw}) = 0 : \phi_{rtw}^{SD} \\
& + \sum_n \pi (\zeta_{rtnw} + \bar{\zeta}_{rtnw}) = 0 : \phi_{rtw}^{SD}
\end{aligned} \tag{32}$$

### 9.3. Primal constraints (block 4 in Figure 4)

TransactivePrimal constraints use primal-dual transformation including Eqs. (33) to (45). These equations are the same as third level constraints equations with new duals variable for using in the next stage (where KKT conditions are again applied to these constraints). The dual constraints of each constraint has been written in front of it.

$$\sum_d P_{rtdw}^D + \sum_{m \in \Psi \Phi_n} B_{nm} (\theta_{rtnw} - \theta_{rtmw}) - \sum_y \sum_i P_{yrtiw}^S - \sum_{y,r'} \sum_i P_{yrr'tiw}^{Sr} - \sum_y \sum_k P_{yrtkw}^E = 0 : \lambda'_{rtnw} \tag{33}$$

$$0 \leq P_{yrtiw}^S \leq X_{yrti} : \underline{\mu}_{yrtiw}^S, \bar{\mu}_{yrtiw}^S \quad \forall y, \forall i, \forall t, \forall r, \forall w \tag{34}$$

$$0 \leq P_{yrr'tiw}^{Sr} \leq X_{yrr'i}^r : \underline{\mu}_{yrr'tiw}^{Sr}, \bar{\mu}_{yrr'tiw}^{Sr} \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall r, \forall w \tag{35}$$

$$0 \leq P_{yrtkw}^E \leq P_{yk}^E : \underline{\mu}_{yrtkw}^E, \bar{\mu}_{yrtkw}^E \quad \forall y, \forall k, \forall t, \forall r, \forall w \tag{36}$$

$$0 \leq P_{rtdw}^D \leq Q_{wr} P_{td}^D : \underline{\mu}_{rtdw}^D, \bar{\mu}_{rtdw}^D \quad \forall d, \forall t, \forall r, \forall w \tag{37}$$

$$-\bar{F}_{nm} \leq B_{nm} (\theta_{rtnw} - \theta_{rtmw}) \leq \bar{F}_{nm} : \underline{V}'_{rtnmw}, \bar{V}'_{rtnmw}, \forall n, \forall m, \forall t, \forall r, \forall w \tag{38}$$

$$-\pi \leq \theta_{rtnw} \leq \pi : \underline{\zeta}'_{rtnw}, \bar{\zeta}'_{rtnw}, \forall n, \forall t, \forall r, \forall w \tag{39}$$

$$\theta_{rtnw} = 0 : \zeta'_t \quad n = 1, \forall t, \forall r, \forall w \tag{40}$$

$$P_{yrtiw}^S = P_{yrtiw}^S : \underline{\mu}_{yrtiw}^S \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall i \tag{41}$$

$$P_{yrtkw}^E = P_{yrtkw}^E : \underline{\mu}_{yrtkw}^E \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall k \tag{42}$$

$$P_{yrr'tkw}^{Sr} = P_{yrr'tkw}^{Sr} : \underline{\mu}_{yrr'tkw}^{Sr} \quad \forall y, \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall i \tag{43}$$

$$P_{rtdw}^D = P_{rtdw}^D : \underline{\mu}_{rtdw}^D \quad \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall d \tag{44}$$

$$\theta_{rtnw} = \theta_{rtnw} : \zeta'_{rtnw} \quad \forall r, \forall w, \forall w' \in \{Q_{wr} = Q_{w'r}\}, \forall t, \forall n \tag{45}$$

