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# Updated analytical solutions of continuity equation for electron beams precipitation – II. Mixed energy losses

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## ABSTRACT

In this paper we consider simultaneous analytical solutions of continuity equations for electron beam precipitation (a) in collisional losses and (b) in ohmic losses, or mixed energy losses (MEL) by applying the iterative method to calculate the resulting differential densities at given precipitation depth. The differential densities of precipitating electrons derived from the analytical solutions for MELs reveal increased flattening at energies below 10–30 keV compared to a pure collisional case. This flattening becomes stronger with an increasing precipitation depth turning into a positive slope at greater precipitation depths in the chromosphere resulting in a differential density distribution with maximum that shifts towards higher energies with increase in column depth, while the differential densities combining precipitating and returning electrons are higher at lower energies than those for a pure collisional case. The resulting hard X-ray (HXR) emission produced by the beams with different initial energy fluxes and spectral indices is calculated using the MEL approach for different ratios between the differential densities of precipitating and returning electrons. The number of returning electrons can be even further enhanced by a magnetic mirroring, not considered in the present model, while dominating at lower atmospheric depths where the magnetic convergence and magnitude are the highest. The proposed MEL approach provides an opportunity to account simultaneously for both collisional and ohmic losses in flaring events, which can be used for a quick spectral fitting of HXR spectra and evaluation of a fraction of returning electrons versus precipitating ones. The semi-analytical MEL approach is used for spectral fitting to Reuven High Energy Solar Spectroscopic Imager observations of nine C, M and X class flares revealing a close fit to the observations and good resemblance to numerical FP solutions.

**Key words:** plasmas – Sun: atmosphere – Sun: flares – Sun: particle emission – Sun: X-rays, gamma rays.

## 1 INTRODUCTION

It is well established that during solar flares, non-thermal electron beams are accelerated somewhere at coronal atmospheric levels, and precipitate into the lower atmosphere depositing their energy into the ambient plasma. The non-thermal electrons lose their energy in collisions and ohmic energy losses while producing bremsstrahlung hard X-ray (HXR) emission. Also a massive amount of non-thermal energy released by accelerated particles is converted into the ambient plasma heating producing strong soft X-ray, ultra-violet and optical emission.

Numerous observations of solar flares in a wide range of electromagnetic spectra have provided a valuable insight on the internal processes within the solar flares, leading to the development of different particle transport and energy-loss models. The most

energetic emission produced by solar flares, HXR and  $\gamma$ -ray spectra, are investigated with Reuven High Energy Solar Spectroscopic Imager (RHESSI) (Lin et al. 2002) with the finest angular and spectral resolutions. Imaging spectroscopy analysis of these data is used as a powerful tool to explore the underlying physics of particle acceleration and transport in solar flares.

Different methods are used to interpret the observed spectra of energetic particles; the most accepted are the analytical solutions for energy spectra of precipitating electrons derived from, for pure collisional energy losses, the inversion of HXR bremsstrahlung spectra of precipitating electron beams (Brown 1971, 1972) or from the direct solutions of continuity equations (Syrovatskii & Shmel'eva 1972). Electron beams precipitating into flaring atmosphere are also found to induce an electrostatic electric field leading to substantial ohmic losses for precipitating electrons (Knight & Sturrock 1977; Emslie 1980; Diakonov & Somov 1988). This electric field at the same time was assumed to produce a return current mostly from the ambient electrons.

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Although Zharkova & Gordovskyy (2006) have shown analytically that the self-induced electric field are mainly formed from the precipitating ones, which are turned back to the corona by the electric field induced by these precipitating electrons. The energy losses in electric field are found to substantially decrease a precipitation depth of precipitating electrons (Zharkova & Gordovskyy 2006); so that a number of low-energy ( $<100$  keV) electrons are also substantially decreased, when ohmic losses are considered (Diakonov & Somov 1988; Zharkova, Brown & Syriavskii 1995; Zharkova & Gordovskyy 2006; Sui, Holman & Dennis 2007). This can lead to the electron beams with a greater initial energy flux to induce a stronger electric field and, thus, to have higher ohmic losses (Zharkova & Gordovskyy 2006; Sui et al. 2007).

In order to consider mixed energy losses (MEL) for electron precipitation the numerical methods are required. This includes the solution of a Fokker–Planck (FP) equation for collisional losses in converging magnetic field by Leach & Petrosian (1981) or for combined collisional and ohmic losses in converging magnetic field (McClements 1992; Syriavskii & Zharkova 1994; Zharkova et al. 1995).

However, full understanding of the role of a self-induced electric field in electron beam dynamics came only from the simulation in the Fokker–Planck approach (FPA) of the time-dependent injection of power-law electrons (Siversky & Zharkova 2009). The authors for the first time derived that the main part of returning electrons is formed from precipitating electrons scattered to pitch angles with negative cosines (or returning beam electrons). This approach is essentially different from the earlier one (Knight & Sturrock 1977) considering a return current formed by the ambient electrons.

Using the FPA to calculate distribution functions of electron beams precipitating into a flaring atmosphere, Siversky & Zharkova (2009) showed that precipitating and returning beam electrons form an electric circuit from the corona to the photosphere where the beam electrons are recycled, or, travel many times up and down while injection continues. The characteristic time-scale for the creation of this electric circuit is 0.07–0.1 s, while the travel time of relativistic beam electrons from the corona to the photosphere is about 0.01–0.1 s. This means that after the circuit is established the beam electrons can make up to 10–100 journeys downward and upward resolving the particle number problem because only (1–10) per cent of accelerated electrons can account for the observed HXR photon numbers.

Further progress with high-resolution observations, made by the RHESSI payload, has provided the data leading to better understanding of photon and electron spectra emitted during solar flares and to deriving the exact locations and shapes of HXR sources on the solar disc where they occur (Brown et al. 2006; Krucker et al. 2008). Many powerful flares are found to produce the elbow-type HXR photon spectra with double power-law energy distributions (Kontar, Brown & McArthur 2002; Conway et al. 2003; Holman et al. 2003), which cannot be completely explained by purely collisional energy losses.

