Influence of Water Depth on the Hydrodynamics of Deep-water Mooring Characteristics

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Abstract: This paper studies the behavior of a mooring system, in water depths from 300m to 3000m, using an indirect time-domain method. The boundary element method and lumped mass method are used to investigate the floating body hydrodynamics and mooring line characteristics, respectively. Results from coupled analysis are compared with experimental data in intermediate water depth. The results from coupled low frequency (LF) and fully coupled analysis are compared and discussed. Results from parametric studies are compared to offer guidance to mooring system designers on the suitability of particular approaches.

Key words: Water depth variation; coupled analysis; coupled LF analysis; mooring system; deep-water

1. INTRODUCTION

Water depth has long been recognised by the offshore industry as a key parameter in mooring system design; it affects mooring line length, material and configuration. The time domain approach (for example, as per the design codes of API (2005) or DNV (2010)) is considered to be a reliable design tool, by incorporating the inherent nonlinearities and coupling effects of the mooring system characteristics.

An ever present challenge is to improve the accuracy of the formulation and modelling, while decreasing computational demands. For larger water depths, coupled analysis has long been considered necessary to obtain accurate values of the motion response of a floating body. It is generally accepted that in deep water the weight of a chain mooring line becomes comparable with that of the floating body, and so it may have a large effect on the of the floating body. To this end, a fully coupled analysis is necessary to obtain accurate values for motions and mooring line tensions. Ormberg and Larsen (1998) studied a turret-moored FPSO,
and showed that, for large water depth, the LF response becomes dominant and a non-coupled analysis failed to predict the motions accurately. However, a fully coupled time-domain simulation is usually much more time-consuming and memory-intensive, which is not helpful for (preliminary) design. Many moored floating platforms are therefore assessed using a quasi-static coupled method. Under certain approximations, other methods are available which can generate reasonably accurate results without excessive computational demands. Their accuracy varies with water depth, and so it seems appropriate to re-examine and quantify approaches as the water depth increases.

Sensitivity studies have been undertaken by many researchers for a range of environmental conditions, mooring line hydrodynamic coefficients and configurations, and different methods of analysis (e.g. Kim, et al, 2001; Guedes Soares, et al, 2001; Wichers and Devlin, 2001; Qiao, et al. 2012; Astrup, et al, 2001; Kim, et al, 2001; Kim and Sclavounos, 2001; Tahar and Kim, 2003). These studies tended to concentrate on the effects of a range of parameters for a particular value of water depth. A few looked at the effects of water depth (e.g. Chen, et al (2001); Luo and Baudic (2003)); it is timely to re-evaluate this problem.

On examining standard design codes, e.g. API(2005) and DNV(2010), there is limited guidance on the effects of water depth variation. DNV-OS-E301 states that a dynamic analysis has to be carried out for water depths larger than 100m. Given that platforms are currently being developed for operation in water depths up to 3000m, it may happen that not all dynamic simulations generate comparable accuracies for similar computational effort.

2. DESCRIPTION OF THE MOORING SYSTEM

The mooring system used in our case studies is a classic Spar platform. Fig. 1 shows a schematic of the four-point mooring system. The main particulars of the cylinder are shown in Table 1.

3. NUMERICAL MODELLING

The diffraction/radiation computing package WAMIT was used to generate the hydrodynamic coefficients. In WAMIT, two types of Boundary Element Method (BEM) 1) lower order method and 2) higher order method (based on B-spline function) are applied for solving the integral equation. In this paper, considering of the large volume of the cylinder, the B-spline based higher order BEM method was applied for solving the integral equation. By solving the linear boundary value problem, the added mass, radiation
damping and wave exciting forces were calculated with different frequencies. Considering the long simulation time for calculating full QTFs (Quadric Transfer Functions), which are almost impossible for running hundreds of cases in design process, only mean drift force was calculated. The second-order difference-frequency compounds of wave loading were calculated with Newman’s Approximation, which is considered as a good and efficient method of approximation under deep-water circumstance.

3.1 MESH OF THE SPAR

Figure 2 shows the mesh generation of the Spar. Because of a B-spline function based the higher-order method was used in this paper, the geometry of the body surface was first determined by one or more smooth patches. A patch is a continuous surface (WAMIT). The cylinder was divided into two patches, one was the side and the other one was the bottom of the cylinder. Because of the symmetry of the cylinder, meshes were generated only on a quarter of the cylinder.

