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# On the Performance of a Mixed RF/MIMO FSO Variable Gain Dual-Hop Transmission System

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**Abstract:** In this work, we propose a mixed radio frequency (RF) and multiple-input-multiple-output (MIMO) free-space optical (FSO) system based on a variable-gain dual-hop relay transmission scheme. The RF channel is modeled by Rayleigh distribution and Gamma-Gamma turbulence distribution is adopted for the MIMO FSO link, which accounts for the equal gain combining diversity technique. Moreover, new closed-form mathematical formulas are obtained including the cumulative distribution function, probability density function, moment generating function, and moments of equivalent signal-to-noise ratio of the dual-hop relay system based on Meijer's G function. As such, we derive the novel analytical expressions of the outage probability, the higher-order fading, and the average bit error rate for a range of modulations in terms of Meijer's G function. Furthermore, the exact closed-form formula of the ergodic capacity is derived based on the bivariate Meijer's G function. The evaluation and simulation are provided for system performance, and the effect of spatial diversity technique is discussed as well.

**Keywords:** Dual-hop relay, mixed radio frequency/multiple-input-multiple-output free-space optical, outage probability (OP), bit-error rate (BER), ergodic capacity, binary modulation schemes.

## 1. Introduction

In recent years, more attention has been paid to the free space optical (FSO) communication systems as a complementary alternative technology to the conventional radio frequency (RF) wireless systems in a range of applications including the last mile back-bone network and airborne communications. FSO offers advantages such as high security at the physical layer, easy installation and deployment and high transmission data compare to the RF wireless communications. However, FSO links are susceptible to the intensity and phase fluctuations induced by the atmospheric turbulence, especially over a long transmission spans (a few km) [1],[2],[3]. In order to reduce the influence of turbulence, a hybrid RF/FSO airborne communication cooperative system has been proposed with the advantages of both RF and FSO transmission techniques [4],[5],[6],[7]. In the mixed RF/FSO airborne communications application scenarios, the RF channel is used for the access sub-network, and the wide bandwidth FSO technology is utilized for the airborne backbone link. In order for the RF sub-network to achieve a seamless access to the FSO backbone network and improve the reliability of the hybrid RF/FSO system, multi-hop relay-based scheme have been adopted to effectively reduce the performance degradation due to the atmospheric turbulence.

Recently, several types schemes investigating a hybrid RF and FSO system with the dual-hop relay have been reported [8],[9],[10]. The dual-hop relay system is generally categorized into decode and forward (DF) and amplify and forward (AF) protocols. In the DF relay, which is a regenerative scheme, the received signal is first decoded and then transmitted to the next node. In AF relay, the incoming signal is amplified and then forwarded to the intermediate node with no decoding. The

AF relay is simpler to implement with reduced level of complexity compared to the DF relay [11]. Furthermore, the AF relay can be fixed-gain and variable-gain. The variable-gain relay depends on the knowledge of instantaneous channel state information (CSI) of the first hop to control the gain of the amplifier at the relay. Whereas, the AF system with the fixed-gain relay uses a fixed-gain factor at the receiver regardless of the instantaneous CSI, which is inferior in performance compared with the variable-gain relay [12].

A number of research works have investigated the hybrid RF/FSO system with the variable-gain dual-hop AF relay system [13],[14],[15]. In [13], the outage probability (OP) and the average bit error rate (BER) performance of the hybrid RF/FSO system with the fixed-gain AF relay and CSI-assisted AF relay schemes were investigated. In [14], a hybrid RF/FSO system with a variable-gain dual-hop AF relay scheme was proposed and its OP performance investigated. The channel was characterized by generalized-K and Gamma-Gamma fading model for RF and FSO links. In [15], a variable-gain dual-hop relay based RF/FSO system was proposed, where the impact of pointing errors for the heterodyne detection and the IM/DD technique was investigated. The authors assumed that the RF and FSO channels were operating over Rayleigh fading and Gamma-Gamma fading distributions, respectively. A number of research works have been proposed on investigation of the hybrid RF/FSO system with spatial diversity [16], [17]. In [16], a dual-hop mixed RF/FSO system with a variable-gain AF relay was proposed, where the transmit diversity technique and the selection combining technique were adopted at the source and destination nodes, respectively. What's more, RF and FSO links were characterized by Rayleigh distribution and M-distributed fading, respectively. The work in [17] investigated the multi-diversity combining and selection schemes for the relay-assisted mixed RF/FSO system with RF and FSO link being considered by Rayleigh and Gamma-Gamma fading channels, respectively.

