Monte Carlo Simulation of Supply and Demand for Payload Limited Routes

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Abstract: Large commercial aircraft by design are typically not capable of transporting maximum fuel capacity and maximum payload simultaneously. Beyond the maximum payload range, fuel requirements reduce payload capability. Varying environmental conditions further impact payload capability noticeably. An airline's commercial department requires prior knowledge of any payload restrictions, to restrict booking levels accordingly. Current forecasting approaches use monthly average performance, at, typically, the 85% probability level, to determine such payload capability. Such an approach can be overly restrictive in an industry where yields are marginal, resulting in sellable seats remaining empty. Monte Carlo simulation principles were applied to model the variance in environmental conditions, as well as in the expected payload demand. The resulting forecasting model allows the risk of demand exceeding supply to be assessed continually. Payload restrictions can then be imposed accordingly, to reduce the risk of demand exceeding supply to a required risk level.

Additional keywords: Fuel, payload, forecasting, performance, environment.

1 Introduction

Large commercial aircraft design requires compromise to contain operating and capital costs, whilst providing performance capability that accommodates the requirements of the majority of intended customers. One such compromise is the trade-off between range capability and payload capability: A large commercial aircraft, when lifting maximum fuel capacity, is unable to carry maximum payload simultaneously and vice versa.

Airlines operating aircraft on routes longer than the design range for maximum payload capability therefore seek to maximise their sellable payload capacity on each flight. Continually varying environmental conditions challenge the performance analysts to provide accurate payload capability predictions for such routes. The airline's revenue team, however, needs to know months in advance how many seats are sellable to potential customers. The risk of flying with empty seats unnecessarily is as taxing to the airline as is the risk of denied boarding and dissatisfied customers.

Traditional approaches to this conundrum apply monthly average environmental conditions at a predetermined probability level, typically at 85%. Annual payload memoranda, depicting predicted monthly load capabilities, are published twice a year. The intent is to ensure that the predicted payload capability is equal to or better than published, at the predetermined probability level. Such an approach does not minimise the inherent risk, though, of flying empty seats nor of having to turn passengers away on any particular day of operation. Nor can a monthly average prediction really be deemed representative of continually varying environmental conditions.

Currently, to establish the payload memoranda, operational flight plans are calculated by commercially available flight planning systems, utilizing the monthly average temperature and wind profiles, at the predetermined probability level. Then, the payload capability is calculated manually, given the fuel requirements per flight. Clearly, the approach and methodology are rudimentary and far from optimal. Nor is the process dynamic. The aim, therefore, is to establish an improved dynamic forecasting methodology that minimises the risk of unfilled seats, respectively of denied boardings.

2 Background

The advances in computational methods concomitant with increased computational power, allow for the modelling and simulation of increasingly detailed aircraft components up to even complete fully configured aircraft behaviour. Filippone [5] found that such advances have not been fully integrated into the Flight Performance discipline seeking to support aircraft in service. Rather, perhaps resulting from aircraft technical data seldom being available in professional journals, the multi-disciplinary analysis of the in-flight performance of in-service aircraft still suffers from over-simplifications and closed-form solutions developed in the 1970s.

Where aerodynamics and propulsion are in themselves advanced disciplines capable of providing accurate predictions, flight performance is not, relying instead on empirical flight data, as far as available, for performance predictions. Fundamentally, flight planning is performed by utilizing an incremental table look-up routine that provides for typical flight profiles.

3 The Payload Range Trade-Off

Ackert [1] reflected on how an understanding of the payload range capability assists both operators and financiers in matching the intended airline network with the optimum payload range of the aircraft to be deployed. With the growth in air transport requirements operating within constrained air traffic structures, ultrahigh capacity aircraft are increasingly becoming necessary. Martinez-Val *et al*. [8] found that, within the current design capabilities, the most constraining factors to producing such aircraft are the wingspan limit imposed by on-airport manoeuvrability and the wing loading resulting from maximum zero fuel weight (maximum payload). Depending on the span wise position of fuel tanks

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and the wing structure arrangement, bending moments at maximum zero fuel weight can become limiting, even below maximum take-off weight.

Fuel capacity is primarily constrained by the available space within the wing structure (other than auxiliary tanks in the tail) which in turn is constrained by airport limitations. The combination of these two factors therefore affects the payload range capability of any large aircraft.

Within this context, Martinez-Val *et al*. [7] traced the development of civilian jet transport aircraft with reference to the payload range diagram, as representative of the range equation: Jet transport aircraft range capability increased from around 20,000 km in the 1970s (L1011, DC10-40, 747-200) to over 30,000 km by 1995 onwards (777-200, A340-600, A380-800). Martinez-Val *et al*. [7] established that, in addition to the constant trend in improved performance with time, wide body (long range) aircraft types further added to performance improvements through size.