3.2 VALIDATION OF MEAN WAVE DRIFT FORCE

Wave drift forces are important in the mooring system design because the mean position of the mooring system is partly determined by the mean drift forces. Although mean drift forces can be evaluated by solving the first-order boundary value problem, it is not easy to evaluate compared first-order compounds. So, in this paper, a specific mooring system and a specific water depth were selected because of the availability of experimental data.

In order to check the accuracy of mean drift forces, a comparison of current WAMIT results and available experimental data (Spar Model Test Joint Industry Project, 1995) is provided in Figure 3. From the comparison we can see that a good trend is shown between numerical results and available experimental data, which offer good and accurate preparation for the time domain whole system dynamic response simulation. A good agreement is shown between present results and the experimental data for frequency less than 0.6 rad/s.

3.3 CALCULATION OF CURRENT FORCES

The current drag loads due to translational relative velocity were calculated in the standard manner:

\[ F = \frac{1}{2} C \rho V^2 A \]  

where
$C = \text{drag coefficient}$

$\rho = \text{water density}$

$V = \text{current velocity}$

$A = \text{projected area}$

In this paper, a current speed of 0.95 m/s was chosen at the surface, linearly decreasing to zero on the seabed.

### 3.4 WAVE DRIFT DAMPING

Wave drift damping is an important potential flow effect for LF motions (DNV-RP-F205, 2010). In OrcaFlex, the effect of wave drift damping is evaluated by modifying the wave drift QTF values. As Newman’s Approximation was used in current study, the modified wave drift QTFs were based on Newman’s Approximation. The modified value only applied on surge and sway. The wave drift damping in the case studies were time-dependent, instead of constant value. The modified diagonal QTF $Q_{de}$ is given by

$$Q_{de}(\beta, \beta, \tau, \tau) = A_e Q_d(\beta_e, \beta_e, \tau_e, \tau_e)$$  

(2)

where

- $A_e = \text{Aranha scaling factor (Aranha, 1994)}$
- $\beta_e = \text{encounter heading}$
- $\tau_e = \text{encounter period}$

### 3.5 MOORING SYSTEM METHODS OF ANALYSIS

- **Coupled LF method**

  In this method, the global motion is split into WF (displacement RAOs) and LF (solving time-domain motion equation) parts. A time-domain coupled analysis is performed under the LF effect only condition while the WF parts of motion response are evaluated by the displacement RAOs. The WF motion response is imposed on the LF motion response to get the total floating body motion response.

  The LF motion equation in the time domain can be expressed as:

$$[M + M_e(\infty)]\ddot{X}_{LF} + \int_0^\infty R(t-\tau)\dot{X}_{LF}d\tau + KX_{LF} = F_{\text{diff}} + F_{\text{current}} + F_{\text{mooring}} + F_{\text{vol}} + F_{v}$$  

(3)
where

\[ M_a = \text{Added mass matrix} \]
\[ M = \text{mass matrix} \]
\[ X_{LF} = \text{floating body low frequency motion response} \]
\[ K = \text{hydrostatic restoring matrix} \]
\[ R = \text{retardation function} \]
\[ F_{wd} = \text{wave drift damping} \]
\[ F_{drift} = \text{wave drift exciting force} \]
\[ F_{current} = \text{current force} \]
\[ F_{mooring} = \text{mooring force} \]

The displacement RAOs were calculated for a range of frequencies by solving the following frequency domain motion equation:

\[
\sum_{j=1}^{6} \left[ -\omega^2 (M_y + A_y) + i\omega B_y + K_y \right] x = F_{ext}^{(1)}
\]  

where

\[ x = \text{motion response} \]
\[ \omega = \text{circular frequency} \]
\[ A_y = \text{added mass matrix} \]
\[ B_y = \text{radiation damping matrix} \]
\[ K_y = \text{hydrostatic restoring matrix} \]
\[ F_{ext} = \text{1st-order wave exciting force} \]

- **Fully-coupled method**

The added mass and radiation damping are calculated in frequency domain for a range of frequencies and transferred from the frequency domain using an Inverse Fourier Transform Cummins (1962). The whole motion response no longer split into WF and LF parts. Instead, the output of motion response will be a combination of WF, LF and mean drift response. Coupling effects between floating body and mooring lines.
are included automatically by solving the time-domain equation of motion iteratively for each time step. Due to the small range of WF motion compared with LF motion, a relatively small time step is required.

