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Antiphase chaotic synchronization enhancement in VCSEL

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In this paper, we demonstrate experimentally that a vertical cavity surface emitting laser (VCSEL) exhibits an enhancement of nonlinear polarization dynamics, i.e., antiphase chaos synchronization and mode hopping with optical feedback (OF). Rotating orthogonal polarization of a VCSEL is used as an external parameter to generate chaotic signal when it is reflected back with a fixed bias current and a strong OF signal. The intensity of the two linear polarizations modes of VCSEL is measured for a range of bias current, which provides a detailed investigation for its dynamics dependence. The results show that, the antiphase chaotic synchronization enhances with increasing the angle of orthogonal polarization of OF. Polarization modes are oscillated entirely in the chaotic regime in antiphase synchronization with no time delay at the bias currents of 1.2 (low) and 1.7 mA (high). However, the synchronization quality of the two modes is completely deteriorated when the bias current is increased to 1.7 mA at a polarization angle of 70°, where power modes differences are increased. The result shows that, the power mode difference affects antiphase synchronization dynamics and thus destroys the mode computation effects. In addition, polarization switching take place at the polarization angles of 60° and about 33° at low and high bias currents, respectively. Polarization mode hopping is also observed, which is associated with improvement of the antiphase synchronization dynamics.

OCIS codes: (230.0230) Optical devices, (190.4360) Nonlinear optics, devices, (140.7260) Vertical cavity surface emitting lasers, (260.5430) Polarization, (140.1540) Chaos

1. Introduction

Vertical-cavity surface-emitting lasers (VCSELs) are seen as a potential light source in high-speed optical data transmission due to low costs, power efficiency and packaging [1, 2]. Since 1996 with introduction of the first commercial VCSELs, we have seen a growing use of VCSELs, which are low cost, in a wide range applications including optical sensing, optical switching, short and long data transmission and networks, optical storage, optical HDMI cables, intra-chip communications, network security etc. [2-6]. Recently, VCSELs have been used as a promising candidate for photonic neuron networks. A neuron-like response dynamic based on polarization switching (PS) and nonlinear dynamics was reported in VCSEL by apply orthogonal polarization injection [7, 8]. Most VCSEL devices usually display stable output polarization emission with steady intensity. However, with VCSEL being sensitive to the optical feedback (OF), which is due to inherent structure of high mirror's reflectivity, the internal or external light reflections can introduce changes in the VCSEL emission and its properties. VCSEL's dynamical state is affected by the OF, optical injection, bias current and temperature, which limits its use in some applications. Therefore, the complexity of VCSEL dynamics is the most active research area, which needs further investigation within the context of secure data communication networks.

In [9], polarization instability in VCSELs commonly reported with PS phenomenon, mode hopping and broadband chaos in the dynamics of the output polarization modes was reported. In a numerical study [10],

dual-channel data communications and synchronization using unidirectionally coupled VCSEL with polarization-rotated optical injection and OF was investigated. Rotating both polarization OF and optical injection have been used to investigate unidirectional coupled VCSELs and the possibility of realizing complete synchronization between the linear polarization modes of the master and slave lasers. OF-based chaotic dynamics have been proposed as a powerful candidate for generating chaotic signals for high-speed and long distance data transmission [11, 12]. VCSEL exhibits interesting polarization dynamics with the polarization rotation feedback compared with an isotropic feedback [10, 13, 14]. In [13], polarization mode competition is evaluated quantitatively in VCSEL for improved performance as a chaotic source in communication systems. Very recently, investigation of the rotated OF showed improvement in the reservoir computer performance [14].

VCSEL as a chaotic source have been used with injection current as a sensitive parameter, which influences VCSEL properties. Its show that, under an the increased level of pumped current high-quality chaos synchronization, high bandwidth chaos and complexity are realized with optical injection [15]. OF and optical injection simultaneously have displayed a larger value of chaotic bandwidth with lower bias currents [16]. In mutually coupled VCSELs, in-phase polarization dynamics induced by the similar mode of the optical injection and antiphase dynamics in the slave VCSEL produced by the orthogonal optical injection were reported [17]. Alternating the emission of power modes in VCSELs, which simultaneously take place with the PS, also indicates the antiphase dynamics. VCSEL based chaos oscillator in

optical communications is an attractive option for providing a high level security at the physical layer [18, 19].

