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A constrained broadband nonlinear energy harvester

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Abstract

This paper investigates the nonlinear electromechanical behaviour of a constrained bimorph piezoelectric energy harvester utilising a coupled multi-modal fully nonlinear model, for the first time. The electromechanical system is modelled via using the nonlinear beam theory of Euler-Bernoulli, while assuming an inextensible centreline, and the piezoelectric constitutive equations. The motion constraints are modelled as nonlinear springs, consisting of linear and cubic terms. The coupled electromechanical model, including the electrical circuit equation, is obtained for both parallel and series connections of the piezoelectric layers. The Galerkin scheme is used to discretise the partial differential-type model while retaining a large number of modes to guarantee converged results. Next, the electromechanical model consisting of geometric, inertial, impact, and piezoelectric-type nonlinearities is solved using a well-optimised continuation code. The proposed model and numerical method are verified through comparison to experimental data in the literature. It is shown that adding a tip mass of 50% of the bimorph cantilever mass increase the power output by 263%. Additionally, it is shown that for a power output threshold of 15 mW, the addition of motion constraints results in a 66.8% resonance bandwidth (normalised with respect to first natural frequency) which is almost 10 time the bandwidth of the unconstrained system.

Keywords: Energy harvester; Broadband; Piezoelectric; Nonlinear energy

1. Introduction

Energy harvesters aim to capture the otherwise wasted ambient energy in the environment and convert it into useful energy. The energy captured by small energy harvesters is not much, but enough to power small electronics as a step toward development of true power-autonomous wireless sensors [1-19]. One of the main limitations vibration energy harvesters is their narrow operating frequency bandwidth, which means they are effective only for a narrow frequency band [20-22]. The environmental vibration frequency, on the other hand, is random and does not necessarily lie in a specific range. Hence, this limitation has motivated a large amount of research on vibration energy harvesters over the last decade [23-26]. In what follows, a concise literature review is conducted; more detailed review of vibration energy harvesters can be found, for instance, in Refs. [27-30].

Dutoit et al. [31] presented considerations for design of piezoelectric-based energy harvesters for micro-scale sensors. Beeby et al. [32] proposed a design for a small electromagnetic generator optimized for a low ambient vibration making use of discrete components. Further investigations were conducted by Soliman et al. [33], who developed a design for an electromagnetic vibration energy harvester, while replacing a linear oscillator with a piecewise-linear oscillator as the element of energy harvesting; they conducted experimental investigations and theoretical analysis using a linear single-mode model. Erturk and Inman [34] obtained the closed-form steady state solutions for harmonic

excitations using a linear single mode model of a bimorph energy harvester; they conducted experimental analysis and obtained good agreement between theoretical results and experiments. Mann and Sims [35] examined the design and analysis of a magnetic-based energy harvesting device operating based on magnetic levitation; they used a single-mode duffing-type model to analyse the response of the system and compared the theoretical analysis to experiments. Further investigations were conducted by Pasharavesh et al. [36], who developed a nonlinear electromechanical mode for beam-piezoelectric energy harvesters; they utilized a single-mode approximation together with the method of multiple scales to study the behaviour of the system.

Further investigations were performed by Kim et al. [37], who studied the effects of proof mass on performance of a vibration energy harvester using a linear single-mode model; Gu and Livermore [38], who presented experiments and models of an impact-driven vibration energy harvesting device; Nguyen and Halvorsen [39], who investigated the suitability of softening springs in a micro energy harvester under noise vibrations and showed that performance enhancement occurs when the input frequency varies in the frequency range of the softening response; Ju et al. [40], who proposed a vibration energy harvester design based on magnetoelectric laminate composite together with a permanent magnet as a proof mass; Firoozy et al. [41], who studied the energy harvesting response of a unimorph piezoelectric energy harvester with a tip magnet subject to a harmonic base excitation using a single-mode nonlinear model.

In this study, for the first time, a fully nonlinear multi-modal coupled electromechanical model of the bimorph piezoelectric cantilever beam is developed taking into account various sources of nonlinearity while retaining a large number of modes in

discretisation procedure. This work aims at contributing to the field by improving *two aspects* of the available single-model linear bimorph energy harvester models through: (i) taking into account all sources of nonlinearity, and (ii) accounting for multiple degrees of freedom. The effect of each of these aspects is clearly highlighted in this study and it is shown that it is essential to have a *multi-modal nonlinear* model to be able to predict the response of constrained and unconstrained energy harvesters accurately. The developed model is verified via comparison to experimental and theoretical results in the literature. The frequency responses of the bimorph piezoelectric energy harvester are constructed in the absence and presence of motion constraints. The effects of various system parameters are examined. It is shown that the presence of motion constraint significantly improve the resonance bandwidth.

2. Coupled electromechanical model development

In this section, the fully nonlinear electromechanical model of a bimorph cantilever-piezoelectric energy harvester is developed through use of the nonlinear Euler-Bernoulli beam theory [42-45] together with inextensibility condition and the piezoelectric constitutive equations. The bimorph configuration is illustrated in Fig. 1, where a substrate of thickness t_s is partially sandwiched between two piezoelectric patches of same thickness t_p . In this section, partial piezoelectric layers coverage is assumed to keep the derivation procedure as general as possible. Both substrate and piezoelectric patches are of width b while the beam is of length L and piezoelectric patches are of length l_2-l_1 . The whole system is under base excitation $z_b \sin(\omega_b t)$ with z_b and ω_b denoting the amplitude and frequency of the base motion. In what follows, to make the derivation procedure easier to follow, the

parameters of the top and bottom piezoelectric patches are differentiated by the superscripts (1) and (2). The layered cantilever displacement in the transverse direction, z , is denoted by $w(x,t)$.