The floating body motion equation in the time domain can be expressed as:

\[
[M + M_a(\infty)]\ddot{X} + \int_0^\infty R(t - \tau)\dot{X}\,d\tau + KX = F_{\text{wave}} + F_{\text{current}} + F_{\text{mooring}} + F_{\text{v}}
\]  

(5)

where

\[X = \text{floating body motion response}\]

\[F_{\text{wave}} = \text{wave exciting force}\]

4. COMPARISON WITH AVAILABLE EXPERIMENTAL DATA

In order to check our numerical model, motion response results for a mooring system in intermediate water depth (318m) were compared with available experimental and numerical data (Chen et al., 2001). The main characteristics of mooring line are shown in table 3.

4.1 IRREGULAR SEA-STATE MODELLING

For validation, the incident random wave was simulated using a JONSWAP spectrum with a significant wave height \(H_s=13.1\text{m}\) and peak period \(T_p=14\text{s}\), as selected by Chen et al., (2001). The expression of JONSWAP spectrum follows the equation below (OrcaFlex)

\[
S(f) = \frac{\alpha g^2}{16\pi^2} \left(\frac{2\gamma}{\alpha}\right)^\frac{\gamma}{2} \frac{1}{\sqrt{\gamma^2 - 1}} e^{-\frac{\gamma}{\sqrt{\gamma^2 - 1}}} \left(\frac{f}{f_m}\right)^\gamma
\]

(6)

where \(g\) is the gravitational constant. \(f\) and \(f_m\) are the frequency and peak frequency, respectively. \(\alpha\) and \(\gamma\) are data items as describes in OrcaFlex user’s Manual.

4.2 COMPARISON OF HORIZONTAL RESTORING FORCE AND WORST LOADED LINE TENSION

In order to check our modelling, horizontal restoring force and worst loaded line tension results (Figs. 4 and 5) for a mooring system in 318m water depth were compared with available experimental data (Chen et al., 2001). They compare well with available experimental data.

4.3 COMPARISON OF MOTION RESPONSE BETWEEN EXPERIMENTAL DATA AND NUMERICAL SIMULATION

Fully-coupled time domain motion responses of surge, heave and pitch are shown in Table 2. It is
generally believed that for such a high wave height nonlinear simulation of incident wave is required. But in present simulation we simulated the incident wave with superposition theory which means the nonlinearity of incident wave and their effects on the response of the cylinder were ignored. From the comparison we can see that the mean value still compares well with the experimental data. This is probably because of the nonlinearity of incident wave in current case studies is less important.

5. PARAMETRIC STUDY AGAINST ANALYSIS METHOD

Table 3 and 4 show the properties of mooring line in different water depths from 318m to 3000m. The two analysis methods mentioned in section 3.5 were applied under a wave only condition and a wave plus current condition, respectively. For the parametric studies, the incident random wave had a significant wave height $H_s=3.25m$ and peak period $T_p=9.7s$, which has an over 20% percentage probability of sea (Lee, et al, 1985). The simulation time was 3 hours. Table 5 shows a list of case studies and environmental conditions.

5.1. LOAD-OFFSET GRAPH FOR DIFFERENT WATER DEPTHS

In view of the large number of parameters, a comparison was undertaken using the static load-offset data in Fig. 6.1, selecting the static horizontal stiffness according to water depth so that the mean horizontal offsets were similar. For taut lines, the mooring system contains 12 lines with 3 mooring line in a group. The layout of each group of mooring line is shown in Fig. 1(b). The angle between each mooring line is 5degree.

5.2. SIMULATION RESULTS

Figs. 7~10 show the one-sided power spectral density (PSD) of motion response and line tension time histories obtained by Fourier Transform. The spectral density was smoothed to reduce any noise (as recommended in the OrcaFlex user manual). Statistical values of motion response and line tension are given in Figs. 11 and 12. Static results of mooring system response are shown in the figures of statistical values for comparison. The first column in the legend shows water depth while the second column represents statistical value or case studies. The maximum heave value in Figs. 12.2 and 14.1 are absolute maximum value.

Computed surge, pitch and heave natural periods are around 250s, 41s and 28s, respectively, for all water depths. Although the static load-offset graph is similar for each water depth, some discrepancies can be seen when dynamic aspects are taken into consideration. The mean value of the surge motion, using the coupled LF method, is about 1% larger than the value from the fully-coupled method (Fig 11.1). Current loading
increases the floating body mean and maximum position, but only affects the LF part of the surge motion, as can be seen from the spectra in Figs. 7.1-7.8. It is further observed that maximum surge response is under-predicted by the coupled LF method, compared with the fully coupled method. In contrast to the mean surge response, the difference between results from the coupled LF and fully-coupled method increase with increasing water depth. However, the horizontal motion responses are highly influenced by the additional viscous damping, and so the maximum motion response results from coupled LF analysis are not necessary larger than the fully coupled analysis. Pitch responses follow a similar trend, particularly for the mean values. Surge and pitch are coupled, but, in contrast to the results for surge, the maximum amplitudes of pitch computed by the coupled LF method are almost double those from the fully-coupled method.

