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Simple harmonic and damped motions of dissipative solitons in two-dimensional complex Ginzburg-Landau equation supported by an external V-shaped potential

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Abstract

Dissipative solitons based on the complex Ginzburg-Landau (CGL) model show many novel dynamic properties. In this paper, a series of novel simple harmonic and damped motion dynamics of soliton supported by induced V-shaped potential in the cubic-quintic CGL model was investigated. Without viscosity, the role of these potential wells in stabilizing dissipative soliton forms periodic oscillation, just like simple harmonic motion. The influence of potential slope and oscillating amplitude on the period and momentum of simple harmonic motion were numerically analyzed. By adding a small diffusivity term (viscosity) into the CGL model, a significant damping effect is applied to the simple harmonic motion of dissipative solitons. The evolution mechanism of the energy and momentum during the simple harmonic motion and the damped motion was numerically studied. In addition, the energy gain/loss in the CGL model has no impact on the dynamical evolution of simple harmonic motion and damped motion of dissipative solitons.

Keywords: optical soliton; dissipative system; Ginzburg-Landau; Simple harmonic motion; damped motion

I. INTRODUCTION

The Complex Ginzburg-Landau (CGL) equations are well known as the basic model of pattern formation in the various nonlinear dissipative media [1-3]. In the nonlinear physics area, CGL equations are universally applicable nonlinear dynamics model that introduces a cubic-quintic term to provide saturation for growth in any achievable physical system [4,5]. The CGL equations have been widely used in nonlinear optical system, where passively mode-locked laser systems and optical transmission lines are two of the massive important applications [6]. At present, the dynamics of spatial optical soliton in dissipative media is a hot research topic, which has great potential in all-optical switching pattern recognition and data processing [7]. Dissipative soliton is a kind of stable local structure in nonlinear non-equilibrium system [8], which needs to not only satisfy the balance of nonlinear and other effects, but also realize the balance of loss and gain. It is well known that the CGL equation with cubic-quintic (CQ) nonlinearity can support stable dissipative solitons in any dimension. Dissipative solitons based on the CQ CGL equation show many unique physical characteristics, including the formation of self-localized state or soliton in chaotic mode [9]. Dissipative soliton phenomenon has a wide range of applications in the fields of optics, photonics, biology and medicine. The viscosity (diffusion) term plays an important role in the transmission stability of dissipative solitons.

Many complex modes can be localized in the CQ CGL models, such as dissipative spatial soliton, vortex soliton, necklace ring soliton, bound state, and so on [10-13]. For example, Rosanov et al. recently studied the three-dimensional (3D) topological dissipative optical soliton in a uniform laser medium [10]. Akhmediev et al. studied the dissipative solitons with extreme peaks in the positive and negative dispersion regions [11]. The transition between coexisting waveforms of different types of steady-state and moving dissipative solitons is revealed in the CQ CGL model [12]. The two-dimensional (2D) CQ CGL model is used to research the new dynamic mechanism of dissipative vortexes supported by radial azimuthal potential (RAP) [13]. The viscosity (diffusion) term in the CGL model plays an important role in the transmission stability of dissipative solitons and bound states. Woo-Pyo Hong reported the existence conditions for stable stationary solitons in one-dimensional CQ CGL equation with a viscosity term [14]. Tabi et al. numerically studied modulational instability and pattern formation in deoxyribonucleic acid dynamics with viscosity in the discrete CGL equation [15]. Doelman et al. demonstrated that slow diffusion acts as a control mechanism that influences the (in)stability of the pulse in a real Ginzburg-Landau System [16] and revealed the existence and stability of pulse solutions in a system with interacting instability mechanisms in a CGL equation with a neutrally stable mode [17].