Initially, the flattening of HXR photon spectra was explained by purely collisional model with an increased lower cut-off energy by shifting it to the turning point where HXR emission flattens (Sui, Holman & Dennis 2005) or by the partial HXR albedo effect at its reflection from the photosphere (Massone et al. 2004). However, these previous studies have emphasized the importance of considering multiple energy-loss mechanisms, particularly, inclusion of the self-induced electric field of precipitating electrons (Sui et al. 2007).

Zharkova & Gordovskyy (2006), Sui et al. (2007) and Siversky & Zharkova (2009) showed that ohmic losses can be the most

likely reason causing this flattening because electron beams with the greater initial energy flux induce stronger electric fields and, thus, have higher ohmic losses for lower energy electrons. This leads to a greater flattening of their photon spectra at lower energies. Since a number of low-energy ( $<100$  keV) precipitating electrons are substantially decreased if both Coulomb collisions and ohmic losses are considered, this causes flattening of energy spectra of electrons at lower energy, which can be a rather logical interpretation of an elbow-type HXR spectrum. This, in turn, allows us to estimate photon spectra, directivity and polarization with close resemblance to the HXR and microwave (MW) observations (Zharkova, Kuznetsov & Siversky 2010; Zharkova et al. 2011).

Of course, in a converging magnetic field of loop legs more electrons can be returned back to the corona by magnetic mirroring effect (Leach & Petrosian 1981, 1983; Zharkova et al. 1995; Siversky & Zharkova 2009). Indeed, Siversky & Zharkova (2009) have indicated (see their figs 7 and 8) that the beam electrons are returned back to the corona by a converging magnetic field with the number of returned electrons increasing with a precipitation depth when the convergence factor and absolute magnitude of magnetic field are progressively increased following the convergence formulae (36)–(37) in Siversky & Zharkova (2009). In other words, a number of the mirrored electrons in the upper corona are much smaller than those from the upper or middle chromosphere. Moreover, from all the four forms of the definition of magnetic convergence available in the current literature, the only one derived from the observations seems to fit best the observed magnetic field magnitudes measured in the chromosphere and photosphere (Siversky & Zharkova 2009).

Using this approach, it was shown from a comparison of the theoretical and observed HXR and MW emission (Zharkova et al. 2010, 2011) that the magnetic mirroring affects noticeably only those electrons with higher energy above 100 keV which reach deeper atmospheric levels where the magnitude and convergence factor of magnetic field are higher. At the same time, despite the mirroring of lower energy electrons occurs at upper atmospheric depths where the magnetic field and convergence factor magnitudes are low, which results in a fraction of the mirrored electrons with lower energies at these depths to be rather low. While the return current is produced by the self-induced electric field of beam electrons immediately after the beam injection in the top of the corona, meaning that this return current at these depths is mostly formed by the beam electrons with the energies below 80 keV (Zharkova et al. 1995; Siversky & Zharkova 2009; Zharkova et al. 2011). It becomes evident that a return current and magnetic mirroring are the complementary effects defining electron distributions and their high-energy emission at different precipitating depths (Siversky & Zharkova 2009; Zharkova et al. 2011).

Given the fact that the beam electrons are assumed to have power-law energy distributions, ( $E^{-\gamma}$ ), the bulk of beam electrons is defined by those electrons with lower energies below 100 keV. These lower energy electrons, in turn, primarily define the electric circuit and return current formed by precipitating and returning electrons, while only a small fraction ( $\approx E^{-\gamma}$ ) of higher energy electrons can be affected by magnetic mirroring. Hence, the magnetic mirroring effects can increase a number of higher energy electrons in the population of returning electrons and, thus, can reduce a spectral index of their resulting HXR emission at higher energies (Zharkova et al. 2010, 2011), contrary to the ohmic losses reducing the HXR spectral indices at lower energies (Siversky & Zharkova 2009).

With all the significant progress in the interpretation of HXR and MW emission, the numerical FPA is rather complex and requires extensive calculations, while observers of photon spectra with RHESSI or FERMI payload would prefer to have a quick

analytical tool for their interpretation. Hence, an alternative semi-analytical approach considering Coulomb collisions and ohmic energy losses with a reduced computational time can be very useful. This approach can be also extended in the future interpretation by adding the analytical solutions for magnetic mirroring of beam electrons, similar to the analytical approach considered by Zharkova & Gordovskyy (2005).

In the current paper we propose an iterative semi-analytical method for the initial investigation of electron beam parameters by combining the analytical solutions for MEL. e.g. pure collisional and pure ohmic losses reported Dobranskis & Zharkova (2015, hereafter Paper I). The outcome of this method for electron distribution functions is benchmarked by a FP approach and compared with the HXR spectra observed in a few flares.

The iterative method for calculation of differential densities (DD) of beam electrons for MEL is described in Section 2, their resulting HXR intensities are presented in Section 3, a comparison of simulated spectra with HXR photon spectra observed for a number of flares are discussed in Section 3.2 and conclusions are drawn in Section 4.

## 2 DIFFERENTIAL SPECTRA OF ENERGETIC ELECTRONS FOR MIXED ENERGY LOSSES

HXR is a bremsstrahlung emission produced by collisions of precipitating beam electrons with the ambient plasma electrons or ions. The loss of energy of precipitating electrons in ohmic losses, on the one hand, reduces the energy of precipitating electrons, and, on the other hand, it changes the direction of motion of low-energy electrons so that some part of them is reversed to a return current (Zharkova & Gordovskyy 2006; Paper I). These can have a significant effect on HXR intensity distribution, producing spectral flattening at lower energies (Zharkova & Gordovskyy 2006; Siversky & Zharkova 2009).

However, for simplicity, the classic collisional approach (Brown 1971, 1972; Syrovatskii & Shmeleva 1972) is implemented in the RHESSI software to fit the observed photon energy spectra, which gives the first views of electron precipitation in flares. It would be a great advantage for observers to have an analytical or semi-analytical method considering not only collisional but also ohmic losses that can be used for the initial estimation of the contribution of these mechanisms into electron beam distributions over a precipitation depth.

This can give researchers an initial idea on the role of ohmic energy losses in the particular flare, while the outcome of the analytical estimations can be later compared with the results found from the numerical FP solutions. This can be achieved using the analytical solutions found from continuity equations for pure collisional and pure ohmic losses reported in Paper I by applying them iteratively as described below.

