off $\mathcal{O}$ such that $(x_1(0), x_2(0)) \neq (0,0), x_3(0) > 0$, the corresponding trajectory is bounded, but its positive limit set is the unit circle on $\mathcal{O}$, and therefore it is not a subset of Γ; see Fig. 2. In conclusion, Γ is not attractive for the closed-loop system (and neither is it stable). This example illustrates the fact that, when $\Gamma \subsetneq V^{-1}(0)$ is compact, simply requiring condition (11) in place of Γ-detectability may not be enough for attractivity of Γ.

In the light of Theorem V.2 and the example above, it is clear that the addition of the stability requirement on Γ, relative to $\mathcal{O}$, is a crucial enhancement to the notions of detectability in [7] and [10].

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High Gain Observer for Structured Multi-Output Nonlinear Systems

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***Abstract*—In this note, we present two system structures that characterize classes of multi-input multi-output uniformly observable systems. The first structure is decomposable into a linear and a nonlinear part while the second takes a more general form. It is shown that the second system structure, being more general, contains several system structures that are available in the literature. Two high gain observer design methodologies are presented for both structures and their distinct features are highlighted.**

***Index Terms*—High gains, nonlinear observers, nonlinear systems.**

I. INTRODUCTION

The synthesis of nonlinear observers is generally a difficult problem due to the fact that the observability property of nonlinear systems is input dependent [17]. In effect, the dependence of the inputs on the observability of nonlinear systems has led several authors to study the problem of characterization of systems that are observable for all inputs; that is, uniformly observable systems (see, e.g., [7] and [18]). This characterization is well established in the single output case for control affine systems. As a matter of fact, a diffeomorphism has been proposed that allows to transform such systems in a well-defined observable canonical form—which is commonly referred to as the triangular observable canonical form, due to the triangular structure of the nonlinearity in the new coordinates system (see, e.g., [7] and [8]). In [8] this triangular observable canonical form has been employed to design a high gain observer for single output uniformly observable control affine systems. Various other observer design approaches, ranging from extended Kalman filter to sliding-mode observers, have been proposed for subclasses of single output uniformly observable systems (see, e.g., [3], [4], [6], and [14]). In the same context, observer design with linearizable error dynamics for such classes of systems has been widely studied ([13]). With regards to the multi-output case, the characterization of the structure of multi-input multi-output (MIMO) uniformly observable systems is still an open problem. However, there are some special observable structures in the MIMO case that are easily recognizable. In effect, suppose that one can find a diffeomorphism that transforms the original MIMO system into a cascade of subsystems; then, it is fairly obvious that if each subsystem is in the triangular observable canonical form (as in the single output case) then the overall system is uniformly observable. This particular structure is not the only structure that characterizes every (MIMO) uniformly observable system. There are indeed other structures, with intricate nonlinear coupling between the subsystems, that characterize MIMO uniformly observable systems. One such structure, which characterizes a subclass of MIMO uniformly observable systems, was proposed in [1] and for

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which a high gain observer was also derived. By using the graph approach, the same authors ([2]) have proposed a structure of class of nonlinear systems containing the class (S1) described in the next section. However, the observer synthesis was not addressed.

Several other subclasses of MIMO uniformly observable systems have been proposed for which a corresponding observer has been designed (see, e.g., [5], [10], [11], and [16]).

In this note, we give an observer synthesis for two nonlinear canonical forms which are characterized by structures (S1) and (S2) described in the following section. The first structure (S1) contains systems that can be decomposed into a linear part and a nonlinear part that is dependent on the inputs. The overall system is displayed in the form of p subsystems corresponding to the number of outputs. The proposed system structure easily leads to the design of a high gain observer for the system. The second canonical structure takes a more general form and is not necessarily decomposable into a linear and nonlinear part. It is a generalization of the first system structure and, as such, characterizes a more general class of MIMO uniformly observable systems. An observer design is also proposed for the second canonical structure. Its design is not as straightforward as for the first canonical form but combines a structured high gain observer with a constant gain, as proposed in [9] and [11]. As we have mentioned above, the problem of finding normal forms for general multi-output uniformly observable nonlinear systems is not solved and is, in effect, a very difficult task. The classes of nonlinear systems proposed in this note contain most classes of uniformly observable systems studied in the literature; in particular, those stated in [1], [5], [10], [15], and [16].

The note is organized as follows. In the next section, we present the two classes of MIMO systems under consideration. We set the assumptions and give the structural conditions on the nonlinearity involved. Some examples are provided to clarify the nonlinear coupling between the various subsystems. Afterwards, in Section III, observer design methodologies are proposed for the considered classes of systems.

II. The Class of Systems Considered

In this section, we present two structures, $(\mathcal{S}_1)$ and $(\mathcal{S}_2)$, of MIMO nonlinear systems for which a high gain observer can be designed. These systems possess a normal form that generalizes most observable normal forms existing in the literature. Even though the first class $(\mathcal{S}_1)$ is contained in $(\mathcal{S}_2)$, the implementation of the observer design for $(\mathcal{S}_1)$ is much simpler compared to that for $(\mathcal{S}_2)$. Consequently, we present these two system structures separately.

