Fault Estimation and Fault Tolerant Control for Discrete-Time Dynamic Systems

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***Abstract*—In this paper, a novel discrete-time estimator is proposed, which is employed for simultaneous estimation of system states, and actuator/sensor faults in a discrete-time dynamic system. The existence of the discrete-time simultaneous estimator is mathematically proved. The systematic design procedure for the derivative and proportional observer gains is addressed, enabling the estimation error dynamics to be internally proper and stable, and robust against the effects from the process disturbances, measurement noises and faults. On the basis of the estimated fault signals and system states, a discrete-time fault-tolerant design approach is addressed, by which the system may recover the system performance when actuator/sensor faults occur. Finally, the proposed integrated discrete-time fault estimation and fault-tolerant control technique is applied to the vehicle lateral dynamics with real data, which demonstrates the effectiveness of the developed techniques.**

***Index Terms*—Discrete-time systems, fault estimation, fault tolerant control, robustness, vehicle lateral dynamics**

I. INTRODUCTION

ENGINEERING systems are usually safety-critical systems, as any faults in actuators, sensors and processes may lead to system performance degradation, system breakdown, economic loss, and even disastrous situations. Therefore, the reliability plays a crucial role in the system design and operation. The evident solution to the reliability is to add the redundancy of the system. Except for the hardware redundancy in some key components, information redundancy has gained more and more attention in both academic community and industries for the last four decades owing to the convenience for implementation and significant saving in the cost. The fruitful theoretic results produced by a variety of fault diagnosis methods such as model-based methods [1-5], signal based methods [6-8] and data-driven methods [9-11], and their applications in wind energy systems, robotic manipulators, power electronics, motor drive, power quality, vehicles and so forth [12-17], have been reported in the above mentioned references and the references therein. It is noted that all the above approaches can be unified within a framework from the viewpoint of data processing [18].

Generally, there are three tasks for fault diagnosis, that is, fault detection, fault isolation and fault identification. Fault detection is to find a fault at the very early stage and trigger an alarm. Fault isolation is to find out which component is being subjected to malfunction or deviation from its normal working status. Fault identification is to determine the size and shape of the fault concerned. It is noticed that fault estimation is an interesting and powerful technique, which may accomplish the tasks of the fault detection, fault isolation and fault identification within a step. The well-known fault estimation methods include adaptive fault estimation method [19, 20], sliding mode fault estimation approach [21, 22], proportional and integral (PI) and proportional and multiple-integral (PMI) observer method [23, 24]. Recently, descriptor observer approach was addressed by [25, 26] to simultaneously estimate system states and system faults, which much facilities fault tolerant control design. In [27], an integrated high-gain descriptor observer based fault diagnosis and fault-tolerant design method is proposed for a gas turbine engine system. The estimation accuracy can be ensured by selecting reasonable high-gains of the estimator to effectively attenuate the effects from the process disturbances. The fault tolerant design avoids the on-line actuator/sensor switching, enabling a satisfactory operation performance even when a fault occurs. However, the work in [25-27] is for continuous systems. It is evident that some fault estimation methods for continuous systems cannot be transplanted to discrete-time systems. In particular, there has not got a clue on how to derive a discrete-time high-gain descriptor simultaneous state/fault observer following the design way of that for continuous system. On the other hand, real-time monitoring and control are essentially on the basis of discrete-time dynamic systems. Recent developments on fault estimation and fault-tolerant control for discrete-time systems can be found in [28, 29]. It is worthy to point out the results reported were either focused on actuator faults [28] or sensor faults [29]. Moreover, sensor noises were not taken into account in [28], and measurement noises were assumed to be the same as the process disturbances in [29]. In [30], a discrete-time PI observer was addressed to estimate both input and output disturbances, where the disturbances were assumed to be in the same types and robustness issues were not taken into account. Therefore,

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the results [28-30] have a limit capacity for applications. This motivates us to reformulate fault estimation and fault tolerant design for discrete-time dynamic systems with multiple faults (including actuator faults and sensor faults) subjected to measurement noises and process disturbances where the measurement noises and process disturbances are allowed to be in different types.

In this paper, a novel simultaneous state and fault discrete-time estimator is proposed by synthesizing descriptor system theory and linear matrix inequality technique, enabling the internal properness and stability of the estimation error dynamics and robustness against the effects from process disturbances and faults. The fault-tolerant design method is then addressed by using actuator/sensor signal compensation. A vehicle dynamic system with real data is finally employed to demonstrate the effectiveness of the proposed methods. The symbols used in this paper are rather standard. R denotes the set of all real numbers; Z_+ denotes the set of all positive integers; A^- denotes the inverse of A ; A^+ denotes the generalized inverse of A ; A^T denotes the transpose of A ; $block-diag\ (A1, A2)$ denotes $\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$; $P > 0$ (or $P < 0$) indicates the symmetric matrix P is positive (or negative) definite; $|a|$ denotes the modulus or absolute value of the scalar a. $\|.\|$ denotes the standard norm symbol; L_2 is the Lebesgue space consisting of all discrete-time vector-valued function that are square-summable over Z_+; $\|z\|_2$ denotes the L_2 norm of a discrete-time signal z, which is defined as $\|z\|_2^2 = \sum_{k=0}^{\infty} z^T(k)z(k)$.

II. DISCRETE-TIME FAULT ESTIMATION

A. The Novel Discrete-Time State and Fault Estimator

Consider a discrete-time dynamic system subjected to actuator faults, sensor faults, process disturbances and sensor noises in the form of

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_a f_a(k) + B_d d(k) \\ y(k) = Cx(k) + Du(k) + D_a f_a(k) + D_s f_s(k) + \omega(t) \end{cases} \quad (1)$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ represents the control input vector, $y(k) \in R^p$ is the measured output vector, $d(k) \in R^{l_d}$ is the process disturbance vector, $\omega(k) \in R^p$ is the measurement noise, and $f_a(k) \in R^{l_a}$ and $f_s \in R^{l_s}$ are the actuator fault and sensor fault, respectively. The discrete-time instant k is a simplified representation of kT, where T is the sampling period.

In this section, a novel discrete-time estimator design technique is to be developed in order to simultaneously estimate the system state, actuator fault, sensor fault, and measurement noise, and to attenuate the process disturbance. For this purpose, we define

$$\begin{aligned} \Delta f_a(k) &= f_a(k+1) - f_a(k), \\ \Delta f_s(k) &= f_s(k+1) - f_s(k) \end{aligned} \quad (2)$$

and denote

$$x_e(k) = \begin{bmatrix} x(k) \\ f_a(k) \\ f_s(k) \\ \omega(k) \end{bmatrix}$$

$$d_{de}(k) = \begin{bmatrix} d(k) \\ \alpha f_a(k) + \Delta f_a(k) \\ \beta f_s(k) + \Delta f_s(k) \\ \omega(k) \end{bmatrix}$$

$$E_e = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & I_{l_a} & 0 & 0 \\ 0 & 0 & I_{l_s} & 0 \\ 0 & 0 & 0 & 0_{p\times p} \end{bmatrix}$$

$$A_e = \begin{bmatrix} A & B_a & 0 & 0 \\ 0 & (1-\alpha)I_{l_a} & 0 & 0 \\ 0 & 0 & (1-\beta)I_{l_s} & 0 \\ 0 & 0 & 0 & -I_p \end{bmatrix}$$

$$N_e = \begin{bmatrix} 0_{n\times p} \\ 0_{l_a\times p} \\ 0_{l_s\times p} \\ I_p \end{bmatrix} \qquad B_e = \begin{bmatrix} B \\ 0_{l_a\times m} \\ 0_{l_s\times m} \\ 0_{p\times m} \end{bmatrix}$$

$$B_{de} = \begin{bmatrix} B_d & 0 & 0 & 0 \\ 0 & I_{l_a} & 0 & 0 \\ 0 & 0 & I_{l_s} & 0 \\ 0 & 0 & 0 & I_p \end{bmatrix}$$

$$C_e = [C \quad D_a \quad D_s \quad I_p] \quad (3)$$

where α and β are scalars which are not equal; and $x_e(k) \in R^{n_e}$ is the extended state vector with the dimension of $n_e = n + l_a + l_s + p$. In this context, the disturbance signals and fault terms $d(k), \omega(k), f_a(k), f_s(k), \Delta f_a(k)$ and $\Delta f_s(t)$ are all assumed to be bounded in the sense of L_2 norm. As a result, the signal $d_{de}(k)$ is thus bounded in the sense of L_2 norm.