In this paper, similar to [16] and [17], we propose and investigate a hybrid RF/FSO system with the spatial diversity scheme and using a variable-gain AF relay. Unlike [16] and [17], we have multiplex signal in the RF domain prior to transmission over a MIMO FSO link with the equal gain combining (EGC) technique adopted at the Rx. As outlined in [18], the maximal-ratio combining (MRC) diversity technique offers slightly improved performance compared to the EGC technique but higher the performance than the selection combining technique in the FSO system. However, this improved in the performance is achieved at the cost of increased complexity since the MRC technique requires the information on the fading channel for each link. More specifically, the RF and FSO links are characterized by Rayleigh and Gamma-Gamma fading channel, respectively. As such, new closed-form expressions of cumulative distribution function (CDF), probability density function (PDF), moment generating function (MGF) and the moment of end-to-end signal-to-noise ratio (SNR) of the system are derived. In addition, we derive expressions for the OP, higher-order amount of fading, ergodic capacity, and BER for a range of binary modulations. Finally, we provide numerical results to verify and illustrate the carried out analysis.

The remainder of this paper is structured as followed. Section 2 presents the system and channel model of the mixed RF/MIMO FSO. In section 3, the expressions for CDF, PDF, MGF and moments of the system are derived. In section 4, mathematical presentations for OP, the higher-order amount of fading, average BER for a range of binary modulations are derived. Section 5 presents several analytical evaluation and simulation results. Section 6 summarizes analytical results.

## **2. System and Channel Model**

The variable-gain dual-hop relay communication system, composed of mixed RF and MIMO FSO links, is present in Fig. 1. The source node (S) aggregates and transmits data to the destination node (D) through the variable-gain AF relay node (R) at which subcarrier intensity modulation (SIM) technique is adopted to convert the input RF signal to the optical signal. As shown in the Fig. 1, the input signals are multiplexed prior to being applied to the R.

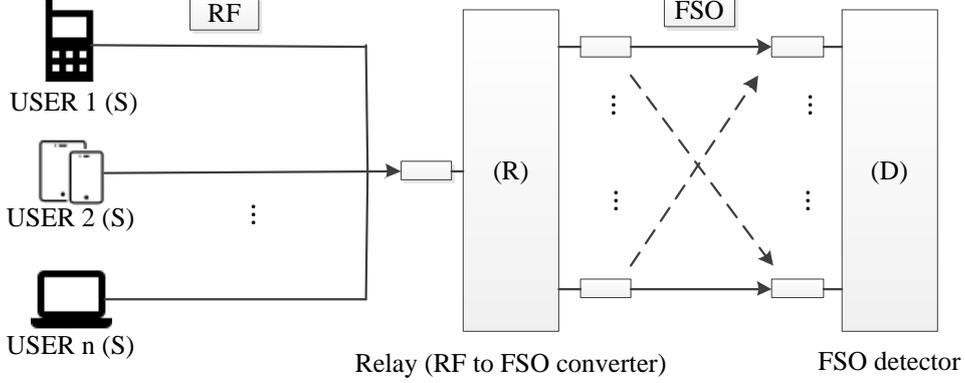


Fig. 1. System model block diagram of a dual-hop relay system composed of RF and MIMO FSO

## 2.1. RF link

For the RF link, the received SNR of the received aperture at relay node (R) can be given as  $\gamma_{SR}$ . We consider Rayleigh fading distribution with PDF of  $\gamma_{SR}$  given by [19]:

$$f_{\gamma_{SR}}(\gamma_{SR}) = 1/\bar{\gamma}_{SR} \exp(-\gamma_{SR}/\bar{\gamma}_{SR}), \quad (1)$$

where  $\gamma_{SR}$  and  $\bar{\gamma}_{SR}$  denote the instantaneous and the average SNR of RF link, respectively. The

average SNR of the RF link  $\bar{\gamma}_{SR} = E[\gamma_{SR}]$ , where  $E[\cdot]$  is the expectation operator. Using the

expression of  $F_{\gamma_{SR}}(\gamma_{SR}) = \int_0^{\gamma_{SR}} f_{\gamma_{SR}}(x) dx$ , we can obtain the CDF of Rayleigh distribution as:

$$F_{\gamma_{SR}}(\gamma_{SR}) = 1 - \exp(-\gamma_{SR}/\bar{\gamma}_{SR}). \quad (2)$$

## 2.2. FSO link

As shown in the Fig. 1, we investigate a FSO system with spatial diversity technique with  $M$  light source (i.e., lasers) and  $N$  photodetectors. The EGC technique is adopted to further mitigate the fading effect at the destination node. In order to ensure that the received irradiance is uncorrelated, we assume that the spatial diversity separation of the photodetectors to be greater than the irradiance spatial coherence distance. In addition, the MIMO FSO transmission link is assumed to follow a unified Gamma-Gamma fading distribution for which the PDF of the SNR is given by [20]:

