Oleg N. Kirillov

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To the memory of my parents Alevtina and Nikolay

Preface

Hubs are special. They dominate the structure of all networks in which they are present, making them look like small worlds. While two pages on the Web are 19 clicks away, Yahoo.com, a giant hub, is reachable from most Webpages in 2 to 3 clicks. From the perspective of the hubs the world is indeed very tiny.

A.-L. Barabási, *Linked*

Almost half a century ago, the seminal book by V. V. Bolotin, *Nonconservative problems of the theory of elastic stability*, came into print [115]. It summarized the experience of engineers in the analysis of nonconservative systems accumulated since the 1920s when the first attempts to take into account dissipative effects in rotor dynamics and to explain such dangerous instabilities as aerodynamic flutter and shimmy in aircraft landing gear were undertaken. Despite the many excellent books on stability theory that were published during the next fifty years [37, 91, 185, 198, 222, 238, 256, 265, 283, 332, 366, 507, 533, 535, 537, 575, 586, 599, 618, 642, 688, 735, 762, 777, 838, 875, 932], Bolotin's monograph still remains the only one devoted solely to the methods and challenges of nonconservative stability. Its only drawback is that this book cannot take into account the dramatic developments in mathematics, engineering and physics that have been made since 1963.

The book [115] was motivated mainly by the problems of stability of slender structures under follower forces and of rotating flexible shafts. Already these models deal with the two fundamental nonconservative perturbations – viscous dissipation and nonpotential positional forces [487, 488].

Last five decades extended considerably the range of applications in which such nonconservative forces play a crucial role. We mention friction-induced instabilities causing the flipping of the tippe top [129] and the problems of the acoustics of friction related to the excitation of audible vibrations in brakes and clutches [403], paper calenders [147, 792], prostheses of hip joints [386] and even in the singing glasses of Benjamin Franklin's glass harmonica [13, 428].

Nonconservative models appear in modern studies of landslides on gentle slopes when the constitutive relation of a nonassociated geomaterial, such as loose sand, is described by a nonsymmetric matrix [90, 91, 180].

In hydrodynamics and plasma physics, the counter-intuitive destabilizing influence of dissipation on negative energy waves [177, 265, 654, 810] is an important ingredient in the theories of boundary layer [512], flow control [171, 172, 271] and stability of wave propagation [141]. In rotating fluids, interplay of the nonconservative and gyroscopic forces may lead to the paradoxical discontinuous change in the stability boundary as happens in the case of the baroclinic instability when the Ekman layer dissipation is infinitesimally small [489, 733, 828].

In magnetohydrodynamics, the Velikhov–Chandrasekhar paradox [184, 860] occurs in the theory of magnetorotational instability when for infinite electrical conduc-

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tivity of the differentially rotating fluid the limit of the vanishing axial magnetic field does not trace back to the Rayleigh threshold of hydrodynamics [5, 46, 305]. Nonconservative forces play an increasing role in celestial mechanics, e. g., in the modeling of tethered satellite systems, satellite and planetary rings [66, 67, 136].

A rich source of nonconservative problems is modern robotics and automatic control [64] and biomechanics [337] – for example, the spine is frequently modeled as a column loaded by a distributed follower force [731]. Of course, traditional areas such as aerospace engineering and structural mechanics remain one of the biggest consumers of the nonconservative stability theory [346, 367].

Already in the 1960s Bolotin emphasized that progress in the nonconservative stability theory depends on developments in the theory of nonself-adjoint operators. Important contributions to the latter motivated mainly by mechanical applications were made, e. g., in the 1940s by S. L. Sobolev [786] and L. S. Pontryagin [708], in the 1950s by M. V. Keldysh [393] and M. G. Krein [491, 492] and since then by many other authors.

The needs of optimal design and rational experiment planning required consideration of multiparameter families of nonself-adjoint boundary eigenvalue problems. In the 1970s, the studies by V. I. Arnold and his co-workers established a sharp correspondence between the multiple eigenvalues of nonsymmetric matrices and geometric singularities on the boundary of the asymptotic stability domain of a matrix family [30, 33, 543, 544]. An immediate consequence of this result is the resolution of the famous Ziegler's paradox (1952) of destabilization of a reversible system by small dissipation [928] by means of the Whitney umbrella singularity, which was done independently by a number of authors starting with O. Bottema in 1955–56 [125, 126].