In terms of (1)-(3), an augmented descriptor system can be obtained as follows:

$$\begin{cases} E_e x_e(k+1) = A_e x_e(k) + B_e u(k) + B_{de}\, d_{de}(k) \\ y(k) = C_e x_e(k) + Du(k). \end{cases} \quad (4)$$

It is noted that the augmented descriptor state vector $x_e(k)$ is composed of the original system state $x(k)$, actuator fault $f_a(k)$, sensor fault $f_s(k)$, and measurement noise $\omega(k)$. As a result, we can get the simultaneous estimations of the original system state, actuator fault, sensor fault and measurement noise if we can construct an estimator to estimate the augmented state vector.

In this context, the discrete-time descriptor estimator can be constructed in the form of:

$$\begin{cases} S_e \eta(k+1) = (A_e - K_e C_e)\eta(k) + B_e u(k) \\ \qquad\qquad -N_e(y(k) - Du(k)) \\ \hat{x}_e(k) = \eta_e(k) + S_e^{-1} L_e(y(k) - Du(k)) \end{cases} \quad (5)$$

where $\eta(k) \in R^{n_e}$ is the descriptor state vector of the above dynamic system, $\hat{x}_e(k) \in R^{n_e}$ is the estimate of the augment state $x_e(k) \in R^{n_e}$, $S_e = E_e + L_e C_e$, and $L_e \in R^{n_e\times p}$ and $K_e \in R^{n_e\times p}$ are respectively the derivative gain and proportional gain of the estimator to be designed.

Theorem 1: If the pair (A, C) is observable, that is,

$$rank \begin{bmatrix} zI_n - A \\ C \end{bmatrix} = n \text{, for any complex number } z \quad (6a)$$

and there are scalars α and β satisfying

$$rank \begin{bmatrix} A + (\alpha-1)I_n & B_a \\ C & D_a \end{bmatrix} = n + l_a \quad (6b)$$

$$rank\begin{bmatrix} A+(\beta-1)I_n & 0 \\ C & D_s \end{bmatrix} = n + l_s \tag{6c}$$

where $\alpha \neq \beta$, there exists an estimator in the form of (5) such that the estimation error dynamics $e_e(k) = x_e(k) - \hat{x}_e(k)$ is internally stable for any k when $d_{de}(k) = 0$, that is, the error dynamics $e(k) \to 0$ as $k \to \infty$.

Proof:

(i) *The existence of L_e for the internal properness of the error dynamics.*

Since

$$rank\begin{bmatrix} E_e \\ C_e \end{bmatrix} = rank\begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & I_{l_a} & 0 & 0 \\ 0 & 0 & I_{l_s} & 0 \\ 0 & 0 & 0 & 0_{p\times p} \\ C & D_a & D_s & I_p \end{bmatrix}$$
$$= n + l_a + l_s + p$$
$$= n_e, \tag{7}$$

then there is a $L_e \in R^{n_e\times p}$ such that $S_e = E_e + L_e C_e$ is nonsingular. Specifically, we can select

$$L_e = \begin{bmatrix} 0_{n\times p} \\ 0_{l_a\times p} \\ 0_{l_s\times p} \\ M \end{bmatrix} \tag{8}$$

where $M \in R^{p\times p}$ is a nonsingular matrix. We can thus calculate

$$S_e^{-1} = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & I_{l_a} & 0 & 0 \\ 0 & 0 & I_{l_s} & 0 \\ -C & -D_a & -D_s & M^{-1} \end{bmatrix} \tag{9}$$

$$C_e S_e^{-1} L_e = \begin{bmatrix} 0 & 0 & 0 & M^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ M \end{bmatrix} = I_p \tag{10}$$

$$A_e S_e^{-1} L_e = -N_e. \tag{11}$$

Substituting the second equation into the first equation in (5) and using (10) and (11), one can obtain

$$\begin{aligned} &S_e\hat{x}_e(k+1) \\ &= S_e\eta(k+1) + L_e\big(y(k+1) - Du(k+1)\big) \\ &= (A_e - K_eC_e)\hat{x}_e(k) + K_e\big(y(k) - Du(k)\big) \\ &\quad + B_eu(k) + L_e\big(y(k+1) - Du(k+1)\big). \end{aligned} \tag{12}$$

Noting that $K_eC_ex_e(k) = K_e\big(y(k) - Du(k)\big)$ and $L_eC_ex_e(k+1) = L_e\big(y(k+1) - Du(k+1)\big)$, the first equation in (4) can be rewritten as

$$\begin{aligned} &S_ex_e(k+1) \\ &= (A_e - K_eC_e)x_e(k) + K_e\big(y(k) - Du(k)\big) \\ &\quad + B_eu(k) + L_e\big(y(k+1) - Du(k+1)\big) + B_{de}d_{de}(k). \end{aligned} \tag{13}$$

Letting $e_e(k) = x_e(k) - \hat{x}_e(k)$, and subtracting (12) from (13), we can obtain the error dynamic equation as follows:

$$S_ee_e(k+1) = (A_e - K_eC_e)e_e(k) + B_{de}d_{de}(k). \tag{14}$$

Since S_e is nonsingular, the error dynamic can be rewritten as

$$e_e(k+1) = S_e^{-1}(A_e - K_eC_e)e_e(k) + S_e^{-1}B_{de}d_{de}(k) \tag{15}$$

which indicates the error dynamics is internally proper.

(ii) *The existence of K_e for the internal stability of the error dynamics.*

Observe that for any complex number z,

$$\begin{aligned} &rank\begin{bmatrix} zI_{n_e} - S_e^{-1}A_e \\ C_e \end{bmatrix} \\ &= rank\begin{bmatrix} zE_e - A_e \\ C_e \end{bmatrix} \\ &= rank\begin{bmatrix} zI_n - A & -B_a & 0 & 0 \\ 0 & (z-1+\alpha)I_{l_a} & 0 & 0 \\ 0 & 0 & (z-1+\beta)I_{l_s} & 0 \\ 0 & 0 & 0 & I_p \\ C & D_a & D_s & I_p \end{bmatrix} \\ &= \begin{cases} rank\begin{bmatrix} zI_n - A \\ C \end{bmatrix} + l_a + l_s + p, \\ \qquad when\ z \neq 1-\alpha \neq 1-\beta \\ rank\begin{bmatrix} A+(\alpha-1)I_n & B_a \\ C & D_a \end{bmatrix} + l_s + p, \\ \qquad when\ z = 1-\alpha, z \neq 1-\beta \\ rank\begin{bmatrix} A+(\beta-1)I_n & 0 \\ C & D_s \end{bmatrix} + l_a + p, \\ \qquad when\ z = 1-\beta, z \neq 1-\alpha. \end{cases} \end{aligned} \tag{16}$$

Substitution (6a)-(6c) into (16) yields

$$rank\begin{bmatrix} zI_{n_e} - S_e^{-1}A_e \\ C_e \end{bmatrix} = n + l_a + l_s + p = n_e \tag{17}$$

indicating the pair $(S_e^{-1}A_e, C_e)$ is observable. Therefore there exists a gain $K_{e*} \in R^{n_e\times p}$ such that $S_e^{-1}A_e - K_{e*}C_e$ is internally stable. Therefore, K_e can be calculated as $K_e = S_eK_{e*}$ such that the error dynamics in (15) is internally stable.