$$f_{\gamma_{RD}}(\gamma_{RD}) = \frac{(\alpha_S \beta_S)^{(\alpha_S + \beta_S)/2} \gamma_{RD}^{(\alpha_S + \beta_S)/4 - 1}}{2\Gamma(\alpha_S)\Gamma(\beta_S)\bar{\gamma}_{RD}^{(\alpha_S + \beta_S)/4}} \times G_{0,2}^{2,0} \left[ \alpha_S \beta_S \sqrt{\frac{\gamma_{RD}}{\bar{\gamma}_{RD}}} \left| \frac{\alpha_S - \beta_S}{2}, \frac{\beta_S - \alpha_S}{2} \right. \right], \quad (3)$$

where  $\gamma_{RD}$  and  $\bar{\gamma}_{RD}$  are the instantaneous and the average SNR of the MIMO FSO link,

respectively. Note that, the instantaneous SNR is given as  $\gamma_{RD} = \bar{\gamma}_{RD} I^2$ , where  $I = \sum_{N=1}^N \sum_{M=1}^M I_{mn}$ ,

$I_{mn}$  denotes the normalized irradiance of the link between the  $m$ th laser and the  $n$ th photodetector.

$\Gamma(\cdot)$  is the gamma function,  $G(\cdot)$  is the Meijer's G function,  $\alpha_S = MN\alpha + \varepsilon$ ,  $\beta_S = MN\beta$ .

The adjustment parameter  $\varepsilon = (MN - 1) \frac{-0.127 - 0.95\alpha - 0.0058\beta}{1 + 0.00124\alpha + 0.98\beta}$ ,  $\alpha$  and  $\beta$  are the PDF

parameters describing the turbulence experienced by propagating optical signals, and in the case of zero-inner scale they are given by [21]:

$$\alpha = \left\{ \exp \left[ \frac{0.49\sigma_0^2}{(1 + 1.11\sigma_0^{12/5})^{7/6}} \right] - 1 \right\}^{-1}, \quad (4)$$

$$\beta = \left\{ \exp \left[ \frac{0.51\sigma_0^2}{(1 + 0.69\sigma_0^{12/5})^{5/6}} \right] - 1 \right\}^{-1},$$

where  $\sigma_0^2$  is Rytov variance, assuming plane wave propagation it is given by:

$$\sigma_0^2 = 1.23 C_n^2 k^{7/6} L_{RD}^{11/6}, \quad (5)$$

where  $L_{RD}$  is FSO link length,  $k = 2\pi/\lambda$  is the optical wave number,  $\lambda$  is the wavelength,

and  $C_n^2$  is the altitude-dependent index of refractive structure parameter determining the

turbulence strength. By substituting Eq. (3) into  $F_{\gamma_{RD}}(\gamma_{RD}) = \int_0^{\gamma_{RD}} f_{\gamma_{RD}}(x) dx$ , the CDF of

instantaneous SNR  $\gamma_{RD}$  can be obtained by utilizing [[22], Eq. (07.34.21.0084.01) and Eq.

(07.34.17.0011.01)], which is given as:

$$F_{\gamma_{RD}}(\gamma_{RD}) = A G_{1,5}^{4,1} \left[ \frac{B}{\bar{\gamma}_{RD}} \gamma_{RD} \left| \frac{1}{k_1}, 0 \right. \right], \quad (6)$$

where  $A = \frac{2^{\alpha_s + \beta_s - 2}}{\pi \Gamma(\alpha_s) \Gamma(\beta_s)}$ ,  $B = \frac{(\alpha_s \beta_s)^2}{16}$ ,  $k_1 = \frac{\alpha_s}{2}, \frac{\alpha_s + 1}{2}, \frac{\beta_s}{2}, \frac{\beta_s + 1}{2}$ .

### 2.3. Relay system

For a dual-hop relay system with a variable-gain relay, the link's end-to-end SNR is given as:

$$\gamma = \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1}. \quad (7)$$

Since the closed-form analysis of the statistical characteristics of  $\gamma$  is complex, instead we have used the standard approximation as given by [23]:

$$\gamma = \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD} + 1} \approx \min(\gamma_{SR}, \gamma_{RD}). \quad (8)$$

## 3. Statistical Characteristics

### 3.1. Cumulative distribution function

The CDF of  $\gamma = \min(\gamma_{SR}, \gamma_{RD})$  can be expressed as  $F_\gamma(\gamma) = \Pr(\min(\gamma_{SR}, \gamma_{RD}) < \gamma)$

which can be rewritten as [[24], Eq. (4)]:

$$F_\gamma(\gamma) = F_{\gamma_{SR}}(\gamma) + F_{\gamma_{RD}}(\gamma) - F_{\gamma_{SR}}(\gamma) F_{\gamma_{RD}}(\gamma). \quad (9)$$

Following some simple algebraic manipulations and simplifications, the CDF of  $\gamma$  can be written as:

$$F_\gamma(\gamma) = 1 - \exp(-\gamma/\bar{\gamma}_{SR}) \left( 1 - AG_{1,5}^{4,1} \left[ \frac{B}{\bar{\gamma}_{RD}} \gamma \middle| \begin{matrix} 1 \\ k_1, 0 \end{matrix} \right] \right). \quad (10)$$