### 2.1 The iterative method of solution

The continuity equation for MELs cannot be analytically integrable. However, the approximated solution of CE for mixed energy losses can be found with using iterative method proposed here, by deploying the analytical solutions found in the Part I of this research (Paper I) for each energy-loss mechanism. The DDs for MELs are derived from the updated solutions of continuity equations for collisional and ohmic losses as follows.

#### 2.1.1 Differential densities for pure collisions

DDs obtained from the updated CE solutions for Coulomb collisions are described by the expression (25) in Paper I:

$$N(E, \xi)_{\text{Coll}} = K_N \sqrt{E} (E^2 + 2a\xi)^{-\frac{\gamma+1}{2}} \times \Theta \left( \sqrt{E^2 + 2a\xi} - E_{\text{low}} \right) \times \Theta \left( E_{\text{upp}} - \sqrt{E^2 + 2a\xi} \right), \quad (1)$$

where  $K_N$  is a normalization constant,  $a$  is the collisional coefficient,  $\Theta$  represents the Heavyside step function, and  $\xi$  is the column depth:

$$\xi(x) = \int_0^x n(t) dt, \quad (2)$$

which defines a total number of particles in the line of sight, within the area of one square centimetre.

#### 2.1.2 Differential densities of precipitating electrons for pure ohmic losses

DDs of precipitating electrons obtained from the updated CE for ohmic losses can be described by the expression (40) in Paper I:

$$N(E, s)_{\text{OhmP}} = K_N E^{-\frac{1}{2}} (E + e\mathcal{E}s_i)^{-\gamma} \times \Theta [(E + e\mathcal{E}s_i) - E_{\text{low}}] \Theta [E_{\text{upp}} - (E + e\mathcal{E}s_i)], \quad \mu = +1, \quad (3)$$

where  $\mathcal{E}$  is the self-induced electric field and  $s$  is a linear precipitation depth.

#### 2.1.3 Differential densities of returning electrons for pure ohmic losses

DDs of 'returning electrons' derived from the updated CE solutions for purely ohmic losses is described by the expression (41) in Paper I:

$$N(E, s)_{\text{OhmR}} = K_N E^{-\frac{1}{2}} (E + e\mathcal{E}s_i)^{-\gamma} \times \Theta [E_{\text{upp}} - E - e\mathcal{E}s_i], \quad \mu = -1. \quad (4)$$

#### 2.1.4 Iterative steps

(i) As the first step, let us use the updated CE equation (1) for purely collisional losses to estimate the DD ( $N(E, \xi)_{\text{Coll}}$ ) at the initial column depth ( $\xi$ ).

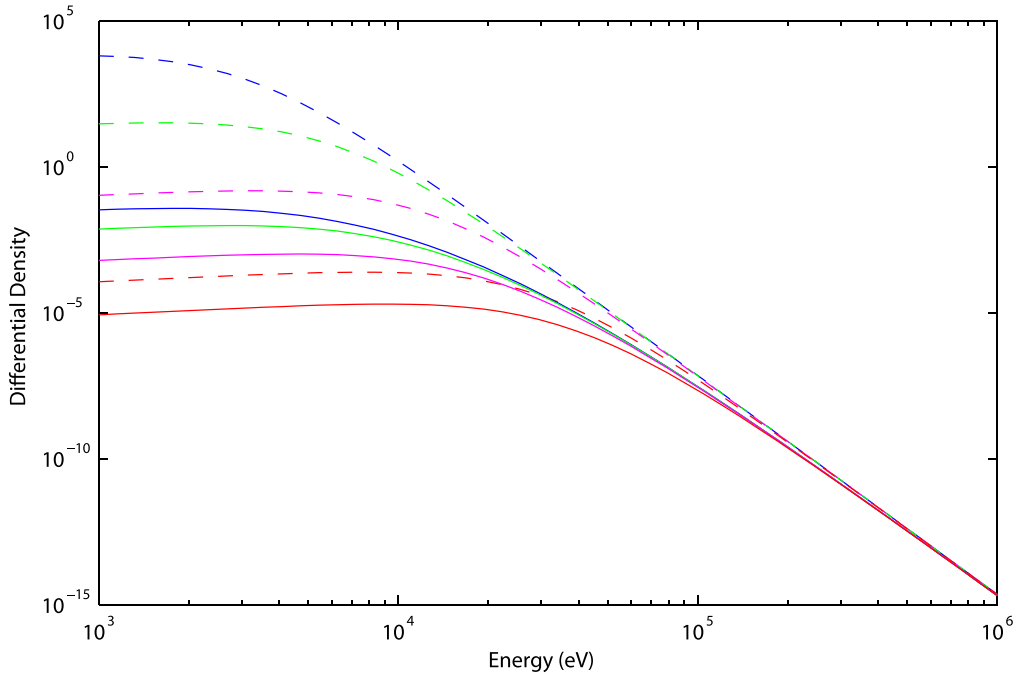
(ii) This DD ( $N(E, \xi)_{\text{Coll}}$ ) is used to calculate electron beam energy flux  $F(E, \xi)$  which is required to estimate the induced electric field at a column depth ( $\xi$ ) as follows:

$$\mathcal{E}(\xi) = \frac{e}{\sigma} \int_{E_{\text{low}}}^{E_{\text{upp}}} F(E, \xi) dE, \quad (5)$$

where  $\sigma$  defines the ambient plasma conductivity, and  $F(E, \xi)$  refers to an electron beam energy flux, found from the DD at a given column depth.

Thus, we estimate the electric field from the updated CE solution (equation 3) for collisional losses, which is then used as a parameter in the DD solution for ohmic losses.

(iii) At each step the electron energies used in the updated CE solutions for ohmic losses are adjusted to reflect at the same step the energy losses in collisions.



**Figure 1.** Differential densities (DDs) of precipitating electron beams calculated for MEL (Coulomb collisions + ohmic losses) (the solid lines) for beam electrons with energies in a range of 16–384 keV, at the column depths of  $\xi$  of  $2.8 \times 10^{18} \text{ cm}^{-2}$  (blue),  $1.3 \times 10^{19} \text{ cm}^{-2}$  (green),  $7.6 \times 10^{19} \text{ cm}^{-2}$  (magenta) and  $5.1 \times 10^{20} \text{ cm}^{-2}$  (red). For comparison, DDs of electron beams with the same parameters losing their energy in Coulomb collisions are plotted by dashed lines.