It is interesting to note that, unlike surge and pitch, the statistical properties of the heave motion are independent of water depth, analysis method and loading condition. With a catenary chain mooring, the heave motion takes its overall largest value for 1000m water depth. One possible reason is that for water depths less than 1000m the chain has approximately 10% of the weight of the floating body, whereas it is less than 0.5% for 318m water depth. The phenomena investigated in current case studies may come from the large heave natural period (28s), which is far from the incident wave peak period (9.7s). To this end, a FPSO with a smaller heave natural period (8.33s) was selected for case study. The FPSO is an internal turret-moored vessel with four mooring lines. Main particular of the FPSO and mooring line characteristics are given in Table 6 and 7. Three water depths, 1000m 2000m and 3000m were studied to investigate water depth variation on heave motion. Similar to the cylinder case studies, in order to make results from different water depth more comparable, the static load-offset graph was similar for all the water depth, as shown in Fig. 6.2. From the heave motion spectrum in Fig. 13 we can see that the heave motion is complete determined by WF response. Under wave only condition, heave spectrum of coupled LF analysis and fully coupled analysis are almost identical. But on the contrary, current effect increases the difference between coupled LF and fully coupled method. In 1000m water depth, maximum value of heave response predicting by fully coupled method is less than 10% larger than the counterpart of coupled LF method. But this difference decrease to less than 2% in 3000m water depth. However, mean heave motion is still less sensitive to water depth, especially for wave-only condition.

For catenary chain and taut line, both mean and maximum mooring line tension increases with the
increasing of water depth. Among all the water depths, the maximum line tension comes from 1000m water depth with catenary chain, instead of 3000m water depth. One of the reasons is the mooring system initial tension accounts for a large amount of total maximum line tension. For slack mooring line, the difference between coupled LF and fully coupled method on predicting mooring line mean tension is 0.04% in 318m water depth, decreasing to 0.01% in 1000m water depth. But for taut mooring line, the difference was 0.04% in 1000m, slightly decreasing to 0.01% in 3000m water depth. For catenary chain, current effect becomes less important on mooring line tension with the increasing of water depth, as can be seen from Figs. 11.4(a) and 12.4(a). Mean line tension under a wave plus current loading is 2.02% larger than wave only condition in 318m water depth, decreasing to 0.75% in 1000m water depth. But for taut line, the discrepancy is less than 1% for all the water depths.

Both surge motion and mooring line (taut line) tension are completely determined by LF response, as we can see from Figs. 7 and 10. LF surge response varies with different water depth. But water depth variation does not appreciably impact surge WF response, as shown in Fig. 7. In the LF range, peak value of surge spectral density predicting by fully coupled method shows a fairly good agreement with the coupled LF method. It also can be seen that WF surge spectrum shows a significant difference between coupled LF and fully coupled method and the difference is less sensitive to water depth. But as surge motion is primarily dominated by LF response, the difference of WF response under different method does not significantly affect the whole motion response.

Pitch motion spectral density follows the same trend as surge motion, but determining by both LF and WF response. But unlike surge and pitch motion response, heave motion spectral density seems independent of water depth, especially for water depth less than 1000m where catenary was applied. Current effect and method of analysis have an effect on heave LF motion response, but not significantly varied.

Fig. 10 shows the spectral density of worst loaded line (line1 for catenary chain and line 2 for taut line) for different water depths. For catenary chain (between 300m and 1000m), LF response slightly increases with the increasing of water depth while WF mooring line response becomes increasingly important. But when considering greater water depths up to 3000m, for which a taut mooring line is applied, the LF range mooring line tension spectral density increases with water depth up to 1500m water depth and decreases slightly in 3000m water depth. Current effect has an impact on mooring line LF response, but results of
mooring line LF response spectrum does not vary significantly by using coupled LF and fully coupled method. Unlike mooring line LF response, there is a significance difference between WF response predicted by coupled LF and fully coupled method. For water depth larger than 1500m, peak value of mooring line WF spectrum predicting by coupled LF method is much smaller than the counterpart predicted by fully coupled method.