As a result, when $d_{de}(k) = 0$, the error dynamics $e(k) \to 0$ as $k \to \infty$. This completes the proof.

Remark 1: The novelty of the estimator is to introduce the scalars α and β to ensure the simultaneous estimation capability of the original system states, actuator faults, sensor faults and measurement noises.

Remark 2: It is desired to find scalars α and β which satisfy (6b) and (6c) and have small sizes, for instance, $0 < |\alpha| < 1$ and $0 < |\beta| < 1$. In this case, the components $\alpha f_a(k)$ and $\beta f_s(k)$ in the disturbance/fault signal $d_e(k)$ may be reduced.

Remark 3: The matrix M in the derivative gain L_e in (8) provides more design freedom. The no-singularity of M can ensure that the matrix $S_e = E_e + L_eC_e$ is non-singular, leading to the properness of the estimation error dynamics. Furthermore, a high-gain matrix M may reduce the effect from the measurement noise which will be shown in the next subsection.

Remark 4: In order to find a proportional gain K_e such that the estimation error dynamics is internally stable, one can either use the eigenvalue assignment method or Lyapunov equation solving method. It is noticed that the error dynamic equation (15) is subjected to the disturbance/fault signal $d_{de}(k)$. Therefore there is an incentive to find a gain K_e not only to ensure the error dynamics to be stable, but also to attenuate the adverse effect from the disturbance/fault, which will be dealt with in the next subsection.

B. Discrete-Time Robust State and Fault Estimator

In this subsection, we will discuss how to design observer gains to attenuate the effect from the disturbance/fault signals to the estimation error dynamics, which is called *robust* observer design.

Let

$$N_{de} = \begin{bmatrix} B_d & 0 & 0 & 0 \\ 0 & I_{l_a} & 0 & 0 \\ 0 & 0 & I_{l_s} & 0 \\ -CB_d & -D_a & -D_s & I_p \end{bmatrix}, \omega_e = \begin{bmatrix} d(k) \\ \alpha f_a(k) + \Delta f_a(k) \\ \beta f_s(k) + \Delta f_s(k) \\ M^{-1}\omega(k) \end{bmatrix},$$

One can obtain

$$S_e^{-1} B_{de} d_{de}(k) = N_{de}\omega_e(k). \tag{18}$$

Form $\omega_e(k)$ in (18), one can see the effect from measurement noise can be reduced by selecting a high-gain constant matrix M. In order to further attenuate the effect from $\omega_e(k)$, the proportional gain K_e will play a key role.

From (15) and (18), the error dynamic equation can be rewritten as:

$$e_e(k+1) = S_e^{-1}(A_e - K_e C_e)e_e(k) + N_{de}\omega_e(k). \tag{19}$$

The plant (19) is internally stable if $\omega_e(k)$ is bounded and the matrix $S_e^{-1}(A_e - K_e C_e)$ is stable, i.e., the eigenvalues of the matrix $S_e^{-1}(A_e - K_e C_e)$ are within the unit circle. The design goal here is to ensure the estimation error dynamics in (19) to be robustly stable against the effect from the disturbance/fault signal $\omega_e(k)$, that is,

$$\|e_e\|_2 \le \gamma_o \|\omega_e\|_2 \tag{20}$$

Theorem 2: The estimation error dynamic system (19) is internally stable, and the robust performance index (20) is met if the following optimization problem is solvable:

$$\text{minimize } \gamma_o$$

$$\text{subject to } \; 0 < \gamma_o, 0 < P_e \in R^{n_e \times n_e},\; Y_e \in R^{n_e \times p}, \text{and}$$

$$\begin{bmatrix} -P_e + I & 0 & A_e^T S_e^{-T} P_e - C_e^T Y_e^T \\ 0 & -\gamma_o^2 I & N_{de}^T P_e \\ P_e S_e^{-1} A_e - Y_e C_e & P_e N_{de} & -P_e \end{bmatrix} < 0 \tag{21}$$

where $S_e = E_e + L_e C_e$ and L_e is in the form of (8).

The gain K_e can thus be calculated as $K_e = S_e P_e^{-1} Y_e$.

Proof.

(i). Internal stability.

Noticing that $Y_e = P_e S_e^{-1} K_e$ and pre-multiplying and post-multiplying $block - diag(I, I, P_e^{-1})$ on both sides of (21), one has equivalently

$$\begin{bmatrix} -P_e + I & 0 & (A_e - K_e C_e)^T S_e^{-T} \\ 0 & -\gamma_o^2 I & N_{de}^T \\ S_e^{-1}(A_e - K_e C_e) & N_{de} & -P_e^{-1} \end{bmatrix} < 0. \tag{22}$$

Let

$$\Omega_e = \begin{bmatrix} \Omega_{e11} & (A_e - K_e C_e)^T S_e^{-T} P_e N_{de} \\ N_{de}^T P_e S_e^{-1}(A_e - K_e C_e) & N_{de}^T P_e N_{de} - \gamma_o^2 I \end{bmatrix},$$

$$\Omega_{e11} = (A_e - K_e C_e)^T S_e^{-T} P_e S_e^{-1}(A_e - K_e C_e) - P_e + I. \tag{23}$$

Applying the well-known Schur complement theory [31] to (22), one can obtain:

$$\begin{aligned} 0 > &\begin{bmatrix} -P_e + I & 0 \\ 0 & -\gamma_o^2 I \end{bmatrix} \\ &- \begin{bmatrix} (A_e - K_e C_e)^T S_e^{-T} \\ N_{de}^T \end{bmatrix} (-P_e^{-1})^{-1} [S_e^{-1}(A_e - K_e C_e) \quad N_{de}] \\ &= \Omega_e \end{aligned} \tag{24}$$

Define a Lyapunov function as

$$V_o(e_e(k)) = e_e^T(k) P_e e_e(k). \tag{25}$$

For $\omega_e(k) = 0$, using (19) and (25), one has

$$\begin{aligned} &\Delta V_o(e_e(k)) \\ &= e_e^T(k+1) P_e e_e(k+1) - e_e^T(k) P_e e_e(k) \\ &= e_e^T(k) \Gamma_e e_e(k) \end{aligned} \tag{26}$$

where

$$\Gamma_e = (A_e - K_e C_e)^T S_e^{-T} P_e S_e^{-1}(A_e - K_e C_e) - P_e \tag{27}$$

From (23) and (24), it is evident that $\Gamma_e < 0$. Therefore, one has

$$\Delta V_o(e_e(k)) \le -\epsilon_o \|e_e(k)\|^2, \text{ when } \omega_e(k) = 0 \tag{28}$$

where $\epsilon_o = \lambda_{min}(-\Gamma_e)$.