A. System Structure $(\mathcal{S}_1)$

We consider nonlinear systems that are equivalent by diffeomorphism to systems of the form

$$\begin{cases} \dot{z} = A + \varphi(u, z) \\ y = Cz \end{cases} \tag{1}$$

where $z = \left(z_1^T, \ldots, z_p^T\right)^T \in R^n$; $z_k = \left(z_{k1}^T, \ldots, z_{kn_k}^T\right)^T \in R^{n_k}$ with $\sum_{i=1}^p n_i = n$ and $n_k \geq 2$; $u \in R^m$; $y = (y_1, \ldots, y_p)^T \in R^p$

$$A = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_p \end{bmatrix}, \quad A_k = \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ 0 & \ldots & 0 & 1 \\ 0 & \ldots & 0 & 0 \end{bmatrix}$$

is $n_k \times n_k$ matrix, $C = \mathrm{Diag}(C_1, \ldots, C_p)$ is a $p \times n$ block diagonal matrix, where $C_k = [1, 0, \ldots, 0] \in R^{n_k}$; $\varphi = \left(\varphi_1^T, \ldots, \varphi_p^T\right)^T$ and $\varphi_k = (\varphi_{k1}, \ldots, \varphi_{kn_k})^T$, the φ_{kj} are of class $\mathcal{C}^1$ w.r.t. (z, u).

We assume the following:

A1) There exist two sets of real numbers $\{\sigma_1, \ldots, \sigma_p\}$ and $\{\delta_1, \ldots, \delta_p\}$, with $\delta_k > 0$, $k = 1, \ldots, p$, such that for $k, l = 1, \ldots, p$; $i = 1, \ldots, n_k$ and $j = 2, \ldots, n_l$, we have

$$\frac{\partial \varphi_{ki}}{\partial z_{lj}}(u, z) \not\equiv 0 \Rightarrow \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^l - \frac{\delta_l}{2} \tag{2}$$

where $\sigma_i^k = \sigma_k + (i-1)\delta_k$, and $\partial\varphi_{ki}/\partial z_{lj}(u, z) \not\equiv 0$ means that there exists $(u^0, z^0) \in R^m \times R^n$, s.t. $\partial\varphi_{ki}/\partial z_{lj}(u^0, z^0) \neq 0$.

Remark 1: For $k = l$, Condition (2) of Assumption **A1)** is equivalent to the following.

For $1 \leq i \leq n_k - 1$, $(j \geq i + 1 \Rightarrow (\partial\varphi_{ki}/\partial z_{kj})(u, z) \equiv 0)$

1) System Structure $(\mathcal{S}_2)$: Using the same notations as for structure $(\mathcal{S}_1)$, consider the following system:

$$\begin{cases} \dot{z} = F(u, z) \\ y = Cz \end{cases} \tag{3}$$

where $u \in U$ is a bounded Borelian subset of R^m

$$F(u, z) = \begin{bmatrix} F_1(u, z) \\ \ldots \\ F_p(u, z) \end{bmatrix}; \quad F_k(u, z) = \begin{bmatrix} F_{k1}(u, z) \\ \ldots \\ F_{kn_k}(u, z) \end{bmatrix}.$$

For the sake of simplicity, F is assumed to be of class $\mathcal{C}^1$ w.r.t. (z, u). System (3) can be rewritten as follows:

$$\begin{cases} \dot{z}_k = F_k(u, z) \\ y_k = C_k z_k \\ k = 1, \ldots, p. \end{cases} \tag{4}$$

A1') There exist two sets of real numbers $\{\sigma_1, \ldots, \sigma_p\}$ and $\{\delta_1, \ldots, \delta_p\}$, with $\delta_k > 0$, $k = 1, \ldots, p$, such that the following conditions are satisfied:

$$\begin{cases} \text{for } k, l = 1, \ldots, p;\ k \neq l;\ \text{for } i = 1, \ldots, n_k;\ j = 2, \ldots, n_l, \\ \qquad \frac{\partial F_{ki}}{\partial z_{lj}}(u, z) \not\equiv 0 \Rightarrow \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^l - \frac{\delta_l}{2} \\ \text{for } k = 1, \ldots, p;\ \text{for } i = 1, \ldots, n_k;\ j = 3, \ldots, n_k, \\ \qquad \frac{\partial F_{ki}}{\partial z_{kj}}(u, z) \not\equiv 0 \Rightarrow \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^k - \frac{3\delta_k}{2} \end{cases} \tag{5}$$

$$\begin{cases} \frac{\partial F_{ki}}{\partial z_{k,i+1}}(u, z) & \text{does not change the sign and satisfy:} \\ & \text{for} 1 \leq i \leq n_k - 1 \\ \exists \tau > 0;\ \forall (u, z) \in U \times R^n, & \left|\frac{\partial F_{ki}}{\partial z_{k,i+1}}(u, z)\right| \geq \tau \end{cases} \tag{6}$$

where $\sigma_i^k = \sigma_k + (i-1)\delta_k$.

Remark 2:

1- The second implication of (5) is equivalent to: $\forall\ J \geq I + 2$, $(\partial f_{ki}/\partial z_{kj})(u, Z) \equiv 0$.

2- If $n_k = 2$, then the second inequality of (5) must be dropped.

3- For systems having structure $(\mathcal{S}_1)$:

i) we have $F_{ki}(u, z) = z_{k,i+1} + \varphi_{ki}(u, z)$, for $i = 1, \ldots, n_k - 1$; in particular $(\partial F_{ki}/\partial z_{k,i+1})(u, z) = 1$.

ii) Conditions (5) and (6) become equivalent to Condition (2).

Remark 3: Notice that if the set of real numbers $\{\sigma_1, \ldots, \sigma_p, \delta_1, \ldots, \delta_p\}$ satisfies Assumption **A1')**, so does $\{\sigma_1 + r, \ldots \sigma_p + r, \delta_1, \ldots, \delta_p\}$, for every $r \in R$.