(iv) Then in the next step electron energies for collisional solutions are adjusted to fit the DDs found for ohmic losses at the previous step and so on.

(v) This process is repeated for every column depth.

Using this approach one obtains the simultaneous solutions for precipitating and returning electrons considering analytical solutions for pure collisions and ohmic losses for precipitating electrons (case 1) and for returning electrons (case 2). The results of DD calculations are discussed in the subsections below.

## 2.2 Results: differential densities for precipitating electrons

DDs for precipitating beam electrons losing energy in Coulomb collisions and ohmic losses are plotted in Fig. 1. The injected electron beams have the following parameters: the initial energy flux of  $F_0 = 10^{10} \text{ ergs}^{-1}$  (supplementary Figs 1a and b) and  $F_0 = 10^{12} \text{ erg cm}^{-2}$  (supplementary Figs 1c and d), and the spectral indices  $\gamma = 3$  (supplementary Figs 1a and c) and 7 (Fig. 1 and supplementary Figs 1b and d).

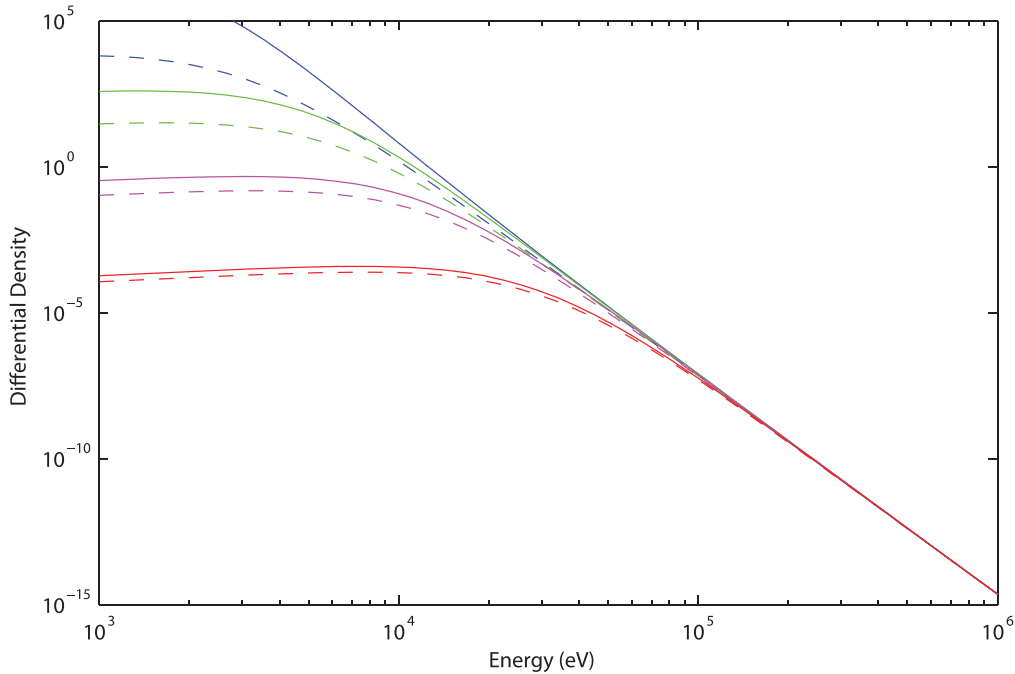
The electric field is considered to be variable, having the initial value  $\mathcal{E}_0 = 5 \times 10^{-5} \text{ V/cm}$  for less intense electron beams (top row), and increasing to  $\mathcal{E}_0 = 3 \times 10^{-3} \text{ V/cm}$  for more powerful beams (bottom row). The energy spectra are constrained by the lower  $E_{\text{low}} = 12 \text{ keV}$  and the upper  $E_{\text{upp}} = 2 \text{ MeV}$  cut-off energies. The results are plotted for the column depth range from  $\xi = 2.8 \times 10^{18}$  (upper curve) to  $5.1 \times 10^{20} \text{ cm}^{-2}$  (bottom curve).

As expected, the DDs, estimated for electron beams precipitating in MEL, considering the additional energy losses in the electric field, are significantly lower than the ones obtained for purely collisional case and strongly depend on the beam's initial flux and spectral index. The DDs of a moderately intense electron beams with a lower initial flux  $F_0$  (supplementary Figs 1a and b) show stronger (by an

order of the magnitude) decrease of electrons with lower energies at the smaller precipitating depth, while at deeper atmospheric depths the differences are nearly negligible for a harder beam ( $\gamma=3$ ) and still noticeable for the softer one ( $\gamma=7$ ). And for a stronger electron beams with the higher initial flux  $F_0$  (supplementary Figs 1c and d), which have much greater energy losses caused by the self-induced electric field, there is a significant (by a few orders of the magnitude) decrease of DDs of beam electrons at lower energies calculated for MEL losses compared to the pure collisional ones.

In general, the DDs calculated for MELs have distributions with the positive slopes towards lower energies than for pure collisional losses with the slopes increasing with the precipitation depth [compare the curves at energies below 3 keV (blue) or below 30 keV (red)] forming wide maxima at greater depths in the chromosphere. The electron beams losing their energy in MELs have steeper positive slopes and more pronounced maxima than the ones losing their energy in purely Coulomb collision that resembles the findings of numerical solutions with FP (Siversky & Zharkova 2009). The maximum shifts towards higher energies with the increase in a column depth, e.g. approximately the two orders of a magnitude increase in column depth (from  $2.8 \times 10^{18}$  to  $5.1 \times 10^{20} \text{ cm}^{-2}$ ), would shift the maximum from 4 to 21 keV (Fig. 1).