Meanwhile, it must be noted that at this stage that the dual variables of inequality constraints have positive values:

$$\underline{\mu}_{yrtiw}^S \geq 0; \bar{\mu}_{yrtiw}^S \geq 0; \underline{\eta}_{yrtiw}^S, \bar{\eta}_{yrtiw}^S \quad \forall y, \forall i, \forall t, \forall r, \forall w \tag{46}$$

$$\underline{\mu}_{yrr'tiw}^{Sr} \geq 0; \bar{\mu}_{yrr'tiw}^{Sr} \geq 0; \underline{\eta}_{yrr'tiw}^{Sr}, \bar{\eta}_{yrr'tiw}^{Sr} \quad \forall y, \forall i, \forall t, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall w \tag{47}$$

$$\underline{\mu}_{yrtkw}^E \geq 0; \bar{\mu}_{yrtkw}^E \geq 0; \underline{\eta}_{yrtkw}^E, \bar{\eta}_{yrtkw}^E \quad \forall y, \forall k, \forall t, \forall r, \forall w \tag{48}$$

$$\underline{\mu}_{rtdw}^D \geq 0; \bar{\mu}_{rtdw}^D \geq 0; \underline{\eta}_{rtdw}^D, \bar{\eta}_{rtdw}^D \quad \forall d, \forall t, \forall r, \forall w \tag{49}$$

$$\underline{V}_{rtnmw} \geq 0; \bar{V}_{rtnmw} \geq 0; \underline{\eta}_{rtnmw}^V, \bar{\eta}_{rtnmw}^V \quad \forall r, \forall t, \forall n, \forall m, \forall w \tag{50}$$

$$\underline{\eta}_{rtnw} \geq 0; \bar{\eta}_{rtnw} \geq 0; \underline{\mu}_{rytnw}^{\eta}, \bar{\mu}_{rytnw}^{\eta} \quad \forall r, \forall t, \forall n, \forall w \tag{51}$$

## 10. Part 2: Converting third and second level to Equivalent MPEC

At this stage, a two-level problem exists such that the first level objective function includes Eq. (23) while first level constraints include Eqs. (24) to (26). Now the third level and second level problem have been merged with a problem as second level problem and have formed MPEC. Second level problem objective function includes Eq. (15) and second level problem constraints include Eqs. (16)-(22) and Eqs. (27)-(52) which include blocks 3, 4, 5 and 8 in Figure 4.

### Second-level objective function:

$$\text{Eq. (15)} \tag{52}$$

### Second-level new constraints

$$\text{Eqs. (27) - (52), Eqs. (16) - (22)} \tag{53}$$

### First level objective function:

$$\text{Eq. (23)} \tag{54}$$

### First-level constraints:

$$\text{Eqs. (24) - (26)} \tag{55}$$

## 11. Part 3: Enforcing Karush-Kuhn-Tucker (KKT) conditions to MPEC (equivalent problem of second and third levels)

Due to the non-linearity of MPEC at this stage, we cannot use primal-dual transformation but to use two level problem KKT conditions to transform Eqs. (52)-(55). Such that KKT conditions are applied to Eqs. (52)-(53). KKT conditions include three groups of primal equality constraints, equality constraints obtained from differentiating the corresponding Langrangian and complimentary constraints obtained from inequality constraints.

### 11.1. Primal equality Constraints (blocks 3,4,5,8 in Figure 4)

These constraints include the equality constraints of Eq. (53) which is as follows:

$$\text{Eqs. (20) - (22), Eqs. (27) - (33), Eqs. (41) - (45)} \tag{56}$$

### 11.2. Equality constraints obtained from differentiating the corresponding Lagrangian (block 6 in Figure 4)

$l_y$  is the Langarangian function of the new second-level problem.

$$\begin{aligned}
\frac{\partial l_y}{\partial P_{yrtiw}^S} &= -W_{wr} \left(\frac{1}{1+f}\right)^r \delta_{rt} (\lambda_{rtn(i \in n)_w} - C_i^S) - \lambda'_{rtn(i \in n)_w} + \bar{\mu}_{yrtiw}^S - \\
&+ \sum_{w' \in \{Q_{wr} = Q_{w'r}\}} \mu_{yrtiw}^S - O_{yrtiw}^S \phi_{rtw}^{SD} = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w
\end{aligned} \tag{57}$$

$$\begin{aligned}
\frac{\partial l_y}{\partial P_{yrtkw}^E} &= -W_{wr} \left(\frac{1}{1+f}\right)^r \delta_{rt} (\lambda_{rtn(k \in n)_w} - C_k^E) - \lambda'_{rtn(k \in n)_w} + \bar{\mu}_{yrtkw}^E - \\
&+ \sum_{w' \in \{Q_{wr} = Q_{w'r}\}} \mu_{yrtkw}^E - O_{yrtkw}^E \phi_{rtw}^{SD} = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w
\end{aligned} \tag{58}$$