2.1 Bimorph cantilever equation of motion

Since the piezo patches are of the same thickness and material, the neutral axis of the cantilever remains unchanged throughout the length. Hence, based on the inextensibility condition, the axial strain of the cantilever (i.e. both substrate and piezo patches) can be formulated as

$$\varepsilon_1 = -z \frac{\partial}{\partial x} \left[\frac{\partial w}{\partial x} + \frac{1}{6} \left(\frac{\partial w}{\partial x} \right)^3 \right], \quad (1)$$

with subscript 1 denoting the x direction.

The axial stress developed in the substrate is given by $\sigma_s = E_s \varepsilon_1$, with E_s representing the substrate's Young's modulus. For the piezoelectric patches, the one-dimensional constitutive equations are given by

$$\begin{cases} \sigma_p^{(k)} = c_{11}^{(k)} \varepsilon_1^{(k)} - h_{31}^{(k)} D_3^{(k)}, \\ E_3^{(k)} = -h_{31}^{(k)} \varepsilon_1^{(k)} + \beta_{33}^{(k)} D_3^{(k)}, \end{cases} \quad k=1,2, \quad (2)$$

in which the x and z directions are denoted by subscripts 1 and 3, respectively; additionally, as mentioned earlier, superscripts 1 and 2 denote the top and bottom piezoelectric patches, respectively. $c_{11}^{(k)}$ is the piezoelectric elastic stiffness, $\beta_{33}^{(k)}$ is the impermeability constant, and

$h_{31}^{(k)}$ is a piezoelectric constant; $D_3^{(k)}$ and $E_3^{(k)}$ stand for the electric displacement and the electric field in the z direction, respectively. The poling direction for both piezoelectric patches is considered to be in the positive z direction.

The variation of the strain energy of the substrate and piezoelectric layers are given by

$$\delta \Pi_s = \int_0^L \int_{A_s} E_s \varepsilon_1 \delta \varepsilon_1 dA dx = \int_0^L \left\{ E_s I_s \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] \delta \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] \right\} dx, \quad (3)$$

$$\delta \Pi_p^{(k)} = \int_{l_1}^{l_2} \int_{A_p^{(k)}} \left[(c_{11} \varepsilon_1 - h_{31} D_3^{(k)}) \delta \varepsilon_1 + (-h_{31} \varepsilon_1 + \beta_{33} D_3^{(k)}) \delta D_3^{(k)} \right] dA dx, \quad k=1,2, \quad (4)$$

in which A_s and I_s denote the substrate cross-sectional area and its second-moment and δ stands for the variational operator. $A_p^{(k)}$ denotes the piezoelectric layers' cross-sectional area. Knowing that the properties and dimensions of both piezoelectric patches are the same, Eq. (4) can be rewritten for each piezoelectric layer as

$$\begin{aligned} \delta \Pi_p^{(1)} = & \frac{1}{2} (t_s + t_p) h_{31} A_p \int_{l_1}^{l_2} D_3^{(1)} \delta \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] dx \\ & + \frac{1}{3} c_{11} b \left[(t_p)^3 + 3(t_p)^2 \left(\frac{t_s}{2} \right) + 3t_p \left(\frac{t_s}{2} \right)^2 \right] \int_{l_1}^{l_2} \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] \delta \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] dx \quad (5) \\ & + \frac{1}{2} (t_s + t_p) h_{31} A_p \int_{l_1}^{l_2} \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] \delta D_3^{(1)} dx + \beta_{33} A_p \int_{l_1}^{l_2} D_3^{(1)} \delta D_3^{(1)} dx, \end{aligned}$$

$$\begin{aligned} \delta \Pi_p^{(2)} = & -\frac{1}{2} (t_s + t_p) h_{31} A_p \int_{l_1}^{l_2} D_3^{(2)} \delta \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] dx \\ & + \frac{1}{3} c_{11} b \left[(t_p)^3 + 3(t_p)^2 \left(\frac{t_s}{2} \right) + 3t_p \left(\frac{t_s}{2} \right)^2 \right] \int_{l_1}^{l_2} \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] \delta \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] dx \quad (6) \\ & - \frac{1}{2} (t_s + t_p) h_{31} A_p \int_{l_1}^{l_2} \left[\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right] \delta D_3^{(2)} dx + \beta_{33} A_p \int_{l_1}^{l_2} D_3^{(2)} \delta D_3^{(2)} dx. \end{aligned}$$

Writing the axial motion in terms of the transverse motion via the inextensibility condition, the variation of the kinetic energy of the bimorph piezoelectric cantilever under base excitation $z_b \sin(\omega_b t)$, with a tip mass M_t , can be formulated as

$$\delta KE = \int_0^L M(x) \left[\frac{\partial}{\partial t} \left(\int_0^x \frac{\partial w}{\partial x} \delta \left(\frac{\partial w}{\partial x} \right) dx \right) \int_0^x \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x \partial t} dx + \delta \left(\frac{\partial w}{\partial t} \right) \left(\frac{\partial w}{\partial t} + z_b \omega_b \cos(\omega_b t) \right) \right] dx, \quad (7)$$

in which

$$M(x) = \rho_s A_s + 2\rho_p A_p G(x) + M_t \delta_D(x-L), \quad (8)$$

where ρ_s is the substrate mass density, while ρ_p denotes the mass density of the piezoelectric layers; δ_D denotes the Dirac delta function. Additionally,

$$G(x) = H_s(x-l_1) - H_s(x-l_2), \quad (9)$$

in which H_s is the Heaviside function.