As a result, the error dynamic system (19) is internally stable when $\omega_e(k) = 0$.

(ii). Robust performance.

Now we consider the case when $\omega_e(k) \ne 0$. In terms of (19) and (25), one has

$$\begin{aligned} &\Delta V_o(e_e(k)) \\ &= e_e^T(k+1) P_e e_e(k+1) - e_e^T(k) P_e e_e(k) \\ &= e_e^T(k)[(A_e - K_e C_e)^T S_e^{-T} P_e S_e^{-1}(A_e - K_e C_e) - P_e + I] e_e(k) \\ &\quad + 2 e_e^T(k)(A_e - K_e C_e)^T S_e^{-T} P_e N_{de} \omega_e(k) \\ &\quad + \omega_e^T(k)(N_{de}^T P_e N_{de} - \gamma_o^2 I)\omega_e(k) \\ &\quad - e_e^T(k) e(k) + \gamma_o^2 \omega_e^T(k)\omega_e(k) \\ &= (e_e^T(k) \quad \omega_e^T(k)) \Omega_e (e_e^T(k) \quad \omega_e^T(k))^T - e_e^T(k) e(k) \\ &\quad + \gamma_o^2 \omega_e^T(k)\omega_e(k) \end{aligned} \tag{29}$$

where Ω_e is given in (23).

Substitution (23) into (29) yields

$$\Delta V_o(e_e(k)) \le -e_e^T(k) e(k) + \gamma_o^2 \omega_e^T(k)\omega_e(k). \tag{30}$$

Under zero initial conditions, it is followed from (30)

$$\begin{aligned} 0 &\le V_o(e_e(n+1)) \\ &\le -\textstyle\sum_{k=0}^n e_e^T(k) e_e(k) + \gamma_o^2 \sum_{k=0}^n \omega_e^T(k)\omega_e(k) \end{aligned} \tag{31}$$

which implies $\|e_e\|_2 \le \gamma_o \|\omega_e\|_2$. This completes the proof.

C. Design Procedure of State and Fault Estimator

The design procedure of the proposed discrete-time estimator can be summarized as follows.

Procedure 1: Discrete-time state and fault estimation

(i). Select the scalars $\alpha = diag(\alpha_1, \; \alpha_2, \; \cdots \; \alpha_{l_a})$ and $\beta = diag(\beta_1, \; \beta_2, \; \cdots \; \beta_{l_s})$ such that (6b) and (6c) are satisfied where α and β have reasonably small amplitudes. For instance, α_i and β_j may be selected as $0 < |\alpha_i| < 1, i = 1, 2, \dots, l_a$ and $0 < |\beta_j| < 1,\; j = 1, 2, \dots, l_s$ such that (6b) and (6c) are satisfied.

(ii). Calculate the augmented matrices E_e, A_e, B_e, C_e, N_e and B_{de} in terms of (3). Therefore, the augmented plant (4) has been formed.

(iii). Select the derivative gain L_e of the estimator in the form of (8), where the matrix M is chosen as a reasonably high-gain nonsingular matrix. For instance, the matrix M can be selected as θI_p, where $\theta > 1$. As a result, the matrix $S_e = E_e + L_e C_e$ can be ensured to be nonsingular and the effect of the measurement noise to the error dynamics can be attenuated to some extent.

(iv). Calculate the modified disturbance/fault matrix N_{de} in (18) and compute $K_e = S_e P_e^{-1} Y_e$, where P_e and Y_e can be obtained by solving the linear matrix inequality (21).

(v). Build the estimator (5) where the parameters are obtained from the steps (i)-(iv), and implement the estimation to get the estimated vector $\hat{x}_e(k)$. As a result, the estimated signals for system state, actuator fault, sensor fault, and measurement noise can be readily formulated as follows:

$$\begin{cases} \hat{x}(k) = [I_n \quad 0_{n\times l_a} \quad 0_{n\times l_s} \quad 0_{n\times p}]\hat{x}_e(k) \\ \hat{f}_a(k) = [0_{l_a\times n} \quad I_{l_a} \quad 0_{l_a\times l_s} \quad 0_{l_a\times p}]\hat{x}_e(k) \\ \hat{f}_s(k) = [0_{l_s\times n} \quad 0_{l_s\times l_a} \quad I_{l_s} \quad 0_{l_s\times p}]\hat{x}_e(k) \\ \hat{\omega}(k) = [0_{p\times n} \quad 0_{p\times l_a} \quad 0_{p\times l_s} \quad I_p]\hat{x}_e(k). \end{cases} \tag{32}$$

III. Discrete-Time Fault-Tolerant Design

A. *Fault Compensation-Based Fault-Tolerant Method*

On the basis of the estimated signals in the previous section, we will deal with fault-tolerant design issues in this section. Using the estimated state vector $\hat{x}_e(k)$, a closed-loop feedback control strategy can be employed:

$$u(k) = -F_e\hat{x}_e(k), \quad F_e \in R^{m\times n_e} \tag{33}$$

where $F_e = [F \quad F_a \quad F_s \quad F_\omega]$; and $F \in R^{m\times n}$, $F_a \in R^{m\times l_a}$, $F_s \in R^{m\times l_s}$ and $F_\omega \in R^{m\times p}$ are the gain matrices for estimated state, estimated actuator fault signal, estimated sensor fault signal and estimated measurement noise signal, respectively.

In terms of (1) and (33) and noticing that $\hat{x}_e(k) = x_e(k) - e_e(k)$, the closed-loop dynamic plant can be described by

$$\begin{aligned} &x(k+1) \\ &= Ax(k) - BF_e\hat{x}_e(k) + B_a f_a(k) + B_d d(k) \\ &= (A - BF)x(k) + (B_a - BF_a)f_a(k) - BF_s f_s(k) \\ &\qquad -BF_\omega\omega(k) + BF_e e_e(k) + B_d d(k), \end{aligned} \tag{34}$$

and

$$\begin{aligned} &y(k) \\ &= Cx(k) - DF_e\hat{x}_e(k) + D_a f_a(k) + D_s f_s(k) + \omega(t) \\ &= (C - DF)x(k) + (D_a - DF_a)f_a(k) \\ &\quad +(D_s - DF_s)f_s(k) + (I - DF_\omega)\omega(k) + DF_e e_e(k). \end{aligned} \tag{35}$$

Suppose

$$rank\begin{bmatrix} B_a & B \\ D_a & D\end{bmatrix} = rank\begin{bmatrix} B \\ D\end{bmatrix} \tag{36}$$

and select

$$F_a = \begin{bmatrix} B \\ D\end{bmatrix}^+ \begin{bmatrix} B \\ D\end{bmatrix}. \tag{37}$$

Therefore one has

$$B_a - BF_a = 0, \;\; D_a - DF_a = 0. \tag{38}$$

Furthermore, we choose

$$F_s = 0_{m\times l_s}, \;\; F_\omega = 0_{m\times p}. \tag{39}$$

As a result, the system can be written as

$$\begin{cases} x(k+1) = (A - BF)x(k) + BF_e e_e(k) + B_d d(k) \\ y(k) = (C - DF)x(k) + D_s f_s(k) + \omega(k) + DF_e e_e(k). \end{cases} \tag{40}$$

From (40), the effects from the actuator faults to the closed-loop plant have been removed successfully provided that the estimation error $e_e(k)$ is small enough. The technique above is called actuator fault compensation, which is employed to remove the adverse effects from actuator faults to the system dynamics and output.

