CVaR-Based Retail Electricity Pricing in Day-ahead Scheduling of Microgrids

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Abstract

One of the most important methods for implementing the demand response (DR) schemes is to allocate the time-varying prices to the power demand by consumers. Indeed, the effective implementation of DR programs passes from a successful pricing way of retail electricity. However, the massive influence of renewable energy sources with unpredictable generation in today's power systems such as Micro-grids has made any planning encounter serious challenges. Therefore, in this paper, a new retail electricity pricing method has been proposed to reduce the effects of risk resulting from the uncertain generation of renewable energy sources and the wholesale electricity market's hourly estimated prices. To this end, a CVaR (Conditional Value at Risk) optimization framework is used to determine the next day's energy management planning of Micro-grid and retail electricity prices. Sim-

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ulation results demonstrate that the use of risk-averse conditions, in comparison with non-risk conditions, results in lessening the standard deviation of optimal retail prices and the expected cost. In more detail, the standard deviation of optimal retail prices and the expected cost is decreased by 29.94% and 24.63%, respectively. Moreover, the results show a 5.92% reduction in the peak value of demand.
Keywords: Demand Response (DR), Retail Electricity Pricing (REP), Conditional Value-At-Risk (CVaR), Energy Management (EM), Micro-grid (MG).

Nomenclature

Acronyms

ARIMA	Auto-Regressive Integrated Moving Average
ARMA	Auto-Regressive Moving Average
BDT	Benders Decomposition Technique
CVaR	Conditional Value at Risk
DG	Distributed Generation
DR	Demand Response
EM	Energy Management
ICT	Information & Communication Technology
LCOE	Levelized Cost of Energy
MG	Micro-Grid
MINLP	Mixed Integer Non-Linear Programming
PV	Photo-Voltaic
REP	Retail Electricity Pricing
RTP	Real-time pricing
RES	Renewable energy source
TOU	Time of Use
VaR	Value at Risk

Indices

t,h	Sets of hours
i,j	Sets of buses
s	Sets of scenarios
DG	Sets of dispatchable DGs
SG	Sets of stochastic DGs
BES	Sets of battery
GSP	Sets of grid supply point
D	Sets of demand (load)

Variables

M	Number of paths generated by scenario generation
X	Decision vector
Y	Stochastic vector
$M_{DG,i}^{s,t}$	binary decision variable for dispatchable DG unit start-up decision in scenario s at hour t
$N_{DG,i}^{s,t}$	binary decision variable for dispatchable DG unit shut-down decision in scenario s at hour t
$L_{DG,i}^{s,t}$	binary decision variable for dispatchable DG unit status in scenario s at hour t
$\rho_{D,i}^{s,t}$	optimal hourly prices announced to consumers in the MG in scenario s at hour t (\$/MWh)
$P_{D,i}^{s,t}$	active power demand for load at bus i in scenario s and hour t in response to optimal hourly prices (MWh)
$P_{DM,i}^{s,t}$	active power demand for load at bus i in scenario s and hour t in response to DA energy market prices (MWh)
$P_{BES,i,Ch}^{s,t}$	amount of power charged in BES in scenario s at hour t (MWh)
$P_{BES,i,Dch}^{s,t}$	amount of power discharged from BES in scenario s at hour t (MWh)
$E_{BES,i}^{s,t}$	BES level in scenario s at hour t (MWh)
$P_{DG,i}^{s,t}$	active power generation of dispatchable DG unit of bus i in scenario s at hour t (MW)
$P_{SG,i}^{s,t}$	active power generation of stochastic DG unit of bus i in scenario s at hour t (MW)
$P_{G,i}^{s,t}$	net active power injection in bus i in scenario s at hour t (MW)
$P_{GSP,i}^{s,t}$	active power exchange with upstream network of bus i in scenario s at hour t (MW)
$Q_{DG,i}^{s,t}$	reactive power generation of dispatchable DG unit of bus i in scenario s at hour t (MW)
$Q_{SG,i}^{s,t}$	reactive power generation of stochastic DG unit of bus i in scenario s at hour t (MW)

$Q^{s,t}_{GSP,i}$	reactive power exchange with upstream network of bus i in scenario s at hour t (MW)
$Q^{s,t}_{G,i}$	net reactive power injection in bus i in scenario s at hour t (MW)
$P^{s,t}_{Loss}$	total active power loss of the MG in scenario s at hour t (MW)
$Q^{s,t}_{Loss}$	total reactive power loss of the MG in scenario s at hour t (MW)
$E^{s,t}_{D,i}$	Energy demand of load of bus i in scenario s at hour t
$E^{day}_{D,i,min}$	Minimum daily energy demand of load of bus i (MWh)
$V^{s,t}_{i}$	Voltage magnitude of bus i in scenario s at hour t (kV)
ξ	Value-at-risk (VaR)
π_s	Probability of scenario s
η_s	Auxiliary variable for CVaR calculation
ω	CVaR weighting factor
β	CVaR confidence level
$\delta^{s,t}_{i}$	Voltage angel of bus i in scenario s at hour t (rad)
$S^{s,t}_{GSP,i}$	Apparent power exchange with upstream network
$S^{s,t}_{i,j}$	Apparent power flow from bus i to bus j in scenario s at hour t (MVA)

Parameters

$C^{s,t}_{SG,i}$	Cost of generated power by SG from bus i in scenario s at hour t ($/MWh)
$C^{s,t}_{DG,i}$	Cost of generated power by DG from bus i in scenario s at hour t ($/MWh)
$SUC_{DG,i}$	Start-up cost of dispatchable DG in bus i ($)
$SDC_{DG,i}$	Shut-down cost of dispatchable DG in bus i ($)
$B_{BES,i}$	Positive constant for BES cost function
$A_{BES,i}$	Positive constant for BES cost function
$C^{s,t}_{BES,i}$	Operational cost of BES ($/h)
$\theta_{i,j}$	Angles of complex admittance matrix elements
V^{max}_{i}	Maximum limit of bus voltage (kV)

V_i^{min}	Minimum limit of bus voltage (kV)
$P_{DG,i}^{max}$	Maximum active power generation og DG unit at bus i (MW)
$P_{DG,i}^{min}$	Minimum active power generation og DG unit at bus i (MW)
$S_{i,j}^{max}$	Upper limit for apparent power flow from bus i to bus j (MVA)
$S_{GSP,i}^{max}$	Maximum allowed apparent power exchange with upstream network (MWA)
$RUP_{DG,i}$	Ramp-up limit for DG unit at bus i (MW)
$RDN_{DG,i}$	Ramp-down limit for DG unit at bus i (MW)
$MUP_{DG,i}$	Minimum up time limit for DG unit (h)
$MDT_{DG,i}$	Minimum down time limit for DG unit (h)
$\rho_{D,i}^{\max}$	price cap for optimal hourly prices
ρ_{D0}	base energy price ($/MWh)
$\rho_{GSP,i}^{s,t}$	forecasted DA market price of energy in scenario s at hour t ($/MWh)
$P_{D0,i}^{t}$	initial active power demand for load at bus i in hour t (MWh)
$P_{BES,Ch}^{\max}$	minimum rate of discharge in an hour (MWh)
$P_{BES,Dch}^{\max}$	maximum rate of discharge in an hour (MWh)
$E_{BES}^{\min}$	minimum level of BES (MWh)
$E_{BES}^{\max}$	maximum level of BES (MWh)
μ_{Ch}	charging efficiency of BES
μ_{Dch}	discharging efficiency of BES
ϕ	payment factor
$Q_{D,i}^{s,t}$	reactive power demand of load at bus i in scenario s and hour t (MVAr)
$P_{D,i}^{\min}$	minimum active power demand level for load at bus i (MW)
$P_{D,i}^{\max}$	maximum active power demand level for load at bus i (MW)
$e_{D,i}^{t}, e_{D,i}^{t,h}$	self and cross price elasticity of load at bus i in hour t
Y_{ij}	magnitude of admittance matrix element

1. Introduction

Implementing Demand Response (DR) programs aim to reduce or shift the peak power demand of consumers. Indeed, the effective implementation of DR programs passes from the successful pricing path of retail electricity. The diverse structure of pricing methods has led to the various effect of each of them on the grid from different technical, economic, and social aspects [1]. Each Retail Electricity Pricing (REP) method must follow some principles. In general, the more a pricing method is in line with tariff design principles, the more effective it can be [2]. The pricing principles are usually based on cost-reimbursement while protecting the rights and interests of consumers [3].

In general, the concept of electricity pricing during peak hours has been considered by various authors for many years [4]. In reference [5], for instance, the concept of pricing mechanisms for electricity transmission has been investigated in a competitive market based on the time-varying total cost. However, in [6], the peak pricing, as well as a fundamental model, namely Time of Use (TOU), have been evaluated extensively.

The Real-Time Pricing (RTP) and applying various tariffs have been made in very short timeframes in [7]. Of course, in this pricing method, the producer's and customer's profits have not been considered. In [8], a TOU tariff plan has been applied to customers in a residential Micro-grid (MG), in order to maximize the profit of the distribution company, regardless of the uncertainty caused by distributed generations (DGs). In [9], RTP has been utilized as one of the new methods of REP in a residential MG. Of course, in this paper, the retail prices are based on the wholesale market prices and have not been considered as decision variables. In [10], a REP method has been proposed by emphasizing the increase of applied prices on consumers at peak times. However, the modification of demand profile has been achieved without prices optimization. All in all, in the abovementioned papers, the applied retail prices have not been optimized. Simultaneously, optimization can lead to a more consistent tariff plan with pricing principles and increase its efficiency. In [11], an optimal daily RTP for MG has been utilized to manage consumers' consumption. In this study, the Benders Decomposition Technique (BDT) has been used to simplify the optimization problem.