When the cantilever is under base excitation, voltages $V^{(1)}(x, t)$ and $V^{(2)}(x, t)$ are generated on the top and bottom piezoelectric patches, respectively; the resultant electrical virtual work of these induced voltages can be written as

$$\delta W_e = b \int_0^L [V^{(2)}(x, t) \delta D_3^{(2)}(x, t)] dx - b \int_0^L [V^{(1)}(x, t) \delta D_3^{(1)}(x, t)] dx. \quad (10)$$

In this study, the motion constraints are modelled as nonlinear springs. The elastic constraints, as shown in Fig. 1, limit the tip motion of the cantilever. The virtual work of the tip motion constraints can be modelled as

$$\delta W_c = - \int_0^L F_c \delta w dx, \quad (11)$$

where

$$F_c = \delta_D (x-L) H_s(\Delta) \text{sgn}(w) (k_3 \Delta^3 + k_1 \Delta), \quad (12)$$

in which $\Delta = |w| - g$, with g denoting the gap width, as shown in Fig. 1; additionally, H_s and sgn denote the Heaviside and sign functions, respectively. k_1 and k_3 are the linear and cubic coefficients of the spring. The expression given in Eq. (12) ensures that a non-zero force is obtained only when contact is initiated; this is done through use of the Heaviside function. The sign function in the contact force expression ensures that the correct direction of the force applied by the constraints. The linear and cubic springs represent the rigidity of the elastic constraint.

Using generalised Hamilton's principle, the following equations of motion can be derived

$$\begin{aligned} & M(x) \frac{\partial^2 w}{\partial t^2} - M(x) z_b \omega_b^2 \sin(\omega_b t) + \frac{\partial}{\partial x} \left\{ \frac{\partial w}{\partial x} \int_L^x M(x) \int_0^x \left[\left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial^3 w}{\partial x \partial t^2} \right] dx dx \right\} \\ & + \frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} \left[C(x) \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) \right] + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial}{\partial x} \left[C(x) \left(\frac{\partial^2 w}{\partial x^2} \right) \right] \right\} \\ & + \frac{\partial}{\partial x} \left\{ \left(1 + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial}{\partial x} [h_1^{(1)}(x) D_3^{(1)} + h_1^{(2)}(x) D_3^{(2)}] \right\} \\ & + \delta_D (x-L) H_s(|w| - g) \text{sgn}(w) (k_3 (|w| - g)^3 + k_1 (|w| - g)) = 0 \end{aligned} \quad (13)$$

$$h_1^{(1)}(x) \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) + \beta^{(1)}(x) D_3^{(1)} + b V^{(1)}(x, t) = 0, \quad (14)$$

$$h_1^{(2)}(x) \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) + \beta^{(2)}(x) D_3^{(2)} - b V^{(2)}(x, t) = 0, \quad (15)$$

in which

$$C(x) = E_s I_s + 2G(x) \left\{ \frac{1}{3} c_{11} b \left[(t_p)^3 + 3(t_p)^2 \left(\frac{t_s}{2} \right) + 3t_p \left(\frac{t_s}{2} \right)^2 \right] \right\}$$

$$h_1^{(1)}(x) = h_{31} A_p \left[\frac{1}{2} (t_s + t_p) \right] G(x), \quad h_1^{(2)}(x) = -h_{31} A_p \left[\frac{1}{2} (t_s + t_p) \right] G(x), \quad (16)$$

$$\beta^{(1)}(x) = \beta^{(2)}(x) = A_p \beta_{33} G(x)$$

$$V^{(1)}(x, t) = V^{(1)}(t) G(x), \quad V^{(2)}(x, t) = V^{(2)}(t) G(x),$$

assuming the same material and dimensions for both piezo patches.

Equations (13)-(15) can be reduced into one equation as

$$M(x) \frac{\partial^2 w}{\partial t^2} - M(x) a_b \sin(\omega_b t) + \frac{\partial}{\partial x} \left\{ \frac{\partial w}{\partial x} \int_L^x M(x) \int_0^x \left[\left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial^3 w}{\partial x \partial t^2} \right] dx dx \right\}$$

$$+ \frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} \left[\left(C(x) - \frac{2(h_1(x))^2}{\beta(x)} \right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) \right] \right.$$

$$\left. + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial}{\partial x} \left[\left(C(x) - \frac{2(h_1(x))^2}{\beta(x)} \right) \left(\frac{\partial^2 w}{\partial x^2} \right) \right] \right\} \quad (17)$$

$$+ \frac{\partial}{\partial x} \left\{ \left(\frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 + 1 \right) \frac{\partial}{\partial x} \left[-\frac{b h_1(x)}{\beta(x)} (V^{(1)}(x, t) + V^{(2)}(x, t)) \right] \right\}$$

$$+ \delta_D(x - L) H_s(|w| - g) \operatorname{sgn}(w) \left(k_3(|w| - g)^3 + k_1(|w| - g) \right) = 0,$$

in which the substitution $a_b = z_b \omega_b^2$ is made with a_b denoting the base acceleration, and

$$h_1(x) = h_1^{(1)}(x) = -h_1^{(2)}(x), \quad \beta(x) = \beta^{(1)}(x) = \beta^{(2)}(x). \quad (18)$$