The application of spatial solitons in all-optical devices has been discussed extensively in conservative systems, given their particle-like properties in collision and interaction [18,19]. Recently, the particle-like behavior of dissipative solitons become a research hotspot. In particular, the dynamic behavior of the interaction between the dissipative soliton and the external potential has also received close attention. Three alternative outcomes of collisions were discovered between dissipative solitons or vortices [21,22]. Furthermore, the variation of the interaction between participating dissipative solitons and vortices has an important impact on the collisional outcomes by modulating relative phase [23,24]. The dissipative solitons and vortices, whose behave like classical-mechanical particles in weak external potentials, can penetrate the potential wall, even if the kinetic energy is smaller than the potential height, the scattering of the dissipative soliton was investigated by a potential wall and a kind of tunneling of the dissipative soliton was discovered [25]. By increasing the external potential to affect the waveform of the soliton, the evolutionary behavior of the soliton could be changed [26]. The CQ CGL equation with potential energy term is a model used to study far-non-equilibrium nonlinear phenomena in physical systems, including mode-locking and fiber laser nonlinear optical waveguide semiconductor devices Bose-Einstein condensate reaction-diffusion system, etc. [12,20,27]. Kochetov et al. studied the evolution of dissipative structures to regime with spontaneous transformation of the topological excitations and logic gates on stationary dissipative solitons in the form of the CGL equation with a potential term [28-29]. Based on the analysis of the high-order (3+1)-dimensional CQ CGL equation, the fourth, sixth, eighth and tenth dissipative bullet complexes were observed using the variational method and the Lyapunov method, and the scalar potential was analyzed to study the stability of optical photon bombs through the medium [30]. Based on the (3+1)-dimensional CQ CGL equation with lateral 2D trap potential, a passive mode-locked laser model based on gradient index nonlinear multimode fiber was studied [31]. Although many novel particle-like dynamical properties of dissipative soliton have been reported recently, but there are still many classical particle-like behaviors of dissipative soliton to be discovered and studied, such as harmonic motion, damped motion, circular motion and so on.

In this paper, an external V-shaped potential was introduced into the 2D CQ CGL model. The role of these potential wells in stabilizing dissipative soliton forms periodic oscillations or damped oscillations, corresponding to the absence/viscosity, respectively, just like simple harmonic motion or damped motion of particles. A detailed numerical study is carried on the dynamical phenomena of dissipative soliton simple harmonic motion or damped motion. In addition, the influence of gain/loss term on the simple harmonic motion and damped motion were theoretically studied.

II. THE MODEL

The 2D CQ CGL equation was considered in terms of nonlinear optics, as the evolution equation for the amplitude of electromagnetic wave in an active bulk optical medium [13,32,33]:

$$iu_z - i\delta \cdot u + (1/2 - i\beta)(u_{xx} + u_{yy}) + (1 - i\varepsilon)|u|^2 u - (\nu - i\mu)|u|^4 u = V(x,y)u, \tag{1}$$

where ν δ is the linear loss coefficient,

μ $\varepsilon > 0$

β

The dependence of the collision result on the relative phase can be fully observed under the following parameter values: $\mu = 1$, $\nu = 0.1$, δ =0.4, and ε =1.85.

The last term on the right-hand side of Eq. (1) introduces the V-shaped potential (VSP) that depends on the x coordinate. The analytical form of $V(x,y)$ is

$$V(x,y) = a\left|x\right|, \tag{2}$$

where a is the slope of the VSP. The desirable patterns of the refractive-index modulation in materials described by the CGL equation may induce an effective potential, which can be achieved by various techniques, such as optics induction [35] and writing by ultrashort laser pulse stream [36]. A stable soliton solution of the CQ CGL equation slides freely down the slope of VSP. The following isotropic ansatz for soliton solution in the general case was used [37]:

$$u = A(z)\exp\left\{-\frac{\left(x\text{-}x_0\right)^2 + y^2}{2w^2(z)} + ic(z)\left[\left(x\text{-}x_0\right)^2 + y^2\right] + i\phi(z)\right\}, \tag{3}$$

where, A, w, c, and ϕ represent the amplitude, width, wavefront curvature, and overall phase, respectively. x_0 is initial position of dissipative soliton. The periodic motion dynamics of a series of dissipative soliton were studied, and a new type of simple harmonic and damped motion of dissipative solitons was revealed. The viscosity term in Eq. (1), $\sim\beta$, plays an important role in generating damping effect for the simple harmonic motion of dissipative soliton. $\beta = 0$ (without viscosity) can make the soliton move freely.