In the above mentioned valuable studies, the objective functions are applied to either maximize or minimize the MG's profit and total cost, respectively. The mutual effects between the uncertainties and optimal value of prices have not been included in planning. However, in recent years, the intermittent and unpredictable electrical energy generation has been significantly raised by penetrating renewable energy sources (RESs) such as wind and solar. Consequently, the power system has been faced with uncertainty in predicting the optimal operation and planning. In [12], the energy management of a smart building in the presence of the energy storage system in the MG is performed considering the uncertainties of the generated power from photo-voltaic (PV) and electricity sales prices to consumers. Although the impact of the uncertainties has been examined using the CVaR index, the demand response (DR) has been neglected. In [13], the impact of uncertainty derived from renewable generation and prices has been evaluated on the implementation of price-based DR programs such as TOU and RTP. Still, the CVaR index has not been used to analyze the risk of solution, and the utilized prices in DR programs have not been optimized. While optimizing these prices to increase profits and decrease the adverse effects of uncertainties can be significantly useful in implementing DR programs.

In general, risk management in electricity markets is influenced by various factors such as the structure of shiftable loads and contracts in the markets.In paper [14], the effect of risk caused by the implementation of DR programs on the actors of markets has been investigated in case studies related to Northern Europe . However, in this paper, the retail electricity pricing and popular DR programs are not examined. The paper [15] presents an optimal electricity purchase model by retailers. Although this paper makes the use of the CVaR index to examine the risk resulting from the uncertainty resources in a spot market, the implementation of DR programs and retail electricity pricing applying to consumers has not been investigated. Some recent studies has considered the CVaR risk management index in the objective function (retail electricity sales) aiming at either maximizing the profit of the utility grid [16] or optimizing the utility grid's bid prices [17]. In addition, risk management in the context of electricity markets can be caried out with the aim of benefiting other market agents such as retailers and aggregators [18]. In several papers, risk management due to various factors such as the existing uncertainty in nodal prices on the bidding strategy for each production unit has been taken into account [19]. For instance, in reference [20], an optimal bidding strategy is investigated for a MGs' producers with DR programs considering the uncertainty and failure of renewable generation units. In some recent studies such as the paper [21], a risk optimization framework and CVaR index for the energy and reserve markets with DR programs are presented. In [22], the effects of uncertainty derived from renewable energy sources and consumer load on implementation of DR programs by retailers are studies. However, in this paper, the amount of cost and risk resulting from these uncertainties are not studied by various indicators. Moreover, the effects of this risk on the optimal prices applied to consumers and MGs' profits before and after the implementation of DR programs has not been considered [22].

The purpose of this paper is to propose a new optimization framework for determining the optimal hourly prices considering uncertainties related to RES units and prices of the wholesale market, in order to enhance the effectiveness of DR programs.

Implementing the proposed optimal prices can reduce the peak of power demand and encourage customers to shift their consumption pattern from peak hours to off-peak hours. Also, all of the battery constraints, the technical constraints of the distribution grid, and the dispatchable DGs have been taken into consideration. The proposed optimization model is a Mixed Integer Non-Linear Programming (MINLP) problem, and BDT is applied to simplify its solution [23]. The significant contributions of this paper can be summarized as follows:

- Determining the optimal hourly retail prices with the aim of reducing the total costs of MG and the effect of the uncertainty caused by the generation of RES and wholesale market prices.

- Providing an optimization framework for MG day-ahead energy management based on the CVaR risk measurement index.

The remainder of this paper is organized as follows: In Section 2, the description of the proposed MG and the required information regarding the optimization framework are presented. In Section 3, the scenarios associated with the uncertainty of the RES and CVaR optimization framework are described. The problem formulation, which includes the objective function, the economic DR model, and the technical and economic constraints are specified in Section 4. In Section 5, the results obtained by the numerical studies are given. Finally, the paper is concluded in Section 6.

2. MG Concept

In this paper, the MG has the ICT (Information & Communication Technology) infrastructure of the communication system. In addition, consumers in this MG are equipped with smart meters, an energy management system, storage devices, generation resources, and flexible demand, as shown in Figure (1). The energy management system is responsible for scheduling the energy consumption based on hourly prices. This MG includes wind turbines and solar power plants as non-dispatchable units and microturbines as a dispatchable unit. The proposed MG has been designed in a way to be able to receive the estimated hourly prices from the wholesale market for energy exchanges (purchasing and selling). By an appropriate internet protocol/gateway the MG receives desired information associated with the estimated wholesale market prices, network information, and battery data. The hourly load profiles and the hourly generation of wind and solar units have been obtained based on historical data and weather forecasts. Those calculations are out of the scope of this paper. Also, the uncertainties regarding the generation of renewable units, and wholesale prices have been considered throughout the planning process. Ultimately, the amount of power generated by wind and solar units has been regarded based on, respectively, wind speed and solar irradiance. Although the maximum production capacity for these units has been considered, other constraints have been ignored.

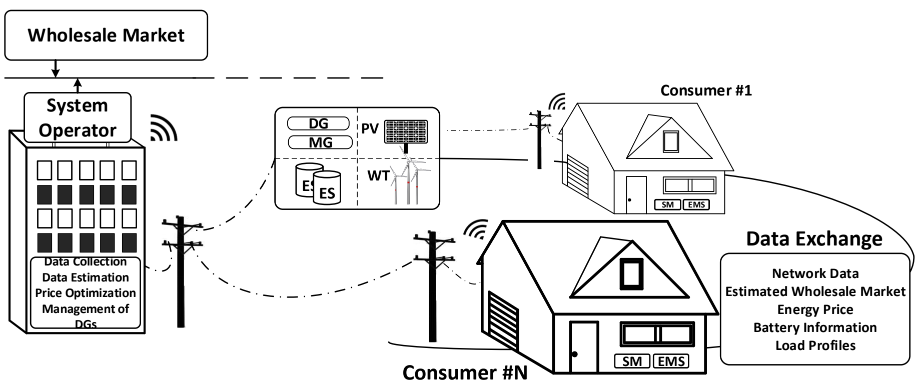

Figure 1: The conceptual model of MG

3. CVaR-based Optimization to Estimate the Uncertainty

3.1. Scenario Generation: ARMA (Auto-Regressive Moving Average) and ARIMA (Auto-Regressive Integrated Moving Average) Models

The uncertainties of RESs and the wholesale market prices make the planning and operation of power systems complex. Therefore, it is necessary to apply stochastic programming in order to model and solve problems facing uncertainty [24]. In this paper, the prices of wholesale markets, wind speed, and solar radiationare considered as uncertainty variables. Therefore, the problem's scenarios should include stochastic information about prices, wind speed, and solar radiation [25]. For this purpose, the ARIMA method is utilized for estimating the prices. Besides, to handle the uncertainty of wind speed and solar radiation, the ARMA approach is used [26]. Y's probabilistic stochastic process is defined by detecting the distribution of its random variables. This stochastic process explains both the random variables' probabilistic manner on their marginal distributions and the interrelations among whole variables. The description of distribution incorporating a set of random parameters needs complex calculations. Therefore, the random nature of realization scenarios should be finalized to cover the other data's internal relationship.

3.2. CVaR-based Risk Measurement

Assume that $f(x, y)$ be a loss function depending on a decision vector $X = (x_1, x_2, ..., x_n), x \in X \in R^n$ and a stochastic vector $Y = (y_1, y_2, ..., y_m), y \in Y \in R^m$. One of the most widely used Risk Measurement Index is VaR (Value at Risk) that is extensively used for loss function with fat tail manners [27]. According to Eq. (1), VaR has the lowest loss rate for the confidence interval β in a given time interval with the probability of (1 − β).

$$VaR_\beta = min\{\alpha \in R : P\{f(x, y) \leqslant \alpha\} \geqslant \beta\}, \quad for \;\; 0 \leqslant \beta \leqslant 1 \tag{1}$$

The main goal of the problem is to obtain a value for x, for which the cost function has its lowest value due to the stochastic value of y. Although VaR is known as one of the risk measurement indicators used in economic affairs, it may be non-convex. In order to avoid this problem, the CVaR risk rating index, also known as average value at risk and mean shortfall, is used. Given the confidence interval β, CVaR is defined as follows [24]:

$$\beta - CVaR = E_y(f(x, y) \mid f(x, y) \geqslant \beta - VaR) \tag{2}$$

Eq. (2) shows the expected conditional value of the cost function with the condition that its amount is greater than $(\beta - percentile)$. Regarding the principles of optimization, CVaR minimization has usability as the objective function.

This is due to the fact that considering the uncertainty in parameter y results in the performance improvement of the optimization problem. This improvement actually means that optimal retail prices get closer to the real conditions of the optimization problem in a MG.