Remark 4: For $1 \leq k, l \leq p$; $1 \leq i \leq n_k$, set $I(k, l, i) = \{j, 2 \leq j \leq n_l, \text{ such that } (\partial F_{ki}/\partial z_{lj})(u, z) \not\equiv 0\}$ and $j(k, l, i) = \max I(k, l, i)$ if $I(k, l, i) \neq \emptyset$. Then, Condition (5) is equivalent to the following.

i) For $k = l$, for $1 \leq i \leq n_k$, $I(k, l, i) \neq \emptyset$ implies $j(k, l, i) \leq i + 1$.

ii) For $k \neq l$, for $1 \leq i \leq n_k$, $I(k, l, i) \neq \emptyset$ implies $\sigma_k + (i - (1/2))\delta_k > \sigma_l + (j(k, l, i) - (3/2))\delta_l$.

We end this subsection by giving a procedure that permits to calculate the $\sigma_1, \ldots, \sigma_p, \delta_1, \ldots, \delta_p$ whenever they exist. To do so, we calculate all $I(k, l, i)$, $1 \leq k, l \leq p$, $1 \leq i \leq n_k$ and we proceed as follows.

a) For $k = l$, condition i) of Remark 4 can be obviously checked.

b) **Condition ii)** for $k \neq l$, we consider the set $E = \{(k,l), 1 \leq k,l \leq p, k \neq l$ for which there exist $i, 1 \leq i \leq n_k$, s.t. $I(k,l,i) \neq \emptyset\}$. We discuss two cases:

1) If $E = \emptyset$, then the structure $(\mathcal{S}_2)$ is a trivial one. It means that for $1 \leq k \leq p$, $\dot{z}_k = F_k(u, y, z_{k2}, \ldots, z_{kn_k})$ and $y_k = C_k z_k$. From the observation point of view, the observer synthesis becomes equivalent to the single output one.

2) If $E \neq \emptyset$, for $(k,l) \in E$, we set $I(k,l) = \{i, 1 \leq i \leq n_k$, s.t. $I(k,l,i) \neq \emptyset\}$. Then, condition ii) of Remark 4 becomes equivalent to

$$\begin{cases} \sigma_k + (i - \frac{1}{2})\delta_k > \sigma_l + (j(k,l,i) - \frac{3}{2})\delta_l, \\ \delta_k > 0, \delta_l > 0 \\ (k,l) \in E, i \in I(k,l) \end{cases} \quad (7)$$

Using Remark 3, it follows that system (7) admits a solution, iff (8)

$$\begin{cases} \sigma_k + \left(i - \frac{1}{2}\right)\delta_k > \sigma_l + \left(j(k,l,i) - \frac{3}{2}\right)\delta_l \\ \sigma_k \geq 0, \sigma_l \geq 0, \delta_k > 0, \delta_l > 0 \\ (k,l) \in E, i \in I(k,l) \end{cases} \quad (8)$$

also admits a solution.

Let $u, v \in R^k$, the notation $u \geq v$ (resp. $u > v$) means that for every i, $1 \leq i \leq k$, $u_i \geq v_i$ (resp. $u_i > v_i$). Setting $X = (\sigma_1, \ldots, \sigma_p, \delta_1, \ldots, \delta_p)^T$, $X_1 = (\sigma_1, \ldots, \sigma_p)$, $X_2 = (\delta_1, \ldots, \delta_p)$, system (8) can be rewritten in the form

$$LX > 0; \quad X_1 \geq 0; \quad X_2 > 0 \quad (9)$$

where L is an $N \times (2p)$ constant matrix, and N is the cardinality of the set $\{(k,l,i); (k,l) \in E, i \in I(k,l)\}$.

Denoting by D the set of X satisfying (9). Set $r = (r, \ldots, r)^T \in R^p$ and $D_r = \{X; LX > 0; X_1 \geq 0; X_2 \geq r\}$. Then, $D \neq \emptyset$ iff for every $r > 0$, $D_r \neq \emptyset$. Indeed, let $r > 0$ and assume that $D \neq \emptyset$. Let $X^0 \in D$ and $\rho > 0$ such that $\rho X_2^0 \geq \mathbf{r}$, thus $\rho X^0 \in D_r$. The converse is trivial. Hence, system (9) admits a solution if, and only if, the following linear program admits a solution:

$$\begin{cases} LX > 0, \ X_1 \geq 0, \ X_2 \geq \mathbf{r} \\ \min HX \end{cases} \quad (10)$$

where $H = (h_1, \ldots, h_{2p})$, is any vector, such that $h_i > 0$ and $\mathbf{r} > 0$.

Consequently, the simplex algorithm can be used in order to obtain a solution of (10).

For small dimensions or for some particular structure, the use of the linear programming is not necessary, as we will show in the following examples.

B. Some Examples

1) Example 1: For $k = 1,2$ and $n_k \geq 2$, set $z_k = [z_{k1}, \ldots, z_{kn_k}]$, $z = [\, z_1 \quad z_2 \,]$ and consider the following pseudo triangular structure:

$$\begin{cases} \text{for } 1 \leq i \leq n_1 - 1 \\ \dot{z}_{1i} = F_{1i}(u, z_{11}, \ldots, z_{1,i+1}, z_{21}) \\ \dot{z}_{1n_1} = F_{1n_1}(u, z) \\ \text{for } 1 \leq j \leq n_2 - 1 \\ \dot{z}_{2j} = F_{2j}(u, z_1, z_{21}, \ldots, z_{2,j+1}) \\ \dot{z}_{2n_2} = F_{2n_2}(u, z) \\ y = [\, z_{11} \quad z_{21} \,]. \end{cases} \quad (11)$$

We will show that system (11) satisfies Condition (5) of Assumption **A1')**, or equivalently Conditions i) and ii) of Remark 4.

- Condition i) of Remark 4 is obviously satisfied.
- Let us check Condition ii) of Remark 4:

1) For $k = 1, l = 2, 1 \leq i \leq n_1 - 1, I(k,l,i) = \emptyset$, and for $i = n_1$, generically, we have $I(k,l,n_1) = \{n_2\}$.