Mostly though, civil transport aircraft are operated well inside their payload range capability, implying that airlines are incurring extra costs, operating aircraft oversized in payload and / or fuel tank capacity. The difficulty here is that the payload range requirements vary vastly between differing air transport organisations, whilst the developmental costs of a transport aircraft prohibit the development of a multitude of aircraft with differing payload range capabilities. Conversely, when aircraft are operated at payload range limits, such operations necessarily require an optimised operation.

Figure 1 graphically illustrates the limits affecting payload as a function of range: A full load of revenue generating load (passengers and / or cargo) can only, subject to winds and other atmospheric conditions, be carried so far. The aircraft is operated at maximum structural payload capability until the combination of fuel and payload carried reaches maximum take-off limit. Further destinations can only be served with a reduced payload, limited either by the maximum take-off weight (MTOW), trading off fuel required against payload carried, or by tank capacity limits beyond that. The ultimate limit in range is the ferry range capability, positioning an aircraft to a different destination without any payload.

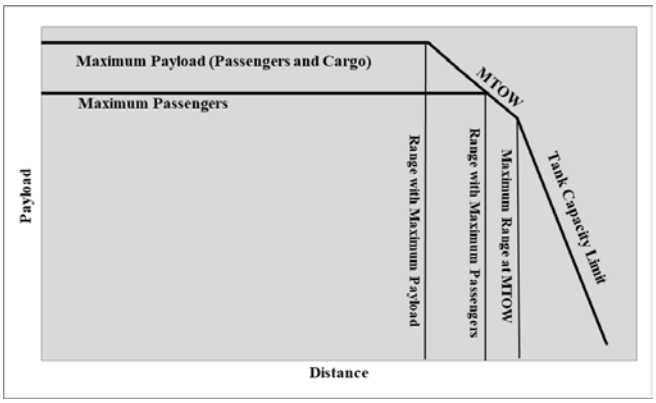

Figure 1 Payload Range Diagram

4 The Payload Memoranda

Whilst last minute travel does occur, typically either for urgent business or as a result of last minute holiday package offers, this is not the norm at regular schedule airlines. Low cost carriers have a different business model and are currently only found in the short to medium range market segments.

Rather, given the cost, effort and distance of travel involved, passengers tend to book well in advance. This creates a number of challenges for the air carriers. An aircraft seat is a perishable commodity: once flown it cannot be recovered. Airlines therefore seek to fill all seats on the aircraft for each flight.

Concurrently, airlines wish to maximise their revenue, e.g. the perishable commodity necessitates being priced "just right", by pricing competitively but with due consideration of whether a market is under capacity, over capacity or balanced in supply and demand [4]. Airlines consequently utilize yield management systems to control demand through differential pricing.

With yield management systems, pricing can be differential on a route depending on time of day (where there are multiple flights per day), day of week and season, to match supply and demand optimally. Supply, however, is only adjustable in discrete batches, number of seats per aircraft. Where demand is hugely variable, airlines can adjust using different types of aircraft. On long and very long routes, such flexibility diminishes, constrained by the range capability of the aircraft types available. On ultra-long range flights only one aircraft type in the fleet might be capable of servicing a particular route. Flexibility then only exists around the number of flights per day / week to match demand.

Complicating matters further are routes that are at the payload range limits of the aircraft types available. Such additional constraints are typically seasonally variable. When flying westbound into the globally prevailing westerly wind system around the numerous permanent westerly jet streams, the payload range capability can be impacted noticeably. In order to utilize the yield management systems effectively, though, the airlines' revenue departments require, well in advance, the number of sellable seats for each flight. On regular scheduled air carriers, flights are available for booking up to a year in advance.

The flight performance department of the air carrier therefore regularly produces payload memoranda, estimating the likely capability of each route over a set period. How this is achieved varies between airlines. Typically, these payload memoranda get calculated bi-annually, analysing flight capabilities on a monthly basis over the review period, based on expected average conditions at a pre-set probability level. A higher probability of conditions being at or better than forecast reduces the risk of denied boarding of an overbooked flight, but increases the airline's risk of flying empty seats on the day of operation.

5 Accuracy of Current Forecasting Methodology

To validate the necessity for an improved forecasting tool for route performance, a comparison is needed of the current forecasting methodology to actual flight plans. Over the period 1st September 2015 to 31st August 2017, 724 actual flights were conducted between Johannesburg and New York on an A340-600, out of a possible 731. Seven flights did not

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operate due to either technical defects or extreme weather conditions [9].

Operational flight plans, optimized for best economic speed and flight level according to environmental conditions were reviewed [2,3]. During the analysis, the correlation coefficient (R-Squared) between trip fuel requirements and the average wind component for the route was found to be in excess of 99% for the A340-600 operated at Maximum Take-off Weight (MTOW). Route average wind component therefore presents an excellent predictor of fuel requirements, negating the need to calculate large numbers of complete operational flight plans. Figure 2 therefore plots the actual average wind component on a day of operation, obtained from the operational flight plans, versus the monthly average wind component at the 85% probability level (as used for forecasting). Average headwind component is shown as positive.