2.2 Electrical circuit equation

To calculate the power generated by the energy harvester, another equation is needed to couple the dynamic oscillation of the system to changes in the induced voltage. In what follows, the electrical circuit equation is obtained for both parallel and series connections. To derive the electrical circuit equation, first the generated electric current in each piezoelectric layer is calculated as

$$i^{(k)} = \frac{\partial}{\partial t} \left[\int_{A_e^{(k)}} (\mathbf{D} \cdot \mathbf{n}) dA \right], \quad k = 1, 2, \quad (19)$$

where $\mathbf{D}$ denotes the vector of electric displacement while $\mathbf{n}$ represents the unit vector normal to the surface area of the electrode A_e . Substitution of Eqs. (14) and (15) into Eq. (19) gives the current generated in each piezoelectric layer. In what follows, the electrical circuit equation is obtained for both parallel and series connections. For each case, it is assumed that the electrodes fully cover the piezoelectric layers' top and bottom surfaces and they are connected to a resistive load R .

2.3 Constrained bimorph energy harvester model for parallel connection

Figure 2(a) shows the bimorph cantilever energy harvester electrical circuit for parallel connections of the piezoelectric layers. For this case

$$V^{(1)}(t) = V^{(2)}(t) = V_p(t), \quad i^{(1)} + i^{(2)} = \frac{V_p(t)}{R}. \quad (20)$$

Utilising Eqs. (17), (19), and (20), one can derive the following coupled nonlinear equations for a constrained bimorph cantilever energy harvester connected in parallel

$$\begin{aligned}
& M(x) \frac{\partial^2 w}{\partial t^2} - M(x) a_b \sin(\omega_b t) + \frac{\partial}{\partial x} \left\{ \frac{\partial w}{\partial x} \int_l^x M(x) \int_0^x \left[\left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial^3 w}{\partial x \partial t^2} \right] dx dx \right\} \\
& + \frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} \left[\left(C(x) - \frac{2(h_1(x))^2}{\beta(x)} \right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) \right] \right. \\
& \quad \left. + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial}{\partial x} \left[\left(C(x) - \frac{2(h_1(x))^2}{\beta(x)} \right) \left(\frac{\partial^2 w}{\partial x^2} \right) \right] \right\} \\
& + \frac{\partial}{\partial x} \left\{ \left(1 + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial}{\partial x} \left[-\frac{2bh_1(x)V_p(t)G(x)}{\beta(x)} \right] \right\} \\
& + \delta_b(x-L)H(|w|-g)\text{sgn}(w) \left(k_3(|w|-g)^3 + k_1(|w|-g) \right) = 0,
\end{aligned} \tag{21}$$

$$\frac{d}{dt} V_p(t) + \frac{t_p \beta_{33}}{2b(l_2 - l_1)} \left[\frac{V_p(t)}{R} + \frac{b(t_s + t_p)h_{31}}{\beta_{33}} \int_{l_1}^{l_2} \frac{\partial}{\partial t} \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) dx \right] = 0. \tag{22}$$

2.4 Constrained bimorph energy harvester model for series connection

The bimorph cantilever energy harvester electrical circuit for series connection of the piezoelectric layers is shown in Fig. 2(b). For this case

$$i^{(1)} = i^{(2)} = \frac{V_s(t)}{R}, \quad V^{(1)}(t) + V^{(2)}(t) = V_s(t). \tag{23}$$

Utilising Eqs. (17), (19), and (23), the coupled nonlinear electromechanical model of the constrained bimorph cantilever energy harvester connected in series can be derived as

$$\begin{aligned}
& M(x) \frac{\partial^2 w}{\partial t^2} - M(x) a_b \sin(\omega_b t) + \frac{\partial}{\partial x} \left\{ \frac{\partial w}{\partial x} \int_l^x M(x) \int_0^x \left[\left(\frac{\partial^2 w}{\partial x \partial t} \right)^2 + \frac{\partial w}{\partial x} \frac{\partial^3 w}{\partial x \partial t^2} \right] dx dx \right\} \\
& + \frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} \left[\left(C(x) - \frac{2(h_1(x))^2}{\beta(x)} \right) \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) \right] \right. \\
& \quad \left. + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial}{\partial x} \left[\left(C(x) - \frac{2(h_1(x))^2}{\beta(x)} \right) \left(\frac{\partial^2 w}{\partial x^2} \right) \right] \right\} \\
& + \frac{\partial}{\partial x} \left\{ \left(1 + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) \frac{\partial}{\partial x} \left[- \frac{b h_1(x) V_s(t) G(x)}{\beta(x)} \right] \right\} \\
& + \delta_D (x-L) H(|w|-g) \operatorname{sgn}(w) \left(k_3 (|w|-g)^3 + k_1 (|w|-g) \right) = 0,
\end{aligned} \tag{24}$$

$$\frac{d}{dt} V_s(t) + \frac{2t_p \beta_{33}}{b(l_2 - l_1)} \left[\frac{V_s(t)}{R} + \frac{b(t_s + t_p) h_{31}}{2\beta_{33}} \int_{l_1}^{l_2} \frac{\partial}{\partial t} \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \right) dx \right] = 0. \tag{25}$$