Moreover, the risk of the system that faces high losses is minimized if CVaR is reduced [28]. For optimization problems with the linear cost function, CVaR can be minimized linearly in the objective function [20]. By using the samples generated by the distribution of stochastic parameter y, CVaR is approximated in Eq. (3):

$$CVaR_{\beta} = \min(\alpha + \frac{1}{M \times (1-\beta)} \sum_{n=1}^{M} [Z]^{+}), \quad Z = f(x,y) - \alpha \tag{3}$$

Where Z^{+} indicates the positive values of Z. The value of M is the number of paths generated by the scenario generation mechanism in order to find the expected value in $\beta - CVaR$ in the cost function. Finally, y_n represents the n-th path from the generated stochastic variable in which the value $[0^{+}]$ usually are entered in the constraint to solve this problem. Therefore, the main equation for minimizing CVaR is formulated as follows:

$$CVaR_{\beta} = \min(\alpha + \frac{1}{M \times (1-\beta)} \sum_{n=1}^{M} [Z_n]), \quad Z_n \geqslant 0, Z_n \geqslant f(x,y) - \alpha \qquad (4)$$

3.3. CVaR-based Optimization Framework

The uncertainty resources cause the risk of total cost. Therefore, the optimal hourly prices have been modeled by using the CVaR with the confidence interval of β . CVaR, as shown in Eq. (4), is the expected value of the number of $(1-\beta) \times 100$ from all scenarios with the highest costs. Figure (2) demonstrates the framework of CVaR-based optimization.

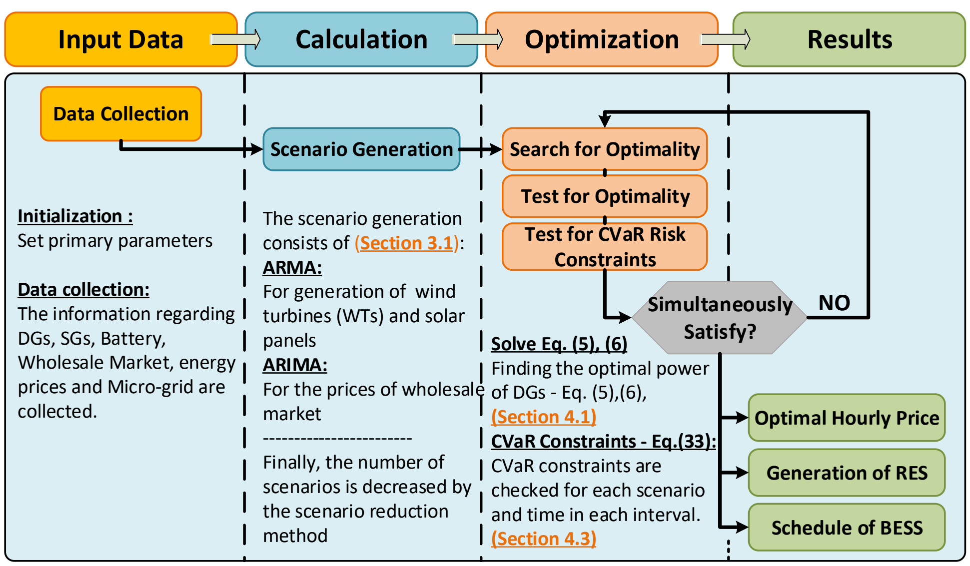

Figure 2: Optimization framework of CVaR-Based

4. Problem Formulation

In this paper, the proposed model aims to minimize the costs of MG, as well as to reduce the negative impacts of existing uncertainties on the optimization results. The DR model has also been designed to evaluate consumers' responsiveness to optimized hourly retail prices. Additionally, various constraints have been considered, such as minimum energy consumption and maximum electricity price of consumers to boost the consumers' welfare. Finally, all technical and economic constraints related to DG units (including dispatchable and non-dispatchable), battery, and the power flow have been considered in the problem formulation.

4.1. Objective Function

The desired problem's objective function is consists of two parts, as mentioned in Eq. (5). The first part revolves around the total cost of the MG, including the cost of generated power by DGs, the cost of using the battery, and the cost of purchasing energy from the dispatchable DG units and the wholesale market. Secondary, Eq. (6) is the revenue function for MG, which consists of two parts: (i) the first part is the sale of energy to consumers/end-users, and (ii) the second part is the sale of energy to the upstream network.

$$
\begin{aligned}
Min\ \sum_{s=1}^{S}\pi_s\sum_{t=1}^{T}&\left[\begin{array}{l}\sum\limits_{DG}(C^{s,t}_{DG,i}.P^{s,t}_{DG,i}.L^{s,t}_{DG,i}+SUC_{DG,i}.M^{s,t}_{DG,i}+SDC_{DG,i}.N^{s,t}_{DG,i})\\ +\sum\limits_{SDG}C^{s,t}_{SG,i}.P^{s,t}_{SG,i}+C_{BES,i}.P^{s,t}_{BES,i}-REV\end{array}\right]\\
+\omega\left(\xi+\frac{1}{1-\alpha}\sum_{s=1}^{S}\pi_s\eta_s\right)&
\end{aligned}
\tag{5}
$$

$$
REV^s=\sum_t(P^{s,t}_{D,i}.\rho^{s,t}_{D,i}+P^{s,t}_{GSP,i}.\rho^{s,t}_{GSP,i}) \tag{6}
$$

The sign $P^{s,t}_{GSP,i}$ shows the status of purchase or sale of electricity at time t. The positive/negative sign means that the MG purchases/sells power from/to the wholesale market or upstream grid. The generation cost of dispatchable and non-dispatchable DG units have been obtained based on the Levelized Cost of Energy (LCOE) [29]. Also, the cost of exploiting the battery, which is usually considered equal to its maintenance costs, is calculated as a linear function of the charging and discharging battery per hour as $C_{BES,i}(P_{BES,i})=A_{BES,i}\times P_{BES,i}+B_{BES,i}$, in which the values of $A_{BES,i}$ and $B_{BES,i}$ are positive. Eq.(6) indicates the revenue resulting from the sales of power to consumers in MG and upstream grid. The second part of Eq.(5) is the CVaR risk measurement index, which is considered in order to examine the effect of uncertainty in the problem, where ξ is the VaR. VaR is the lowest cost, with the probability of scenarios at a cost higher than ξ, less than or equal to $(1-\alpha)$. In addition, η_S is an auxiliary variable that is considered while solving the problem. This parameter creates the condition that the extra value of ξ can be positive.

4.2. DRP Model

DR programs are one of the Demand-Side Management (DSM) methods that refer to the modification of consumption patterns of consumers by altering the electricity prices (real-time pricing). The primary purpose of utilizing these schemes is to control the utility grid's demand during on-peak hours. They are offered to consumers in the form of incentivized schemes aiming at consumers' active participation. By following these programs, the electricity bill for consumers dramatically decreases. DR schemes mainly include two types of (i) Time-based such as Time of Use (TOU), Real-time Pricing (RTP), Critical Peak Pricing (CPP), Critical Peak Rebates (CPR), and (ii) Incentive-based such as e Direct Load Control (DLC), Interruptible/curtailable service (I/C), and Ancillary Service Markets (A/S).

The existing consumers can reduce their bills by participating in DR programs and changing their consumption patterns. However, all consumers cannot shift their power consumption at same time or, if they make changes in their demand, this amount is not the same with other consumers for the same applied prices. For this reason, residential consumers have more flexibility toward the industrial and commercial customers [30]. Correspondingly, a comprehensive economic model for DR has been considered based on demand characteristics, including price flexibility and load demand profile [31]. Eq. (7) demonstrates an economical DR model for consumer D at time t [31].

$$P_{D,i}^{s,t} = P_{D0,i}^{s,t}.\left\{1 + \frac{e_{D,i}^{s,t}.[\rho_{D,i}^{s,t} - \rho_{D0}]}{\rho_{D0}} + \sum_{\substack{h=1 \\ h \neq t}}^{24} e_{D,i}^{s,t,h}.\frac{[\rho_{D,i}^{s,t} - \rho_{D0}]}{\rho_{D0}}\right\} \tag{7}$$

The consumers could consider Eq. (7) to minimize their costs subject to the following constraints.

- **Constraints regarding DR model**

– The maximum and minimum amount of peak demand by the consumer in the bus i at time t. This constraint is intended to bring the amount of load consumed by consumers after the implementation of the DR program closer to reality.

$$P_{D,i,min}^{s,t} \leqslant P_{D,i}^{s,t} \leqslant P_{D,i,max}^{s,t} \tag{8}$$

– Minimum amount of energy needed per day for consumers. Normally, the daily energy consumption of consumers will exceed a fixed amount.

$$\sum_{t=1}^{24} E_{D,i}^{s,t} \geqslant E_{D,i,min}^{day} \tag{9}$$

– Maximum amount of applied prices to consumer to take into account the customers' welfare and avoid applying high energy prices on them.

$$\rho_{D,i}^{s,t} \leqslant \rho_{max}^{s,t} \tag{10}$$

– Providing incentive for consumers to participate in the program, the maximum electricity bill of each consumer should be less or equal to the case that wholesale market prices are directly sent to consumers. Although maximum end-use prices have been limited by (10), constraint (11) protects consumers from being offered relatively high prices at most hours of a day by the MG with the aim of maximizing the MG's benefit. In this win-win situation, the benefit of the MG and consumers are simultaneously derived.

$$\sum_{s=1}^{5}\sum_{t=1}^{24} P_{D,i}^{s,t}.\rho_{D,i}^{s,t} \leqslant \phi.\sum_{s=1}^{5}\sum_{t=1}^{24} P_{DM}^{s,t}.\rho_{GSP,i}^{s,t} \tag{11}$$

- **Constraints regarding power flow**

– The AC power flow equations in each node. The constraints (12) and (13) are designed to model the active and reactive power distribution equations in each node.

$$P_{G,i}^{s,t} - P_{D,i}^{s,t} = \sum_j \left|V_i^{s,t}\right| \left|V_j^{s,t}\right| |Y_{ij}| \cos(\theta_{ij} + \delta_j^{s,t} - \delta_i^{s,t}) \tag{12}$$

$$Q_{G,i}^{s,t} - Q_{D,i}^{s,t} = \sum_j \left|V_i^{s,t}\right| \left|V_j^{s,t}\right| |Y_{ij}| \sin(\theta_{ij} + \delta_j^{s,t} - \delta_i^{s,t}) \tag{13}$$