However, the output $y(k)$ is still subjected to the effect from the sensor fault and measurement noise. In order to eliminate the effect caused by the sensor fault and measurement noise, we implement the sensor fault signal compensation as follows:

$$\begin{aligned} &y_s(k) \\ &= y(k) - D_s\hat{f}_s(k) - \hat{\omega}(k) \\ &= (C - DF)x(k) + DF_e e_e(k) + D_{sp}e_e(k) \\ &= (C - DF)x(k) + (D_{sp} + DF_e)e_e(k). \end{aligned} \tag{41}$$

where $D_{sp} = [0_{p\times n} \quad 0_{p\times l_a} \quad D_s \quad I_p]$.

From (41), one can see the sensor signal compensation is carried out by using the actual output to subtract the estimated signals of sensor faults and measurement noises. In the new output $y_s(k)$, the effects from the sensor faults and measurement noises are successfully removed/offset.

According to (40) and (41), the closed-loop after the fault compensation can be described by

$$\begin{cases} x(k+1) = (A - BF)x(k) + BF_e e_e(k) + B_d d(k) \\ y_s(k) = (C - DF)x(k) + (D_{sp} + DF_e)e_e(k). \end{cases} \tag{42}$$

Remark 5: From (42), the effect from actuator faults, sensor faults and measurement noise to the system dynamics and output have been removed via the actuator and sensor signal compensations. The state-feedback gain F can be employed to stabilize the system, and attenuate the effect from process disturbance, which will be investigated by the following two theorems.

Theorem 3: The closed-loop system (42) is internally stable, and satisfies the following robust performance index

$$(\|y_s\|_2)^2 \le \gamma_{c1}^2(\|d\|_2)^2 + \gamma_{c2}^2(\|e_e\|_2)^2 \tag{43}$$

if the following *sequential optimization* problems are solvable:

(a). minimize γ_{c1}

subject to $0 < \gamma_{c1}, 0 < X \in R^{n\times n}, Y \in R^{m\times n}$, and

$$\begin{bmatrix} -X & 0 & XA^T - Y^TB^T & XC^T - Y^TD^T \\ 0 & -\gamma_{c1}^2 I & B_d^T & 0 \\ AX - BY & B_d & -X & 0 \\ CX - DY & 0 & 0 & -I \end{bmatrix} < 0. \tag{44}$$

(b). minimize γ_{c2}

subject to $0 < \gamma_{c2}$, and

$$\begin{bmatrix} -X & 0 & XA^T - Y^TB^T & XC^T - Y^TD^T & 0 \\ 0 & -\gamma_{c1}^2 I & B_d^T & 0 & 0 \\ AX - BY & B_d & -X & 0 & BF_e \\ CX - DY & 0 & 0 & -I & D_{sp} + DF_e \\ 0 & 0 & F_e^TB^T & D_{sp}^T + F_e^TD^T & -\gamma_{c2}^2 I \end{bmatrix} < 0 \tag{45}$$

where the parameters X, Y and γ_{c1} are obtained by solving (44); $F_e = [F \quad F_a \quad F_s \quad F_\omega]$, F_a, F_s and F_ω are given in (37) and (39), respectively; and $F = YX^{-1}$.

Proof.

(i). Internal stability.

We consider the case when $d(k) = 0$, and $\omega_e(k) = 0$.

Noticing that $F = YX^{-1}$, and pre-multiplying and post-multiplying $block - diag(X^{-1}, I, X^{-1}, I)$ on both sides of

(44), and letting $Q = X^{-1}$, one has

$$\begin{bmatrix} -Q & 0 & (A-BF)^T Q & (C-DF)^T \\ 0 & -\gamma_{c1}^2 I & B_d^T Q & 0 \\ Q(A-BF) & QB_d & -Q & 0 \\ C-DF & 0 & 0 & -I \end{bmatrix} < 0 \quad (46)$$

Applying the Schur complement to (46), one has

$$\begin{bmatrix} \Theta_{c11} & (A-BF)^T Q B_d \\ B_d^T Q^T (A-BF) & B_d^T Q B_d - \gamma_{c1}^2 I \end{bmatrix} < 0 \quad (47)$$

where

$$\Theta_{c11} = \underbrace{(A-BF)^T Q (A-BF) - Q}_{\Gamma_c} + (C-DF)^T (C-DF) \quad (48)$$

It is evident that $\Gamma_c < 0$ in terms of (47) and (48).

Define a Lyapunov function as

$$V_c(x(k)) = x^T(k) Q x(k). \quad (49)$$

From (42) and (49), one has

$$\begin{aligned} &\Delta V_c(x(k)) \\ &= x(k+1)Qx(k+1) - x(k)Qx(k) \\ &= x^T(k)\Gamma_c x(k) + 2x^T(k)(A-BF)^T QBF_e e_e(k) \\ &\quad + e_e^T(k)(BF_e)^T Q(BF_e)e_e(k) \\ &\le -\epsilon_c \|x(k)\|^2 + 2x^T(k)(A-BF)^T QBF_e e_e(k) \\ &\quad + e_e^T(k)(BF_e)^T Q(BF_e)e_e(k) \end{aligned} \quad (50)$$

where $\epsilon_c = \lambda_{min}(-\Gamma_c)$.

Let

$$V_{oc}(x(k)) = V_c(x(k)) + \theta V_o(e_e(k)). \quad (51)$$

From (28), (50) and (51), one has

$$\begin{aligned} \Delta V_{oc}(x(k)) &= \Delta V_c(x(k)) + \theta \Delta V_o(e_e(k)) \\ &\le -\epsilon_c \|x(k)\|^2 + \epsilon_x \|x(k)\| \|e_e(k)\| \\ &\quad + \epsilon_e \|e_e(k)\|^2 - \theta \epsilon_o \|e_e(k)\|^2 \end{aligned} \quad (52)$$

where

$$\epsilon_x = 2\|(A-BF)^T QBF_e\| \quad (53)$$

$$\epsilon_e = \|Q\| \|BF_e\|^2 \quad (54)$$

Selecting

$$\theta \ge \frac{\epsilon_x^2 + \epsilon_c \epsilon_e}{\epsilon_c \epsilon_o}, \quad (55)$$

it is followed from (52):

$$\Delta V_{oc}(x(k)) \le -\frac{\epsilon_c}{2}\|x(k)\|^2 - \frac{\theta}{2}\left(\epsilon_o - \frac{\epsilon_e}{\theta}\right)\|e_e(k)\|^2 \quad (56)$$

which indicates $e_e(k) \to 0, x(k) \to 0$ as $k \to \infty$ for $d(k) = 0$ and $\omega_e(k) = 0$.

(2). Robust performance index.

Now we consider the case when $\omega_e(k) \neq 0$ and $d(k) \neq 0$.