$$\frac{\partial l_y}{\partial P_{rtdw}^D} = \lambda'_{rtnw} + \bar{\mu}_{rtdw}^D - \underline{\mu}_{rtdw}^D - \mu_{yrtdw}^D + \sum_{w' \in \{Q_{wr} = Q_{w'r}\}} \mu_{yrtdw}^D + \phi_{rtw}^{SD} \tag{59}$$

$$\frac{\partial l_y}{\partial P_{yrtiw}^{Sr}} = W_{wr} \left( \frac{1}{1+f} \right)^r \delta_{rt} (\lambda_{rtn(i \in n)w} - C_{yrti}^r) - \lambda_{rtn(i \in n)w} + \bar{\mu}_{yrtiw}^{Sr} \frac{\partial l_y}{\partial \mu_{yrtiw}^{Sr}} - \rho_{yrtiw}^{Sr} - \bar{\eta}_{yrtiw}^{Sr} = 0 \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall w$$

$$+ \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \mu_{yrtiw}^{Sr} = 0 : -O_{yrtiw}^{Sr} \phi_{rtw}^{SD} \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall i, \forall w \quad (76)$$

$$\frac{\partial l_y}{\partial \theta_{rtmw}} = \sum_m B_{nm} (\lambda'_{rtmw} - \lambda_{rtmw}) + \sum_m B_{nm} (\bar{V}'_{rtmw} - \bar{V}_{rtmw})$$

$$+ \sum_m B_{nm} (V'_{rtmw} - V_{rtmw}) + \bar{\zeta}'_{rtmw} - \zeta'_{rtmw} - \zeta_{rtmw}$$

$$\sum_m \zeta'_{rtmw} + (\zeta'_{rtmw})|_{n=1} = 0 \quad \forall r, \forall t, \forall n, \forall w$$

$$w'(w, w' \in \{Q_{wr} = Q_{w'r}\}) \quad (60)$$

$$\frac{\partial l_y}{\partial \theta_{yrtiw}} = -\eta_{yrtiw} - \eta_{yrtiw}^S + \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \eta_{yrtiw}^S + \rho_{yrtiw}^S - \phi_{rtw}^{SD} \rho_{yrtiw}^S = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w$$

$$\frac{\partial l_y}{\partial \bar{V}_{rtnmw}} = B_{nm} (\rho_{rtmw}^\theta - \rho_{rtmw}^\theta) - \bar{\eta}_{rtnmw}^V + \phi_{rtw}^{SD} \bar{F}_{nm} = 0 \quad \forall r, \forall t, \forall n, \forall m, \forall w \quad (77)$$

$$\frac{\partial l_y}{\partial \bar{V}_{rtnmw}} = -B_{nm} (\rho_{rtmw}^\theta - \rho_{rtmw}^\theta) - \eta_{rytnmw}^V + \phi_{rtw}^{SD} \bar{F}_{nm} = 0 \quad \forall r, \forall t, \forall n, \forall m, \forall w \quad (78)$$

$$\frac{\partial l_y}{\partial O_{yrtiw}^E} = -\eta_{rytkw}^E - \eta_{rytkw}^E + \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \eta_{rytkw}^E + \rho_{rytkw}^E - \phi_{rtw}^{SD} \rho_{rytkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w$$

$$\frac{\partial l_y}{\partial \bar{V}_{rtnmw}} = -B_{nm} (\rho_{rtmw}^\theta - \rho_{rtmw}^\theta) - \eta_{rytnmw}^V + \phi_{rtw}^{SD} \bar{F}_{nm} = 0 \quad \forall r, \forall t, \forall n, \forall m, \forall w \quad (79)$$

$$\frac{\partial l_y}{\partial O_{yrtiw}^E} = -\eta_{rytkw}^E - \eta_{rytkw}^E + \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \eta_{rytkw}^E + \rho_{rytkw}^E - \phi_{rtw}^{SD} \rho_{rytkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (80)$$