These following constraints actually represent the total production capacity per bus. In Eqs. (12) and (13), the values $P_{G,i}^{s,t}$ and $Q_{G,i}^{s,t}$ are calculated as follows:

$$P_{G,i}^{s,t} = P_{GSP,i}^{s,t} + P_{DG,i}^{s,t} + P_{SG,i}^{s,t} \tag{14}$$

$$Q_{G,i}^{s,t} = Q_{GSP,i}^{s,t} + Q_{DG,i}^{s,t} + Q_{SG,i}^{s,t} \tag{15}$$

– The voltage limit for each bus. To maintain stability, the voltage of each bus is considered to be between 0.95 and 1.05.

$$V_i^{min} \leqslant V_i^{s,t} \leqslant V_i^{max} \tag{16}$$

– Transformer Capacity Limit. The apparent power capacity of the bus in which the transformer is installed should not exceed the maximum nominal capacity of the transformer.

$$S_{GSP}^{s,t} \leqslant S_{GSP}^{max} \tag{17}$$

– Power flow limitation. The apparent transfer capacity of each line should not exceed the maximum transfer capacity of each line.

$$S_{ij}^{s,t} \leqslant S_{ij}^{max} \tag{18}$$

- **Constraints regarding Adequacy**

– Active and reactive power balance per hour. The total amount of active and reactive power generation in each bus must be equal to the total consumption and losses of active and reactive power in the same bus, per hour.

$$\sum_{DG} P_{DG,i}^{s,t} + \sum_{SDG} P_{SDG,i}^{s,t} + P_{GSP,i}^{s,t} = \sum_{D} P_{D,i}^{s,t} + P_{Loss}^{s,t} \tag{19}$$

$$\sum_{DG} Q_{DG,i}^{s,t} + \sum_{SDG} Q_{SDG,i}^{s,t} + Q_{GSP,i}^{s,t} = \sum_{D} Q_{D,i}^{s,t} + Q_{Loss}^{s,t} \tag{20}$$

In Eqs. (19) and (20), values $P_{Loss}^{s,t}$ and $Q_{Loss}^{s,t}$ are obtained from the following equations:

$$P_{Loss}^{s,t} = 0.5 \times \sum_{i,j} \left| \begin{array}{l} \left((V_i^{s,t})^2 + (V_j^{s,t})^2 \right) .Y_{ij} \cos(\theta_{ij}) - \\ 2 \times V_i^{s,t}.V_j^{s,t}.Y_{ij} \cos(\theta_{ij}). \cos(\delta_i^{s,t} - \delta_j^{s,t}) \end{array} \right| \tag{21}$$

$$Q_{Loss}^{s,t} = 0.5 \times \sum_{ij} \left| \begin{array}{l} \left((V_i^{s,t})^2 + (V_j^{s,t})^2 \right) Y_{ij} \sin(\theta_{ij}) \\ -2 \times V_i^{s,t}.V_j^{s,t}.Y_{ij} \sin(\theta_{ij}). \cos(\delta_i^{s,t} - \delta_j^{s,t}) \end{array} \right| \tag{22}$$

– Energy Reserve Capacity

Due to the possibility of changes in the stochastic generation of RESs units and customer demands, there is a need for an energy reserve to ensure adequate energy capacity to meet the expected hourly demand. Therefore, according to the coefficient (χ%) (which is approximately equal to 5% of the total generation of non-dispatchable DG units), and the coefficient (γ%) (which is usually equal to 2% of total hourly demand [32]) for total power demand at any time interval, the energy reserve limit for dispatchable DGs is as follows:

$$\sum_{DG}(P_{DG,i,max}^{s,t} - P_{DG,i}^{s,t}) \geqslant \chi \sum_{SDG} P_{SDG,i}^{s,t} + \gamma \sum_{D} P_{D,i}^{s,t} \tag{23}$$

- **Constraints regarding BES**

Eqs. (24) and (25) indicate the limit of rechargeable power and discharge of the battery per hour.

$$0 \leqslant P_{BES,i,Ch}^{s,t} \leqslant P_{BES,Ch}^{max} \tag{24}$$

$$0 \leqslant P_{BES,i,Dch}^{s,t} \leqslant P_{BES,Dch}^{max} \tag{25}$$

The battery cannot operate on both charge and discharge modes at the same time. Therefore, constraint (26) applies this issue in a binary way to the optimization problem.

$$P_{BES,i,Ch}^{s,t} . P_{BES,i,Dch}^{s,t} = 0 \tag{26}$$

Eq. (27) describes the charge and discharge status of the battery according to the amount of initial energy contained in it.

$$E_{BES,i}^{s,t} = E_{BES,i}^{s,t-1} + \mu_{ch}.P_{BES,i,Ch}^{s,t} - P_{BES,i,Dch}^{s,t} \tag{27}$$

Eq. (28) states that the energy capacity of each battery is limited.

$$E_{BES}^{min} \leqslant E_{BES,i}^{s,t} \leqslant E_{BES}^{max} \tag{28}$$

- **Constraints regarding dispatchable DGs**

The following constraints have been intended to consider the physical limitations of each DG as much as possible. Eq. (29) indicates that each DG unit in each bus has a production limitation.

$$P_{DG,i}^{min} \leqslant P_{DG,i}^{s,t} \leqslant P_{DG,i}^{max} \tag{29}$$

Eq. (30) states that the rate of increase and decrease of power production of each DG unit has a certain value.

$$\begin{aligned} & P_{DG,i}^{s,t+1} - P_{DG,i}^{s,t} \leqslant RUP_{DG,i} \\ & P_{DG,i}^{s,t} - P_{DG,i}^{s,t+1} \leqslant RDN_{DG,i} \end{aligned} \tag{30}$$

The following constrain demonstrate that the minimum amount of time to keep the DG units off or on has a certain value.

$$\begin{aligned}
&\sum_{h=1}^{MUT} L_{DG,i}^{t+h-1} \geqslant MUP_{DG,i}, \quad \forall M_{DG,i}^{t} = 1, \ \sum_{h=1}^{MDT} \left(1 - L_{DG,i}^{t+h-1}\right) \\
&\sum_{h=1}^{MUT} L_{DG,i}^{t+h-1} \geqslant MDT_{DG,i}, \quad \forall N_{DG,i}^{t} = 1
\end{aligned} \tag{31}$$

Each DG unit in each bus cannot be both on and off at the same time. The constraint (32) applies this issue in the optimization problem.

$$M_{DG,i}^{t} - N_{DG,i}^{t} = L_{DG,i}^{t} - L_{DG,i}^{t-1}, \ M_{DG,i}^{t} + N_{DG,i}^{t} \leqslant 1 \tag{32}$$

- **Constraints regarding risk aversion**

Eq. (33) indicates the CVaR constraint, where η_s is a variable dependent on the scenario, which equals to the difference in average cost and VaR in the scenario s.

$$\sum_{t=1}^{T}\left[\begin{array}{l}\sum_{DG}(C_{DG,i}^{s,t}.P_{DG,i}^{s,t}.L_{DG,i}^{s,t}+SUC_{DG,i}.M_{DG,i}^{s,t}+SDC_{DG,i}.N_{DG,i}^{s,t})+\\ \sum_{SDG}C_{SG,i}^{s,t}.P_{SG,i}^{s,t}+C_{BES,i}.P_{BES,i}^{s,t}-REV^{s,t}\end{array}\right]$$

$$-\xi\leqslant\eta_s,\ \ \eta_s\geqslant 0 \tag{33}$$

4.3. Stochastic Problem Solution Method

The optimization framework for determining hourly prices based on CVaR risk measure has been formulated as an MINLP problem in the previous section. The objective function of this framework is to minimize the total costs. Additionally, these prices are different for each consumer per hour. As a result, the large number of variables regarding a real-size distribution system needs to be eased by an appropriate decomposition-based approach. In this regard, the proposed MINLP problem is solved by utilizing the decomposition techniques such as BDT [33]. This algorithm divides the problem into two entirely distinct levels (the master and slave problem), and solves the optimization problem by creating an iterative process between the two levels [34]. First, the main problem is to compute the optimal prices by ignoring the network constraints. Secondary, the slave problem covers network constraints. The master problem outputs are transferred to the slave problem, including the charging and discharging of the batteries, the generation of dispatchable units, the power exchanges of MG with the upstream network, and power demand. The slave problem's constraints examine the feasibility of results derived from the master problem and return the marginal values and binary variables to the master problem. The repetitive process used in this technique is iterated in each hour of the created scenarios.

The CVaR Risk Indicator evaluates the extent of the uncertainty effects regarding the created scenarios during each iteration in the objective function for hourly prices. These data have been used in any Benders Cuts. In each iteration, Benders Cuts, the solutions to the master problem, and the marginal data of the slave problem are updated. This duplicate process continues until the marginal data value goes to zero, and the convergence of the problem is achieved. In such a situation, all decision variables are achieved their optimal value per hour. Also, using the CVaR in scenarios, different risk management policies are implemented to minimize the MG's total cost under a stochastic framework. The problem-solving process is shown in Figure (3) using the BDT and CVaR index.