2.5 Discretisation

The coupled electromechanical nonlinear models developed in Sections 2.3 and 2.4 should be discretised in the spatial domain so that they can be solved numerically. The Galerkin modal decomposition method [46] is utilised to transform the partial differential equations into ordinary differential ones. To this end, the transverse motion is defined as a series expansion of generalised coordinates $p_n(t)$ multiplied by spatial basis functions $\Phi_n(x)$ as

$$w(x, t) = \sum_{n=1}^N \Phi_n(x) p_n(t), \tag{26}$$

where

$$\Phi_n(x) = \left[\cosh\left(\frac{\lambda_n x}{L}\right) - \varphi_n \sinh\left(\frac{\lambda_n x}{L}\right) \right] - \left[\cos\left(\frac{\lambda_n x}{L}\right) - \varphi_n \sin\left(\frac{\lambda_n x}{L}\right) \right], \quad (27)$$

$$\varphi_n = \left(\sin(\lambda_n) + \sinh(\lambda_n) \right)^{-1} \left(\cos(\lambda_n) + \cosh(\lambda_n) \right),$$

in which λ_n is the n th root the equation $1 + \cos(\lambda)\cosh(\lambda) = 0$. Application of the Galerkin method gives the discretised equations for both bimorph energy harvester models developed in Sections 2.3 and 2.4. In this study, 6 modes are retained for the transverse displacement resulting in a discretised model consisting of 7 equations. After transforming the equations into first-order form, a modal damping mechanism is added with damping for each mode being proportional to its natural frequency. The resultant coupled nonlinear set of equations is solved numerically through use of a well-optimised pseudo-arclength continuation method [47] capable of handling geometrical- and inertial-type nonlinearities arising in the electromechanical coupled model as well as complex impact-type nonlinearities arising due to presence of motion constraints.

3. Unconstrained energy harvester

This section studies the effect of various system parameters on the unconstrained bimorph cantilever energy harvester. For all cases examined in this section, it is assumed that the piezoelectric layers fully cover the substrate, i.e. $l_1=0$ and $l_2=L$. Additionally, the base acceleration is set to 9.81 m/s^2 and the modal damping ratio is set to 0.0167 for all the results reported in this section.

3.1 Model verification

The model developed in Section 2 is verified through comparison to two studies in the literature [34, 48]. For the first comparison case, the experimental results reported in Fig. 7(c,d) of Ref. [34] are reproduced numerically and compared. The substrate and piezoelectric layers properties and dimensions are set to those given in Ref. [34]. It is worth mentioning that since the cantilever considered in Ref. [34] is very wide ($b=0.6260L$), it is more accurate to replace elastic moduli of substrate and piezoelectric layers by their plain strain counterparts, i.e. $E_s/(1-\nu_s^2)$ and $c_{11}/(1-\nu_p^2)$, respectively, with ν_s and ν_p being the substrate and piezoelectric Poisson's ratio, respectively. The substrate and piezoelectric Poisson's ratio are set to 0.34 and 0.31, for brass and PZT-5A materials, respectively. For this comparison, $R=33 \text{ k}\Omega$, $a_b=9.81 \text{ m/s}^2$, and $M_t=0.012 \text{ kg}$. The results of the comparison are shown in Fig. 3 for (a) voltage and (c) tip velocity; sub-figures (b) and (d) show the result of the error analysis for voltage and tip velocity, respectively. As seen, the voltage and tip velocity frequency responses predicted numerically are very close to those obtained experimentally.

The second comparison case is conducted by reproducing the theoretical results reported in Fig. 5 of Ref. [48] for the parallel connection. The substrate and piezoelectric layers properties and dimensions are set to those given in Ref. [48]. The result of this comparison is shown in Fig. 4; as seen, the power output predicted by the proposed model is in excellent agreement with that reported in Ref. [48].

The two comparisons shown in Figs. 3 and 4 verify the accuracy and reliability of the multi-modal coupled nonlinear electromechanical model developed in this study for bimorph energy harvesters.

3.2 Parallel connection versus series connection

A thorough comparison between bimorph energy harvesters connected in parallel and series is performed in this section. The properties and dimensions of the bimorph system substrate and piezoelectric layers are given in Tables 1 and 2, respectively. Using the piezoelectric properties given in Table 2, the required piezoelectric constants can be evaluated as

$$\begin{aligned} c_{11} &= \left(s_{11} - d_{31}^2 / \epsilon_{33} \right)^{-1} = 69.81 \text{ GPa}, \\ h_{31} &= - \left(d_{31} - s_{11} \epsilon_{33} / d_{31} \right)^{-1} = -579.76 \text{ MV/m}, \\ \beta_{33} &= \left(\epsilon_{33} - d_{31}^2 / s_{11} \right)^{-1} = 38.03 \text{ Mm/F}. \end{aligned} \tag{28}$$

Figure 5 shows the frequency responses of a bimorph energy harvester connected in parallel for various values of load resistance R . More specifically, sub-figures (a-d) show the frequency responses for maximum voltage output, power output, tip transverse displacement, and tip transverse velocity, respectively. As seen, the maximum voltage output increases with increasing the load resistance. The power output, on the other hand, first reaches a maximum value with increasing R and then decreases as R is further increased. The optimum load resistance for maximizing the power output is around 10 k Ω . It is interesting to note that tip displacement and velocity behaviour with increasing load resistance is the opposite of that of power output; in other words, both tip displacement and velocity decrease first with increasing R until reaching a local minimum and then increase as R is increased.