The underlying physics of signal instabilities and chaos dynamics observed in the output power of VCSEL's polarization modes is a nonlinear process, thus higher coupling of co-existing modes with the polarized OF or the optical injection mode within the laser cavity [20, 21]. Gain competition between the laser's co-existing polarization modes is also responsible for the polarization instability in the output power of VCSEL [22]. In our previous works on VCSELs for a range of bias currents and OF levels, we investigated the antiphase dynamics [23-25] and showed that the antiphase dynamics of the two polarization modes were correlated for the preserved OF level of -7.4 dB. It was also shown that, there was high quality antiphase oscillations over a wide range of polarization angle (i.e., $> 45^\circ$) [23]. In [24], using numerical modelling it was shown that, antiphase synchronization with poor and good qualities were achieved for low and high levels of OF, respectively.

Thus, concluding that the antiphase dynamics is an inherent property of VCSELs, which is associated with orthogonal oscillation of the polarization modes. The antiphase property of VCSEL can be enhanced by using an external optical signal considering the polarization states, strength of the optical signal and the injection current. Consent of literature in [22, 26, 27] with our results offer a valid reason for investigating the antiphase properties of VCSELs. Investigation of the underlining principle of complex dynamics of laser with the polarization rotated feedback considering projections at different rotating polarization angles is still necessary.

In this paper we experimentally investigate the antiphase dynamic dependency on both the bias current and the rotating polarization angle for fixed and strong OF levels. The bias current is a key parameter, which effects the VCSEL's dynamics, therefore we can anticipate that the chaos behaviours under different bias currents and OF strength will be more complex and interesting. Increasing current destabilizes the self-pulsing characteristics of the VCSEL thus leading to chaos, where the chaos bandwidth is strongly dependent on the bias current. In addition, it is well-known that, relaxation oscillation of laser follows the changes in the bias current. Furthermore, the polarization mode switching is another dynamics complexity due to the bias current variation [28, 29]. In this work, we consider two operating points of bias currents of 1.2 and 1.7 mA, i.e., low and high bias current, in order to investigate its effect with the rotating polarization angle for the VCSEL, where the XP is the dominant mode with high output power and the YP mode is completely suppressed. Further details about the laser properties of the light-current (L-I) curve can be found in [23]. The results show that, high quality chaotic synchronization with no delay time oscillations appears between the X polarizations (XP) and Y polarization (YP) modes for a range of polarization angles and the bias current.

Mode hopping is also observed in the dynamical states of VCSELs, which is associated with the highly synchronized signals. PS takes place at 60° and at $\sim 33^\circ$ with the bias currents of 1.2 and 1.7 mA, respectively. This means as long as the bias current increases a lower level of polarization is required to achieve PS, which was demonstrated in our previous work [30]. The results also show that, a complex behavior with chaotic synchronization and mode hopping is quite possible for the given values of the polarization angle and OF. This experimental data offer practical insights on the polarization dynamics with OF, particularly dynamic synchronization in VCSEL as a desired feature in optical transceivers. The rest of the paper is organized as follows. The experimental setup used to evaluate the antiphase

property of VCSEL is introduced next followed by results and discussions. Finally, our conclusions are presented.

2. Setup description

The schematic diagram of the experimental setup used for investigating the influence of rotating polarization angle of OF on the dynamic intensity of VCSEL is shown in Fig. 1(a). A commercial 852 nm VCSEL is used, which is lasing in its fundamental mode with two orthogonally polarization modes (i.e., XP and YP) over the entire bias current range of 0-2 mA. The threshold current and the maximum output power of VCSEL are ~ 0.5 mA and 0.95 mW (at a bias current of 2 mA), respectively. The VCSEL is driven by a laser diode driver module (Newport, 505B) and is temperature controlled using a thermoelectric temperature controller (TED 200° C) to within 0.01° C. The output beam of the laser is first collimated using an objective lens prior to being applied to a 50/50 non-polarizer beam splitter (BS). The BS is used to direct $\sim 50\%$ of VCSEL's output power to a mirror (M) through a polarizer (P) and an attenuator (ATT) in order to control the OF level. A half wave plate (HWP) is used to direct the orthogonal polarizations modes to a polarizer beam splitter (PBS) and then to two identical optical receivers (i.e., OR₁ and OR₂). The ORs' outputs are captured using a digital oscilloscope (Agilent, DSO9254A, 20GSa/s) for further analysis. An optical power meter (Anritsu, ML9001A) is used to measure the optical output power of the VCSEL and the OF level. The external cavity length is about 26 cm, which corresponds to a 1.7 ns delay time.

Fig.1. a) Experimental setup for VCSEL's dynamics measurements. BS- non-polarizer beam splitter, P - polarizer, ATT- attenuator, M- mirror, HWP- half wave plate, PBS- polarizer beam splitter, and OR₁ & OR₂- optical receivers; b) Intensity of the XP (black) and YP (red) as a function of time of VCSEL, and c) corresponding correlation profile of the XP and YP modes for the free running VCSEL.