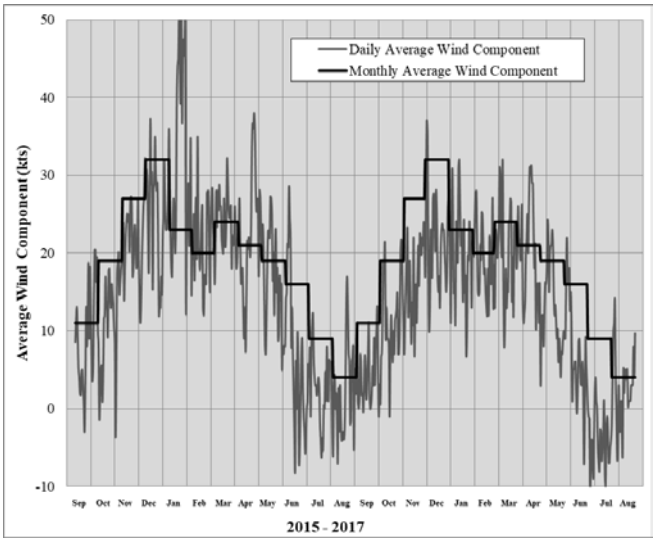

Figure 2 Daily Average Wind Components 2015 to 2017 [9]

From the fuel requirements the payload capability is calculated. Whenever a full passenger load cannot be carried, the number of sellable seats is restricted in the airline's flight reservation system. Cargo carrying capability does not get considered in this instance, as passenger carrying capability is prioritised.

The difference between actual and predicted trip fuel requirements is presented in figure 3, positive results indicating higher actual fuel requirements than predicted. The 85% probability level is presented in figure 3 by the difference being 0%. The 724 data points yielded an average difference of predicted versus actual trip fuel requirements of -0.327% with a standard deviation of 0.526%, with the data testing positively for being normally distributed.

The predicted values are those based on 85% probability wind component level whilst the observed values vary around the mean (50% probability level). On occasion some extreme outliers are observable (December 2015 and January 2016) attributable to the unusually low / high headwind components in December 2015 / January 2016 respectively, as observed in figure 2.

Comparing the 85% predictions to the actual fuel requirements at MTOW over the period September 2015 to August 2017 yields an R Squared of 35.0%. 65% of the variance remains unexplained by the current predictive methodology. Consequently, with payload capability potentially being restricted by fuel requirements, the risk of overstating or understating payload capability predictions remains significantly onerous to the airline.

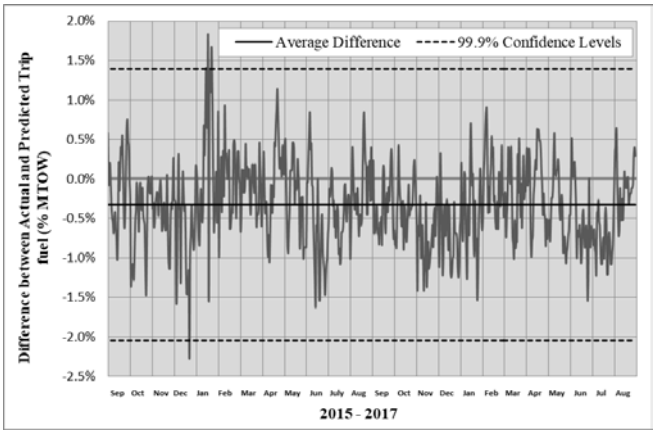

Figure 3 Difference between Actual and Predicted Trip Fuel at MTOW [9]

6 Effect on Passenger Load Carrying Capabilities

With the predictive methodology focused on fuel requirements but the application thereof ultimately being on forecasting passenger load capability, it cannot be inferred that, in 85% of cases passenger load capability is understated. The number of passenger seats on an aircraft is finite and static. Consequently, where theoretical predicted passenger capacity exceeds 100% the over- or underestimation becomes diminished or even inconsequential. (Rather, the extra payload capacity above a full passenger load enables the carrying of ad hoc cargo. Ad hoc cargo demand typically arises at short notice rather than being booked months in advance.)

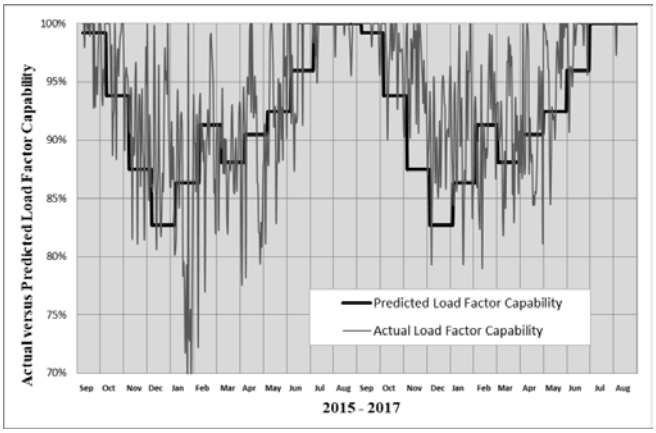

Figure 4 Actual and Predicted Passenger Load Factor Capability [9]

For a given average wind component the trip fuel requirements can be calculated. With the trip fuel plus fuel reserve requirements and the operating empty weight (OEW) of the aircraft subtracted from the maximum take-off weight

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(MTOW), the payload capability can be determined. With average passenger weights known, the predicted versus actual passenger load factor capability (percentage of available seats) is assessed on any given day, as shown in figure 4.