### 3.2. Probability density function

Differentiating (10) with respect to  $\gamma$  and using Meijer's G function property [[22], Eq. (07.34.20.0002.01)], we obtain the PDF of the system as given by:

$$f_\gamma(\gamma) = \exp(-\gamma/\bar{\gamma}_{SR}) \times \left[ \frac{1}{\bar{\gamma}_{SR}} + \frac{A}{\bar{\gamma}_{SR} \gamma} \left( \bar{\gamma}_{SR} G_{0,4}^{3,1} \left[ \frac{B}{\bar{\gamma}_{RD}} \gamma \middle| \begin{matrix} - \\ k_1 \end{matrix} \right] - \gamma G_{1,5}^{4,1} \left[ \frac{B}{\bar{\gamma}_{RD}} \gamma \middle| \begin{matrix} 1 \\ k_1, 0 \end{matrix} \right] \right) \right]. \quad (11)$$

### 3.3. Moment generating function

The MGF is defined as  $M_\gamma(s) = \mathbb{E}[e^{-\gamma s}]$ , which can be rewritten, via integration by parts, in terms of the CDF as:

$$M_\gamma(s) = s \int_0^\infty e^{-\gamma s} F_\gamma(\gamma) d\gamma. \quad (12)$$

Substituting (10) into (12) and using [[25], Eq. (7.813.1)], we obtain MGF of  $\gamma$  as:

$$M_\gamma(s) = 1 - \frac{s}{s+1/\bar{\gamma}_{SR}} \times \left( 1 - AG_{1,4}^{4,1} \left[ \frac{B}{\bar{\gamma}_{RD}(s+1/\bar{\gamma}_{SR})} \middle| \begin{matrix} 1 \\ k_1 \end{matrix} \right] \right). \quad (13)$$

### 3.4. Moments

The moments is defined as  $E[\gamma^n]$ , which can be obtained in terms of the complementary CDF (CCDF)  $F_\gamma^c(\gamma) = 1 - F_\gamma(\gamma)$  as given by:

$$E[\gamma^n] = n \int_0^\infty \gamma^{n-1} F_\gamma^c(\gamma) d\gamma. \quad (14)$$

Then substituting (10) into (14) and employing [[25], Eq. (7.813.1)], the moments can be expressed as:

$$E[\gamma^n] = n \bar{\gamma}_{SR}^n \left( \Gamma(n) - AG_{2,5}^{4,2} \left[ \frac{B \bar{\gamma}_{SR}}{\bar{\gamma}_{RD}} \middle| \begin{matrix} 1-n, 1 \\ k_1, 0 \end{matrix} \right] \right). \quad (15)$$

## 4. Performance Analysis

### 4.1. Outage probability

The outage probability is described in % when the instantaneous SNR  $\gamma$  falls below a certain specified threshold  $\gamma_{th}$ , which can be obtained by replacing  $\gamma$  with  $\gamma_{th}$  in the CDF as given by:

$$P_{out}(\gamma_{th}) = F_\gamma(\gamma_{th}). \quad (16)$$

### 4.2. Higher-order amount of fading

The higher-order amount of fading, utilized to parameterize the distribution of the SNR of the received signal, is an important parameter in assessing the performance of wireless communication systems. More specifically, for the  $n^{th}$ -order amount of fading in terms of the instantaneous SNR  $\gamma$  is defined as [26]:

$$AF_\gamma^{(n)} = \frac{E[\gamma^n]}{E[\gamma]^n} - 1. \quad (17)$$

### 4.3. Average BER

The average BER for a range of binary modulations, which is introduced in [[27], Eq. (12)], is given by:

$$\bar{P}_e = \frac{q^p}{2\Gamma(p)} \int_0^\infty \exp(-q\gamma) \gamma^{p-1} F_\gamma(\gamma) d\gamma, \quad (18)$$

where  $p$  and  $q$  are parameters related to different modulation schemes, which is best described in

[15], are given in Table 1. Substituting (10) into (18) and utilizing [[25], Eq. (7.813.1)], the average BER can be expressed as:

$$\bar{P}_e = \frac{1}{2} - \frac{q^p}{2(q+1/\bar{\gamma}_{SR})^p} \times \left( 1 - \frac{A}{\Gamma(p)} G_{2,5}^{4,2} \left[ \frac{B}{\bar{\gamma}_{RD}(q+1/\bar{\gamma}_{SR})} \middle| \begin{matrix} 1-p, 1 \\ k_1, 0 \end{matrix} \right] \right). \quad (19)$$

Table 1 BER parameters of binary modulations

Modulation	$p$	$q$
Coherent Binary Frequency Shift Keying (CBFSK)	0.5	0.5
Coherent Binary Phase Shift Keying (CBPSK)	0.5	1
Non-Coherent Binary Frequency Shift Keying (NBFSK)	1	0.5
Differential Binary Phase Shift Keying (DBPSK)	1	1