3. Results and discussion

The free running dynamics of VCSEL at 1.2 mA are shown in Fig. 1(b, c), where the dominant (i.e., XP) and the suppressed (i.e., YP) modes are represented in black and red color, respectively. Note, because of

VCSEL's single mode emission feature, YP displays lower intensity compared with XP, where only the higher gain mode is emitted at free running operation, see Fig. 1(b), the fluctuations shown in the XP and YP intensities are due to the spontaneous emission noise of the laser [25]. The correlation plot, see Fig. 1(c), of two polarizations clearly indicates that there is no correlation dynamics between the XP and YP modes for the free running VCSEL. Next, we consider two cases for at the bias currents of 1.2 and 1.7 mA for investigating the influence of the rotating polarization angle on the dynamical states of VCSEL's intensity for a fixed OF level of -7dB.

A. Case 1

The measured intensities of modes (XP and YP) for a fixed bias current of 1.2mA and a range of polarization angles are depicted in Fig. 2. As shown in Fig. 2 (a-d), for the polarization angle $< 60^\circ$, the XP mode is the dominant whereas the YP mode is suppressed with a very low light emission level. However, for the polarization angles of $60^\circ, 70^\circ, 80^\circ$ and 90° the YP is the dominant mode. Note, the antiphase synchronization of XP and YP modes are best shown using the correlation plots, see Fig. 2(a'-h'), where we observe a marked improvement in the antiphase synchronization between the two modes as the polarization angle increase. The results show that, increasing the polarization angle leads to chaos in the polarization dynamics of VCSEL. For the polarization angle of 90° we observe modes hopping due to the rapid antiphase oscillation between the XP and YP modes as displayed in Fig. 2(h), which is supported by improved synchronization as in Fig. 2(h').

Fig. 2. Experimental time traces (black XP mode, red YP mode) and corresponding correlation plots (blue color) of the orthogonal polarization modes for the polarization angles of a) 0° , b) 20° , c) 30° , d) 50° , e) 60° , f) 70° , g) 80° and h) 90° .

Mode competition (i.e., between the two modes) in VCSEL and a fast alternating gain, particularly in mode hopping, could improve the antiphase dynamics [22, 31]. We should note that, a mutual gain mechanism is conveyed with mode hopping between the XP and YP modes of VCSEL [32, 33]. Mode hopping in VCSEL is also accompanied by the rapid anticorrelated dynamics [32] and therefore anticorrelated oscillations lead to synchronization enhancement as demonstrated in [34]. In general, anti-phase oscillations are demonstrated with the chaos dynamics in VCSEL's polarization modes and it has been shown that, synchronization quality increases with the polarization angles of OF. Additionally, no polarization dynamical delays for the antiphase dynamical oscillation are observed between the two polarization modes of the VCSEL.

B. Case 2

Similar to the case 1 but with a bias current of 1.7 mA, the dynamical state of the XP and YP modes are represented in Fig. 3. Fig. 3(a-h) display the time traces of the polarization resolved intensities of VCSEL's modes and Fig. 3(a'-h') show evaluation of the modes synchronization with the correlation profiles. Similar dynamic features of the antiphase oscillations and synchronization improvement with the rotating polarization angle to the case 1 are observed. At a polarization angle of 0° , weak antiphase synchronization is observed

over a range of 0° - 30° where the XP mode is dominant. Nevertheless, the YP mode increases with the polarization angle [35], which leads to PS at $\sim 33^\circ$ as shown in Fig. 3(d-h). Synchronization's quality of dynamics of the XP and YP modes increases faster than the case 1, which is observed at a lower polarization angle of 20° . However at 70° the antiphase synchronization is completely deteriorated. For this angle, the XP and YP modes are unconnectedly oscillated as shown in Fig. 3(g), and demonstrated in the corresponding correlation profile, see Fig. 3(g'). We note that, the intensities difference at 70° is slightly high compared with the polarization angle range of 45° to 90° . The value of the intensity difference is 0.670 at 70° , see Fig. 5(b). For polarization angle range of 45° - 90° , e.g., the intensities differences are 0.660 and 0.615 at 60° and 80° (for comparison), respectively. Thus, the two modes oscillate independently and exhibit low anti-phase oscillations.

In our previous works with a low bias current and a strong OF level [23-25] we showed the chaotic behavior of the VCSEL with anti-phase synchronization over the entire range of polarization angle. The strong mode competition are responsible for oscillations dependency of the two modes [13, 22]. However, mode competition seems to be less effect for the XP and YP modes with higher separation power levels. To best of our knowledge, the power mods difference between the VCSEL polarization modes has not been considered yet, which destroyed mode computation effects and affects the antiphase synchronization dynamics, and therefore need further investigations.