2) Similarly, for $k = 2, l = 1, 1 \leq i \leq n_2, I(k,l,i) = \{2, \ldots, n_1\}$; hence, $j(k,l,i) = n_1$.

Thus, system of inequalities (8) takes the form

$$\begin{cases} \sigma_k \geq 0, \ \delta_k > 0, \ k = 1,2 \\ \sigma_1 + \left(n_1 - \frac{1}{2}\right)\delta_1 > \sigma_2 + \left(n_2 - \frac{3}{2}\right)\delta_2 \\ \sigma_2 + \left(i - \frac{1}{2}\right)\delta_2 > \sigma_1 + \left(n_1 - \frac{3}{2}\right)\delta_1 \\ 1 \leq i \leq n_2 \end{cases} \quad (12)$$

which is equivalent to

$$\begin{cases} \sigma_k \geq 0, \ \delta_k > 0, \ k = 1,2 \\ \sigma_1 + \left(n_1 - \frac{1}{2}\right)\delta_1 > \sigma_2 + \left(n_2 - \frac{3}{2}\right)\delta_2 \\ \sigma_2 + \frac{\delta_2}{2} > \sigma_1 + \left(n_1 - \frac{3}{2}\right)\delta_1 \end{cases} \quad (13)$$

Finally, the system is equivalent to

$$\begin{cases} \sigma_k \geq 0, \ \delta_k > 0, \ k = 1,2 \\ \sigma_1 + \left(n_1 - \frac{3}{2}\right)\delta_1 - \frac{\delta_2}{2} < \sigma_2 < \sigma_1 + \left(n_1 - \frac{1}{2}\right)\delta_1 \\ \qquad - \left(n_2 - \frac{3}{2}\right)\delta_2. \end{cases} \quad (14)$$

Set $\alpha_1 = \sigma_1 + (n_1 - (3/2))\delta_1 - (\delta_2/2)$ and $\alpha_2 = \sigma_1 + (n_1 - (1/2))\delta_1 - (n_2 - (3/2))\delta_2$. It suffices to find $\sigma_1 \geq 0$, $\delta_1 > 0$ and $\delta_2 > 0$ such that $\alpha_1 \geq 0$, $\alpha_2 - \alpha_1 > 0$ and to set $\sigma_2 = (\alpha_1 + \alpha_2/2)$. To do so, it suffices to choose $\delta_1 > 0$, $\delta_2 > 0$ and $\sigma_1 \geq 0$ such that $\delta_1 > \max\{\delta_2, (n_2 - 2)\delta_2\}$

2) Example 2:

$$\begin{cases} \dot{z}_{11} = F_{11}(u, z_{11}, z_{12}, z_{21}, z_{22}) \\ \dot{z}_{12} = F_{12}(u, z_{11}, z_{12}, z_{13}, z_{21}, z_{22}) \\ \dot{z}_{13} = F_{13}(u, z_{11}, z_{12}, z_{13}, z_{21}, z_{22}) \\ \dot{z}_{21} = F_{21}(u, z_{11}, z_{21}, z_{22}) \\ \dot{z}_{22} = F_{22}(u, z_{11}, z_{12}, z_{21}, z_{22}, z_{23}) \\ \dot{z}_{23} = F_{23}(u, z_{11}, z_{12}, z_{13}, z_{21}, z_{22}, z_{23}) \\ y = [\, z_{11} \quad z_{21} \,] \end{cases} \quad (15)$$

As for the above example, we will check Conditions i) and ii) of Remark 4:

- Condition i) is obvious.
- Condition ii) of Remark 4:
  a) For $k = 1, l = 2$: $j(k,l,1) = 2$, $j(k,l,2) = 2$ and $j(k,l,3) = 2$
  b) For $k = 2, l = 1$: $I(k,l,1) = \emptyset$, $j(k,l,2) = 2$ and $j(k,l,3) = 3$.

System (8) takes the form

$$\begin{cases} \sigma_k \geq 0, \ \delta_k > 0, \ k = 1,2 \\ \sigma_1 + \frac{\delta_1}{2} > \sigma_2 + \frac{\delta_2}{2}; \qquad \sigma_1 + \frac{3\delta_1}{2} > \sigma_2 + \frac{\delta_2}{2} \\ \sigma_1 + \frac{5\delta_1}{2} > \sigma_2 + \frac{\delta_2}{2}; \qquad \sigma_2 + \frac{3\delta_2}{2} > \sigma_1 + \frac{\delta_1}{2} \\ \sigma_2 + \frac{5\delta_2}{2} > \sigma_1 + \frac{3\delta_1}{2} \end{cases} \quad (16)$$

Set $\sigma_2 = 0$. Then, the system of inequalities (16) becomes equivalent to

$$\begin{cases} \sigma_1 \geq 0, \ \sigma_2 = 0, \ \delta_k > 0, \ k = 1,2 \\ -\frac{\delta_1}{2} + \frac{\delta_2}{2} < \sigma_1 \\ \sigma_1 < \frac{3\delta_2}{2} - \frac{\delta_1}{2} \\ \sigma_1 < \frac{5\delta_2}{2} - \frac{3\delta_1}{2}. \end{cases} \quad (17)$$

Set $a_1(\delta_1, \delta_2) = -(\delta_1/2) + (\delta_2/2)$, $a_2(\delta_1, \delta_2) = (3\delta_2/2) - (\delta_1/2)$ and $a_3(\delta_1, \delta_2) = (5\delta_2/2) - (3\delta_1/2)$. To obtain a solution $\sigma_1 \geq 0$, $\delta_1 > 0$, $\delta_2 > 0$ satisfying (17), it suffices to find $\delta_1 > 0$, $\delta_2 > 0$ such that $a_2(\delta_1, \delta_2) > 0$, $a_3(\delta_1, \delta_2) > 0$ and $\min\{a_2(\delta_1, \delta_2), a_3(\delta_1, \delta_2)\} - a_1(\delta_1, \delta_2) > 0$. A solution is given by choosing $\delta_2 > (3\delta_1/5) > 0$ and $a_1(\delta_1, \delta_2) < \sigma_1 <$

$\min\{a_2(\delta_1,\delta_2), a_3(\delta_1,\delta_2)\}$ (take for instance, $\delta_1 = 3$, $\delta_2 = 2$ and $\sigma_1 = 0$).