Noticing that $F = YX^{-1}$, and pre-multiplying and post-multiplying $block-diag(X^{-1}, I, X^{-1}, I, I)$ on both sides of (45), and letting $Q = X^{-1}$, one has

$$\begin{bmatrix} -Q & 0 & (A-BF)^T Q & (C-DF)^T & 0 \\ 0 & -\gamma_{c1}^2 I & B_d^T Q & 0 & 0 \\ Q(A-BF) & QB_d & -Q & 0 & QBF_e \\ C-DF & 0 & 0 & -I & D_{sp} + DF_e \\ 0 & 0 & F_e^T B^T Q & D_{sp}^T + F_e^T D^T & -\gamma_{c2}^2 I \end{bmatrix} < 0. \quad (57)$$

Let

$$W = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 \end{bmatrix}. \quad (58)$$

Pre-multiplying W^T and post-multiplying W on the left-hand side and right-hand side of (57), respectively, one can obtain:

$$\begin{bmatrix} -Q & 0 & 0 & (A-BF)^T Q & (C-DF)^T \\ 0 & -\gamma_{c1}^2 I & 0 & B_d^T Q & 0 \\ 0 & 0 & -\gamma_{c2}^2 I & F_e^T B^T Q & D_{sp}^T + F_e^T D^T \\ Q(A-BF) & QB_d & QBF_e & -Q & 0 \\ C-DF & 0 & D_{sp} + DF_e & 0 & -I \end{bmatrix} < 0. \quad (59)$$

Applying the Schur complement to (59), one has

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{12}^T & \Pi_{22} & \Pi_{23} \\ \Pi_{13}^T & \Pi_{23}^T & \Pi_{33} \end{bmatrix} < 0 \quad (60)$$

where

$$\begin{aligned} \Pi_{11} &= (A-BF)^T Q(A-BF) + (C-DF)^T (C-DF) - Q \\ \Pi_{12} &= (A-BF)^T QB_d \\ \Pi_{13} &= (A-BF)^T QBF_e + (C-DF)^T (D_{sp} + DF_e) \\ \Pi_{22} &= B_d^T QB_d - \gamma_{c1}^2 I \\ \Pi_{23} &= B_d^T QBF_e \\ \Pi_{33} &= F_e^T B^T QBF_e + (D_{sp}^T + F_e^T D^T)(D_{sp} + DF_e) - \gamma_{c2}^2 I \end{aligned}$$

From (42), (49) and (60), one has

$$\begin{aligned} &\Delta V_c(x(k)) \\ &= V_c(x(k+1)) - V_c(x(k)) \\ &= x(k+1)Qx(k+1) - x(k)Qx(k) \\ &= [(A-BF)x(k) + B_d d(k) + BF_e e_e(k)]^T Q \\ &\times [(A-BF)x(k) + B_d d(k) + BF_e e_e(k)] - x^T(k)Qx(k) \\ &+ [(C-DF)x(k) + (D_{sp} + DF_e)e_e(k)]^T [(C-DF)x(k) \\ &+ (D_{sp} + DF_e)e_e(k)] - \gamma_{c1}^2 d^T(k)d(k) - \gamma_{c2}^2 e_e^T(k)e_e(k) \\ &- y_s^T(k)y_s(k) + \gamma_{c1}^2 d^T(k)d(k) + \gamma_{c2}^2 e_e^T(k)e_e(k) \\ &= (x^T(k) \quad d^T(k) \quad e_e^T(k))\Pi(x^T(k) \quad d^T(k) \quad e_e^T(k))^T \\ &- y_s^T(k)y_s(k) + \gamma_{c1}^2 d^T(k)d(k) + \gamma_{c2}^2 e_e^T(k)e_e(k) \\ &\le -y_s^T(k)y_s(k) + \gamma_{c1}^2 d^T(k)d(k) + \gamma_{c2}^2 e_e^T(k)e_e(k). \end{aligned} \quad (61)$$

Under zero initial conditions, it is followed from (61)

$$\begin{aligned} 0 &\le V_c(x(n+1)) \\ &\le -\sum_{k=0}^{n} y_s^T(k)y_s(k) + \gamma_{c1}^2 \sum_{k=0}^{n} d^T(k)d(k) \\ &\quad + \gamma_{c2}^2 \textstyle\sum_{k=0}^{n} e_e(k)e_e(k). \end{aligned} \quad (62)$$

From (20) and (62), one has

$$(\|y_s\|_2)^2 \le \gamma_{c1}^2 (\|d\|_2)^2 + \gamma_{c2}^2 (\|e_e\|_2)^2. \quad (63)$$

This completes the proof.

Remark 6: In Theorem 3, the state-feedback gain F is designed to mainly attenuate the effect from the process disturbance $d(k)$ to the dynamic system (42). However, the design of F above seems not to have essential contribution in attenuating the effect from estimation error to dynamic system. It is reasonable for this kind of design if the error dynamics $e_e(k)$ has been made sufficiently small against the disturbance/fault signal $\omega_e(k)$ by the design of the estimator gains K_e and L_e shown in Section II. Nevertheless, we will further discuss how to simultaneously attenuate $d(k)$ and $e_e(k)$ during the design of the state-feedback gain F.

Let

$$e_{Fe(k)} = \begin{bmatrix} F_e e_e(k) \\ e_e(k) \end{bmatrix},\ B_{Fe} = [B \quad 0_{n\times n_e}],$$
$$D_{Fe} = [D \quad D_{sp}]. \tag{64}$$

The system (42) can be written as

$$\begin{cases} x(k+1) = (A - BF)x(k) + B_d d(k) + B_{Fe} e_{Fe}(k) \\ y_s(k) = (C - DF)x(k) + D_{Fe} e_{Fe}(k). \end{cases} \tag{65}$$

Theorem 4: The closed-loop system (65) is internally stable, and satisfies the following robust performance index

$$(\|y_s\|_2)^2 \le \gamma_{m1}^2(\|d\|_2)^2 + \gamma_{m2}^2(\|e_{Fe}\|_2)^2 \tag{66}$$

if there exists scalars γ_{m1} and γ_{m2}, a positive define symmetric matrix $X \in R^{n\times n}$ and a matrix $Y \in R^{m\times n}$ such that

$$\begin{bmatrix} -X & 0 & XA^T - Y^TB^T & XC^T - Y^TD^T & 0 \\ 0 & -\gamma_{m1}^2 I & B_d^T & 0 & 0 \\ AX - BY & B_d & -X & 0 & B_{Fe} \\ CX - DY & 0 & 0 & -I & D_{Fe} \\ 0 & 0 & B_{Fe}^T & D_{Fe}^T & -\gamma_{m2}^2 I \end{bmatrix} < 0 \tag{67}$$

The feedback gain can thus be calculated as $F = YX^{-1}$.

Proof. This proof is similar to Theorem 3, which is omitted for the limit of space.

Remark 7: In Theorem 4, the state-feedback gain F is designed to attenuate the effect from the process disturbance $d(k)$ and the estimation error dynamics $e_e(k)$ to the dynamic system (65).

B. Design Procedure of Fault-Tolerant Control

The design procedure of the proposed discrete-time fault-tolerant controller can be summarized as follows.

Procedure 2: Discrete-time fault-tolerant control

(i). Select $F_s = 0_{m\times l_s}$, $F_\omega = 0_{m\times p}$, and calculate $F_a = \begin{bmatrix} B \\ D \end{bmatrix}^+ \begin{bmatrix} B \\ D \end{bmatrix}$.

(ii). Solve the LMIs (44) and (45) (or solve the LMI (67)) to get $X\, and\, Y$, leading to the state-feedback gain $F = YX^{-1}$.

(iii). Apply the control law $u(k) = -F_e\hat{x}_e(k)$, where $F_e = [F \quad F_a \quad F_s \quad F_\omega]$ to implement actuator fault signal compensation.

(iv). Implement sensor fault signal compensation as follows:

$$y_s(k) = y(k) - D_s\hat{f}_s(k) - \hat{\omega}(k)$$

where $\hat{f}_s(k)$ and $\hat{\omega}(k)$ are the estimated signal of the sensor fault and measurement noise, which are obtained in Procedure 1 of Section II.