The frequency responses for the cases when piezoelectric layers are connected in series are illustrated in Fig. 6. As seen, the behaviour is similar to that of the parallel

bimorph with the main difference being the load resistance magnitude. In fact, for series connection, the power output reaches its maximum at larger magnitudes of load resistance compared the parallel connection case. It is important to note that the maximum possible power output of the bimorph energy harvester is the same for both parallel and series connection; the only difference is the load resistance magnitude at which the power output becomes a maximum. To better show this, Fig. 7 is constructed showing the maximum voltage and power outputs as well as maximum tip displacement and velocity as functions of load resistance when $\omega_b=98.15$ Hz. As seen, the voltage and power outputs for parallel and series connection coincide at $R=20$ k Ω . For load resistance smaller than that, parallel connection gives larger voltage and power output while for load resistance larger than 20 k Ω , the series connection gives larger voltage and power outputs. Additionally, it is seen that the peak power output for both cases is the same.

3.3 Tip mass effect

This section examines the effect of an added tip mass on the voltage and power outputs as well as tip displacement and velocity of the bimorph energy harvester connected in parallel. For convenience, a dimensionless tip mass ratio parameter is defined as

$$\gamma = M_t / (\rho_s A_s L + 2\rho_p A_p L), \quad (29)$$

which represents the ratio of the added mass at tip to the mass of the bimorph cantilever. The frequency responses of the bimorph energy harvester for various tip mass ratios are shown in Fig. 8. The added tip mass has two important effects on the behaviour of the energy harvester. First, as seen, the added tip mass moves the resonance region to smaller

base excitation frequencies. This is significantly important from a design point of view; an added tip mass could be used to tune the resonance region of the energy harvester based on the expected environmental vibration range. The other important effect of the added tip mass is the increased power output of the harvester. More specifically, the maximum power output increases from 3.84 mW to 10.10 mW as γ is increased from 0 to 0.5; in other words, a 50% increase in the total mass of the harvesters results in 263% increase in the maximum power output. For the cases with $\gamma=1.0$, 1.5, and 2.0, the maximum power output is 17.02 mW, 24.07 mW, and 31.01 mW, respectively. Sub-figures 8(c) and 8(d) show that the maximum tip transverse displacement and velocity increase significantly with increasing tip mass ratio.

4. Constrained energy harvester

This section examines the broadband nonlinear response of the bimorph energy harvester in the presence of motion constraints. As seen in Fig. 1, motion constraints are located at both sides of the beam in the transverse direction. In this section, the effect of one-sided motion constraint is examined as well. Similar to the previous section, it is assumed that the piezoelectric layers fully cover the substrate, i.e. $l_1=0$ and $l_2=L$, and the base acceleration is set to 9.81 m/s^2 for all cases examined in the following. Additionally, k_1 and k_3 are set to 10^4 N/m and $5 \times 10^{10} \text{ N/m}^3$, respectively. The tip mass ratio γ is set to 1.5 throughout this section. It should be noted that for this tip mass ratio, the optimum load resistance leading to maximum power output is obtained as 45 k Ω . The substrate and piezoelectric dimensions and properties are the same those given in Section 3. The modal damping ratio is set to 0.0167 for all the cases examined in this section.

4.1 Broadband energy harvesting

The broadband nonlinear response of the constrained energy harvester is shown in Fig. 9. To obtain the frequency-response curves of Fig. 9, the excitation frequency is varied as the control parameter. For this case, the bimorph cantilever is constrained from both sides, as shown in Fig. 1. As seen, the presence of motion constraints widens the resonance band significantly. In particular, considering power output threshold of 5 mW, the unconstrained energy harvester has a resonance band of [34.29 40.01] Hz while the constrained harvester resonance band is [34.29 60.70], i.e. 4.6 times the resonance band of the unconstrained system. In other words, as a result of addition of motion constraints, a 71.4% resonance bandwidth (normalised with respect to the first natural frequency) is achieved, considering a power output threshold of 5 mW, while the unconstrained system shows a 15.5% bandwidth. Increasing the power output threshold to 15 mW, the constrained system displays a 66.8% resonance bandwidth while the unconstrained one exhibits a 6.6% bandwidth. This comparison shows that the addition of motion constraints significantly improves the efficiency of the energy harvester. The bifurcation points in the frequency response are shown in Fig. 9(c). As seen, the harvester shows two saddle-node bifurcations at points S_1 and S_2 and two torus bifurcations at points T_1 and T_2 . The bimorph cantilever displays quasiperiodic motion between points T_1 and T_2 , while the motion is unstable between the points S_1 and S_2 . To show the detailed dynamic behaviour of the energy harvester before and after hitting the constraints, Figs. 10 and 11 are constructed showing the time response of voltage and power outputs and tip displacement and velocity in one period of oscillation. As seen in Fig. 10, before hitting the constraint at $\omega_b=33.93$ Hz,

the energy harvester shows a smooth response for voltage and power outputs as well as tip displacement and velocity.

To better energy harvesting efficiency of the constrained and unconstrained bimorph cantilever, Fig. 12 is constructed showing the comparison of three systems: (i) unconstrained, (ii) constrained from one side, and (iii) constrained from both sides. It is seen that in terms of widening the resonance bandwidth, the energy harvester with constraints at both sides has the best efficiency; however, if maximizing the peak power output is the goal, then the harvester with one-sided constraint has the best efficiency.