III. High Gain Observer Design

In this section, we firstly state the observer structure for systems of the class $(\mathcal{S}_1)$, without giving the proof. Afterwards, we give the proof of the convergence of the observer for the second structure; since $(\mathcal{S}_1)$ is contained in $(\mathcal{S}_2)$.

A. Observer Design for Structure $(\mathcal{S}_1)$

In this subsection, we will assume the following:

A2) The nonlinear term φ of system (1) is a global Lipschitz function; that is: $\forall\, M > 0; \exists \gamma > 0; \forall\, u, \|u\| \le M; \forall\, z, z' \in R^n$, $\|\varphi(u,z) - \varphi(u,z')\| \le \gamma\|z - z'\|$.

The candidate observer for system (1) takes the form

$$\dot{\hat{z}} = A\hat{z} + \varphi(u,\underline{\hat{z}}) + \Delta_\Theta K(C\hat{z} - y) \tag{18}$$

where:

i)
$$\begin{cases} \underline{\hat{z}}_{k1} = y_k \text{ for } k = 1,\ldots,p \text{ (output injection)} \\ \underline{\hat{z}}_{ki} = \hat{z}_{ki}, \text{ for } i \ne 1 \end{cases} \tag{19}$$

ii) u and y are the known output and input of system (1);

iii)
$$\begin{cases} \Delta_{\theta^{\delta_k}} = \mathrm{Diag}(\theta^{\delta_k}, \theta^{2\delta_k}, \ldots, \theta^{n_k\delta_k}) \\ \Delta_\Theta = \mathrm{Diag}(\Delta_{\theta^{\delta_1}}, \ldots, \Delta_{\theta^{\delta_p}}) \end{cases} \tag{20}$$

are diagonal matrices of dimension $n_k \times n_k$ and $n \times n$ respectively, and $K = \begin{bmatrix} K_1 & & \\ & \ddots & \\ & & K_p \end{bmatrix}$ is a diagonal matrix with K_k being a n_k-column vector such that $A_k + K_k C_k$ is Hurwitz.

Theorem 1: Assume that system (1) satisfies Assumptions **A1)–A2)** and set $e(t) = \hat{z}(t) - z(t)$. Then: $\forall\, M > 0; \exists \theta_0 > 0; \forall\, \theta \ge \theta_0; \exists \lambda_\theta > 0; \exists \mu_\theta > 0$ such that $\|e(t)\|^2 \le \lambda_\theta e^{-\mu_\theta t}\|e(0)\|^2$ for every admissible control u such that $\|u\|_\infty \le M$ and for every initial conditions $\hat{z}(0)$, $z(0)$.

This means that system (18) is an exponential observer for system (1) which works for bounded inputs. Moreover, $\lim_{\theta \to +\infty} \mu_\theta = +\infty$.

The proof of Theorem 1 can be obtained in a similar way as the proof of Theorem 2 below.

B. Observer Design for Structure $(\mathcal{S}_2)$

Consider systems of the form (3) satisfying Conditions (5)–(6) of Assumption **A1')**. By using the continuity of $(\partial F_{ki}/\partial z_{k,i+1})(u,z)$; then, from Condition (6), either one of the two inequalities hold for (k,i):

- $\forall\,(u,x) \in U \times R^n$, $(\partial F_{ki}/\partial z_{k,i+1})(u,z) \ge \tau$;
- $\forall\,(u,x) \in U \times R^n$, $(\partial F_{ki}/\partial z_{k,i+1})(u,z) \le -\tau$.

From a trivial change of coordinates, we can assume that

$$\text{For } 1 \le k \le p, \quad 1 \le i \le n_k - 1, \quad \left(\frac{\partial F_{ki}}{\partial z_{k,i+1}}\right)(u,z) \ge \tau. \tag{21}$$

As in the above subsection, we will make the following assumption:

A2') $\exists \gamma > 0; \forall\, u \in U; \forall\, z, z' \in R^n$, $\|F(u,z) - F(u,z')\| \le \gamma\|z - z'\|$.

Combining this assumption with (21), we deduce that

$$\tau \le \frac{\partial F_{ki}}{\partial z_{k,i+1}}(u,z) \le \gamma. \tag{22}$$

In order to design a high gain observer for system (3), some preliminary technical results will be required.

In effect, consider the following $n_k \times n_k$ matrix:

$$A_k(t) = \begin{bmatrix} 0 & a_1(t) & 0 & & 0 \\ \vdots & & a_2(t) & & \\ 0 & & & \ddots & a_{n_k-1}(t) \\ 0 & 0 & 0\ldots & & 0 \end{bmatrix} \tag{23}$$

where the $a_i(\cdot)$'s might be unknown and satisfy the following inequality:

$$\tau \le a_i(t) \le \gamma. \tag{24}$$

Let S_k be a $n_k \times n_k$ symmetric matrix of the form:

$$S_k = \begin{bmatrix} s_{11} & s_{12} & 0 & & 0 \\ s_{12} & s_{22} & s_{23} & \ddots & \\ 0 & \ddots & & \ddots & 0 \\ \vdots & & & \ddots & s_{n_k-1,n_k} \\ 0 & \ldots & 0 & s_{n_k-1,n_k} & s_{n_k,n_k} \end{bmatrix} \tag{25}$$

and where $C_k = (1,0,\ldots,0)$ is the n_k-row vector. The following lemma is stated in [9] (and further improved in [12]).