$$\frac{\partial l_y}{\partial O_{yrtiw}^E} = -\eta_{rytkw}^E - \eta_{rytkw}^E + \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \eta_{rytkw}^E + \rho_{rytkw}^E - \phi_{rtw}^{SD} \rho_{rytkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (81)$$

$$\frac{\partial l_y}{\partial O_{yrtiw}^{Sr}} = -\eta_{yrtiw}^{Sr} - \eta_{yrtiw}^{Sr} + \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \eta_{yrtiw}^{Sr} + \rho_{yrtiw}^{Sr} - \phi_{rtw}^{SD} \rho_{yrtiw}^{Sr} = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w$$

$$\frac{\partial l_y}{\partial \bar{V}_{rtnmw}} = -B_{nm} (\rho_{rtmw}^\theta - \rho_{rtmw}^\theta) - \eta_{rytnmw}^V + \phi_{rtw}^{SD} \bar{F}_{nm} = 0 \quad \forall r, \forall t, \forall n, \forall m, \forall w \quad (82)$$

$$\frac{\partial l_y}{\partial \lambda_{rtmw}} = -W_{wr} \left( \frac{1}{1+f} \right)^r \delta_{rt} \sum_{i(i \in n)} P_{yrtiw}^S + \sum_{i(i \in n)} \sum_{r'} P_{yrtiw}^{Sr} + \sum_{k(k \in n)} P_{yrtkw}^E$$

$$+ \sum_i \rho_{yrtiw}^S + \sum_i \sum_{r'} \rho_{yrtiw}^{Sr} + \sum_k \rho_{yrtkw}^E - \sum_d \rho_{rtdw}^D + \sum_m B_{nm} (\rho_{rtmw}^\theta - \rho_{rtmw}^\theta) = 0 \quad \forall y, \forall r, \forall t, \forall n, \forall w$$

$$\frac{\partial l_y}{\partial \bar{V}_{rtnmw}} = -B_{nm} (\rho_{rtmw}^\theta - \rho_{rtmw}^\theta) - \eta_{rytnmw}^V + \phi_{rtw}^{SD} \bar{F}_{nm} = 0 \quad \forall r, \forall t, \forall n, \forall m, \forall w \quad (83)$$

$$\frac{\partial l_y}{\partial \mu_{yrtiw}^S} = -\rho_{yrtiw}^S - \bar{\eta}_{rytiw}^S + \phi_{rtw}^{SD} X_{yri} = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w \quad (65)$$

$$\frac{\partial l_y}{\partial \mu_{yrtiw}^S} = -\rho_{yrtiw}^S - \eta_{rytiw}^S = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w \quad (66)$$

$$\frac{\partial l_y}{\partial \mu_{yrtiw}^S} = -\rho_{yrtiw}^S - \eta_{rytiw}^S = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w \quad (67)$$

$$\frac{\partial l_y}{\partial \mu_{yrtiw}^S} = \rho_{yrtiw}^S - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtiw}^S = 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w \quad (68)$$

$$\frac{\partial l_y}{\partial \bar{\eta}_{yrtkw}^E} = -\rho_{yrtkw}^E - \bar{\mu}_{yrtkw}^E + \phi_{rtw}^{SD} \bar{P}_{ytk}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (69)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = -\rho_{yrtkw}^E - \eta_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (70)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = \rho_{yrtkw}^E - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (71)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{rtdw}^D} = \rho_{rytdw}^D - \bar{\eta}_{rytdw}^D + \phi_{rtw}^{SD} Q_{wr} \bar{P}_{td}^D = 0 \quad \forall r, \forall t, \forall d, \forall w \quad (72)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = \rho_{yrtkw}^E - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (73)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{rtdw}^D} = \rho_{yrtdw}^D - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtdw}^D = 0 \quad \forall r, \forall t, \forall d, \forall w \quad (74)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = \rho_{yrtkw}^E - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (75)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = -\rho_{yrtkw}^E - \bar{\eta}_{yrtkw}^E + \phi_{rtw}^{SD} X_{ytk}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (76)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = -\rho_{yrtkw}^E - \eta_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (77)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = \rho_{yrtkw}^E - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (78)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = \rho_{yrtkw}^E - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (79)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = -\rho_{yrtkw}^E - \bar{\eta}_{yrtkw}^E + \phi_{rtw}^{SD} X_{ytk}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (80)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = -\rho_{yrtkw}^E - \eta_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (81)$$

$$\frac{\partial l_y}{\partial \bar{\mu}_{yrtkw}^E} = \rho_{yrtkw}^E - \sum_{w'(w, w' \in \{Q_{wr} = Q_{w'r}\})} \rho_{yrtkw}^E = 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (82)$$