#### 4.4. Ergodic capacity

The ergodic capacity is defined as  $\bar{C} @E[\log_2(1+\gamma)]$ , which can be written in terms of the CCDF of  $\gamma$  as [[28], Eq. (15)]:

$$\bar{C} = \frac{1}{\ln(2)} \int_0^\infty \frac{F_\gamma^c(\gamma)}{1+\gamma} d\gamma. \quad (20)$$

Utilizing the identity [[29], Eq. (15)]  $(1+az)^{-b} = \frac{1}{\Gamma(b)} G_{1,1}^{1,1} \left[ az \middle| \begin{matrix} 1-b \\ 0 \end{matrix} \right]$  and employing the integral identity [[27], Eq. (20)], we obtain the ergodic capacity in terms of the extended generalized bivariate Meijer's G function as given by [27]:

$$\bar{C} = \frac{1}{\ln(2)} \left( \bar{\gamma}_{SR} G_{2,1}^{1,2} \left[ \bar{\gamma}_{SR} \middle| \begin{matrix} 0, 0 \\ 0 \end{matrix} \right] - \bar{\gamma}_{SR} A G_{1,0,1,1,1,5}^{1,0,1,1,4,1} \left[ 1 \middle| \begin{matrix} 0 & 1 \\ 0 & k_1, 0 \end{matrix} \right] \bar{\gamma}_{SR}, \frac{B\bar{\gamma}_{SR}}{\bar{\gamma}_{RD}} \right]. \quad (21)$$

## 5. Results and Discussion

In this section, we present simulation results for different performance metrics for a hybrid RF/MIMO FSO with a variable-gain AF relay for a dual-hop relay transmission system using the mathematical expressions outlined above. Note that, the RF link is subject to Rayleigh fading, and for the FSO link we have used the unified Gamma-Gamma fading channel for weak and strong turbulent conditions. The system parameters used are the operating wavelength  $\lambda=1550 \text{ nm}$ , the propagation span for FSO link  $L_{RD} = 10 \text{ km}$  and the diameter of transmitting and receiving aperture are  $D = 20 \text{ cm}$ . Using Eq.(4) and Eq.(5), we obtained Rytov variance  $\sigma_0^2$ ,  $\alpha$  and  $\beta$ , which are shown in Table 2.

Table 2 Parameters for atmospheric turbulence

Distance / km	$C_n^2 / m^{-2/3}$	$\sigma_0^2$	$\alpha$	$\beta$
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10	$4.24 \times 10^{-15}$	5.75	4.76	1.20
10	$8.05 \times 10^{-14}$	109.20	14.63	1.00

Table 3 System parameters

<i>Parameter</i>	<i>Symbol</i>	<i>Value</i>
Distance of RF link	$L_{SR}$	1 km
Distance of FSO link	$L_{RD}$	10 km
Wavelength	$\lambda$	1550 nm
Diameter of transmitting and receiving aperture	$D$	20 cm

To ensure that the channel is independent, the spacing between the photodetectors must be greater than the irradiance spatial coherence distance  $\sqrt{\lambda L_{RD}} \approx 13 \text{ cm}$ . Base on the above parameters which is shown in Table 2 and Table 3, the average BER and ergodic capacity performance of proposed system for a range of turbulence regimes and diversity orders are evaluated. Fig. 2 and Fig. 3 show the average BER performance against the average SNR per hop for CBPSK and for a range of diversity orders under strong and weak turbulence conditions, respectively. Note that, we have assumed that the average SNRs for each link is the same. As shown in both figures, the atmosphere turbulence conditions get severe, the average BER improves with the SNR and the order of the diversity. For example, at the SNR of 40 dB and  $M=N=1$  the BER values are  $1.70 \times 10^{-3}$  and  $3.00 \times 10^{-3}$  for the strong and weak turbulence regimes, respectively. With high-order diversity, the BER performance improves further. For example, for the same average SNR of 40 dB and for  $(M=N=1)$  to  $(M=N=4)$ , the average BER is decreased from  $3.00 \times 10^{-3}$  to  $2.50 \times 10^{-5}$  under the strong turbulence regimes, see Fig. 2. The same trend also applies to the case with weak turbulence. Note that, the decrease in the average BER is less pronounced for  $M$  and  $N$  being higher than 2. This is because increasing the number of transmitting and receiving apertures beyond a certain value results in saturation of the diversity gain, which limits the improvement in system performance. Hence, the number of transmitting and receiving aperture cannot be increased indefinitely as this will increase the system complexity and cost.