Lemma 1: ([9], [12]): Assuming that (24) holds, then for every $\rho > 0$; there exist $\eta_k > 0$ and a symmetric positive definite (SPD) matrix S_k of the form (25), such that

$$\forall\, t \ge 0, \quad A_k^T(t)S_k + S_kA_k(t) - \rho C_k^T C_k \le -\eta_k I_k. \tag{26}$$

Notice that the constant matrix S_k depends only on ρ and the constants τ, γ given in (24).

Now, consider the $n \times n$ SPD diagonal matrix $S = \mathrm{Diag}(S_1,\ldots,S_p)$. Our candidate observer takes the following form:

$$\dot{\hat{z}} = F(u,\underline{\hat{z}}) - \Omega\Delta_\Theta S^{-1}C^T(C\hat{z} - y) \tag{27}$$

where

$$\begin{cases} \underline{\hat{z}}_{k1} = y_k = z_{k1}, & \text{for } k = 1,\ldots,p \\ \underline{\hat{z}}_{ki} = \hat{z}_{ki}, & \text{for } i \ne 1. \end{cases} \tag{28}$$

Δ_Θ and $\Delta_{\theta^{\delta_k}}$ are the diagonal matrices given in (20); $\Omega = \mathrm{Diag}(\omega_1 I_1,\ldots,\omega_p I_p)$ is a $n \times n$ diagonal matrix, where I_k is the $n_k \times n_k$ identity matrix, and $\omega_1 > 0,\ldots,\omega_p > 0$ are constants which must be judiciously chosen.

Theorem 2: Assume that system (3) satisfies Assumptions **A1')–A2')** and set $e(t) = \hat{z}(t) - z(t)$. Then $\exists \theta_0 > 0$; $\exists \omega_{10} > 0,\ldots,\exists \omega_{p0} > 0$; $\forall\,\theta \ge \theta_0$; $\forall\,\omega_k > \omega_{k0}$, $k = 1,\ldots,p$; $\exists \lambda_\theta > 0$; $\exists \mu_\theta > 0$ such that $\|e(t)\|^2 \le \lambda_\theta e^{-\mu_\theta t}\|e(0)\|^2$, for every admissible control u taking its values in the bounded set U, and for every initial conditions $\hat{z}(0)$, $z(0)$. Moreover, $\lim_{\theta\to+\infty} \mu_\theta = +\infty$.

Proof of Theorem 2: Setting $e = \widehat{z} - z$, $e_k = \widehat{z}_k - z_k$, we obtain

$$\dot{e}_k = F_k(u,\underline{\widehat{z}}) - F_k(u,z) - \omega_k\Delta_{\theta^{\delta_k}}S_k^{-1}C_k^TC_ke_k. \tag{29}$$

Set $\widetilde{z}(k,i) = \begin{bmatrix} \widetilde{z}_1(k,i) \\ \vdots \\ \widetilde{z}_p(k,i) \end{bmatrix} \in R^n$, where $\widetilde{z}_l(k,i) = \begin{bmatrix} \widetilde{z}_{l1}(k,i) \\ \vdots \\ \widetilde{z}_{ln_l}(k,i) \end{bmatrix} \in R^{n_l}$ is defined by

$$\begin{cases} \text{for } 1 \le i \le n_k - 1, \\ \text{for } l = k, \text{ and } j = i+1, \quad \widetilde{z}_{k,i+1}(k,i) = z_{k,i+1} \\ \text{otherwise} \quad \widetilde{z}_{l,j}(k,i) = \widehat{\underline{z}}_{lj} \\ \text{for } i = n_k, \quad \tilde{z}(k,n_k) = \widehat{\underline{z}} \end{cases} \tag{30, 31}$$

Setting $\widetilde{F}_k(u,z,\widehat{z}) = \begin{bmatrix} \widetilde{F}_{k1}(u,z,\widehat{z}) \\ \vdots \\ \widetilde{F}_{kn_k}(u,z,\widehat{z}) \end{bmatrix}$, where $\widetilde{F}_{ki}(u,z,\widehat{z}) = F_{ki}(u,\widetilde{z}(k,i))$, then (29) becomes (32), as shown at the bottom of the page.

$$\begin{cases} \dot{e}_k = (F_k(u,\widehat{\underline{z}}) - \widetilde{F}_k(u,z,\widehat{z})) \\ +(\widetilde{F}_k(u,z,\widehat{z}) - F_k(u,z)) - \omega_k \Delta_{\theta^{\delta_k}} S_k^{-1} C_k^T C_k e_k. \end{cases} \tag{32}$$

Setting $\widetilde{e}_{lj} = \widetilde{z}_{lj}(k,i) - z_{lj}$ and using the definition of $\widehat{\underline{z}}$ and $\widetilde{z}(k,i)$, then

$$\begin{cases} \widetilde{e}_{l1} = 0, \text{ for } 1 \le l \le p, \\ \widetilde{e}_{lj} = e_{lj} \text{ for } l \ne k, j \ne i+1 \text{ and } j \ne 1 \\ e_{l,j}\widetilde{(k,i)} = 0 \text{ for } l = k, \text{ and } j = i+1. \end{cases} \tag{33}$$

From the mean value theorem, we obtain $\widetilde{F}_{ki}(u,z,\widehat{z}) - F_{ki}(u,z) = F_{ki}(u,\widetilde{z}(k,i)) - F_{ki}(u,z) = \sum_{l=1}^{p}\sum_{j=1}^{n_l} b_{lj}^{ki}(t)\widetilde{e}_{lj}$, where $b_{lj}^{ki}(t) = (\partial F_{ki}/\partial z_{lj})(u, z + h\widetilde{e})$, for some $h \in [0,1]$.