In the following simulations, Eq. (1) was solved using the split-step Fourier method with typical transverse and longitudinal step sizes $\Delta x = \Delta y = 0.1$ and $\Delta z = 0.1$. The second-order derivative terms in x and y were solved with the periodic boundary conditions. Adopting this algorithm, we can successfully reproduce the corresponding results of the previous literature on both the dissipative system and the passive fiber, which can reflect reliability and correctness of this work.

III. Simple harmonic motion

Firstly, in the absence of a viscous term in Eq. (1), it is assumed that the dynamical characteristics of the dissipative soliton are freely tilted from x_0=20 along the VSP in the dissipative system. An obvious dynamic evolution of the periodic oscillating of the dissipative soliton near the centre of the VSP was observed in Fig. 1(a) at VSP' slope a=1, by propagating distance z = 100. The oscillating period was calculated as $z_p = 25.5$. The detailed influence of a on the oscillating period was studied. Figure 1(b) shows the displacement trajectories of the dissipative soliton with different slopes a = 0.25, 0.5 and 1, respectively. The trajectory of the periodic motion is a sine waveform, just like simple harmonic motion. The oscillating period z_p decreases significantly as the slope of VSP increases. The simulated relationship between the slope a and the period z_p is illustrated in Fig. 1(c). For periodic oscillating of particles, an important parameter is the momentum. After analyzing the inverse relationship between period of simple harmonic motion and the slope of the VSP, it is found that their relationship can be completely fitted by function:

$$Z_p = m / \sqrt{a}, \qquad (4)$$

where m = 25.38 is the fitting coefficient, the $R2$ of function fitting is 1. The property in Eq. (4) is consistent with the formula of the period of simple harmonic motion of a particle. Due to the refractive index modulation of the VSP along the *x-direction*, the dissipative soliton periodically oscillates along the *x-direction*. For dissipative system in Eq. (1), the x component of the momentum of the dissipative soliton can be expressed as follow [25,37]:

$$P_x = \left(-\frac{i}{2} \right) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left(u_x u^* - u_x^* u \right) dxdy, \quad (5)$$

The evolution of the momentum P_x was calculated by propagating z=100 in Fig. 1(d) at x_0=20 with different slopes a=0.25, 0.5 and 1, respectively. The momentum P_x also forms period oscillations, and its motion waveform is close to a triangular wave instead of sine wave, which is different from the momentum change of the simple harmonic motion of a particle. In addition, the maximum momentum $P_{\max}$ during oscillating will increase with increase of a. The relationship between the maximum momentum and a is shown in Fig. 1(e). It is also different from the simple harmonic motion of a physical particle.

For general simple harmonic motion of a physical particle, the period of the motion is independent of the amplitude. We numerically studied the influence of oscillating amplitude x_0 on the period and momentum of dissipative soliton's simple harmonic motion drove by the VSP. The evolutions of the dissipative soliton displacement were numerically simulated with different x_0 of 10, 20 and 30 at same slope of VSP a=1 [shown in Fig. 2(a)]. Obviously, the oscillating period increases as the oscillating amplitude x_0 increases, which is completely different from the simple harmonic motion of a physical particle. The relationship between oscillating period and x_0 is shown in Fig. 2(b). Figure 2(c) shows the evolutions of momentum with x_0 = 10, 20 and 30 at slope a=1. The variety amplitude of momentum P_x

$P_{\max}$ and x_0 is shown in Fig. 2(b). The trend of their relationship curve and the relationship is very similar to the relationship between oscillating period and x_0. If $P_{\max}$ and x_0 as potential energy directly related to kinetic energy and potential energy of physical particle, respectively, the characteristics of maximum momentum $P_{\max}$ increasing with oscillating amplitude x_0 is similar to the simple harmonic motion of a physical particle.