IV. Fault Estimation and Fault-Tolerant Deign for Vehicle Lateral Dynamics

Vehicle lateral dynamics plays a key role in the stability, safety and maneuverability of the vehicle. The vehicle dynamics can be modelled as the second order system, which is formulated as follows:

$$\begin{cases} \begin{bmatrix} \beta_s(k+1) \\ r_y(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{c_{av}+c_{ah}}{mv_{ref}} & \frac{l_hc_{ah}-l_vc_{av}}{mv_{ref}^2} - 1 \\ \frac{l_hc_{ah}-l_vc_{av}}{I_z} & -\frac{l_v^2c_{av}+l_h^2c_{ah}}{I_zv_{ref}} \end{bmatrix}}_{A} \begin{bmatrix} \beta_s(k) \\ r_y(k) \end{bmatrix} \\ \quad + \underbrace{\begin{bmatrix} \frac{c_{av}}{mv_{ref}} \\ \frac{l_vc_{av}}{I_z} \end{bmatrix}}_{B} \delta_l(k) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_d} d(k) \\ \begin{bmatrix} \alpha_y(k) \\ r_y(k) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{c_{av}+c_{ah}}{m} & \frac{l_hc_{ah}-l_vc_{av}}{mv_{ref}} \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} \beta_s(k) \\ r_y(k) \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{c_{av}}{m} \\ 0 \end{bmatrix}}_{D} \delta_l(k) \end{cases} \tag{68}$$

where $\beta_s(k)$ denotes the vehicle side slip angle, $r_y(k)$ is the yaw rate, $\delta_l(k)$ is the steering wheel angle as the input, $\alpha_y(k)$ is the lateral acceleration, v_{ref} is the vehicle reference velocity, m is the total mass, c_{av} is the front tire cornering stiffness, c_{ah} is the rear tire cornering stiffness; l_v is the distance from the vehicle centre of the gravity to the front axle, l_h is the distance from the vehicle centre of the gravity to the rear axle, l_z is the moment of the inertia about the z-axis of the vehicle. In addition, $d(k)$ is the process disturbance, denoted by

$$d(k) = -\frac{g}{v_{ref}\sin(\alpha_x)}$$

where g is the gravity constant, and α_x is the road bank angle.

When the vehicle speed is 150km/hour and the sampling time is 0.01s, the discrete-time dynamic model can be described as follows.

$$\begin{cases} \begin{bmatrix} \beta_s(k+1) \\ r_y(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 0.9617 & -0.0091 \\ 0.4328 & 0.9544 \end{bmatrix}}_{A} \begin{bmatrix} \beta_s(k) \\ r_y(k) \end{bmatrix} + \\ \quad \underbrace{\begin{bmatrix} 0.009586 \\ 0.3692 \end{bmatrix}}_{B} \delta_l(k) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_d} d(k) \\ \begin{bmatrix} \alpha_y(k) \\ r_y(k) \end{bmatrix} = \underbrace{\begin{bmatrix} -153.9 & 2.413 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} \beta_s(k) \\ r_y(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 48.07 \\ 0 \end{bmatrix}}_{D} \delta_l(k) \end{cases} \tag{69}$$

(a). Robust fault estimator design.

Here we consider the scenario when the actuator of the steering angle and the sensor of the lateral acceleration both have faults. The actuator fault occurs at 50s with 80% offset of the input signal. The acceleration sensor fault happens at 10s with the slope rate -0.1, then keeps the value at -1 from 20s to 30s, next increases at 30s with the slope rate 0.1, and finally disappears at 40s.

In terms of the original system matrices A, B, C, D and B_d defined by (68), we can easily construct the augmented matrices $E_e, A_e, B_e, C_e, B_{de}, N_e,$ and N_{de} in the form of (3) and (18).

Choose the derivative observer gain as

$$L_e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 50 \end{bmatrix}^T. \tag{70}$$

Selecting $\alpha = 0.001, \beta = \mathrm{diag}(0, 0.01)$ and solving the matrix inequality (21), we can obtain the proportional gain:

$$K_e = \begin{bmatrix} -0.0103 & -0.1786 \\ -0.0102 & 7.3043 \\ -0.0110 & 1.6314 \\ 6.4656 & -85.8152 \\ 0.0160 & -0.2123 \\ 6.6208 & 28.7726 \\ 0.0236 & 5.7595 \end{bmatrix} \quad (71)$$

Therefore, using the estimator in the form of (5) and real data from a vehicle company, we can get simulated curves of the states, faults and their estimates. Fig. 1 and Fig. 2 are states β_s and r_y and their estimates, which have shown excellent state estimation performance.

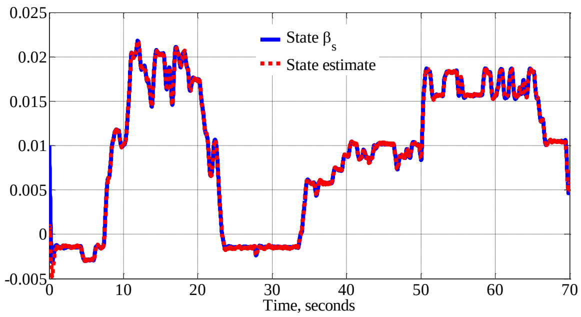

Fig. 1. State β_s and its estimation.

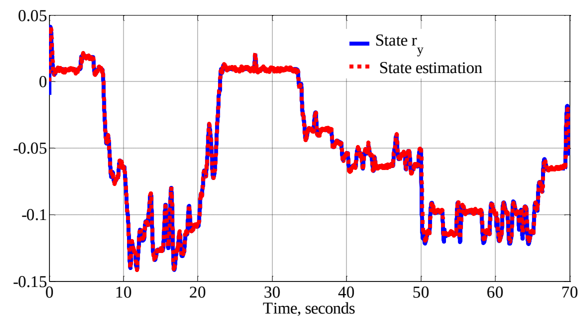

Fig. 2. State r_y and its estimation.

Fig. 3 and Fig. 4 are the actuator fault, sensor fault and their estimates, respectively. The curves have shown that the faults have been tracked successfully. The lateral acceleration sensor noise is a band-limited noise signal, and Fig. 5 exhibits the noise signal and its estimation.

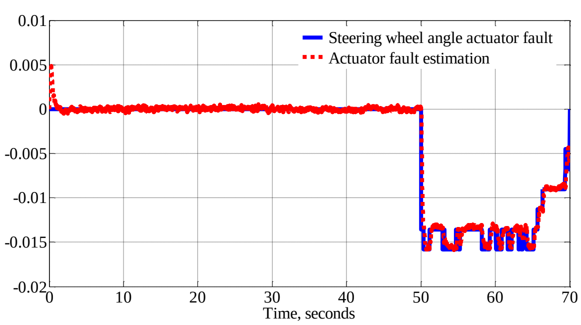

Fig. 3. Steering wheel angle actuator fault and estimation.

From Fig. 6, one can see the actuator fault and sensor fault have seriously distorted the system output signal $\alpha_y(k)$. In the meanwhile, the actuator fault has significantly distorted the output signal $r_y(k)$, seen from Figure 7. Therefore, there is a motivation for fault tolerant control.