Since the 1950s, the concept of the symplectic or Krein signature of eigenvalues has been widely used in hydro- and magnetohydrodynamics to describe waves of positive and negative energy [396, 473, 570, 572, 903, 904]. In the 1990s, the influence of non-Hamiltonian perturbations on the stability of Hamiltonian systems became a topic for a systematic investigation [571, 576] that gave birth to the area of research known as 'dissipation-induced instabilities' [110] – a concept that touches a broad variety of physical applications [488].

Though very excellent, a network of these results is chaotically scattered at present throughout the specialized journals. Many brilliant physical phenomena that could crown the nonconservative stability theory are almost unknown to the stability theorists with a classical mechanical background. On the other hand, achievements of the theory of nonself-adjoint operators [615, 909], the theory of operators in the spaces with indefinite metric (Krein and Pontryagin spaces) [308], Lidskii–Vishik–Lyusternik perturbation theory for multiple eigenvalues [631], theory of multiparameter eigenvalue problems [42, 869], modern results of applied linear algebra [850] as well as singularity theory appear to be still not familiar to many engineers, physicists and even stability theorists working with nonconservative stability problems.

However, the combination of these approaches of modern applied mathematics with the complex fundamental nonconservative phenomena of physics and mechanics seems to be the only way to understand the latter and to create a rather complete and unified constructive theory of stability of nonconservative systems and of dissipation-induced instabilities. That is why there is a strong need for a detailed exposition which would bring together the scattered results of the last fifty years and which would endeavor to unify and to systematize both the results and the methods of treatment of the nonconservative stability problems of modern physics. This book is an attempt to fill this need.

In it an effort is made to present the subject of nonconservative stability from the modern point of view as completely as possible within the allotted space. It presents relevant mathematical concepts, both already familiar and the new ones for this subject as well as rigorous stability results and numerous classical and contemporary examples from mechanics and physics. The book is substantially based on the results of the author; although by necessity it contains some results of other authors without which it is impossible to create a self-consistent exposition. It is hoped that this book will serve the present and prospective specialist in the field by acquainting him with the current state of knowledge in this actively developing field.

The book has 12 chapters. After a number of examples accompanied by a historical overview in the Introduction, the first six chapters deal with the finite dimensional nonconservative systems, while the rest of the book is dedicated to the infinite dimensional ones. Naturally, the first part of the book contains fundamentals of the theory and more general results because of the wide variety of mathematical tools available in finite dimensions. The center of gravity in the second part is shifted to studies of concrete physical problems. All chapters contain illustrative physical examples.

I would like to express my warmest gratitude to all the colleagues and collaborators whose support, friendly advice, encouraging discussions and fruitful joint research were among the main inspirational factors driving my work on this project: Abd Rahim Abu Bakar, Sergei Agafonov, Vadim Anischenko, Teodor Atanackovic, Vladimir Beletsky, Carl Bender, Michael Berry, Noël Challamel, Gengdong Cheng, Richard Cushman, Felix Darve, Barbara Dietz, Mikhail Efroimsky, Yasuhide Fukumoto, Gunter Gerbeth, Valentin Glavardanov, Yuanxian Gu, Eva-Maria Graefe, Samvel Grigorian, Uwe Günther, Peter Hagedorn, Hanns-Ludwig Harney, Daniel Hochlenert, Norbert Hoffmann, Igor Hoveijn, Wolfhard Kliem, Anthony Kounadis, Yuri Leschinski, Alexei Mailybaev, Vadim Marchenko, Jerrold Marsden, Maxim Miski-Oglu, Oliver O'Reilly, Huajiang Ouyang, Pauli Pedersen, Dmitry Pelinovsky, Alexandra Perlova, Karl Popp, Achim Richter, Ingrid Rotter, Florian Schäfer, Guido Schneider, Alexander Seyranian, Sergei Sorokin, Gottfried Spelsberg-Korspeter, Frank Stefani, Ferdinand Verhulst, Alexander Zevin, and Miloslav Znojil.

I learned a lot from the lectures and seminars given during my studies at the Moscow Institute of Physics and Technology in 1989–1995 by Alexander Abramov, Oleg Besov, Boris Fedosov, Victor Galactionov, Victor Lidskii, Boris Rauschenbach, Victor Zhuravlev, and other brilliant professors and lecturers of my Alma Mater whose

enthusiastic work in the ‘stormy 1990s’ in Russia was an impressive example of a noble dedication to students and the profession that deeply influenced my formation as a scientist.

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I am indebted to my wife Ksenia and to my daughter Marina and son Nikolay who generously gifted me time, support, and understanding.