As a dissipative system, the stable soliton in CQ CGL model should satisfy the balance between energy gain and loss. Therefore, the effect of gain or loss coefficient on the simple harmonic motion of the dissipative soliton was considered. Figures 3(a) an (b) show the evolutions of simple harmonic motion trails with ε=2 and 1.85, respectively. The oscillating period is not affected by the change of the cubic-gain coefficient ε. In addition, the evolution of total amount of energy E carried was calculated [25,37]:

$$E(z)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left|u(x,y)\right|^2 dxdy, \tag{6}$$

which as a function of propagating distance z in Fig. 3(c). The energy E of dissipative soliton also changes periodically during simple harmonic motion, and its change period is half of the simple harmonic motion period. Comparing the evolution of E by propagating z=100 under different ε of 1.85, 1.9 and 2, the period of energy change will not be affected, but the energy of soliton will increase with the gain increases. The corresponding evolution of the momentum P_x under different ε was calculated by propagating z=100 in Fig. 3(d). Obviously, the evolutions of momentum are almost the same, only the maximum momentum increases slightly as increasing of the gain term ε. The reason is that the energies of dissipative soliton slightly raise.

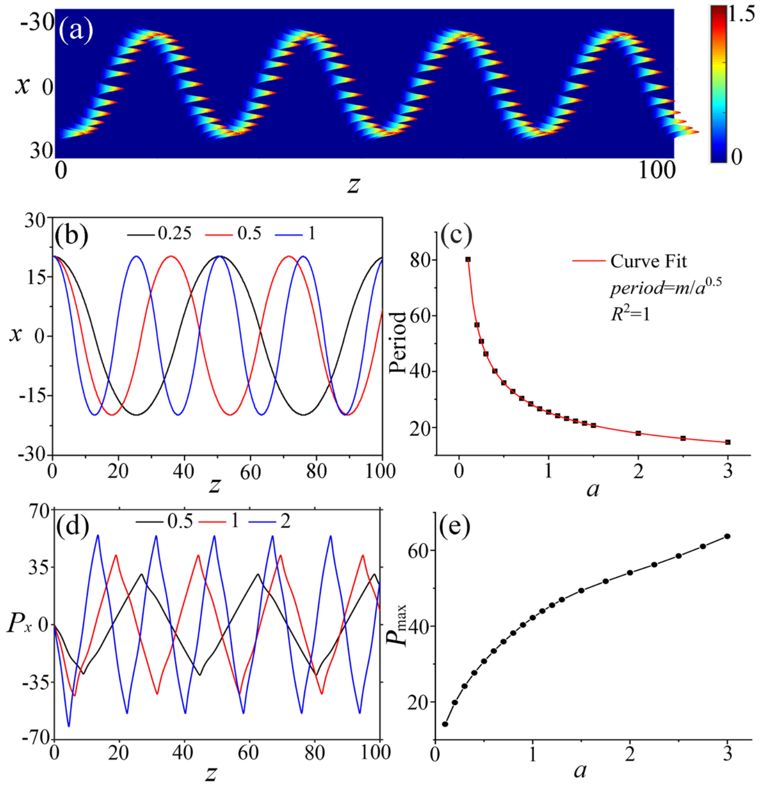

FIG. 1. (Color online). (a) Simple harmonic motion of dissipative soliton for a=1 and x_0 =20; (b) Evolutions of motion trails of dissipative soliton for a=0.25, 0.5, and 1 at x_0=20; (c) Relationship between Periods of simple harmonic motion with a at x_0=20; (d) evolutions of P_x for a=0.5, 1 and 2 at x_0=20; (e) Relationship between max of momentum $P_{\max}$ of simple harmonic motion and a.