4.3.1. Master problem

The objective function (Eq. (5)) has two parts: the first part is the total cost of MG and the second part represents the costs of problem-solving feasibility of slave problem in different scenarios at each hour.

$$\mathrm{Cost}^{st} = \sum_{s=1}^{S} \pi_s \sum_{t=1}^{T} \left[\begin{array}{l} \sum\limits_{DG} (C_{DG,i}^{s,t}.P_{DG,i}^{s,t}.L_{DG,i}^{s,t} + SUC_{DG,i}.M_{DG,i}^{s,t} \\ +SDC_{DG,i}.N_{DG,i}^{s,t}) + \sum\limits_{SDG} C_{SG,i}^{s,t}.P_{SG,i}^{s,t} + C_{BES,i}.P_{BES,i}^{s,t} \\ -\sum\limits_{t} (P_{D,i}^{s,t}.\rho_{D,i}^{s,t}) + P_{GSP,i}^{s,t}.\rho_{GSP,i}^{s,t} \end{array} \right]$$
$$+\beta \left(\xi + \frac{1}{1-\alpha} \sum_{s=1}^{S} \pi_s \eta_s \right) + \sum_{s} \sum_{t} \psi_{sub}^{s,t} \tag{34}$$

$$\psi_{sub}^{s,t} \geqslant \psi_{n-1}^{s,t} + \sum_{D} k_{D,i,n-1}^{s,t}.(P_{D,i}^{s,t} - \tilde{P}_{D,i,n-1}^{s,t}) + \sum_{DG} k_{DG,i,n-1}^{s,t}.(P_{DG,i}^{s,t} - \tilde{P}_{DG,i,n-1}^{s,t}) +$$
$$\sum_{GSP} k_{GSP,i,n-1}^{s,t}.(P_{GSP,i}^{s,t} - \tilde{P}_{GSP,i,n-1}^{s,t}) + \sum_{BES} k_{BES,i,n-1}^{s,t}.(P_{BES,i}^{s,t} - \tilde{P}_{BES,i,n-1}^{s,t}) \tag{35}$$

In Eq. (35), $\psi_{sub}^{s,t}$ is the cost of the slave problem and the values $\tilde{P}_{D,i,n-1}^{s,t}$,$\tilde{P}_{DG,i,n-1}^{s,t}$,$\tilde{P}_{GSP,i,n-1}^{s,t}$, and $\tilde{P}_{BES,i,n-1}^{s,t}$ are the answers obtained from the iteration $n-1$ of the slave problem in whole scenarios. To boost the speed and accuracy of convergence in the BDT method, the values of Benders Normal Cuts in Eq. (35) are replaced with the values of Benders Strong Cuts in Eqs. (36) and (37).

$$\psi_{sub}^{s,t} \geqslant \psi_{b}^{s,t} + \sum_{D} k_{D,i,b}^{s,t}.(P_{D,i}^{s,t} - \tilde{P}_{D,i,b}^{s,t}) + \sum_{DG} k_{DG,i,b}^{s,t}.(P_{DG,i}^{s,t} - \tilde{P}_{DG,i,b}^{s,t}) + \sum_{GSP} k_{GSP,i,b}^{s,t}.(P_{GSP,i}^{s,t} - \tilde{P}_{GSP,i,b}^{s,t}) + \sum_{BES} k_{BES,i,b}^{s,t}.(P_{BES,i}^{s,t} - \tilde{P}_{BES,i,b}^{s,t}) \tag{36}$$

$$\begin{bmatrix} \psi_{b}^{s,t} + \sum_{D} k_{D,i,b}^{s,t}.(P_{D,i}^{s,t} - \tilde{P}_{D,i,b}^{s,t}) + \sum_{DG} k_{DG,i,b}^{s,t}.(P_{DG,i}^{s,t} - \tilde{P}_{DG,i,b}^{s,t}) \\ + \sum_{GSP} k_{GSP,i,b}^{s,t}.(P_{GSP,i}^{s,t} - \tilde{P}_{GSP,i,b}^{s,t}) + \sum_{BES} k_{BES,i,b}^{s,t}.(P_{BES,i}^{s,t} - \tilde{P}_{BES,i,b}^{s,t}) \end{bmatrix} = Max \begin{bmatrix} \psi^{st} + \sum_{D} k_{D,i}^{st}.(P_{D,i}^{st} - \tilde{P}_{D,i}^{st}) + \sum_{DG} k_{DG,i}^{st}.(P_{DG,i}^{st} - \tilde{P}_{DG,i}^{st}) \\ + \sum_{GSP} k_{GSP,i}^{st}.(P_{GSP,i}^{st} - \tilde{P}_{GSP,i}^{st}) + \sum_{BES} k_{BES,i}^{st}.(P_{BES,i}^{st} - \tilde{P}_{BES,i}^{st}) \end{bmatrix} \tag{37}$$

In Eq. (36) and (37), the parameter b is the iteration number, which has the maximum amount of Eq. (38). This value is selected among all the iterations that make possible the first part of the slave problem in the shortest time, for each scenario and per hour.

$$\left[\begin{array}{l} \psi^{s,t} + \sum\limits_{D} k^{s,t}_{D,i}.(P^{s,t}_{D,i} - \tilde{P}^{s,t}_{D,i}) + \sum\limits_{DG} k^{s,t}_{DG,i}.(P^{s,t}_{DG,i} - \tilde{P}^{s,t}_{DG,i}) \\ + \sum\limits_{GSP} k^{s,t}_{GSP,i}.(P^{s,t}_{GSP,i} - \tilde{P}^{s,t}_{GSP,i}) + \sum\limits_{BES} k^{s,t}_{BES,i}.(P^{s,t}_{BES,i} - \tilde{P}^{s,t}_{BES,i}) \end{array}\right] \tag{38}$$

4.3.2. Slave problem

The slave problem as a Non-Linear Programming (NLP) examines the feasibility of the master problem outputs by solving the AC power flow problem [35]. Then the existence of any conflict in constraints Eq. (12),Eq. (13),Eq. (16),Eq. (24),Eq. (25), and Eq. (29) is compensated by changes in charging and discharging of BES unit, power exchanges with the upstream network, power flow of DG units, and the consumers' demand regulation. The objective function in Eq. (39) minimizes the existence of any dispute in the outputs of the master problem in each scenario on an hourly basis.

$$\sum_{i} r^{s,t}_{P,UP,i} + r^{s,t}_{P,DN,i} + r^{s,t}_{Q,UP,i} + r^{s,t}_{Q,DN,i} \tag{39}$$

Where $r^{s,t}_{P,UP,i}$, $r^{s,t}_{P,DN,i}$, $r^{s,t}_{Q,UP,i}$ and $r^{s,t}_{Q,DN,i}$ are the slack bus variables concerning the optimization problem which have been added to the AC power flow equation to feasible the slave problem. The constraints of slave problem including

Eq. (12),Eq. (13),Eq. (16),Eq. (24),Eq. (25), Eq. (29), Eq. (40), Eq. (41), and Eq. (42), check the feasibility of master problem in all scenarios and hours, and then provide marginal data and binary values. These values update the Benders Cuts during each iteration to improve the optimal results of the main problem. This repetitive process continues until the master problem is feasible.

$$P_{G,i}^{s,t} - P_{D,i}^{s,t} + r_{P,UP,i}^{s,t} - r_{P,DN,i}^{s,t} = \sum_j \left|V_i^{s,t}\right| \left|V_j^{s,t}\right| |Y_{ij}| \cos(\theta_{ij} + \delta_j^{s,t} - \delta_i^{s,t}) \quad (40)$$

$$Q_{G,i}^{s,t} - Q_{D,i}^{s,t} + r_{Q,UP,i}^{s,t} - r_{Q,DN,i}^{s,t} = \sum_j \left|V_i^{s,t}\right| \left|V_j^{s,t}\right| |Y_{ij}| \sin(\theta_{ij} + \delta_j^{s,t} - \delta_i^{s,t}) \quad (41)$$

$$\begin{aligned} &P_{D,i}^{s,t} = \bar{P}_{D,i}^{s,t} \leftrightarrow k_{D,i,n-1}^{s,t};\ P_{DG,i}^{s,t} = \bar{P}_{DG,i}^{s,t} \leftrightarrow k_{DG,i,n-1}^{s,t} \\ &P_{GSP,i}^{s,t} = \bar{P}_{GSP,i}^{s,t} \leftrightarrow k_{GSP,i,n-1}^{s,t};\ P_{BES,i}^{s,t} = \bar{P}_{BES,i}^{s,t} \leftrightarrow k_{BES,i,n-1}^{s,t} \end{aligned} \quad (42)$$

In which the values $\bar{P}_{D,i}^{s,t}$, $\bar{P}_{DG,i}^{s,t}$, $\bar{P}_{GSP,i}^{s,t}$, and $\bar{P}_{BES,i}^{s,t}$ are the results derived from the master problem in the same iteration. Moreover, the values $k_{D,i,n-1}^{s,t}$, $k_{DG,i,n-1}^{s,t}$,

$k^{s,t}_{GSP,i,n-1}$, and $k^{s,t}_{BES,i,n-1}$ are the dual variables, which are related to the results of optimization problem.