### 11.3. Complementary constraints: (block 7 in Figure 4)

Complementary constraints of the new second level inequality constraints is as follows. It must be noted that each complementary constraint is in the form of  $a \geq 0, b \geq 0$  equivalent to  $ab = 0$  and  $b \geq 0, a \geq 0$ . These constraints are non-linear which because of the existence of the expressions multiplication of the variables  $b$  each other and complementary constraints, have high non-convergence.

#### Complementary constraints of primary second level problem inequality constraints (Eqs. (16)-(22)).

$$0 \leq (\bar{K}_y - \sum_r \left( \frac{1}{1+f} \right)^r \sum_i K_{yri} X_{yri}) \perp \Delta_y \geq 0 \quad \forall y \quad (84)$$

$$0 \leq O_{yrtiw}^S \perp \eta_{yrtiw}^S \geq 0 \quad \forall y, \forall r, \forall t, \forall i, \forall w \quad (85)$$

$$0 \leq O_{yrtkw}^E \perp \eta_{yrtkw}^E \geq 0 \quad \forall y, \forall r, \forall t, \forall k, \forall w \quad (86)$$

$$0 \leq O_{yrtiw}^{Sr} \perp \eta_{yrtiw}^{Sr} \geq 0 \quad \forall y, \forall r, \forall r' \in \{X_{r'i} > 0, r > r'\}, \forall t, \forall i, \forall w \quad (87)$$

#### Third-level inequality constraints complementary constraints after applying primal-dual transformation (Eqs. (34)-(39)).

$$0 \leq P_{yrtiw}^S \perp \mu_{yrtiw}^{S'} \geq 0 \quad \forall y, \forall i, \forall t, \forall r, \forall w \quad (88)$$

$$0 \leq (X_{yri} - P_{yrtiw}^S) \perp \bar{\mu}_{yrtiw}^{S'} \geq 0 \quad \forall y, \forall i, \forall t, \forall r, \forall w \quad (89)$$

$$0 \leq P_{yrtiw}^r \perp \mu_{yrtiw}^{Sr'} \geq 0 \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall i, \forall w \quad (90)$$

$$0 \leq (X_{yrti}^r - P_{yrtiw}^{Sr'}) \perp \bar{\mu}_{yrtiw}^{Sr'} \geq 0 \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall w \quad (91)$$

$$0 \leq P_{yrtkw}^E \perp \mu_{yrtkw}^{E'} \geq 0 \quad \forall y, \forall k, \forall t, \forall r, \forall w \quad (92)$$

$$0 \leq (P_{yrtkw}^E - \mu_{yrtkw}^{E'}) \perp \bar{\mu}_{yrtkw}^{E'} \geq 0 \quad \forall y, \forall k, \forall t, \forall r, \forall w \quad (93)$$

$$0 \leq P_{rtdw}^D \perp \mu_{rtdw}^{D'} \geq 0 \quad \forall d, \forall t, \forall r, \forall w \quad (94)$$

$$0 \leq (Q_{wr} \bar{P}_{td}^D) \perp \bar{\mu}_{rtdw}^{D'} \geq 0 \quad \forall d, \forall t, \forall r, \forall w \quad (95)$$

$$0 \leq B_{nm}(\theta_{rtnw} - \theta_{rtmw}) + \bar{F}_{nm} \perp \underline{V}_{rtnmw} \geq 0 \quad \forall n, \forall m, \forall t, \forall r, \forall w \quad (96)$$

$$0 \leq \bar{F}_{nm} - B_{nm}(\theta_{rtnw} - \theta_{rtmw}) \perp \bar{V}_{rtnmw} \geq 0 \quad \forall n, \forall m, \forall t, \forall r, \forall w \quad (97)$$

$$0 \leq (\theta_{rtnw} + \pi) \perp \underline{\zeta}'_{rtnw} \geq 0 \quad \forall n, \forall t, \forall r, \forall w \quad (98)$$

$$0 \leq (\pi - \theta_{rtnw}) \perp \bar{\zeta}'_{rtnw} \geq 0 \quad \forall n, \forall t, \forall r, \forall w \quad (99)$$