4.2 Constraint gap effect

The gap width of the motion constraint is one of the key parameters when designing an energy harvester with elastic constraints. To better understand the effect of the gap width g on the behaviour of the energy harvester, Figs. 13 and 14 are constructed. These figures show the gap width effect on frequency responses of energy harvesters with one-sided and two-sided constraints, respectively. As seen in Fig. 13, for an energy harvester with constraints at both sides, decreasing the gap width from 2.4 mm to 2.1 mm almost doubles the resonance bandwidth; however, this comes at the cost of decreased power output level. By further decreasing the gap width from 2.1 mm to 1.8 mm, the resonance bandwidth is not changed much, but the power output level is decreased. The choice of gap width in the design of an energy harvester depends on many parameters. Although a smaller gap gives smaller power output levels, the contact is initiated at a smaller displacement magnitude which is particularly important for relatively small base

accelerations. In general, the best gap width for each application should be selected through an optimisation procedure with objectives of maximizing power output and bandwidth with their respective weights.

The gap width effect on frequency responses of the bimorph cantilever harvester with one-sided constraint is shown in Fig. 14. In this case, the resonance bandwidth increases gradually with decreasing gap width. Again, similar to the previous case, the increased bandwidth comes at the cost of decreased power output level. Since resonance bandwidth and power output level are competing outputs, the optimised gap width depends on the weight and importance of these outputs as well as the base acceleration magnitude.

5. Conclusions

This study investigated the broadband vibration energy harvesting capabilities of a constrained bimorph piezoelectric cantilever. A fully nonlinear beam model was developed accounting for geometric, inertial, impact, and piezoelectric nonlinearities. The nonlinear coupled electromechanical formulations were derived for both parallel and series connections of the piezoelectric layers. The Galerkin modal decomposition method was utilised to discretise the partial differential-type model while retaining a large number of modes to guarantee converged results. A continuation code was developed to handle various nonlinearities and to study the resonance response of the energy harvester.

To verify the accuracy of the proposed model, a comparison was made between the proposed model's predictions and the experimental results in the literature for a bimorph

piezoelectric cantilever energy harvester connected in series, showing excellent agreement between theoretical and experimental results. Conducting numerical simulations on the unconstrained system revealed that the maximum possible power output of the bimorph system is the same for both parallel and series connection. It was shown that the load resistance magnitude at which the power output becomes a maximum for parallel connection is almost 25% of that for series connection. Examining the effect of an added mass at tip on the response of the system showed that by adding a tip mass the resonance frequency region can be changed and the power output of the bimorph vibration energy harvester can be enhanced. In fact, it was shown that adding a tip mass of 50% of the bimorph cantilever mass could increase the power output by 263%.

The numerical results for constrained vibration energy harvester showed that the with a power output threshold of 15 mW, a 66.8% resonance bandwidth (normalised with respect to the first natural frequency) is achieved which is almost 10 time the bandwidth of the unconstrained system. It was shown that the energy harvesting efficiency is best when the system is constrained from both sides. Examining the effect of the constraint gap width on frequency responses of the energy harvester showed that for the case with constraints at both sides, a significant bandwidth improvement is gained as the gap is decreased from 2.4 mm to 2.1 mm; however, further decrease in the gap does not change the resonance band. For the system with one-sided constraint, the bandwidth is gradually increased as the gap width is decreased.

Appendix A. Effect of number of vibration modes

The effect of the number of vibration modes, i.e. degrees of freedom, on the voltage and power outputs of the constrained energy harvester is examined in this section and shown in Fig. A.1. To this end, three discretised models are used for the *nonlinear* model developed in Section 2, namely a 1-degree-of-freedom (1-DOF) model, as well as a 2- and 6-DOF models. The numerical results are obtained for a bimorph energy harvester connected in parallel with $R=45\text{ k}\Omega$, $\gamma=2.5$, $g=2.1\text{ mm}$, and $a_b=9.81\text{ m/s}^2$; the constraint properties are the same as those defined in Section 4. As seen in Fig. A.1, the 1-DOF model predicts the response with significant inaccuracy. More specifically, the 1-DOF model predicts the resonance bandwidth with almost 85% error; additionally, it predicts a very smooth voltage and power outputs which is not the case when higher vibration modes are accounted for. The 2-DOF model does a much better job than the 1-DOF model, but still does not give converged results. The 6-DOF model, i.e. the one used in this study, gives converged results.

Appendix B. Effect of sources of nonlinearity

A comparison of various models of an unconstrained cantilever bimorph energy harvester is conducted in this section and the results are shown in Fig. B.1. The bimorph harvester is of $l_1=0$, $l_2=0.3L$, $L=100$ mm, $R=150$ k Ω , $\gamma=1.5$, and $a_b=1.5\times 9.81$ m/s²; the rest of the dimensions and properties are the same as those given in Tables 1 and 2. The modal damping ratio is set to 0.01 for this case. It is important to note that for all the models examined here, 6 degrees of freedom are retained in the discretisation procedure. As seen, a nonlinear model of the harvester with only inertial nonlinearities predicts a softening-type nonlinear behaviour while that with only geometric nonlinearities predicts a hardening response. For the complete nonlinear model, some of these nonlinear effects cancel each other out, and the system shows a weak hardening behaviour. As seen, the linear model overestimates the transverse displacement by more than 20%. Hence, to ensure reliable predictions for a bimorph energy harvester, it is essential to account for nonlinearities and to retain sufficient number of modes in discretisation procedure.