Figure 5 then shows the potential for denied boarding respectively for flying with empty seats unnecessarily. When predictions are under- or overstated, the magnitudes of such events are often significant, ranging from as much as potentially 18.1% of passengers denied boarding to possibly 17.3% seats remaining unsold. A 1% difference in fuel burn magnifies into approximately a 10% difference in load factor capability.

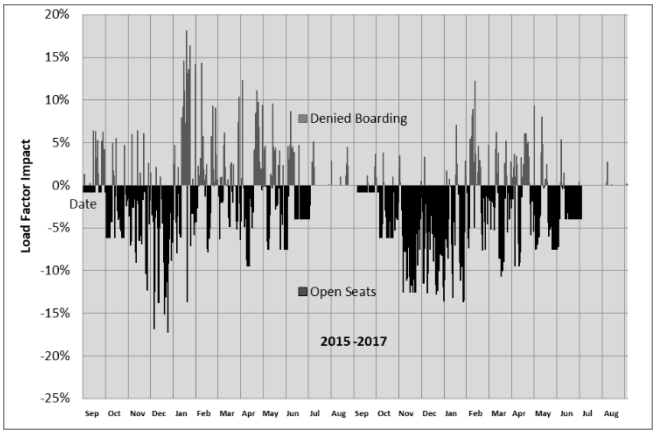

Figure 5 Potential for Denied Boarding and for Seats Available but not Sold as a Result of the 85% Probability Monthly Prediction [9].

7 Monte Carlo Simulation

Monte Carlo simulations tend to follow the following pattern [6]:

1. Define a domain of possible inputs: Here, the predominantly deterministic input variable is the average headwind component along any given route.
2. Generate inputs randomly from a probability distribution over the domain: With the mean and standard deviation known, a random number generator, the essence of any Monte Carlo simulation, then provides a probability distribution from sufficient number of iterations.
3. Perform a deterministic computation on the inputs: For any / every chosen probability level, the fuel requirements, and hence the payload capability can be determined.
4. Aggregate the results of the individual computations into the final result: The daily payload capabilities then aggregate at any desired probability level into payload memoranda for an airline to determine the number of sellable tickets for the next forecast period.

Numerous Monte Carlo Simulation packages are commercially available. Here, Microsoft Excel was used to create the Monte Carlo model. A spreadsheet was designed to predict trip fuel requirements and payload capability for a twelve months period, from a selected starting date. The user is able to choose the required probability level of achieving the predicted performance, or better. Ideally, the required probability level is a considered balance between the risks / costs of flying empty seats versus off-loading overbooked passengers. At the pre-determined probability level, the spreadsheet provides both the expected trip fuel value as a percentage of maximum take-off weight, and the predicted bookable number of passengers.

Further, the user can assess the prospect of the availability of a desired load factor, including the likelihood of being able to sell all seats on the aircraft (100% load factor). The Aircraft Operator needs to be able to achieve a minimum average load factor annually to operate any route profitably.

For instance, entering the break-even load factor on the spreadsheet determines the daily probability level of having the performance capability to carry the break-even load. There may well be days where it may not be guaranteed that a flight can be operated profitably. However, a tool of this nature allows one to assess the risk of the operation over the period of a financial year, when combined with expected load factors (passenger loads). Invariably, such a spreadsheet is highly and easily adaptive to specific needs.

8 Comparison of the Monte Carlo Simulation versus Current Forecasting Approaches

Current forecasting approaches use monthly average winds at a chosen probability level, to predict trip fuel requirements, or better (less), to derive payload capability. Typically, an 85% probability level is chosen, although this may be overly conservative. An immediately apparent shortcoming is that such an approach utilizes the same average wind component for the entire month, followed by a noticeable step change for the next month.

The use of probability as high as 85% partly disguises the reduction in the intended conservative view of this current methodology, but can also partly result in potentially unnecessarily onerous results. The actual probability of the average wind component being the monthly average figure applied (or less) can potentially vary between 65% and 95%. Figure 6 reflects the Monte Carlo simulation of the 85% probability winds, compared to the monthly average wind components, at 85% probability level.