**TransactiveDual variables complementary constraints created after applying primal-dual transformation to third level problem (Eqs. (46)-(51)).**

$$0 \leq \underline{\mu}_{yrtiw}^S \perp \underline{\eta}_{rytiw}^S \geq 0, \quad 0 \leq \bar{\mu}_{yrtiw}^S \perp \bar{\eta}_{rytiw}^S \geq 0, \quad \forall y, \forall i, \forall t, \forall r, \forall w \quad (100)$$

$$0 \leq \underline{\mu}_{yrr'tiw}^{Sr} \perp \underline{\eta}_{rr'ytiw}^{Sr} \geq 0, \quad 0 \leq \bar{\mu}_{yrr'tiw}^{Sr} \perp \bar{\eta}_{rr'ytiw}^{Sr} \geq 0 \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall i, \forall w \quad (101)$$

$$0 \leq \underline{\mu}_{yrtkw}^E \perp \underline{\eta}_{rytkw}^E \geq 0, \quad 0 \leq \bar{\mu}_{yrtkw}^E \perp \bar{\eta}_{rytkw}^E \geq 0, \quad \forall y, \forall k, \forall t, \forall r, \forall w \quad (102)$$

$$0 \leq \underline{\mu}_{rtdw}^D \perp \underline{\eta}_{rtdw}^D \geq 0, \quad 0 \leq \bar{\mu}_{rtdw}^D \perp \bar{\eta}_{rtdw}^D \geq 0, \quad \forall d, \forall t, \forall r, \forall w \quad (103)$$

$$0 \leq \underline{V}_{rtnmw} \perp \underline{\eta}_{rtnmw}^V \geq 0, \quad 0 \leq \bar{V}_{rtnmw} \perp \bar{\eta}_{rtnmw}^V \geq 0, \quad \forall r, \forall t, \forall n, \forall m, \forall w \quad (104)$$

$$0 \leq \underline{\zeta}_{rtnw} \perp \underline{\eta}_{rytnw}^{\zeta} \geq 0, \quad 0 \leq \bar{\zeta}_{rtnw} \perp \bar{\eta}_{rytnw}^{\zeta} \geq 0, \quad \forall r, \forall t, \forall n, \forall w \quad (105)$$

#### 11.4. Part 4: Equivalent MINLP problem of thri-level model and Linearization

TransactiveAfter applying primal-dual transformation consecutively and KKT conditions to three-level problem now a single level problem has been created. The total problem's objective function is Eq. (23) which is non-linear. The constraints of this problem are as Eq. (106).

$$\text{Eqs. (24) - (26), Eqs. (55) - (105)} \quad (106)$$

The above problem is non-linear and is as MINLP. The problem objective function that is Eq. (23) is non-linear because of the multiplication of the variables by the expression  $\sum_i P_{yrtiw}^S \lambda_{rtn(i \in n)_w} + \sum_i \sum_{r'} P_{yrr'tiw}^{Sr} \lambda_{rtn(i \in n)_w} + \sum_k P_{yrtkw}^E \lambda_{rtn(k \in n)_w}$  which for making it linear the dual theory obtained from primal-dual transformation that is Eq. (32) is used. As a result Eq. (23) becomes equivalent with Eq. (107).

$$\begin{aligned} \text{TP} = & \sum_w \sum_r W_{wr} \left( \frac{1}{1+f} \right)^r \sum_t \sigma_{rt} \left[ \sum_d U_{td}^D P_{rtdw}^D \right. \\ & - \sum_d \bar{\mu}_{rtdw}^D Q_{wr} \bar{P}_{td} - \sum_n \sum_m (\underline{V}_{rtnmw} + \bar{V}_{rtnmw}) \bar{F}_{nm} - \sum_n (\underline{\zeta}_{rtnw} + \bar{\zeta}_{rtnw}) \\ & \left. - \sum_r \left( \frac{1}{1+f} \right)^r \sum_{i \in \psi^S} K_{yri} X_{yri} \right] \end{aligned} \quad (107)$$

Three groups of Eq. 106 constraints are non-linear.