The DD distributions presented for lower  $\gamma = 3$  (supplementary Figs 1a and c) and higher  $\gamma = 7$  (supplementary Figs 1b and d) spectral indices show that for harder beams there are smaller differences between the DDs calculated for MEL and pure collisional case than for the beams with higher spectral index. This happens because the electric field responsible for ohmic losses is induced by the precipitating electrons themselves. Thus, it is proportional to the initial energy flux of a beam, leading to higher ohmic losses for more intense beams and has much more complex dependence on their spectral index as shown in formulae (1) and (3). This results in a lower magnitude and increased positive slopes of DD of beam



**Figure 2.** DDs of electron beams derived for precipitating electrons losing their energy in Coulomb collisions plus returning electrons (solid lines) in comparison with DDs calculated for pure collisions (dashed lines). Results are presented at the same column depths as in Fig. 1 for beam electrons with the energy range of 12 keV–2 MeV. The colour schemes of the curves correspond to the depths described in Fig. 1.

electrons at given depths for the MEL losses for harder and more intense beams and leads to the maxima moving to higher energies for softer beams that can be essential for the generation of plasma turbulence.

### 2.3 Results: differential densities for returning electrons

It was demonstrated by FPA (Siversky & Zharkova 2009) that a large fraction of precipitating electrons can be turned by their own electric field back to the top in the corona where they are injected from. As we discovered in Section 2.2, the distributions of returning electrons are substantially different from those of the precipitating electrons losing their energy in electric field.

In order to estimate the effect of returning electrons on HXR emission emitted upwards to the corona, let us investigate the DDs of electrons for a combination of electron beams losing their energy in Coulomb collisions at given depth (equation 1) and returning electrons (equation 4) gaining their energy in a self-induced electric field. The variations of the resulting DDs are presented in Fig. 2 for the initial energy flux  $F_0 = 10^{12} \text{ erg cm}^{-2} \text{ s}^{-1}$  and  $\gamma=7$  and in the supplementary Fig. 2 for hard  $\gamma = 3$  (supplementary Figs 2a and c) and soft  $\gamma = 7$  (supplementary Figs 2b and d) electron beams with the initial energy fluxes of  $10^{10} \text{ erg cm}^{-2} \text{ s}^{-1}$  (supplementary Figs 2a and 2b) and  $10^{12} \text{ erg cm}^{-2} \text{ s}^{-1}$  (supplementary Figs 2c and d).

The resulting DDs at every precipitation depth of combined precipitating and returning electrons are higher than those for purely collisional case because the returning electrons being accelerated by the electric field increase the number of electrons at given depth because they gain energy in this electric field and increase the resulting DD for MEL. The impact of an electric field magnitude on DD distributions is compared in the supplementary Fig. 2 (top row), where the electron beams induce the self-induced electric field of moderate magnitude of ( $\mathcal{E}_0 = 7 \times 10^{-6} \text{ V/cm}$ ), in the supplementary

Fig. 2 (bottom row) calculated for a stronger electric field magnitude ( $\mathcal{E}_0 = 3 \times 10^{-3} \text{ V/cm}$ ).

Again, similar to the precipitating electrons shown in Fig. 1, the increase in DD at given precipitating depth in the ‘returning’ electrons is more significant for more powerful electron beams, due to a very strong self-induced electric field produced by such beams. An increase in the electric field magnitude results in some flattening of the positive slope formed by precipitating electrons at lower energies, especially at the deeper atmospheric depths, from where the returning electrons gain the largest amount of energy. This increase, in fact, is comparable with but slightly lower than the decrease of DDs for precipitating electrons described in Section 2.2.

While the dependence of DDs for precipitating plus returning electrons on spectral indices is much more complicated following the formulae (1) and (4), at higher atmospheric levels, or lower precipitation depths, the increase in returning electrons becomes extremely significant approaching one order of the magnitude for harder beams ( $\gamma=3$ ), 3 orders of magnitude for softer moderate beams ( $\gamma=7$ ) and 4 orders of the magnitude for a softer stronger beam (Fig. 2). Although with the increase in a precipitation depth beyond the stopping depth of electrons with lower cutoff energy (12 keV), the densities of returning electrons become negligible leading to the same DDs as for the precipitating electrons only shown in Fig. 1.

### 2.4 Fractions of precipitating and returning electrons

In order to accurately estimate the ratios between the numbers of precipitating ( $\mu = +1$ ) and returning ( $\mu = -1$ ) electrons from a total number injected, let us use the solutions obtained from numerical FPA (Siversky & Zharkova 2009). The ratio can be expressed as a percentage of precipitating electrons present at a certain column depth from the total number of electrons (100 per cent) including

**Table 1.** Fraction of precipitating electrons at  $\xi = 2.4 \times 10^{17} \text{ cm}^{-2}$ .

$\gamma$	3		5		7	
$\Delta\mu$	0.2	0.02	0.2	0.02	0.2	0.02
$F = 10^8$	77.2 per cent	85.4 per cent	77.4 per cent	85.5 per cent	77.6 per cent	85.6 per cent
$F = 10^9$	70.5 per cent	79.0 per cent	68.0 per cent	76.5 per cent	67.2 per cent	75.6 per cent
$F = 10^{10}$	45.2 per cent	56.7 per cent	43.0 per cent	53.00 per cent	42.3 per cent	51.8 per cent

**Table 2.** Fraction of precipitating electrons at  $\xi = 2.5 \times 10^{18} \text{ cm}^{-2}$ .

$\gamma$	3		5		7	
$\Delta\mu$	0.2	0.02	0.2	0.02	0.2	0.02
$F = 10^8$	73.1 per cent	81.4 per cent	73.0 per cent	81.2 per cent	73.2 per cent	81.1 per cent
$F = 10^9$	67.5 per cent	75.3 per cent	65.6 per cent	72.9 per cent	65.0 per cent	72.0 per cent
$F = 10^{10}$	49.4 per cent	56.4 per cent	48.9 per cent	54.0 per cent	49.1 per cent	53.6 per cent

**Table 3.** Fraction of precipitating electrons at  $\xi = 2.4 \times 10^{19} \text{ cm}^{-2}$ .

$\gamma$	3		5		7	
$\Delta\mu$	0.2	0.02	0.2	0.02	0.2	0.02
$F = 10^8$	70.8 per cent	71.8 per cent	71.5 per cent	72.1 per cent	71.9 per cent	72.4 per cent
$F = 10^9$	69.2 per cent	69.6 per cent	70.3 per cent	70.5 per cent	71.2 per cent	71.3 per cent
$F = 10^{10}$	63.9 per cent	64.5 per cent	68.2 per cent	68.6 per cent	70.7 per cent	71.1 per cent

precipitating and returning ones. This ratio is found to be dependent on electron beam parameters as outlined in Tables 1–3 showing its variations for different initial energy fluxes  $F$ , spectral indices  $\gamma$  and the pitch angle dispersion  $\Delta\mu$ .