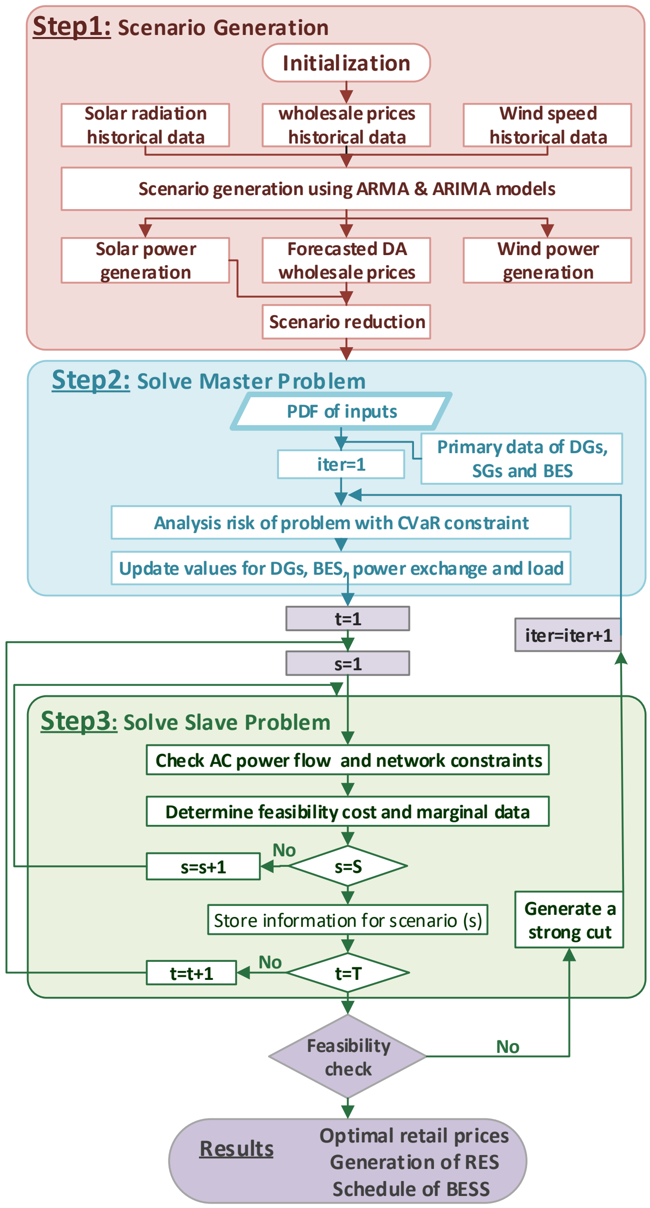

Figure 3: The problem-solving process using BDT and CVaR index.

5. Numerical Studies

5.1. Data

As shown in Figure (4), the MG structure used in this paper is an 18-bus distribution network [36], including a battery storage located at bus 7, a connection point to the upstream grid via a post-transformer, Grid Supply Point (GSP), located at bus 1, two MTs at buses 11 and 12, a wind and solar power plants, respectively, in buses 14 and 18. Additionally, the capacity of transformer at GSP equals to 50 MVA. .

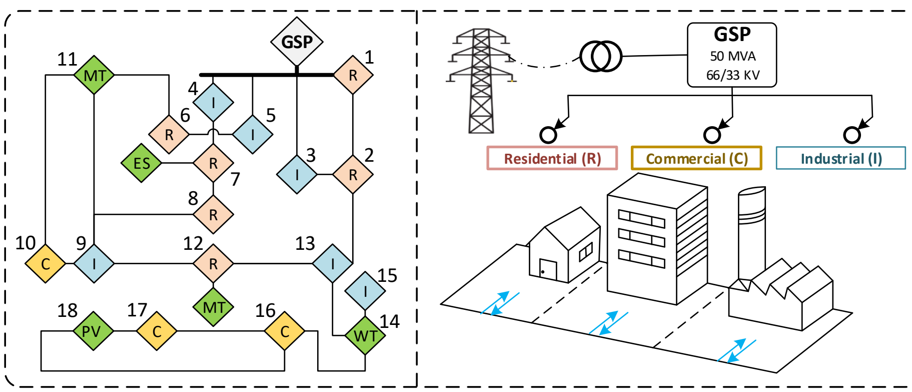

Figure 4: Case study: the 18-bus distribution system

The information regarding SGs' data, and hourly bids of DG units, cost function coefficients, and BES are visible in Tables (1) to (4).

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Table 1: Characteristics of SG units

ine	SG no	Bus no	$P_{D,i}^{max}$ (MW)	$P_{D,i}^{max}$ (MW)	A_{DG} $(\$/MWh)$	B_{DG} $(\$)$
	ine 1	14	0	9	7	22
	2	18	0	2	6	14
	ine					

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Table 2: Characteristics of dispatchable DG units

ine	DG no	Bus no	$P_{D,i}^{max}$ (MW)	$P_{D,i}^{max}$ (MW)	SUC_{DG} $(\$)$	SDC_{DG} $(\$)$
	ine 1	11	1	8	8	7
	2	12	1	7	5	3
	ine					

Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 3: Hourly price offers for dispatchable DGs

ine Hour	A_{DG1} $(\$/MWh)$	B_{DG1} $(\$)$	A_{DG2} $(\$/MWh)$	B_{DG2} $(\$)$
ine t=1,...,6	1.2	72	0.52	60
t=7,...,10	1	86	1.3	82
t=11,...,16	1.1	85	1	86
t=17,18	1.8	79	1.3	75
t=19,...,21	1	90	0.5	82
t=22,...,24	0.6	70	0.3	67
ine				

Table generated by Excel2LaTeX from sheet 'Sheet2'

Table 4: Characteristics of BES unit

ine	Bus no	B_{DG} $(\$)$	A_{DG} $(\$/MWh)$	Initial level (MWh)	η_{Dch}	η_{Ch}	P_{Dch}^{max} (MW)	P_{Ch}^{max} (MW)	Capacity (MWh)
	ine 7	9	17	2	0.9	0.9	1.2	1.2	9
	ine								

By applying the ARMA scenario generation method, the scenarios of the wind power plant have been generated. The data concerning wind speed belong to the time interval (1 January 2017 to 1 January 2018) on the Kingston Site, Ontario [37]. Also, the scenarios of the solar power plant have been derived from the available historical data (1 January 2017 to 1 January 2018) [38]. It should be noted that the predicted hourly prices of the wholesale market are the result of applying the ARIMA scenario method. The initial data related to the hourly prices of the wholesale market have been extracted from the Ontario electricity market for the generation of the scenarios [39]. The base price of energy in numerical studies has also been considered equal to US $ 101, which is the average energy price in November 2018 [40]. Moreover, the load profile is related to the Ontario electricity market, which has been considered on a smaller scale to better match the terms of the desired distribution system. The profile of the power demand is indicated in Figure (5).

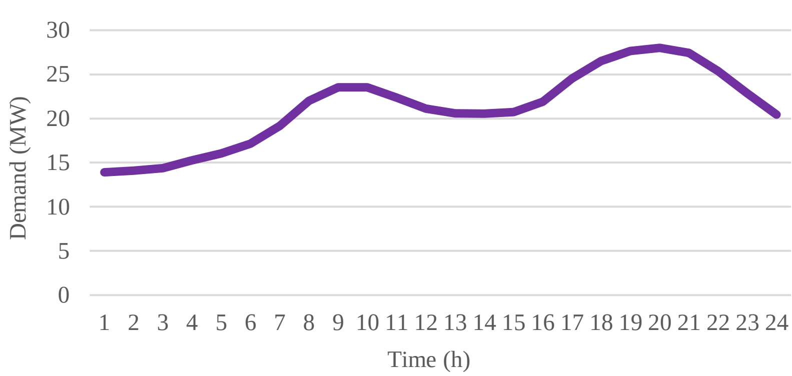

Figure 5: The profile of consumed load

Ultimately, the number of scenarios has been decreased because the number of generated scenarios is very large. As a result, there are five scenarios for each value of wind and solar power plants generations, and the hourly price of the wholesale market.