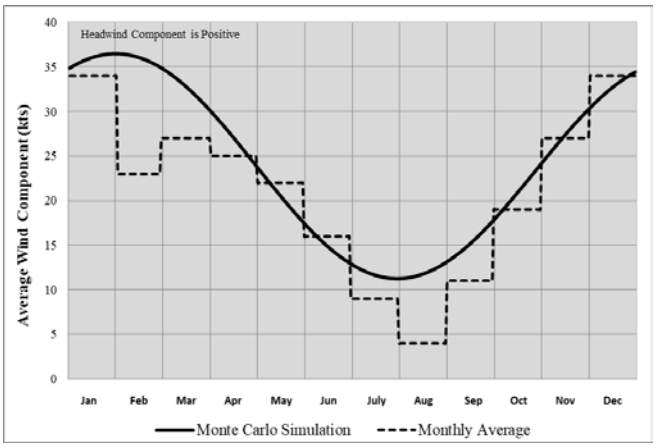

Figure 6 85% Probability Wind Predictions [9]

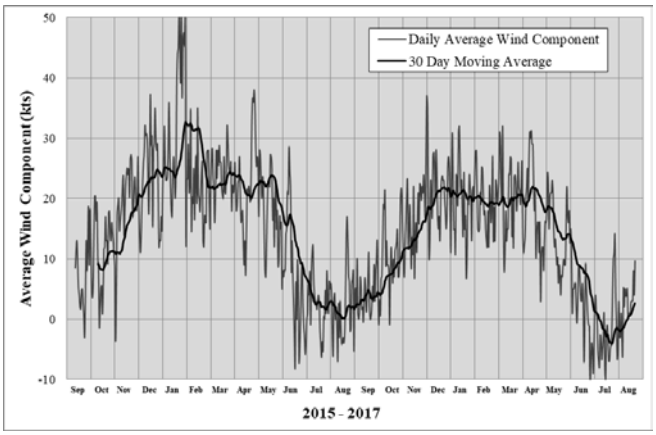

Figure 7 Daily Average Wind Component Variations [9]

Very noticeable are the seeming "anomalies" in the wind patterns for February and August. This raises the question of whether the modelling is sufficiently representative. Figure 7 reproduces figure 2, with a 30-day moving average trend line added.

The 30-day moving average trend line suggests that January and February 2016 indeed experienced unusual average headwind components relative to the surrounding months. This effect is not repeated the following year, 2017, although it can be argued that the whole Northern Hemisphere winter period experienced milder headwind conditions for that season. The August "trough", seen in the monthly-average-wind-component curve in figure 2, is not reflected in the moving average trend line in Figure 7 for 2016 or for 2017.

Clearly, the anomalies seen in figure 6 in the monthly average wind components are not recurring annual events. Rather, they appear to be short-term variations from the long-term trend of the annually cyclical nature of weather patterns. Consequently, a significant shortcoming in the current methodology exists: Using the monthly average wind component experienced during the preceding year can distort the predictions for the following year, when unusual short-term variations occur.

As a result booking levels might become more restricted than necessary resulting in excessive empty seats on the day of operation. Alternatively, the aircraft might end up overbooked requiring denied boardings.

9 Monte Carlo Simulation of the Load Factors

Passenger demand can vary by day of week and by month of year. The commercial departments of airlines make use of commercially available yield management systems to forecast passenger loads on a daily basis taking into account such variability [4]. The purpose here is not to duplicate or replace such yield management system functionality, although the Monte Carlo simulation methodology might well be of value to such underlying analyses. Rather, the purpose here is to assess the interactivity between capability and demand with due regard for variability in both.

Figure 8 plots actual load factors from December 2016 to January 2018, inclusive. Additionally, the Monte Carlo simulation predictions for 2017 of load factor capability are presented for 2017.

Interestingly, only 16% of flights had a load factor better than predicted at the 85% probability level. This compares favorably to the contemporary methodology of using monthly average wind components, where 27% of flights achieved load factors better than predicted capability. Consequently, the risk of flying empty seats unnecessarily is reduced through the simulation depicted in figure 8.

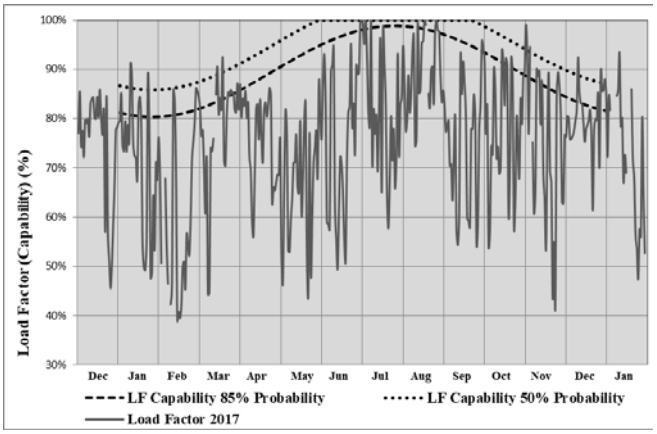

Figure 8: Actual Load Factors versus Monte Carlo Predicted Capability 2017 [9]

In figure 9 the 30-day moving average trend line is added to the actual load factor graph. Seasonal effects are evident: Peaks in passenger loads are seen over the Easter and Christmas holiday periods. The Northern Hemisphere summer holiday period is reflected in the passenger load peaks of July and August.