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Table 1. Properties and dimensions of the substrate of the bimoprph cantilever energy harvester

Properties and dimensions	Substrate (brass)
t_s (mm)	0.25
b (mm)	15
L (mm)	60
ρ_s (kg/m ³)	8490
E_s (GPa)	106

Table 2. Properties and dimensions of the piezoelectric layers of the bimoprph cantilever energy harvester

Properties and dimensions	Each piezoelectric layer (PZT-5H)
t_p (mm)	0.25
b (mm)	15
L (mm)	60
d_{31} (pm/V)	-250
ξ_{33}/ξ_0 ($\xi_0 = 8.8542$ pF/m)	3400
s_{11} (pm ² /N)	16.4
ρ_p (kg/m ³)	7400

Fig.1. Schematic of the constrained bimorph piezoelectric cantilever energy harvester.

(a)

(b)

Fig. 2. Electrical circuit of the bimorph piezoelectric cantilever beam showing (a) parallel and (b) series connections of the piezoelectric layers.

Fig.3. Frequency responses of the bimorph cantilever energy harvester connected in series obtained via the model developed in this study (solid line) and those obtained experimentally in Ref. [34] (circles); (a, b) maximum voltage output and the percentage error of the model predictions, respectively; (c, d) maximum tip transverse velocity and the percentage error of the model predictions, respectively. $R=33.0 \text{ k}\Omega$ and $a_b=9.81 \text{ m/s}^2$.

Fig.4. (a) Comparison between the maximum power output versus the load resistance R of a bimorph cantilever energy harvester connected in parallel obtained via the model developed in this study (solid line) and those obtained theoretically in Ref. [48] (circles); (b) the percentage difference between the predictions of the two models. $\omega_b=80.0$ Hz and $a_b=9.81$ m/s².

(a)

(b)

(c)

(d)

Fig.5. Frequency responses of the bimorph cantilever energy harvester connected in *parallel* for various values of load resistance R ; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $\gamma=0$ and $a_b=9.81 \text{ m/s}^2$.

(a)

(b)

(c)

(d)

Fig.6. Frequency responses of the bimorph cantilever energy harvester connected in *series* for various values of load resistance R ; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $\gamma=0$ and $a_b=9.81 \text{ m/s}^2$.

(a)

(b)

(c)

(d)

Fig.7. (a, b) maximum voltage and power outputs, respectively, versus the load resistance R ; (c, d) maximum tip transverse displacement and velocity, respectively, versus the load resistance R . $\gamma=0$, $a_b=9.81 \text{ m/s}^2$, and $\omega_b=98.15 \text{ Hz}$.

(a)

(b)

(c)

(d)

Fig.8. Effect of tip mass ratio γ on frequency responses of the bimorph cantilever energy harvester connected in parallel; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $R=10$ k Ω and $a_b=9.81$ m/s 2 .

(a)

(b)

(c)

(d)

Fig.9. Frequency responses of the constrained bimorph cantilever energy harvester connected in parallel; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $R=45 \text{ k}\Omega$, $\gamma=1.5$, $g=2.1 \text{ mm}$, and $a_b=9.81 \text{ m/s}^2$.

Fig.10. Detailed responses of the constrained bimorph cantilever energy harvester of Fig. 9 at $\omega_b=33.93$ Hz; (a, b) voltage and power outputs in one period of oscillation, respectively; (c, d) tip transverse displacement and velocity in one period of oscillation, respectively. t_n indicates normalised time relative to oscillation period.

Fig.11. Detailed responses of the constrained bimorph cantilever energy harvester of Fig. 9 at $\omega_b=60.70$ Hz; (a, b) voltage and power outputs in one period of oscillation, respectively; (c, d) tip transverse displacement and velocity in one period of oscillation, respectively. t_n indicates normalised time relative to oscillation period.

(a)

(b)

(c)

(d)

Fig.12. Frequency responses of constrained and unconstrained bimorph cantilever energy harvester connected in parallel; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $R=45 \text{ k}\Omega$, $\gamma=1.5$, $g=2.1 \text{ mm}$, and $a_b=9.81 \text{ m/s}^2$.

(a)

(b)

(c)

(d)

Fig.13. Gap effect on frequency responses of the bimorph cantilever energy harvester connected in parallel and constrained from *both sides*; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $R=45$ k Ω , $\gamma=1.5$, and $a_b=9.81$ m/s².

(a)

(b)

(c)

(d)

Fig.14. Gap effect on frequency responses of the bimorph cantilever energy harvester connected in parallel and constrained from *one side*; (a, b) maximum voltage and power outputs, respectively; (c, d) maximum tip transverse displacement and velocity, respectively. $R=45 \text{ k}\Omega$, $\gamma=1.5$, and $a_b=9.81 \text{ m/s}^2$.

(a)

(b)

Fig. A.1. Effect of number of degrees of freedom on nonlinear resonance response of the constrained energy harvester; (a, b) maximum voltage and power outputs, respectively.

(a)

(b)

Fig.B.1. Comparison of various models of a bimorph energy harvester; (a, b) maximum tip transverse displacement and velocity, respectively. The number of degrees of freedom for all cases is equal to 6.