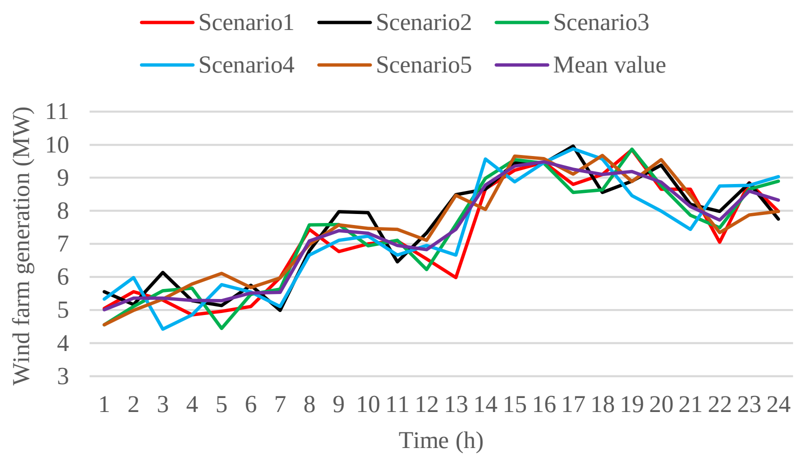

Figure 6: The scenarios of wind power generation

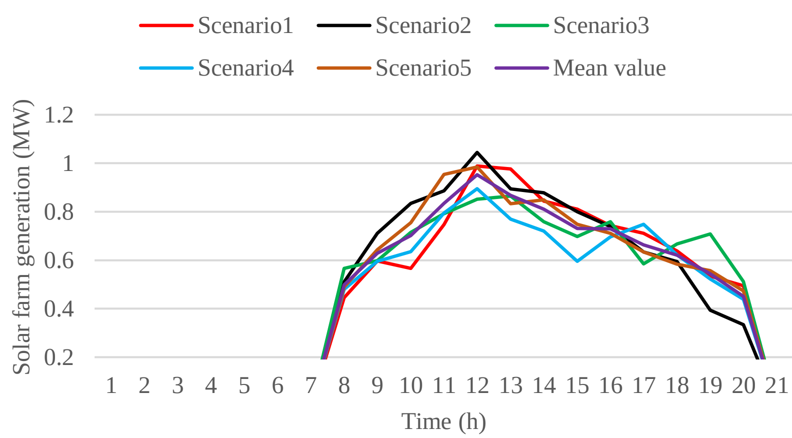

Figure 7: The scenarios of solar power generation

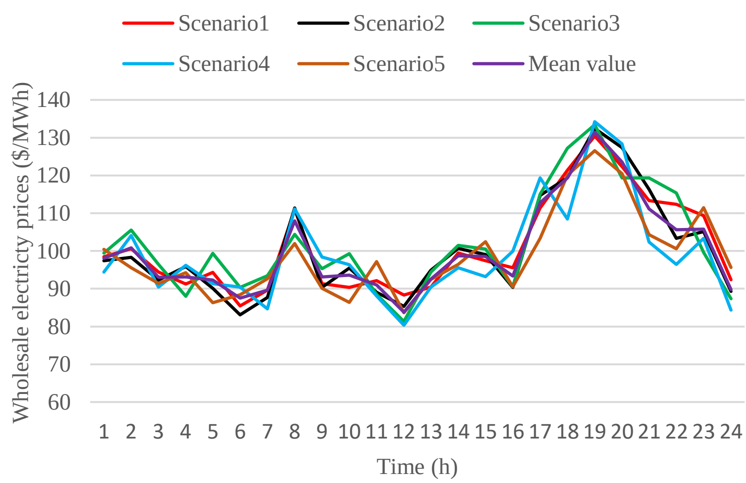

Figure 8: The scenarios of hourly Prices for the wholesale market

In this paper, MG supplies three categories of residential, commercial, and industrial loads. The consumers in each bus have been taken into account as a lumped load, as shown in Figure (4). Because of the fact that the industrial consumers have a fixed load profile, their flexibility to the price can be neglected [41]. On the opposite side, residential and commercial consumers' demand profile has the most flexibility; therefore, these consumers' flexibility has the highest possible amount toward price. However, the flexibility of the commercial consumers is a bit less than the residential consumers and is equal to approximately 80% of the residential sector [42]. The flexibility value of the residential consumers is not the same throughout various hourly and seasonal periods. In this paper, these values have been extracted from the electricity market of Ontario, Canada and OEB. The flexibility value of residential consumers derived from the OEB report, which is demonstrated in Table (5).

Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 5: The residential consumers' flexibility toward price

ine	On-peak	Mid-peak	Off-peak	Off-peak
ine On-peak (7-11 and 17-19)	-0.06	-0.08	0.02	-0.03
Mid-peak (11-17)	-0.11	0.03	-0.01	-0.08
Off-peak (19-21)	0.06	-0.01	-0.12	-0.07
Off-peak (Reminder)	-0.06	-0.03	-0.01	-0.1
ine				

To investigate the effect of the risk caused by uncertainty sources on the optimal prices and expected costs in the objective function, a comparison has been performed among the results of two different studies. In Case Study A, by a risk-neutral policy, the impact of the uncertainties on the total cost and optimal retail prices has been examined. Secondary, in Case Study B, the efficacy of the uncertainties on optimal retail prices and costs have been analyzed utilizing the CVaR risk measurement criterion and applying a risk-averse policy.

In fact, this case examines the impact of uncertainties resulting from renewable energy production and wholesale electricity market prices on optimal retail prices. As mentioned previously, the extent of this effect has been analyzed by the CVaR risk measurement index. In general, case B expresses a risk-averse policy during the optimization problem to enhance optimal retail prices.

As mentioned in section 4, the desired problem-solving method decomposes the optimization problem into two levels of master and slave problems by using CVaR and BDT. The master and slave problems are MINLP and NLP, respectively. It should be noted that the number of iterations in BDT method, in GAMS software environment, in the best possible case equals 15. The proposed algorithm for solving the master and slave problems has been implemented in GAMS environment using SBB and CONOPT solvers for master and slave problems, respectively [43], and executed with a Core i7 Laptop, 2.7 GHz with 8 GB.

5.2. Discussion and Findings

In Case A, the risk in the optimization problem has been regarded by the CVaR risk measurement criterion as a risk-neutral policy with a confidence level of $\beta = 0.95$. In addition, on Case B, an risk averse policy with a confidence level of $\beta = 0.95$ has been considered.

Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 6: Optimal dispatch of DGs and BES and optimal power exchange of MG to UN in case A and B (with $\beta = 0.95$)

Hour	DG1 (MW)		DG2 (MW)		GSP (MW)	
	Case A	Case B	Case A	Case B	Case A	Case B
1	6.6344	6.7344	7	7	0.9324	0.9892
2	6.9345	6.9109	7	7	0.5245	1.3245
3	7.1239	7.4756	7	7	1.3645	1.7017
4	6.7445	6.9881	7	7	2.0333	2.5671
5	6.7843	6.8122	7	7	1.6987	2.345
6	6.3792	6.8411	7	7	3.5784	3.9104
7	2.9876	2.8123	5.0158	5.0158	7.7168	7.7034
8	8	8	7	7	-1.4643	-2.1246
9	6.8345	6.9901	7	7	-0.7645	-1.0532
10	6.4908	6.4789	7	7	-0.5287	-1.1346
11	0	0	3.4435	3.6531	11.433	11.567
12	0	0	1	1	14.983	15.678
13	0	0	5.7659	5.8903	8.4309	9.8011
14	8	8	7	7	-2.4489	-1.5682
15	7.3746	7.5678	7	7	-2.8434	-2.0349
16	5.3412	5.6669	7	7	-1.032	-0.4567
17	8	8	7	7	-2.0543	-2.9813
18	8	8	7	7	-3.4432	-4.3501
19	8	8	7	7	-2.5836	-3.1123
20	8	8	7	7	-0.9284	-0.9791
21	8	8	7	7	4.2021	2.9775
22	8	8	7	7	3.1355	4.2348
23	8	8	7	7	0.8322	1.4586
24	8	8	7	7	-3.3547	-2.6789

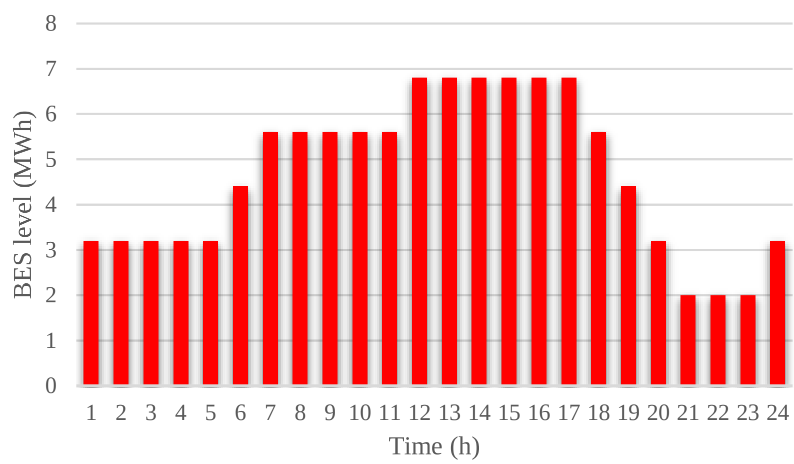

Figure 9: BES level in Case A and B

Table (7) indicates the expected standard deviation resulting from the generation of DG1 and DG2 units, as well as power exchange with the upstream grid. To solve the optimization problem and reduce the risk effect caused by the uncertainty sources, the standard deviation of DG generation and power exchanges with the upstream network has been lessened in the case B in comparison with the case A.

Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 7: The sum of standard deviation expected for DG units and MG power exchanges with the upstream grid at the GSP point

ine	Total expected St.de of DG1		Total expected St.de of DG2		Total expected St.de of GSP	
ine	Case A	Case B	Case A	Case B	Case A	Case B
ine	2.6598	1.0243	0.6534	0.5249	3.2655	2.9568
ine						

5.2.1. Economic Index

In this part, MG's total cost regarding an economic indicator has been investigated in cases A and B. In Table (**??**), the mean values of exploiting cost from the MG, along with the mean standard deviation of the cost and the amount of CVaR have been shown for different values of the weight coefficient ω and the confidence level $\beta = 0.95$. The amount of total cost, which has been achieved for $\omega = 0$, has the lowest possible amount of cost with the highest exploitation risk amount with respect to two terms of standard deviation and CVaR.

In addition, according to Figure (10), a 2% increase in the mean cost has caused a 3.14% decrease of the mean of cost standard deviation and an 8.639% increase in CVaR. It should be pointed out that the small amounts of the standard deviation of the expected cost and large values of CVaR indicate the risk-averse policy.

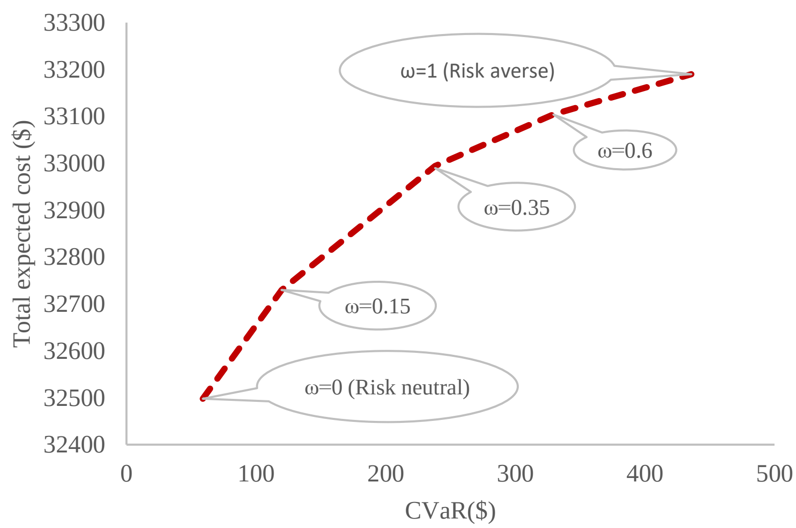

Figure 10: The efficient frontier of expected total cost vs CVaR

5.2.2. Technical Index

In Table (8), the total power demand of consumers is given in peak hours [8 to 11] and [18 to 20]. Moreover, the sum of total voltage deviation of each bus per hour has been compared through the relation $(\sum_i \sum_t |V_i^t - 1|)$, as a technical indicator in two cases A and B. In general, since after taking into account the risk-averse policy in case B, a load of consumers in the peak hours has decreased; therefore, the amount of power demand at peak hours in case B is less than case A.

Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 8: Comparison of technical indices in two case A and B with confidence level $\beta = 0.95$

ine	Total demand at peak hours (MW)		Total expected voltage St.de (pu)	
ine	Case A	Case B	Case A	Case B
ine ine	176.15	164.84	0.0783	0.0425
ine				

5.2.3. Analyzing the impact of uncertainty on optimal hourly prices

Figure (11) demonstrates the optimal hourly prices for both cases A and B. Also, Figure (12) shows the load profile of consumers in four situations prior to the optimal hourly prices program, case A (risk-neutral policy) and case B (risk-averse policy). In general, in case A, applying the DR program causes the customers to reduce their loads in peak hours of consumption or shift it into off-peak or mid-peak hours. According to Figures (11) and (12), the desired MG is considered to minimize the total cost (stated as the objective function) in case B with risk-averse. To this end, MG has applied the lower prices during peak hours on the consumers in comparison with case A. The total daily load before applying the risk in the optimization problem is approximately 540 MW. Also, this amount after applying the risk-neutral neutrals in case A and the risk-averse in case B equals 525 and 529 MW, respectively.

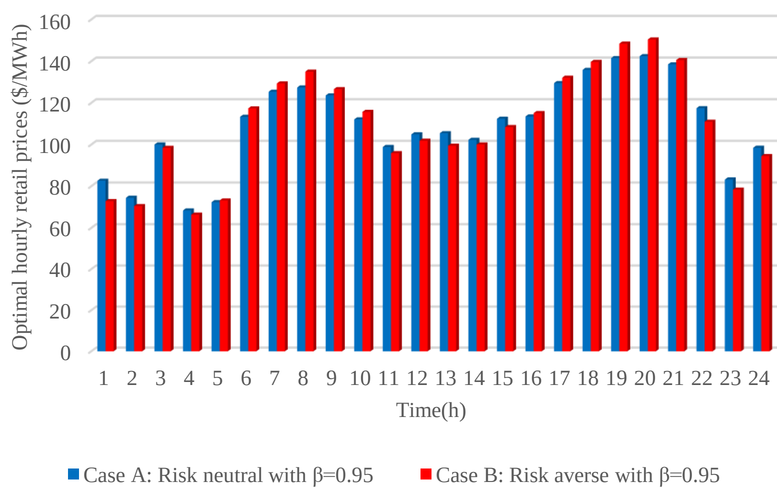

Figure 11: The optimal hourly retail prices in cases A and B with confidence level ($\beta = 0.95$)

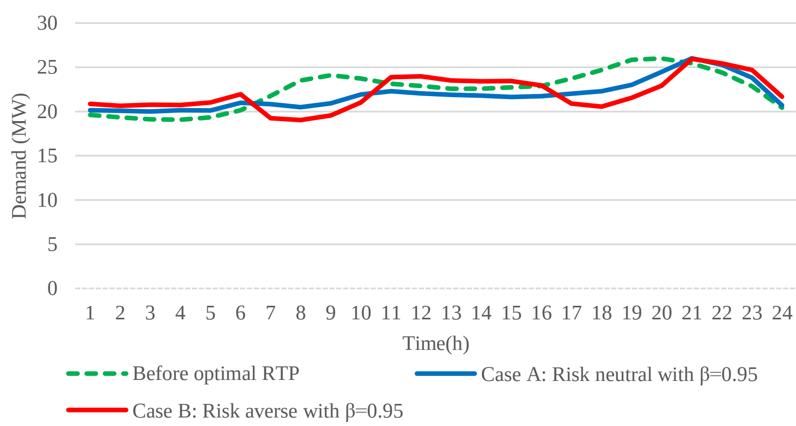

Figure 12: The consumption load profile before applying hourly prices in cases A and B based on average value

5.2.4. The risk analysis regarding total cost, power demand profile, and optimal hourly prices

In Table (9), the average power demand for consumers per hour has been given in case A and Case B with different confidence levels. Also, the amount of load reduction in peak hours for each different confidence levels of case B has been shown. In general, by increasing the confidence level in risk-averse conditions, the amount of load reduction in case B has increased compared to the initial load profile. Generally, by increasing the confidence level in risk-averse policy, the total cost of MG is increased (Table (10)). As a result, the MG increases the prices at peak hours to reduce the consumption load and the cost. Hence, in case B, the total cost has increased because of the increment of CVaR cost in comparison with case A. The MG has improved the optimal prices and the load profile in case B to lessen these costs.

Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 9: the average consumed load of consumers in case A and case B with different confidence levels and comparing the amount of load profile reduction

Hour	Case A: Risk neutral		Case B: Risk averse					
	β=0.95	Red (%)	β=0.90	Red (%)	β=0.95	Red (%)	β=0.99	Red (%)
1	20.1564	-	20.7456	-	20.8746	-	20.9767	-
2	20.0814	-	20.5192	-	20.6594	-	20.8347	-
3	20.0013	-	20.6943	-	20.7764	-	20.9419	-
4	20.1386	-	20.719	-	20.7448	-	20.9872	-
5	20.1162	-	21.0266	-	21.0355	-	21.2305	-
6	21.9761	-	21.8884	-	21.9484	-	21.9867	-
7	20.8339	4.27	19.8331	8.86	19.2542	11.5	19.009	12.6
8	20.7542	11.7	19.4027	16	19.1356	19	18.7685	20.2
9	20.8608	13.4	19.6504	18.4	19.5441	18.8	19.3256	19.8
10	21.9394	7.62	21.2873	10.3	21.0218	12.4	20.7857	12.5
11	23.3023	-	23.8445	-	23.8948	-	23.9857	-
12	22.0643	-	23.7456	-	23.9844	-	24.3354	-
13	21.9053	-	23.4536	-	23.5234	-	23.875	-
14	21.8014	-	23.0459	-	23.4345	-	23.5981	-
15	21.6445	-	23.1085	-	23.4364	-	23.7857	-
16	21.7431	-	22.9146	-	22.9475	-	23.2139	-
17	22.0281	7.12	21.1128	11	20.8939	10.9	20.6784	12.8
18	22.2864	9.79	21.2921	13.8	20.5634	16.7	20.2346	18
19	23.0184	10.9	21.9673	15	21.5467	16.6	21.0988	18.3
20	24.4646	5.92	23.7384	8.73	22.9139	12	22.0394	15.3
21	25.9081	-	26.0056	-	26.0185	-	26.3485	-
22	25.2641	-	25.6453	-	25.4536	-	25.5647	-
23	23.8387	-	24.1379	-	24.6875	-	24.7658	-
24	20.7023	-	20.9384	-	21.6758	-	21.9843	-

Table (10) shows the expected operating costs concerning cases A and B with different confidence levels. According to this table, the cost of exploitation for case A is $ 32,498. Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 10: Comparison of expected exploitation costs in Case A and Case B with different confidence levels

Case studies	Total expected cost ($)	(%) Inc.	Total expected st.de of cost ($)	Red. (%)
Case A (Risk neutral with β=0.95)	32498	-	889.7	-
Case B (Risk averse with β=0.90)	32853	1.08	753.1	15.35
Case B (Risk averse with β=0.95)	33190	2.08	670.5	24.63
Case B (Risk averse with β=0.99)	34071	4.61	572.8	35.61

According to Table (11), the average standard deviation of the optimal hourly prices in case A is $ 97.23. This amount has been decreased by $ 68.11 after applying the CVaR risk measure index through the risk averse policy on case B with a confidence level $\beta = 0.95$. Table generated by Excel2LaTeX from sheet 'Sheet1'

Table 11: Total mean value and st.de of optimal optimal hourly retail prices for case A and different confidence levels in case B

ine Case studies	Total expected st.de of optimal hourly retail prices ($/MWh)	Red(%)
ine Case A (Risk neutral with β=0.95)	97.23	-
Case B (Risk averse with β=0.90)	68.11	29.94
Case B (Risk averse with β=0.95)	56.05	42.35
Case B (Risk averse with β=0.99)	36.84	62.11
ine		

6. Conclusion

In this paper, a stochastic optimization framework based on CVaR has been considered in order to address the effect of risk on optimal retail prices in MG energy management. The simulation results demonstrate that the total costs of MG increase by almost 2.08% after applying the existing risk by the CVaR index. This increment in the total cost of MG is due to the applying risk analyses. On the other hand, in case B, considering the risk-averse policy at the confidence level $\beta = 0.95$, the standard deviation of the expected cost has been decreased by 24.63% compared to case A. Finally, applying the risk-averse policy in case B with a confidence level of $\beta = 0.95$ reduces the standard deviation of optimal retail prices up to 29.94% in comparison with case A. If the confidence level of risk-averse policy increases up to $\beta = 0.99$, this value could be reduced to 62.11%.

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