With the variation around the seasonal pattern, the 30-day moving average curve, known from the actual load factors for the period, a Monte Carlo simulation again becomes possible. The resulting Excel spreadsheet is similar in design to the one described above. Figure 10 portrays, at the 50% and 85% probability levels, the required load factors and the load factor capabilities from the respective Monte Carlo simulations, for 2017.

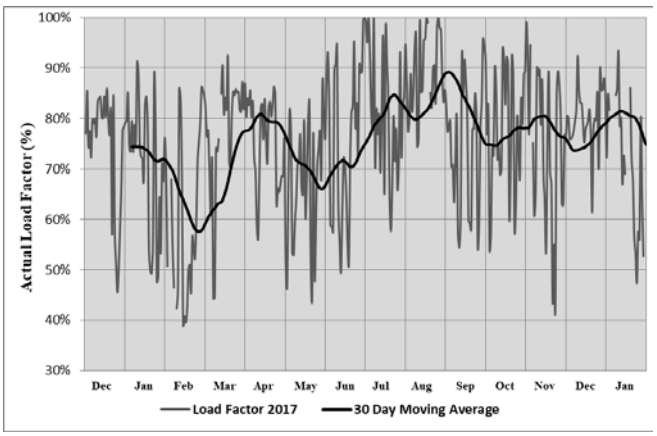

Figure 9 Actual Load Factors 2017 Flight [9]

In this instance, up to 19% of flights are predicted to require a load factor greater than predicted available, at the 85% level both for the load demand and supply capability

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predictions. It must be emphasised that the requirement prediction is for 85% or less demand, whilst the capability is for 85% or more supply. As evident from Figure 10, the risk of demand exceeding supply is predominant only during the peak periods, most noticeably over December / January, when winds are the most adverse.

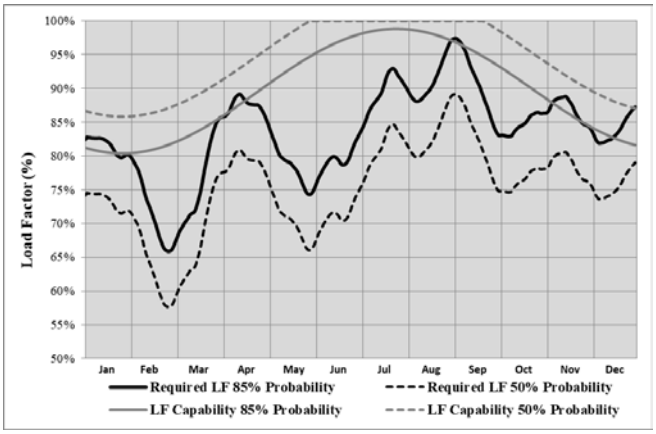

Figure 10 Monte Carlo Load Factor Predictions 2017 [9]

Up to this point, the approach has been to assess the 85% probability wind levels. Figure 10 now suggests that a different approach to dealing with the demand versus supply challenge would be more prudent. With Monte Carlo simulations established for both the supply and demand distributions, it becomes practical to assess the risk factor of demand exceeding supply on a continual basis. Payload restrictions based on risk appetite can then be imposed individually per flight, rather than generically per month, as done historically.

10 The Risk of Demand Exceeding Supply

Figure 11 reveals that the risk of demand exceeding supply only surpasses the 15% probability level over December.

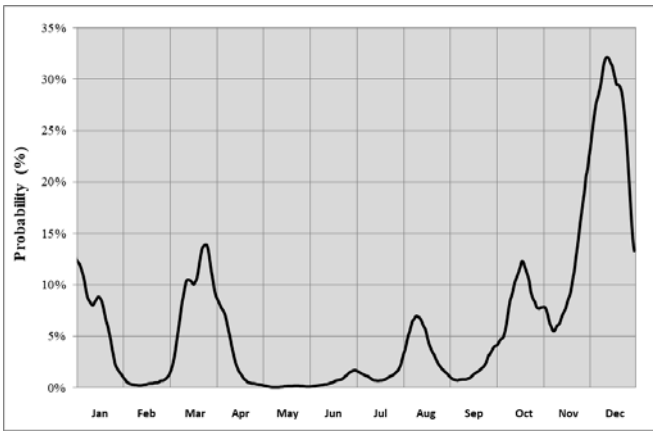

Figure 11 Risk of Demand Exceeding Supply, 2017 [9]

However, from Figure 10, the December period does not reflect the highest demand levels, but does fall within the period where winds are least favourable. Invariably, the pattern in figure 11 differs from that of Figure 10, as figure 11 combines load demand with load supply probability distributions, taking into account that the load factor cannot exceed 100%.

In combination, however, the December period presents the greatest risk by far of demand exceeding supply. For this period, imposing payload restrictions, reducing the number of sellable seats, is certainly warranted to contain the risk. Conversely, though, for the remainder of the period under review, the risk of demand exceeding supply remains well below the 15% risk level, prior to any payload restrictions having been imposed.