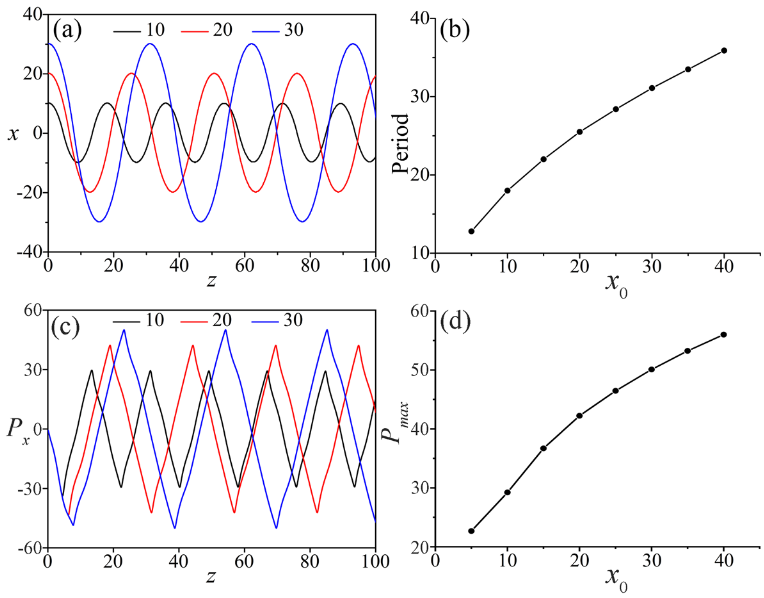

FIG. 2. (Color online). (a) Evolutions of simple harmonic motion trails for x_0 =10, 20, and 30, at a=1; (b)

of P_x for x_0 =10, 20, and 30, at a=1; (d) Relationship between $P_{\max}$ of simple harmonic motion with x_0 at a=1.

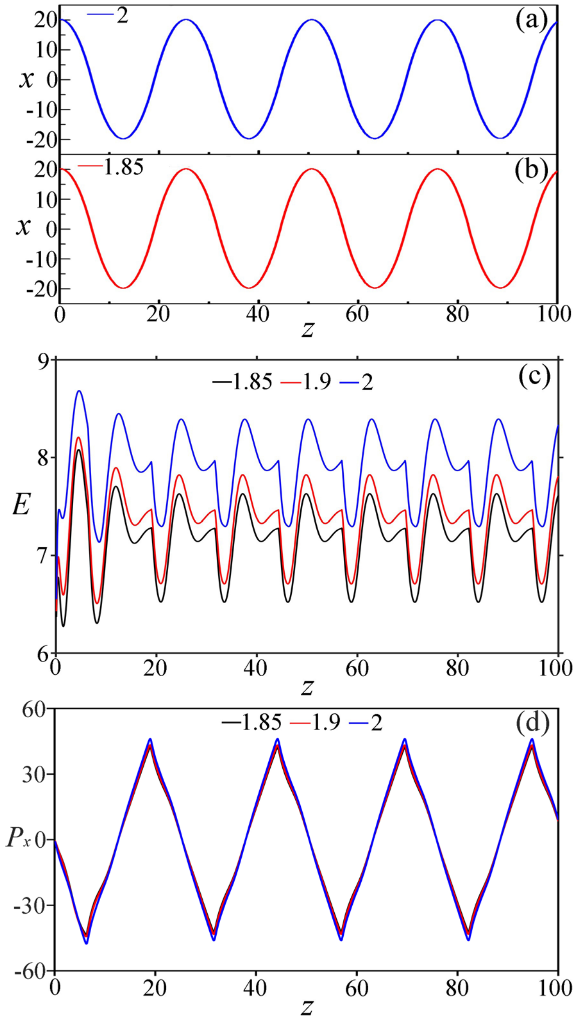

FIG. 3. (Color online). (a)-(b) Evolutions of simple harmonic motions trails at a=1 with ε=2 and 1.85; (c)-(d) Evolutions of energy E and momentum P_x at a=1 with ε=1.85, 1.9, and 2.