From Table 1 one can derive that on the top of the atmosphere ( $\xi = 2.4 \times 10^{17} \text{ cm}^{-2}$ ), weak electron beams ( $F = 10^8 \text{ erg cm}^{-2} \text{ s}^{-1}$ ) have highest percentage of precipitating electrons (77.23 per cent for  $\gamma = 3$ ,  $\Delta\mu = 0.2$ ), which becomes only slightly higher for softer electron beams (77.58 per cent  $\gamma = 7$ ). A smaller pitch angle dispersion ( $\Delta\mu$ ) at injection of beam electrons (defining beams directed closer to the direction  $\mu=1$ ) leads to a further increase by approximately 8 per cent (from 77 per cent to 85 per cent), allowing for 15–23 per cent of the number returning electrons, reducing thus the number of precipitating electrons.

In case of powerful electron beam with the initial energy flux ( $F = 10^{12} \text{ erg cm}^{-2} \text{ s}^{-1}$ ) and small  $\gamma = 3$ , a stronger self-induced electric field is generated. This results in a larger number of returning electrons moving towards the corona, or in a much lower percentage (45.21 per cent) of precipitating electrons. The increase in spectral index leads to an increase in electric field and, thus, to a decrease in the percentage of precipitating electrons (by 3 per cent). While a lower pitch angle dispersion ( $\Delta\mu$ ) increases this percentage by approximately 9–11 per cent (from 45 per cent to 56 per cent for  $\gamma = 3$ , or from 42 per cent to 51 per cent for  $\gamma = 7$ ), allowing for 43–57 per cent of the returning electrons, depending on spectral index and angle of dispersion.

Further investigation in Tables 2 and 3 and Fig. 3 reveals that electron beams with a very weak initial energy flux, e.g.  $F = 10^8$  (Fig. 3a),  $10^9$  (Fig. 3b)  $\text{erg cm}^{-2} \text{ s}^{-1}$ , and smaller pitch angle dispersion  $\Delta\mu = 0.02$  have the highest percentage of precipitating electrons at the upper atmospheric level while this number decreases exponentially with the increase of column depth.

On the other hand, the maximum in the percentage of precipitating electrons shifts deeper into the atmosphere (from  $\xi = 10^{17} \text{ cm}$  to  $5.0^{19} \text{ cm}$ ) with the increase of the initial energy flux  $F$ , which means that a significant number of electrons reach the chromo-

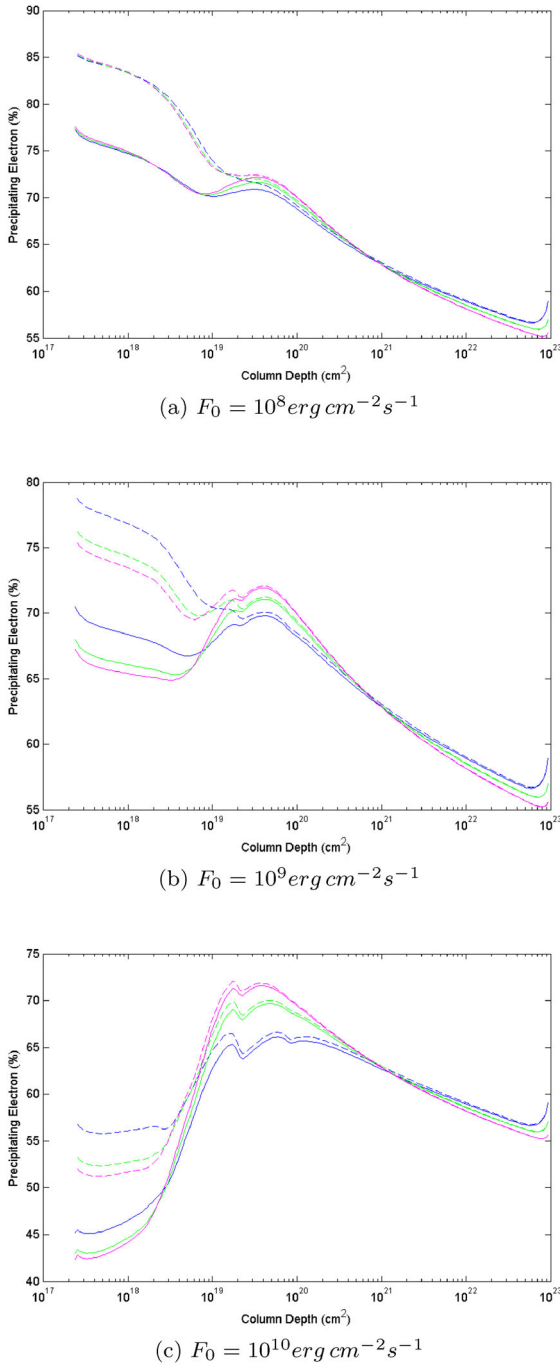
spheric levels. The shifted maximum first becomes clearly visible in Fig. 3(b) ( $F = 10^9 \text{ erg cm}^{-2} \text{ s}^{-1}$ ,  $\Delta\mu = 0.2$ ), while, in the case of  $F = 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1}$  (Fig. 3c) the maximum is well pronounced for both magnitudes of a pitch angle dispersion  $\Delta\mu = 0.2$  and  $0.02$ .

Similar to FP solutions (Siversky & Zharkova 2009), in the iterative use of the analytical solutions for collisional and ohmic losses, a magnitude of an initial energy flux,  $F$ , of beam electrons has the most significant impact on the density ratios of precipitating-to-returning electrons at all column depths. While the variations of a pitch angle dispersion has a significant effect only at higher atmospheric levels up to  $\xi = 2.0 \times 10^{19} \text{ cm}^{-2}$ , and they are 8 per cent higher for narrower beams with  $\Delta\mu = 0.02$ , at deeper atmosphere electrons become strongly thermalized approaching nearly isotropic distribution at the lower chromosphere levels.