Fig. 4 illustrates the average BER performance as a function of the average SNR per hop for different binary modulations for  $M=N=1$  under the strong turbulence regime. Note that, the modulation schemes employed here for demonstration are based on the values of  $p$  and  $q$  as presented in Table 1. Also, the average SNRs of each link is assumed to be the same. The plots show that CBPSK offers superior performance compared with other modulation schemes, whereas CBFPSK outperformance DBPSK and NBFPSK. In Fig. 5 and Fig. 6, the ergodic capacity performance as a function of the average SNR for the FSO link, a range of diversity orders and a SNR of 20 dB for the RF link under strong and weak turbulence conditions, respectively are shown. Compared Fig. 5 with Fig. 6, it can be observed that, the ergodic capacity decreases with the level of turbulence. For example, for SNR of 0 dB with  $M=N=1$  the ergodic capacity is reduced from 2.02 bits/Hz to 1.71 bits/Hz under weak and strong turbulence regimes, respectively. However, it is clearly

illustrated in these figures that, the use of diversity technique can significantly increase the ergodic capacity for the proposed mixed RF/MIMO FSO system. For instance, at an average SNR of 0 dB, increasing  $M$  and  $N$  from 1 to 4, results in increased ergodic capacity from 1.71 bits/Hz to 3.52 bits/Hz under strong turbulence, see Fig. 5. In line with BER performance, the ergodic capacity becomes less pronounced for both  $M$  and  $N$  being higher than 2.

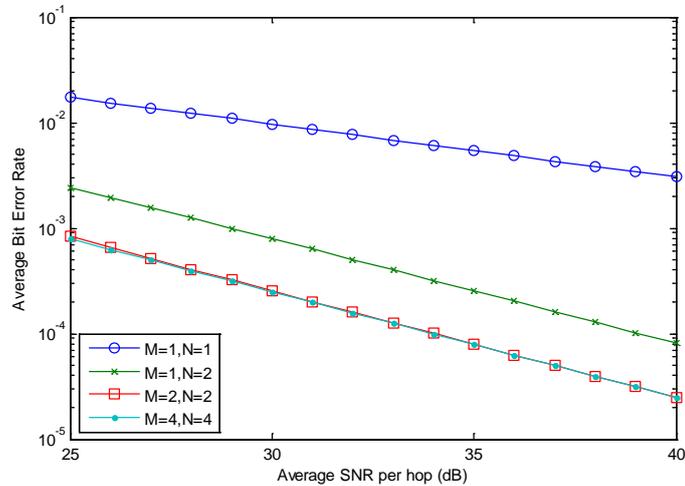


Fig. 2. Average BER as a function of average SNR for CBPSK under the strong turbulence condition.

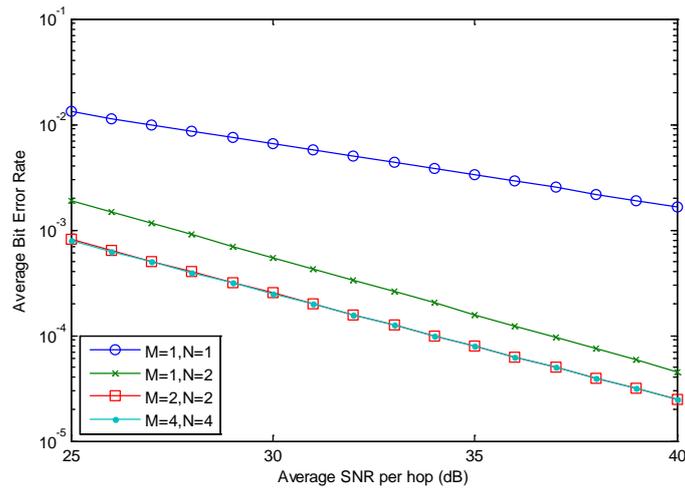


Fig. 3. Average BER as a function of average SNR for CBPSK under the weak turbulence condition.

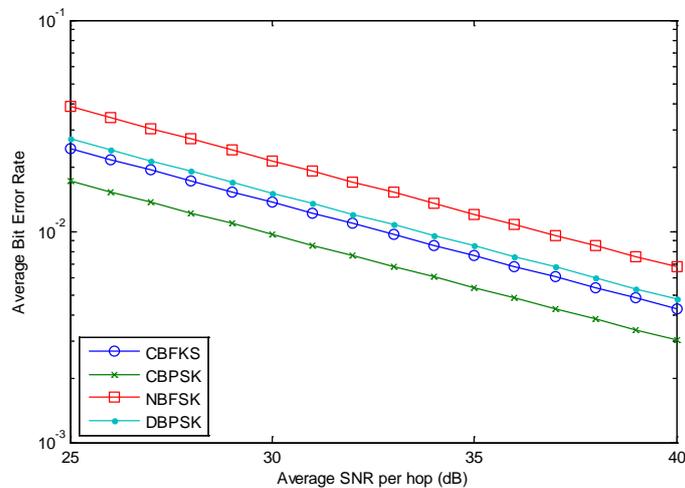


Fig. 4. Average BER as a function of average SNR for different modulations for the performance of

$M=N=1$  and under the strong turbulence condition.