IV. Damped Motions

The spatial-diffusion term β in the Eq. (1), plays a special role in the CQ CGL model, which represents the viscosity that hinders the motion of dissipative solitons. In the absence of viscosity, a new simple harmonic motion of dissipative soliton was observed and investigated in the above section. In order to study the influence of the viscosity, an effective viscosity term β was added to the simple harmonic motion of dissipative soliton. Figures 4(a), (b) and (c) show the dynamical evolutions of dissipative solitons by propagating z=200, with the slope a=0.1 of the VSP at $\beta = 0$, 0.01, and 0.03, respectively. It clearly shows that by inducing the viscosity term, a significant damping effect was observed on the simple harmonic motion of dissipative soliton. Figure 4(d) shows the displacement trajectories of the dissipative soliton when z=200 with different viscosity terms $\beta = 0$, 0.01, 0.03 and 0.05, is propagated, respectively. As β increases from 0, the simple harmonic motion gradually changes from undamped to underdamped, and then to overdamped: for no viscosity with $\beta = 0$, the simple harmonic motion is undamped; for $\beta = 0.01$, the damping effect is not strong; for $\beta = 0.05$, the motion of dissipative soliton is close to overdamped state. In addition, since Fig. 2 reveals that the oscillation period decreases with the oscillation amplitude x_0, the oscillation period will decrease gradually during the damping motion. It can be seen from Fig. 4(d) that, as β increases, the decrease rate of the oscillation period becomes faster.

For different viscosity terms $\beta = 0$, 0.01, 0.03 and 0.05 of damped motion, the evolutions of energy E and momentum P_x of soliton are numerically calculated in Figs. 4(e) and 4(f), respectively. The energy E of soliton can also produce less damped vibration. With the end of the damped motion, the soliton energy tends to stabilize. But due to different β, the final stable soliton energy will be slightly different. The momentum P_x of soliton also forms a damped variant by propagating z. During damped motion, the evolution waveform of momentum gradually evolves from a triangular wave to a sine wave. Then, until the end of the damped vibration, the momentum is reduced to zero.