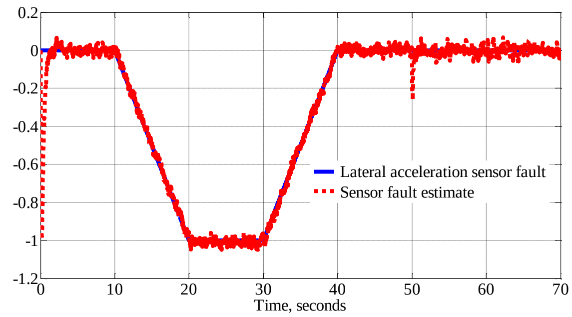

Fig. 4. Lateral acceleration sensor fault and estimation.

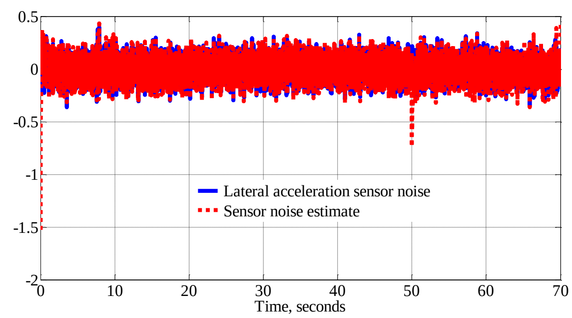

Fig. 5. Lateral acceleration noise and estimation.

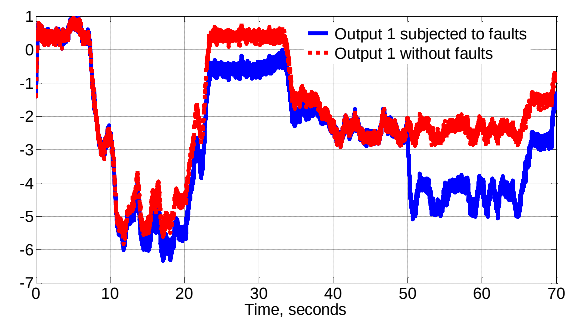

Fig. 6. System output α_y with and without faults.

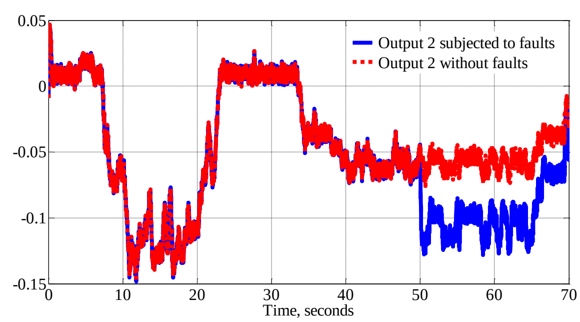

Fig. 7. System output r_y with and without faults.

(b). Robust fault tolerant design.

As the actuator fault is the offset of the input signal, one has

$B_a = B$ and $D_a = D$. Therefore, one can obtain $F_a = 1$ in terms of (37). It is noted the system matrix of (69) is stable, therefore we can simply choose $F_e = [F, F_a, F_s, F_\omega] = [0\ 0\ 1\ 0\ 0\ 0\ 0]$. After implementing actuator and sensor signal compensation following (iii) and (iv) of the procedure 2, we obtain the compensated output response curves in Fig. 8 and Fig. 9. It is shown that the distortion has been removed and the system performance has been recovered after the fault tolerant design.

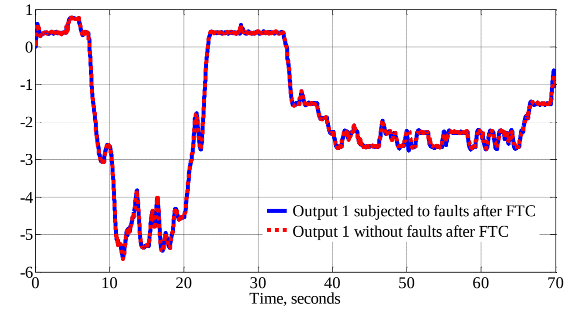

Fig. 8. System output α_y after fault tolerant control.

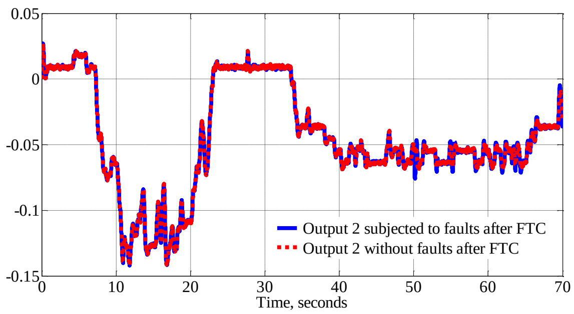

Fig. 9. System output r_y after fault tolerant control.

In order to attenuate the influence of the process disturbance (70), we obtain the gain matrix $F = [-3.1923 \quad 0.0472]$ by solving the matrix inequality (68). Furthermore, the control matrix can be selected as

$$F_e = [-3.1923 \quad 0.0472 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0].$$

After the implementation of the fault tolerant control (see (iii) and (iv) of the procedure 2), we can obtain the compensated system outputs in Fig. 10 and Fig. 11, which indicate the system output performances are consistent with and without faults under the proposed fault-tolerant design schemes.

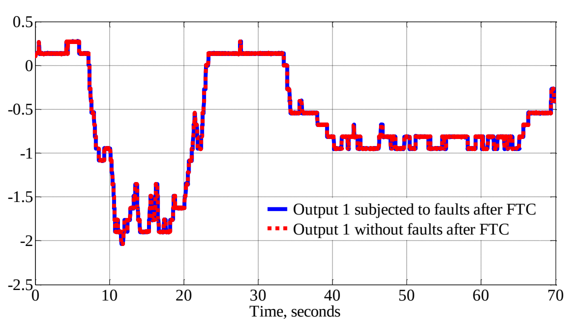

Fig. 10. System output α_y after robust fault tolerant control.

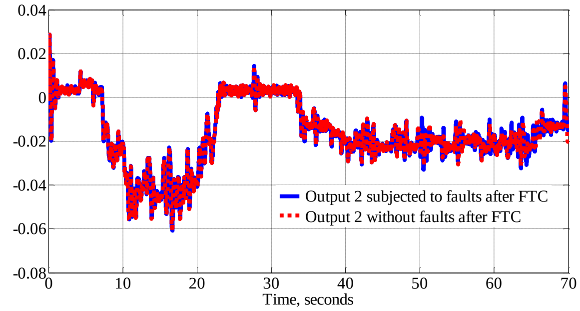

Fig. 11. System output r_y after robust fault tolerant control.

Remark 8: The above simulated results have shown that the estimation and fault-tolerant control methods proposed in the paper have excellent robustness performance against process disturbances and measurement noises, which are in an advantageous position compared with the known techniques that did not take into account the robustness issue or assumed the input disturbances and measurement noises were in the same forms [28-30].

Remark 9: Different selection of the values of α and β may affect the fault estimation performance, which would further affect the quality of fault-tolerant control. Generally speaking, the lower are the values α or β, the better estimates are the concerned faults. For the multiple faults concerned, there are trade-offs of the estimation performance when adjusting α and β.

V. Conclusion

An integrated fault estimation and fault tolerant control approach has been proposed for discrete-time dynamic systems, which has been mathematically proved and real-data demonstrated in a vehicle lateral dynamic system. The proposed design is motived by real-time monitoring and fault-tolerant design, which may find a wide scope of applications in various engineering systems.

Further results are anticipated by extending/applying the proposed fault estimation and fault tolerant control techniques to more complex systems such as Markovian jump processes [32], time-varying systems [33], distributed systems [34], swarm systems [35] and hybrid systems [36].

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