6. DISCUSSION

This paper has examined and quantified the accuracy of the coupled LF method, applied to a Spar platform, by comparing against the fully coupled method. Low and Langley (2006) studied a spread-moored vessel in water depths of 200m and 2000m, showing that the fully coupled frequency domain method generated results as accurate as the fully coupled time domain method, provided the geometric nonlinearity of the mooring line is small. This nonlinearity depends on the motion response, the length of mooring line/water depth and the time-dependency of the mooring line shape. When the geometric nonlinearity becomes important, the hybrid method (which simulates the WF motion response in the frequency domain and the LF response in the time domain) provides an accurate and efficient solution. The coupling effects between the WF response of the floating body and mooring line are captured by a linearized mooring line drag model. The hybrid method is almost as accurate as the fully coupled time domain, but requires only one-tenth of the run time (Low and Langley, 2008).

However, in the present study for a Spar platform, the geometric nonlinearity is small, leading to a difference between the coupled LF method and fully coupled method, especially for the WF pitch motion. The difference in WF surge motion is less significant than pitch, as the total surge motion response is determined by LF response. Two coupling effects (Low and Langley, 2008) contribute to this difference. One is the coupling effect between the floating body and mooring line (type 1 coupling) in the WF range - the WF response was calculated in the frequency domain without mooring line. The second is the effect between the WF, LF and mean position of the floating body (type 2 coupling) - the interpolation of floating body WF motion requires the instantaneous position of the LF response. The WF motion response is inertia dominated and it is often considered that the interaction between the floating body and mooring line is weak. However, as can be seen from the present comparisons (see Figs 15.1 and 15.2), the discrepancy is due to
the interaction between mooring line and WF motion; the coupled WF method refers to a fully coupled method described in section 3.5 but only includes first-order wave forces. This suggests that when the coupling effects are large, the fully coupled method is required, even for small geometry nonlinearity, unless the contributions from mooring lines in the WF range can be accounted for.

There were notable jumps in the surge and pitch motion response spectra and in the mooring line tension spectrum in the frequency range 0.05Hz-0.08Hz. Two reasons contribute for the jumps. One is the viscous damping effect, as can been seen from Fig. 16. Figures 16 show motion response and line tension response spectrum using different linear damping coefficients (i.e. equation (5)). When the linear damping force increases, the peak value of WF response decreases. The relative difference between the jumps in this frequency range and the peak values of WF response also decrease with increased damping. However, as the surge motion is totally determined by LF response, the peaks in the WF parts of motion spectrum are of less importance compared with those of the pitch motion. A second important feature is the wave exciting force, especially for surge and pitch (Figs. 17): the WF motion spectra for surge and pitch have their peak values at 0.09Hz (which is the peak value of the incident wave frequency), but the corresponding wave excitation force has its peak value around 0.07Hz; this contribute to the ‘jumps’ in the motion spectra. The mooring line tension response, shows a similar trend to the surge motion, and it also has variations around 0.08Hz - the line tension response is highly influenced by the instantaneous surge motion.

7. CONCLUSIONS AND GUIDANCE

In this paper, two types of mooring system design methods (coupled LF, fully-coupled) are assessed for water depths between ~300m and 3000m. Statistical results from the fully-coupled method are compared against available numerical and experimental results for validation purposes. The main results and recommendations from parametric studies indicate:

- Both mooring line tension and surge response are completely determined by the LF response, particularly for large water depths. The WF surge response is independent of water depth, in contrast to the LF surge response. Water depth variation has little effect on mean heave motion, even at resonance.
For a catenary chain, the WF response becomes more significant as the water depth increases (see Figs. 10.1~10.3). However, for a taut mooring line, the WF response is significant in intermediate water depths (between 1500m and 2000m), becoming less important in ultra-deep depths (> 2000m).

The static method is recommended for evaluating the mean motion response - it has almost the same accuracy as the coupled LF method, but is less time consuming. The coupled LF method is also recommended for predicting the WF response - the total surge motion response is dominated by the LF response.

Estimation of the mean and maximum values of the heave response and line tension can be determined by the static method when the heave natural period is far from incident wave period. For the maximum values of the surge and pitch responses, a fully coupled analysis is strongly recommended.

The coupled LF method is recommended for predicting mooring line LF response, especially for the wave-only condition, regardless of the mooring line configuration. However, for the mooring line WF response, the fully coupled method is recommended for slack chains in large water depth (e.g. larger than 600m).

The accuracy of the WF motion response predicted by the coupled LF method not only depends on the geometric nonlinearity, but also on the coupling effects - the coupling between WF, LF and mean drift motion response and the coupling between the floating body and mooring line in the WF range.

Acknowledgements

This research was supported by the Department of Naval Architecture, Ocean and Marine Engineering and the Faculty of Engineering at the University of Strathclyde; and by the China Scholarship Council (CSC) (Grant No. 2011606021).

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