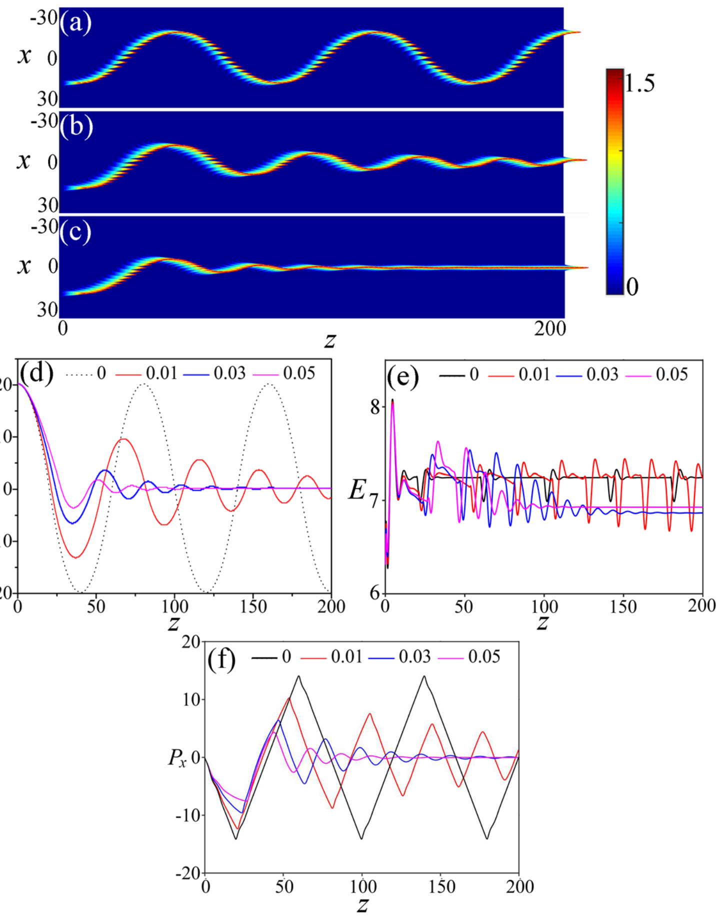

FIG. 4. (Color online). (a)-(c) Damped motions of solitons at $a = 0.1$ with $\beta = 0$, 0.01, and 0.03. (d)-(f) Evolutions of damped motions trails, energy E, and momentum P_x of solitons for at $a = 0.1$ with $\beta = 0$, 0.01, 0.03 and 0.05.

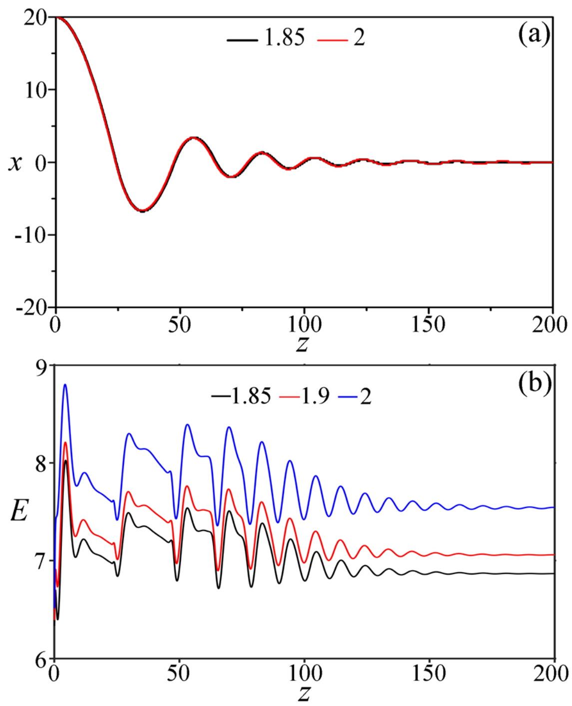

FIG. 5. (Color online). (a) Evolutions of damped motion trails at ε =1.85 and 2. (b) Evolutions of energy E at ε =1.85, 1.9, and 2.

Finally, the influence of gain or loss coefficient on the damped motion of dissipative soliton was studied. Specifically, Figure 5(a) shows the same evolutions of the damping motion trajectory when $\beta = 0.03$ with different cubic-gain coefficient ε = 1.85 and 2. It can prove that the damped motion state will not be affected by the various gains and losses of the dissipative system. Figure 5(b) shows the evolutions of energy E of solitons through propagation z = 200 with different ε of 1.85, 1.9 and 2. During damped motion, for different ε, the evolution law of energy E is almost the same. However, the total energy of soliton will increase slightly with the increase of ε.

Ⅴ. Conclusions

In summary, a systematic dynamical analysis of simple harmonic motion and damped motion of dissipative soliton in the two-dimensional CQ CGL equation with a VSP were conducted. Without the diffusion coefficient, the simple harmonic motion of dissipative solitons will occur near the center of the VSP, revealing the mechanism of the influence of the slope a of the VSP and oscillation amplitude x_0 on the period and momentum of the simple harmonic motion. Furthermore, the evolution mechanism of energy and momentum during simple harmonic motion and damped motion has been numerically studied. However, by adding a small diffusion term in the CQ CGL model, the viscosity will be acted on the simple harmonic motion of the dissipative solitons, resulting in the damped motion of dissipative solitons. The stronger the diffusion term, the more obvious the damping effect. In addition, the gain/loss coefficients (such as cubic-gain ε) in the CGL model, only affects the energy of dissipative solitons, not the dynamic states of simple harmonic motion and damped motion. These results will provide potential applications for the design of all-optical data processing schemes.

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