Additionally, continuity equations for purely collisional (formula 1) and ohmic (formula 3) energy losses are integrated by energy in the following way:

$$\begin{aligned}
 N(\xi) &= K_N \int_{E_{\text{low}}}^{E_{\text{upp}}} \sqrt{E} (E^2 + 2a\xi)^{-\frac{\gamma+1}{2}} dE \\
 &= \frac{K_N \times 2^{\frac{1-\gamma}{2}} E^{\frac{3}{2}} \left( \frac{E^2}{a\xi} + 2 \right)^{\frac{\gamma+1}{2}} (E^2 + 2a\xi)^{-\frac{\gamma+1}{2}}}{3} \\
 &\quad \times \left( \sum_{n=0}^{\infty} \frac{(0.75)_n \left( \frac{\gamma+1}{2} \right)_n \left( -\frac{E^2}{2a\xi} \right)^n}{(1.75)_n n!} \right) \Big|_{E_{\text{low}}}^{E_{\text{upp}}} \quad (6)
 \end{aligned}$$

$$\begin{aligned}
 N(s) &= K_N \int_{E_{\text{low}}}^{E_{\text{upp}}} E^{-0.5} (E + e\mathcal{E}s_i)^{-\gamma} dE \\
 &= 2K_N \sqrt{E} \left( \frac{E}{e\mathcal{E}s_i} + 1 \right)^{\gamma} (E + e\mathcal{E}s_i)^{-\gamma} \\
 &\quad \times \left( \sum_{n=0}^{\infty} \frac{(0.5)_n (\gamma)_n \left( -\frac{E}{e\mathcal{E}s_i} \right)^n}{(1.5)_n n!} \right) \Big|_{E_{\text{low}}}^{E_{\text{upp}}}, \quad (7)
 \end{aligned}$$



**Figure 3.** Variations with a column depth of precipitating electron ratio derived from FPA for electron beams with spectral index  $\gamma = 3$  (blue),  $\gamma = 5$  (green) and  $\gamma = 7$  (magenta). The solid lines are used for the pitch angle dispersion at injection of  $\Delta\mu = 0.2$  and dashed lines for  $\Delta\mu = 0.02$ .

producing the solutions for electron density  $N$  for any given spectral index  $\gamma$  at a given column depth  $\xi$ , for pure collisions (equation 6), and at a linear column depth  $s$ , for pure ohmic losses (equation 7).

It can be seen that, similar to Zharkova & Kobylinskij (1992), the total beam abundances integrated over all energies either for pure collisional losses,  $N(\xi)$ , or for purely ohmic losses,  $N(s)$ , are linearly proportional to the beam's initial energy flux, which is implicitly included into the normalization coefficient  $K$  (Paper I).

In fact, the total densities,  $N(\xi)$ , of precipitating electron beams losing their energy in collisions (equation 6) decrease proportionally to a column depth as  $N(\xi) \approx \xi^{-(\gamma+1)}$  that is similar to the results reported by Zharkova & Kobylinskij (1992) for pure collisional losses, for the beam electrons losing their energy in ohmic losses the total electron abundances,  $N(s)$ , (equation 7) are decreasing with a linear depth  $s$  as  $N(s) \approx s^{-\gamma}$ . Hence, harder electron beams (with smaller  $\gamma$ ) will have a smaller density decrease with depth than the softer ones that enforce a further decrease of the abundances, if both the losses are combined.

### 3 HXR INTENSITY OF ELECTRONS WITH MIXED ENERGY LOSSES

As shown in Section 2, by combining iteratively the solutions of the updated CE for both Coulomb collisions and ohmic losses one can find the semi-analytical solutions for DDs of precipitating beams with MELs. This allows us to evaluate the impact of collisional and ohmic losses on the resulting HXR bremsstrahlung emission.

The DDs, obtained for collisional and ohmic loss cases, are converted into the distribution functions  $f$ ,  $f = \frac{N(E, \xi)}{N(E, 0)}$  (Dobranskis & Zharkova 2014; Paper I). In the calculations we use the fractions of precipitating and returning electrons defined in Section 2.4. For comparison, the distribution functions  $f_N$  are obtained for the same beam parameters from a numerical FPA (Siversky & Zharkova 2009).

In order to estimate the HXR intensity,  $I$ , for given photon energy  $\varepsilon$  and viewing angle  $\psi$ , the corresponding distribution functions  $f$  are integrated by azimuthal angles  $\phi$  (giving the factor  $2\pi$  before the integral), pitch angle cosine  $\mu$ , electron energy  $\eta$  and column depth  $\xi$  as follows (Zharkova et al. 2010):

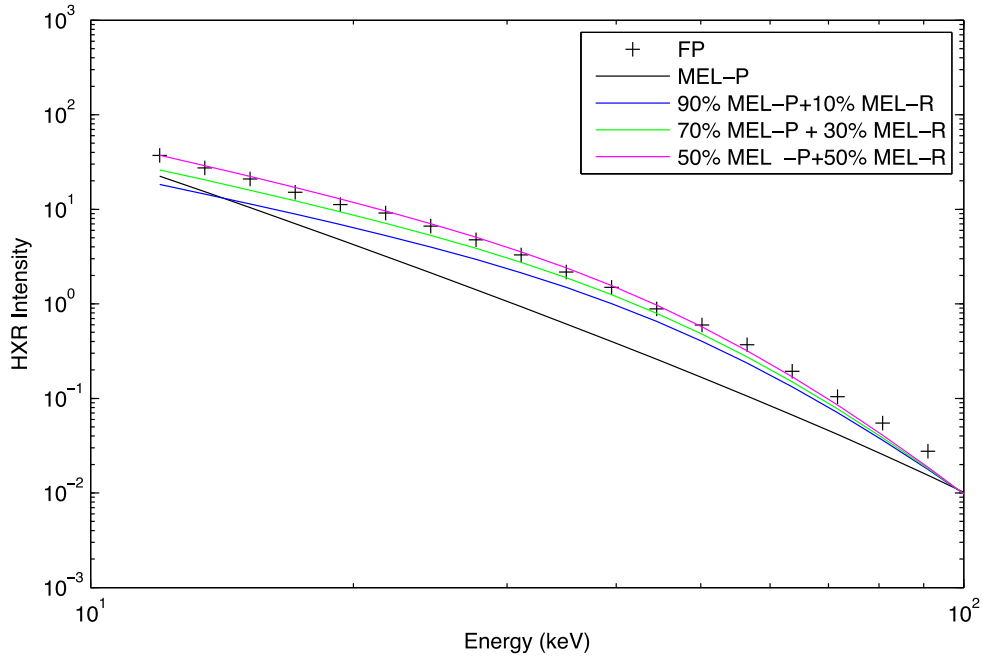
$$I(\varepsilon, \psi) = 2\pi A_x K \times \int_{\xi_{\min}}^{\xi_{\max}} \int_{\varepsilon}^{\infty} \int_{-1}^1 f(\xi, \eta, \mu) \eta \sigma^H(\eta, \varepsilon, \psi, \mu) d\mu d\eta d\xi, \quad (8)$$