Dresden, 12 April 2012

Preface to the second edition

Since 2013, when the first edition of this book came out of print, the interpercolation of ideas between the fields of nonconservative dynamics and non-Hermitian physics only intensified. I mention excellent new reviews [40, 133, 159, 194, 199, 208, 321, 327, 330, 361, 378, 547, 578, 581, 623, 694, 776, 854, 864, 908] and books [74, 86, 92, 119, 140, 142, 203, 223, 224, 271, 320, 358, 388, 452, 511, 580, 600, 665, 680, 717, 732, 820, 868, 879, 883–885, 888] reflecting this process.

In particular, the concept of exceptional points has found applications in new areas of physics and engineering such as thermoacoustics [378, 581, 674].

Nonconservative circulatory forces (sometimes under the name of curl forces [82, 321]) being the natural component of the radiation pressure force of light [821] are discussed more and more in the publications related to optical tweezers [41, 304, 318, 893], otpomechanics [29], and light robotics [699, 827].

The concept of the nonconservative follower force has recently been realized in a series of controlled mechanical experiments [92–96, 178, 820] that also confirmed the Ziegler–Bottema destabilization paradox [93, 95]. Nowadays follower forces find new application in the modeling of subcellular structures (organelles) such as celia and flagella. The latter are made up of microtubules that can exhibit flutter and induce a wave-like propulsion [60, 229]. In its turn, this phenomenon inspires biomimetic design of microrobots exploiting electrohydrodynamic instabilities to reproduce the natural propulsion mechanism [920, 921].

A remarkable discovery of the last decade is the link between the phenomenon of stability loss delay in the systems with adiabatically slow variation of parameters (existing, e. g., in the Ziegler pendulum [650]) and nonadiabatic transitions accompanying encircling of exceptional points in non-Hermitian systems with gain and loss [622] that originated a full new line of research in non-Hermitian physics [263].

Diabolical and exceptional points re-appear in the emerging field of topological mechanics [361, 562, 628, 825, 826, 878, 908] that engineers chiral mechanical metamaterials [151, 648, 659] using the concept of gyroelastic continua [143, 898].

Recent studies of the precession of orbits of Brouwer's particle in a rotating saddle potential [447, 448] have led to an advancement in the classical averaging theory [547, 767, 900] accompanied by the discovery of the ponderomotive magnetism [216, 217].

The works [121, 482, 483, 915, 916] link $\mathcal{PT}$-symmetry, pseudo-Hermiticity, and G-Hamiltonian structure. In particular, [441] finds $\mathcal{PT}$-symmetry and the corresponding G-Hamiltonian structure in the model of the magnetized Taylor–Couette flow with finite viscosity and resistivity subject to the azimuthal magnetic field which helps to treat the azimuthal magnetorotational instability as a double-diffusive phenomenon and place it in the framework of the theory of dissipation-induced instabilities.

Finally, it is worth to mention the topic of radiation damping [788, 879] tracing back to the famous Lamb oscillator coupled to a continuum [54, 503, 669] and related

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to it phenomenon of radiation-induced instabilities [109, 331, 529] that, in its turn, goes back to the classical Chandrasekhar–Friedman–Schutz instability of Maclaurin spheroids due to radiation reaction caused by emission of gravitational waves [187–189, 257, 750, 751]. Note that radiation reaction is discussed as a nonconservative force [1]. The radiation-induced instabilities are connected to the anomalous Doppler effect [299, 300, 654] and, taken wider, to superradiance [63] that includes also Cerenkov radiation [551]. Flutter due to the anomalous Doppler effect [4, 239, 290] is a hot topic in modern aeroelasticity [499, 500, 639] and solid mechanics [619].

The present edition is revised and extended to take into account the new developments and contains more than 50 pages of new material with illustrations as well as an updated list of literature that includes now more than 900 titles.

I thank my colleagues for friendly discussions and generous collaboration during preparation of the second edition: Davide Bigoni, Alexei Borisov, Gert Botha, Michele Brun, Thomas Bridges, Olivier Doare, Yasuhide Fukumoto, Joris Labarbe, Rodrigo Ledesma-Aguilar, Mark Levi, Valerii Kozlov, Glen McHale, James McLaughlin, Ivan Menshikov, Andrei Metrikine, Diego Misseroni, Innocent Mutabazi, Giovanni Noselli, Oliver O'Reilly, Michael Overton, Andy Ruina, Frank Stefani, J. Michael T. Thompson, Mirko Tommasini, Laurette Tuckerman, John Woodward, and Rong Zou.

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Durham, 12 April 2020

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