According to (33), we get

$$\widetilde{F}_{ki}(u,z,\widehat{z}) - F_{ki}(u,z) = \sum_{l=1,l\ne k}^{p}\sum_{j=2}^{n_l} b_{lj}^{ki}(t)e_{lj} + \sum_{j=2,j\ne i+1}^{n_k} b_{kj}^{ki} e_{kj} \tag{34}$$

- if $n_k = 2$, then the last term of the right side of (34) must be dropped.
- From Condition (5), we know that we get

$$\begin{cases} \text{if } l \ne k, & \left(\frac{\partial F_{ki}}{\partial z_{lj}} \not\equiv 0 \Rightarrow \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^l - \frac{\delta_l}{2}\right) \\ \text{if } l = k, \text{ and } j \ge 3, & \left(\frac{\partial F_{ki}}{\partial z_{kj}} \not\equiv 0 \Rightarrow \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^k - \frac{3\delta_k}{2}\right) \end{cases} \tag{35}$$

The second inequality of (35) is equivalent to: $j \ge i+2$ implies $\partial F_{ki}/\partial z_{kj} \equiv 0$ (see Remark 2-1)), thus

$$\sum_{j=2,j\ne i+1}^{n_k} b_{kj}^{ki} e_{kj} = \sum_{j=2}^{i} b_{kj}^{ki} e_{kj}. \tag{36}$$

Now, set

$$\begin{cases} J(k,i) = \{(l,j), 1 \le l \le p; l \ne k; 2 \le j \le n_l, \\ \text{s.t. } \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^l - \frac{\delta_l}{2}\} \cup \{(k,j); 2 \le j \le i\}. \end{cases} \tag{37}$$

It is obvious to see that if $2 \le j \le i$, then $\sigma_i^k + (\delta_k/2) > \sigma_j^k - (\delta_k/2)$. Combining this remark with (34), (36), and (37), we obtain

$$\begin{cases} \widetilde{F}_{ki}(u,z,\widehat{z}) - F_{ki}(u,z) = \sum_{(l,j)\in J(k,i)} b_{lj}^{ki}(t)e_{lj} \\ \text{additionally } \forall\,(l,j) \in J(k,i),\ \sigma_i^k + \frac{\delta_k}{2} > \sigma_j^l - \frac{\delta_l}{2} \end{cases} \tag{38}$$

Finally, using the definition of $\widetilde{F}_{ki}(u,z,\widehat{z})$, we get

$$F_{ki}(u,\widehat{\underline{z}}) - \widetilde{F}_{ki}(u,z,\widehat{z}) = a_{ki}(t)e_{k,i+1} \tag{39}$$

where $a_{ki}(t) = (\partial F_{ki}/\partial z_{k,i+1})(u(t), z(t) + h(t)e(t))$, for some $h(t) \in [0,1]$.

Now, set $A_k(t) = \begin{bmatrix} 0 & a_{k1}(t) & 0 & & 0 \\ \vdots & & a_{k2}(t) & & \\ 0 & & & \ddots & a_{k,n_k-1}(t) \\ 0 & \cdots & & 0 & 0 \end{bmatrix}$ and $B_k(e) = \begin{bmatrix} \sum_{(l,j)\in J(k,1)} b_{lj}^{k1} e_{lj} \\ \vdots \\ \sum_{(l,j)\in J(k,n_k)} b_{lj}^{kn_k} e_{lj} \end{bmatrix}$.

Combining (32), (38), and (39), we obtain

$$\dot{e}_k = A_k(t)e_k - \omega_k \Delta_{\theta^{\delta_k}} S_k^{-1} C_k^T C_k e_k + B_k(e). \tag{40}$$

Now, consider the $n_k \times n_k$ and the $n \times n$ diagonal matrices $\Lambda_k(\theta) = \text{diag}(\theta^{\sigma_1^k}, \ldots, \theta^{\sigma_{n_k}^k})$ and $\Lambda(\theta) = \text{diag}(\Lambda_1(\theta), \ldots, \Lambda_p(\theta))$. One can check that the following equalities hold:

$$\begin{cases} \Lambda_k^{-1}(\theta)A_k(t)\Lambda_k(\theta) = \theta^{\delta_k}A_k(t) \\ \Lambda_k^{-1}(\theta)\Delta_{\theta^{\delta_k}} = \theta^{-\sigma_k+\delta_k} I_k \\ C_k\Lambda_k(\theta) = \theta^{\sigma_k}C_k. \end{cases} \tag{41}$$

Consider the following change of variables: $\bar{e}_{ki} = \theta^{-\sigma_i^k} e_{ki}$, $\bar{e}_k = \Lambda_k^{-1}(\theta)e_k$, $\bar{e} = \Lambda^{-1}(\theta)e$, and using (40) and (41), we obtain

$$\dot{\bar{e}}_k = \theta^{\delta_k}\left(A_k(t) - \omega_k S_k^{-1} C_k^T C_k\right)\bar{e}_k + \Lambda_k^{-1}(\theta)B_k(e). \tag{42}$$

From (22), $\tau \le a_i(t) \le \gamma$, and from Lemma 1, we know that for every $\eta_k > 0$; there exist $\rho_k > 0$ and a SPD matrix S_k such that

$$A_k^T(t)S_k + S_k A_k(t) - \rho_k C_k^T C_k \le -\eta_k I_k. \tag{43}$$

Set $W_k = \bar{e}_k^T S_k \bar{e}_k$, and consider the quadratic positive definite function $W(\bar{e}) = \bar{e}^T S \bar{e} = \sum_{k=1}^{p} W_k(\bar{e}_k)$, where $S = \text{diag}(S_1, \ldots, S_p)$.