11 Load Factor Restrictions based on Risk of Denied Boarding

Using Monte Carlo simulation of the payload demand and supply probability distributions it becomes possible to determine the required payload restriction to achieve a preferred risk profile. The preferred risk profile is presented by the associated load factor restriction, of not having to off load passengers. Figure 12 portrays such required payload restrictions (load factors) at four different risk levels. The lower the desired risk level is, the more likely it will be that a payload restriction will be required. The respective graphs are more angular since payload restrictions can only be done in discrete units (seats), even if expressed as a percentage of total seats on the aircraft.

As already predicted by Figure 10, payload restrictions need only be considered during the various peak periods. Depending on risk appetite, the required restrictions are minimal, with the notable exception over the December period. At the 15% risk level, only the December period requires some intervention. Further, the predicted payload capability for 85% probability winds (or better) is presented in Figure 12. Noticeably, using 85% probability winds to predict payload capability remains highly conservative in this instance. The probability of denied boarding (demand exceeding supply) is mostly far less than 1% except for the July / August period, where the probability touches 2% at peak demand.

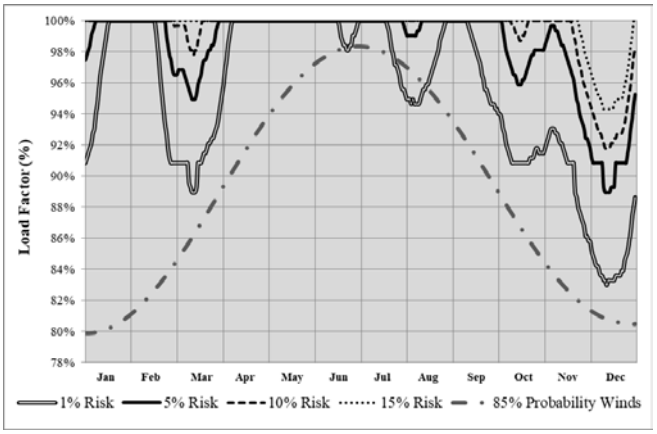

Figure 12 Required Payload Restriction at the Selected Risk Level [9]

Figure 13 repeats Figure 12 but includes a 5% increase in demand, approximating one to two year's market growth. To retain the selected risk levels of denied boarding requires more restrictive load factor limitations, as expected.

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Nonetheless, the load factor limitations based on 85% probability winds remains highly conservative with mostly around 1% risk of denied boarding, except for the period July / August. Here, the risk of denied boarding can be as high as 5% at peak time. Nevertheless, the winds for the July / August period are the most favourable, permitting full passenger capability more often than not.

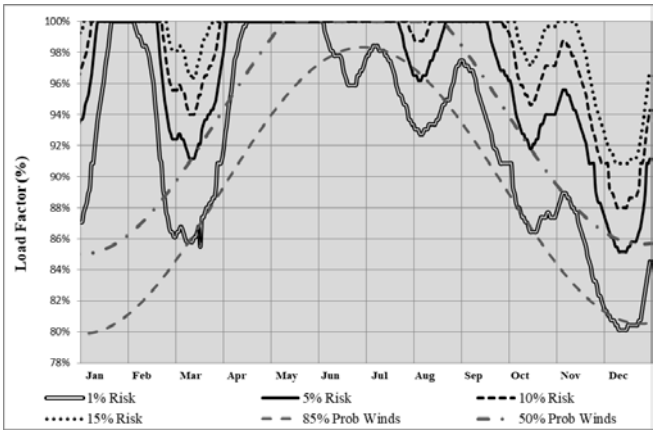

Figure 13 Required Payload Restriction at the Selected Risk Level, with 5% Market Growth [9]

Accordingly, in the absence of availability of utilizing payload demand predictions, restricting payload on the basis of only the predicted wind components at 85% probability level, remains an overly conservative methodology for denied boarding risk management. Figure 10 would suggest that using the average wind component (50% probability) remains sufficiently conservative, even with near term growth above current load demand levels. Only the August peak period would then be exposed to a 10% (current load demand) to 15% (with growth) risk of denied boarding, only when the aircraft is not able to carry 100% passenger load.

In all these graphs depicting load factor on the vertical axis, a 100% load factor remains the maximum achievable. At a desired probability level, payload capability may well be higher than 100% passenger load during periods throughout the year, implying that additionally cargo could be carried. Since this is cargo capability is not available year round, such cargo would be ad hoc, typically at short notice, and thus does not distract from this study: The focus remains on passenger load capability, which cannot exceed 100%, as there are only a finite number of seats that can be filled on the aircraft. Therefore, the graphs are shown capped at 100% load factor.

12 The Probability of Flying Empty Seats Unnecessarily

Reducing the risk of denied boarding, becoming more restrictive in the number of seats made available, invariably increases the risk of flying empty seats that could have been filled. Potential revenue is not realized. Invariably, there is a balance depending on risk appetite.