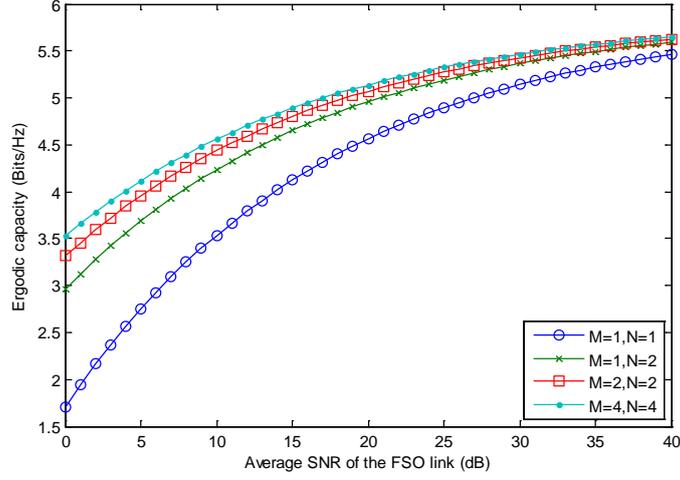


Fig. 5. Ergodic capacity as a function of average for the FSO link spatial diversity under strong turbulence condition and for the average SNR of 20 dB for the RF link.

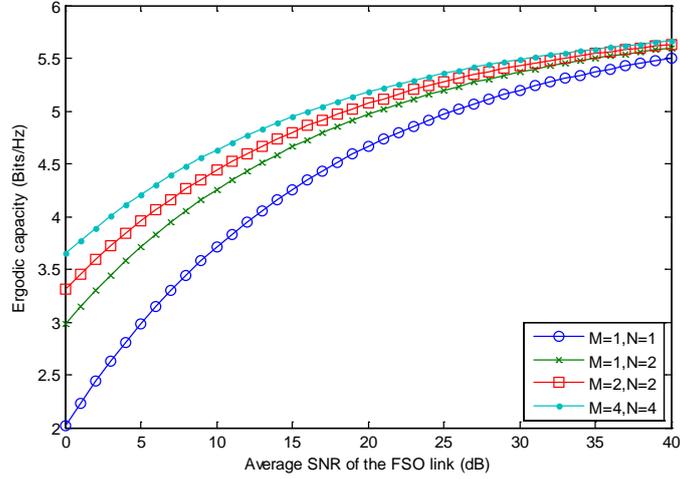


Fig. 6. Ergodic capacity as a function of average SNR for the FSO link with spatial diversity under weak turbulence condition and for the average SNR of 20 dB for the RF link.

## 6. Conclusion

In this work, we provided the performance analysis of the hybrid RF/MIMO FSO communications system operating over a variable-gain dual-hop relay scheme. The RF and FSO component underwent Rayleigh fading distribution and Gamma-Gamma fading model, respectively. We derived the new exact closed-form expressions for the CDF, the PDF, the MGF, and the moments of a mixed RF/MIMO FSO system with a variable-gain AF dual-hop relay. Furthermore, we derived analytical expressions for various performance metrics of the proposed system including the OP, higher order amount of fading, BER for a range of modulation schemes, and the ergodic capacity in terms of bivariate Meijer's G function. Depending on the above expressions, the effects of turbulence under various modulation techniques and spatial diversity technology with EGC technique was simulated. The results showed that the spatial diversity technique could improve the hybrid RF/MIMO FSO system performance including the average BER and ergodic capacity. However, we showed that the improvement of in the average BER and the ergodic capacity of the proposed system became less pronounced for the number of both transmitting and receiving

apertures being higher than 2. Finally yet importantly we showed that, CBPSK binary modulation offered superior performance compare with other modulations in this mixed RF/MIMO FSO system.