Fig. 3. Evolution the time trace of the XP and YP modes with polarization angles at: a) 0° , b) 20° , c) 30° , d) 45° , e) 50° , f) 60° , g) 70° and h) 90° , (a'-h') show the corresponding correlation profiles.

Figs. 4, (a) and (c) show the zoom in time traces at 70° and 90° , respectively clearly displaying the negative correlation between VCSELs' modes at 70° and a complete chaotic synchronization at 90° . Also shown are the corresponding autocorrelation plots, see Figs. (b), (d) and (f). For comparison, Figs. 4 (e) and (f) show the zoom in time traces and autocorrelation at free running operation, where the unit correlation oscillations of the XP and YP modes are similar to the modes tendency of no correlation between the XP and YP modes at 70° .

The average intensities as a function of the polarization angle for the XP and YP modes at the bias current values of 1.2 and 1.7 mA, are depicted in Fig. 5(a, b). For the case 1, the maximum intensities difference between the two modes is ~ 0.39 at 80° , see Fig. 5(a). While for case 2, the dominant modes (XP before 33° and YP after 33°) have higher intensities compared with the case 1, due to the bias current effect. The YP mode reach the maximum intensity value over the range of 45° - 90° , therefore the intensity difference is increased between the XP and YP modes. The maximum intensity differences is recorded at 70° with a value of 0.670, as depicted in Fig. 5(b), where the antiphase dynamics and synchronization is collapsed, see Fig. 3(g) and the corresponding figures (g').

Fig. 4, Zooming time traces of the XP and YP intensities at: a) 70°, c) 90° and e) free running operation, (b), (d) and (f) the corresponding autocorrelation plot respectively.

The intensity difference values at the angles close to 70°, i.e., at 60° and 80°, are 0.660 and 0.615, respectively, which is below the values measured at 70°, as in Fig. 5(b). Fast oscillation occur in the XP and YP modes at polarization angles of 45°, 50°, and 90°, which leads to mode hopping and improved synchronization as shown in Figs. 3(d, e, h). Mode hopping happens in both cases of 1 and 2 following changes in the polarization states of VCSEL, (i.e., PS). Fig. 5(c) illustrates the OF strength as a function of the polarization angle for both XP and YP modes with opposite behaviors, which is related to the polarization modes tendency, see Fig. 5(a, b). Note, at a low polarization angle range of 0° - 20° we observe the maximum (minimum) OF level of XP (YP), which is gradually decreasing (increasing) until reaching the low (high) level of OF at 90°, see Fig. 5(c). More details on the polarization OF behaviors with the polarization angle can be found in [30, 35]. PS occurs at 60° and ~ 33° with bias currents of 1.2 and 1.7 mA, respectively which indicates a lower degree of polarization is required to achieve PS when the bias current is increasing. This was demonstrated theoretically and experimentally in our previous work [30], and theoretically in [35].

Fig. 5. (a) and (b), average intensities of the polarization modes of VCSEL as a function of polarization angle at 1.2 and 1.7 mA, respectively and (c) OF strength as a function of the polarization angle.

4. Comments and Conclusions

This study focused on the antiphase dynamics properties of VCSEL's polarization modes, which offered further insight into VCSEL that could be used in many applications. We showed that, the antiphase dynamics is an inherent feature of the orthogonal polarization modes dynamics of VCSEL, which was gradually enhanced with rotating of the polarization angle of optical feedback at a low bias current of 1.2 mA. The synchronization quality of the dynamical states of VCSEL was vividly improved with increasing the polarization angle, which, was observed at the polarization angle of 20° when the bias current increased to 1.7 mA for the case 2. For the case 2, the synchronization of the antiphase dynamics collapsed at the polarization angle of 70°, where the maximum intensity difference was recorded. The separated intensity value of the XP and YP modes were measured for the antiphase synchronization evaluation. The results demonstrated that, rotation of polarization angles induced polarization switching, which occurred at the polarization angle of 60° at a low bias current of 1.2 mA and reducing to about 33° at a high bias current of 1.7 mA. In addition, a mode hopping phenomenon was observed following changes in polarization states of VCSEL's modes in both cases 1 and 2 at different polarization angles. We also showed that, mode hopping is associated with the synchronization quality of the antiphase dynamics for two polarization modes, where the antiphase and chaos synchronization properties of VCSEL's modes are of greatest interest in chaos-based secure communications and optical switching.

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