In what follows, we will show that $\overline{e}(t)$ exponentially converges to 0.

Differentiating $W_k(t)$, and using (43), we obtain

$$\begin{aligned} \dot{W}_k \le& -\eta_k\theta^{\delta_k}\|\overline{e}_k\|^2 - \theta^{\delta_k}(2\omega_k - \rho_k)(C_k e_k)^2 \\ &+ 2\bar{e}_k^T S_k \Lambda_k^{-1}(\theta)B_k(e) \\ \le& -\eta_k\theta^{\delta_k}\|\overline{e}_k\|^2 - \theta^{\delta_k}(2\omega_k - \rho_k)(C_k e_k)^2 \\ &+ 2\|\bar{e}_k\|\|S_k\|\|\Lambda_k^{-1}(\theta)B_k(e)\| \\ \le& -\eta_k\theta^{\delta_k}\|\overline{e}_k\|^2 - \theta^{\delta_k}(2\omega_k - \rho_k)(C_k e_k)^2 \\ &+ 2\|\bar{e}_k\|\|S_k\|\sum_{i=1}^{n_k}\theta^{-\sigma_i^k}\left|\sum_{(l,j)\in J(k,i)} b_{lj}^{ki}(t)e_{lj}\right|. \end{aligned}$$

Choosing $2\omega_k \ge \rho_k$ and using the fact that $e_{lj} = \theta^{\sigma_j^l}\bar{e}_{lj}$, and that $\left|b_{lj}^{ki}\right| \le \gamma$ (γ is the Lipshitz constant given by Assumption **A2'**)), we get

$$\begin{aligned}\dot{W}_k \le & -\eta_k \theta^{\delta_k} \|\bar{e}_k\|^2 \\ & + 2\|\bar{e}_k\| \|S_k\| \gamma \sum_{i=1}^{n_k} \sum_{(l,j)\in J(k,i)} \theta^{\sigma_j^l - \sigma_i^k} |\bar{e}_{lj}| \\ \le & -\eta_k \theta^{\delta_k} \|\bar{e}_k\|^2 \\ & + 2\|\bar{e}_k\| \|S_k\| \gamma \sum_{i=1}^{n_k} \sum_{(l,j)\in J(k,i)} \theta^{\sigma_j^l - \sigma_i^k} \|\bar{e}_l\| \\ & (|\bar{e}_{lj}| \le \|\bar{e}_l\|).\end{aligned}$$

Let $c_1 > 0$, $c_2 > 0$ be two constants such that $c_1 W_l \le \|\bar{e}_l\|^2 \le c_2 W_l$, $1 \le l \le p$, we obtain

$$\begin{aligned}\dot{W}_k \le & -\eta_k \theta^{\delta_k} c_1 W_k \\ & + 2\|S_k\| c_2 \gamma \sum_{i=1}^{n_k} \sum_{(l,j)\in J(k,i)} \theta^{\sigma_j^l - \sigma_i^k} \sqrt{W_k}\sqrt{W_l} \\ = & -(\sqrt{\eta_k \theta^{\delta_k} c_1 W_k})^2 + 2\|S_k\| c_2 \gamma \\ & \times \sum_{i=1}^{n_k} \sum_{(l,j)\in J(k,i)} \left(\frac{1}{c_1} \frac{\theta^{\sigma_j^l - \sigma_i^k - (\delta_k/2) - (\delta_l/2)}}{\sqrt{\eta_k \eta_l}} \right. \\ & \left. \times \sqrt{\eta_k \theta^{\delta_k} c_1 W_k} \sqrt{\eta_l \theta^{\delta_l} c_1 W_l} \right).\end{aligned}$$

Hence

$$\begin{aligned}\dot{W} \le & -\sum_{k=1}^{p} (\sqrt{\eta_k \theta^{\delta_k} c_1 W_k})^2 \\ & + \sum_{k=1}^{p} \left(2\|S_k\| \frac{c_2}{\sqrt{\eta_k}} \gamma \sum_{i=1}^{n_k} \sum_{(l,j)\in J(k,i)} \frac{\theta^{\sigma_j^l - \sigma_i^k - (\delta_k/2) - (\delta_l/2)}}{c_1 \sqrt{\eta_l}} \right. \\ & \left. \times \sqrt{\eta_k \theta^{\delta_k} c_1 W_k} \sqrt{\eta_l \theta^{\delta_l} c_1 W_l} \right).\end{aligned}$$

From definition of $J(k,i)$ [see (37)], we know that for every $(l,j) \in J(k,i)$, $\sigma_j^l - \sigma_i^k - \delta_k/2 - \delta_l/2 < 0$. Thus, we can choose a constant $\theta_0 > 1$ such that for every $\theta > \theta_0$, we have

$$\dot{W} \le -\frac{1}{2} \sum_{k=1}^{p} (\sqrt{\eta_k \theta^{\delta_k} c_1 W_k})^2 \le -\alpha \theta^{\delta_0} W$$

where $\delta_0 = \min\{\delta_k, k = 1, \dots, p\}$ and $\alpha > 0$ is a constant.

This complete the proof of the theorem.

IV. Conclusion

In this note, we have presented two system structures, with specific structural conditions, that characterize some classes of uniformly observable systems; that is, systems that are observable whatever the applied input. The classes of systems under consideration contain many classes of nonlinear systems existing in the literature. However, the structural conditions given are coordinates dependent, and the transformations that are required to transform a system into the proposed structures are not discussed. Two high gain observer design methodologies are presented for both structures and their distinct features are highlighted.

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