- The complementary constraints of Eqs. (84)-(105). Each complementary constraint in the form of  $b \geq 0 \perp a \geq 0$  is equivalent to  $b \geq 0, a \geq 0, a \leq M\psi$  and  $b \leq M(1-\psi)$ . It's worth mentioning that  $\psi$  and  $M$  are respectively binary variable and a constant number big enough.

- Eq. 31 dual theory constraint: the equality constraint obtained from the dual theory because of its non-linear nature including the multiplication of continues variable is not simply linearized. As was explained in part 5 of the paper (Converting the three-level to one-level problem) the equation obtained from dual theory has been obtained using primal-dual transform method equivalent to the set of commentary constraints using KKT conditions. Because of this linearization, dual theory constraints Eq. (32) are replaced with its equivalent complementary constraints in 107-118 (block 8 in Figure 4 is replaced with block 2). It's worth mentioning that the complementary constraints are linearized using the explanations of linearization of previous section.

Constraints including  $\phi_{rtw}^{sd}$  from the mathematical viewpoint, we can linearize the non-linear expressions including this variable by giving a specific value to  $\phi_{rtw}^{sd}$ .

$$0 \leq P_{yrtiw}^S \perp \underline{\mu}_{yrtiw}^S \geq 0, \quad \forall y, \forall i, \forall t, \forall r, \forall w \quad (108)$$

$$0 \leq (X_{yri} - P_{yrtiw}^S) \perp \bar{\mu}_{yrtiw}^S \geq 0, \quad \forall y, \forall i, \forall t, \forall r, \forall w \quad (109)$$

$$0 \leq P_{yrr'tiw}^{Sr} \perp \underline{\mu}_{yrr'tiw}^{Sr} \geq 0, \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall i, \forall w \quad (110)$$

$$0 \leq (X_{yrr'i}^r - P_{yrr'tiw}^{Sr}) \perp \bar{\mu}_{yrr'tiw}^{Sr} \geq 0, \quad \forall y, \forall r, \forall r' \in \{X_{r's} > 0, r > r'\}, \forall t, \forall i, \forall w \quad (111)$$

$$0 \leq P_{yrtkw}^E \perp \underline{\mu}_{yrtkw}^E \geq 0, \quad \forall y, \forall k, \forall t, \forall r, \forall w \quad (112)$$

$$0 \leq (\bar{P}_{yk}^E - P_{yrtkw}^E) \perp \bar{\mu}_{yrtkw}^E \geq 0, \quad \forall y, \forall k, \forall t, \forall r, \forall w \quad (113)$$

$$0 \leq P_{rtdw}^D \perp \underline{\mu}_{rtdw}^D \geq 0, \quad \forall d, \forall t, \forall r, \forall w \quad (114)$$

$$0 \leq (Q_{wr} \bar{P}_{td}^D) \perp \bar{\mu}_{rtdw}^D \geq 0, \quad \forall d, \forall t, \forall r, \forall w \quad (115)$$

$$0 \leq B_{nm}(\theta_{rtnw} - \theta_{rtmw}) + \bar{F}_{nm} \perp \underline{V}_{rtnmw} \geq 0, \quad \forall n, \forall m, \forall t, \forall r, \forall w \quad (116)$$

$$0 \leq \bar{F}_{nm} - B_{nm}(\theta_{rtnw} - \theta_{rtmw}) \perp \bar{V}_{rtnmw} \geq 0, \quad \forall n, \forall m, \forall t, \forall r, \forall w \quad (117)$$

$$0 \leq \theta_{rtnw} + \pi \perp \underline{\zeta}_{rtnw} \geq 0, \quad \forall n, \forall t, \forall r, \forall w \quad (118)$$

$$0 \leq \pi - \theta_{rtnw} \perp \bar{\zeta}_{rtnw} \geq 0, \quad \forall n, \forall t, \forall r, \forall w \quad (119)$$

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