As expected, Figure 14 shows the risk of sellable seats being blocked to be higher during the peak periods. Further, the risk of revenue not realized increases as the risk of denied boarding decreases, with the 85% probability winds based payload restrictions being most onerous. An approximate balance exists between the 5% off load risk profile, respectively the 50% probability wind profile with respect to the risk of flying empty seats unnecessarily. The risks of offload approximately match the risks of revenue not realized.

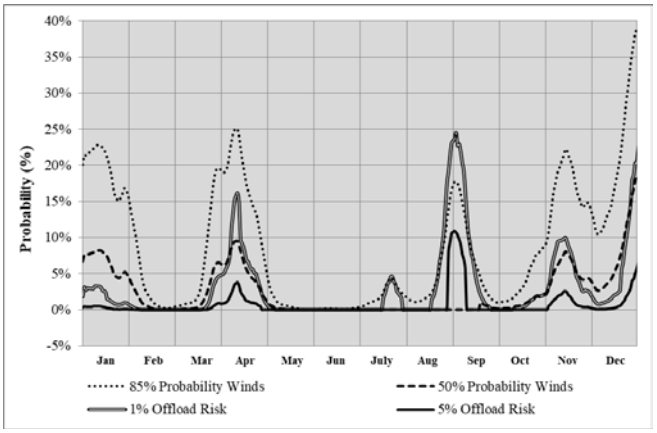

Figure 14 Risk of Revenue not Realized [9]

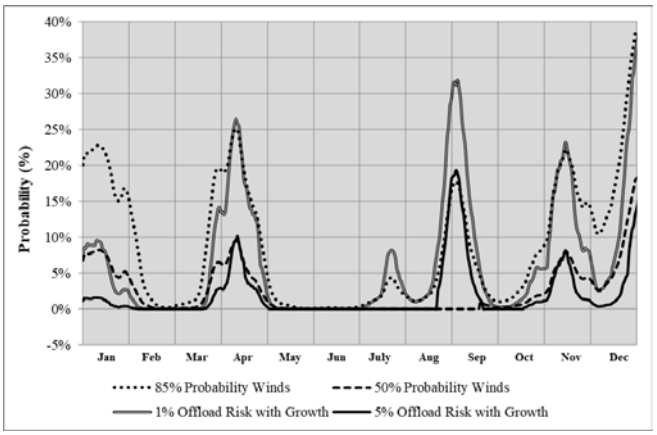

Figure 15 Risk of Revenue not Realized, with 5% Market Growth [9]

As before, figure 15 in turn considers the effect of a 5% market growth. The 5% offload risk profile now moves closer to the 50% probability winds profile. Figure 12 to figure 15 suggest that, in the absence of load demand predictions, the 50% probability winds should be used to restrict the load factors, largely containing the risks of offloads to within 5% and the risk of revenue not realized to within 10%.

13 Conclusion

Current forecasting approaches use monthly average winds at a chosen probability level, to predict trip fuel requirements, or better (less), to derive payload capability. An immediately apparent shortcoming is that such an approach utilizes the same average wind component for the entire month, followed by a noticeable step change for the next month. The required probability level is predetermined and thus fixed. Typically, an 85% probability level is chosen. As such, the current methodology is onerously restrictive with numerous flights operating with open seats that were in fact sellable.

Having established normality of the variance of the independent variable, the average headwind component was then modelled using random number generation, the Monte Carlo simulation. A predetermined probability level

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reflecting risk appetite is no longer required. Rather, the desired probability levels can now be applied as a variable. Further, along the same principles, historic (or projected) demand can be modelled simultaneously with predicted average wind components, determining supply, again using Monte Carlo simulation. Seasonal variations in average wind component with the annual cyclical weather patterns and climate changes can now be compared to the unrelated daily, weekly and seasonal variations in passenger travel patterns. Supply and demand can now be compared and, where necessary, matched.

With Monte Carlo simulations established for both the supply and demand distributions, it becomes practical to assess the actual risk factor of demand exceeding supply on a daily basis, rather than predetermining probability levels. Payload restrictions based on risk appetite can then be imposed individually per flight post analysis, rather than predetermined generically per month, as done historically. By managing supply versus demand on a daily basis, the risk of flying empty seats unnecessarily, respectively the risk of denied boarding can be minimized. Further, predicted growth in demand can easily be incorporated into the forecasting process.

Further, and more importantly, as projected demand is updated closer to time of flight, the analyses can easily be recalculated to achieve the optimum balance between available supply and required demand.

Invariably, as payload demand increases with market growth over time, the risk of denied boarding will increase, with the sinusoidal modelling containing the increased risk with market growth somewhat longer, before it could become problematic. Monte Carlo simulation contains the risk even longer. Fundamentally, through the Monte Carlo simulation capabilities, the forecasting has progressed from rudimentary and conservative supply prediction towards daily scenario planning of matching supply and demand.

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