## References

- [1] Uysal M, Capsoni C, Ghassemlooy Z, Boucouvalas A, Udvary E, Optical wireless communications: an emerging technology. Springer, 2016.
- [2] Stotts L B, Stadler B, Lee G, Free space optical communications: coming of age, Proc. SPIE. 6951 (2008) 69510W.
- [3] Nor N A M, Ghassemlooy Z F, Bohata J, et al., Experimental investigation of all-optical relay-assisted 10 Gb/s FSO link over the atmospheric turbulence channel, *Journal of Lightwave Technology*. 35 (1) (2017)45-53.
- [4] Stotts L B, Andrews L C, Cherry P C, et al., Hybrid optical RF airborne communications, Proceedings of the IEEE. 97 (6) (2009) 1109-1127.
- [5] Jing Z, Shang-hong Z, Wei-hu Z, et al., Performance analysis for mixed FSO/RF Nakagami-m and Exponentiated Weibull dual-hop airborne systems, *Optics Communications*. 392 (2017) 294-299.
- [6] Bagley Z C, Hughes D H, Juarez J C, et al., Hybrid optical radio frequency airborne communications, *Optical Engineering*. 51 (5) (2012) 055006-1-055006-25.
- [7] Abadi M, Ghassemlooy Z, Zvanovec S, et al., Dual Purpose Antenna for Hybrid Free Space Optics/RF Communication Systems, *Journal of Lightwave Technology*. 34 (14) (2016) 3432-3439.
- [8] Miridakis N I, Matthaïou M, Karagiannidis G K, Multiuser relaying over mixed RF/FSO links, *IEEE Transactions on Communications*. 62(5) (2014) 1634-1645.
- [9] Anees S, Bhatnagar M R, Performance evaluation of decode-and-forward dual-hop asymmetric radio frequency-free space optical communication system, *IET Optoelectronics*. 9 (5) (2015) 232-240.
- [10] Ansari I S, Abdallah M, Alouini M S, et al., Outage Analysis of Asymmetric RF-FSO Systems, Vehicular Technology Conference (VTC-Fall), 2016 IEEE 84th. IEEE, (2016) 1-6.
- [11] Bhatnagar M R, Performance analysis of decode-and-forward relaying in gamma-gamma fading channels, *IEEE Photon. Technol. Lett.*, 24 (7) (2012) 545-547.
- [12] Hussain A, Kim S H, Chang S H, Dual-Hop Variable-Gain AF Relaying with Beamforming over  $k$ - $\mu$  Shadowed Fading Channels, Global Communications Conference (GLOBECOM), 2016 IEEE. IEEE, (2016) 1-6.
- [13] Zhao J, Zhao S H, Zhao W H, et al., Performance of mixed RF/FSO systems in exponentiated Weibull distributed channels, *Optics Communications*. 405 (2017) 244-252.
- [14] Khanna H, Aggarwal M, Ahuja S, Outage analysis of a variable-gain amplify and forward relayed mixed RF-FSO system, India Conference (INDICON), 2016 IEEE Annual. IEEE, (2016) 1-6.
- [15] Ansari I S, Yilmaz F, Alouini M S, On the Performance of Mixed RF/FSO Variable Gain Dual-Hop Transmission Systems with Pointing Errors, Vehicular Technology Conference. IEEE, (2014) 1-5.
- [16] Yang L, Hasna M O, Gao X, Performance of mixed RF/FSO with variable gain over generalized atmospheric turbulence channels, *IEEE Journal on Selected Areas in Communications*. 33 (9) (2015) 1913-1924.
- [17] Chen L, Wang W, Multi-diversity combining and selection for relay-assisted mixed RF/FSO system, *Optics Communications*. 405 (2017) 1-7.
- [18] Luong D A, Pham A T, Average capacity of MIMO free-space optical gamma-gamma fading

- channel, Communications (ICC), 2014 IEEE International Conference on. IEEE, (2014)3354-3358.
- [19] Simon M K, Alouini M S, Digital communication over fading channels. John Wiley & Sons, 2005.
- [20] Luong D A, Pham A T, Average capacity of MIMO free-space optical gamma-gamma fading channel, Communications (ICC), 2014 IEEE International Conference on. IEEE, (2014) 3354-3358.
- [21] Al-Habash M A, Andrews L C, Phillips R L. Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media, Optical Engineering, 40 (8) (2001) 1554-1562.
- [22] Wolfram I. Mathematica Edition: Version 8.0. Champaign, Illinois: Wolfram Research Inc, 2010.
- [23] Suraweera H A, Louie R H Y, Li Y, et al., Two hop amplify-and-forward transmission in mixed Rayleigh and Rician fading channels, IEEE Communications Letters. 13 (4) (2009)227-229.
- [24] Ikki S S, Aissa S, A study of optimization problem for amplify-and-forward relaying over Weibull fading channels with multiple antennas, IEEE Communications Letters. 15 (11) (2011) 1148-1151.
- [25] Gradshteyn I S, Ryzhik I M, Table of integrals, series, and products, Academic press, 2014.
- [26] Yilmaz F, Alouini M S, Novel asymptotic results on the high-order statistics of the channel capacity over generalized fading channels, Signal Processing Advances in Wireless Communications (SPAWC), 2012 IEEE 13th International Workshop on. IEEE, (2012) 389-393.
- [27] Ansari I S, Al-Ahmadi S, Yilmaz F, et al., A new formula for the BER of binary modulations with dual-branch selection over generalized-K composite fading channels, IEEE Transactions on Communications. 59 (10) (2011) 2654-2658.
- [28] Annamalai A, Palat R C, Matyjas J, Estimating ergodic capacity of cooperative analog relaying under different adaptive source transmission techniques, Sarnoff Symposium, 2010 IEEE. IEEE, (2010) 1-5.
- [29] Mathai A M, Saxena R K, The H function with applications in statistics and other disciplines, 1978.