where  $\varepsilon$  is a dimensionless photon energy  $h\nu/E_0$ ,  $\sigma^H$  are the cross-sections for HXR emission with energy  $\varepsilon$  formed by electrons with given energy and angle parameters and  $A_x = \frac{S}{4\pi R} \frac{2E}{m_e}$ , where  $S$  defines area of a flare and  $R$  is the distance to the observer ( $\approx 1$  au). In equation (8) the relativistic cross-sections are used following Bai & Ramaty (1978) with the correction proposed by Zharkova et al. (2010).

#### 3.1 HXR intensities from precipitating and returning electrons

Let us calculate the HXR emission (equation 8) produced by electron beams towards an observer by assuming that it is produced (a) directly by the returning electrons and (b) by the precipitating electrons whose emission is fully reflected from the photosphere (a full albedo effect). In this scenario one can utilize the ratios of the numbers of precipitating-to-returning electrons derived from FPA in the previous section (Section 2.4). For this purpose the DDs obtained with the updated CE solution for MEL for both precipitating (Coulomb collisions and ohmic losses) (MEL-P) and returning (Coulomb collisions & returning electrons) (MEL-R) beam electrons are combined using the given ratio for their abundances.

The HXR intensities plotted in Fig. 4 and in the supplementary Fig. 4 are obtained for the electron beams with the same parameters as in Fig. 1 (and supplementary Fig. 1, respectively) for the following ratios of precipitating and returning electrons: 100 per cent MEL-P (black line), 90 per cent MEL-P + 10 per cent MEL-R



**Figure 4.** HXR intensity in relative units, produced by beam electrons losing their energy in MEL for precipitating (MEL-P) and returning electrons (MEL-R): MEL-P (black line), 90 per cent MEL-P + 10 per cent MEL-R (blue line), 70 per cent MEL-P + 30 per cent MEL-R (green line), 50 per cent MEL-P + 50 per cent MEL-R (magenta line), and from FPA (crosses). The HXR intensity variations are plotted for low (top row) and high (bottom row) initial energy fluxes of beam electrons and for the beams with hard (left column) and soft (right column) energy spectra.

**Table 4.** Ratios of electron abundances for MEL obtained for precipitating (MEL-P) and returning (MEL-R) electrons as estimated from numerical simulations with FPA for different initial energy fluxes (Siversky & Zharkova 2009).

$F_0 = 10^8 \text{ erg cm}^{-2} \text{ s}^{-1}$	90 per cent MEL-P + 10 per cent MEL-R
$F_0 = 10^{10} \text{ erg cm}^{-2} \text{ s}^{-1}$	70 per cent MEL-P + 30 per cent MEL-R
$F_0 = 10^{12} \text{ erg cm}^{-2} \text{ s}^{-1}$	50 per cent MEL-P + 50 per cent MEL-R

(blue line), 70 per cent MEL-P + 30 per cent MEL-R (green line), 50 per cent MEL-P + 50 per cent MEL-R (magenta line). The intensities calculated from the FPA are shown by crosses. From Fig. 4 one can see that the inclusion of returning electrons has a significant effect on the resulting HXR intensity spectra produced jointly by precipitating and returning electrons, which is similar to the conclusions derived from the full FPA (Zharkova & Gordovskyy 2006; Siversky & Zharkova 2009).

It can be observed that for weaker beams the intensity distributions derived by FP solutions are closer to the distributions when a fraction of returning electrons ranges between 10 (supplementary Fig. 4a) and 30 per cent (supplementary Fig. 4b), while for more intense beams the FP curve fits the MEL curve for 50 per cent of returning electrons (supplementary Figs 4c and d). It can be seen that an increase (10 per cent, 30 per cent, 50 per cent) in the returning electron percentage would result in a noticeable decrease by 0.2 of the lower energy spectral index  $\delta_1$  (or in steepening of a HXR intensity spectra) for any additional 20 per cent of returning electrons (see Table 4).

For example, weak hard beam precipitating electrons combined with at least 10 per cent of returning electrons result in a considerable flattening of their HXR intensity spectra at lower and steepening at higher energies (supplementary Fig. 4a). In the case of stronger but still hard electron beams ( $\gamma = 3$ , supplementary Fig. 4c) the flattening would extend to approximately 40 keV, while

for stronger soft electron beams ( $\gamma = 7$ , supplementary Fig. 4d) it would reach approximately 60 keV.

This explains in very simple analytical terms the similar findings presented earlier from FP solutions (Zharkova & Gordovskyy 2006; Siversky & Zharkova 2009), describing the HXR intensities, which reflects very non-linear dependence on the parameters of electron beams. In other words, we demonstrate here that the electron distribution functions formed by the redistribution of precipitating and returning electrons caused by the electric field induced by the precipitating beam itself are accountable for the variations of their resulting HXR intensities. This allows us to derive the beam parameters and the ratio of precipitating to returning electrons from fitting their HXR distributions obtained from observations.

### 3.2 Fitting to observations the simulated HXR spectra from beams with MELs

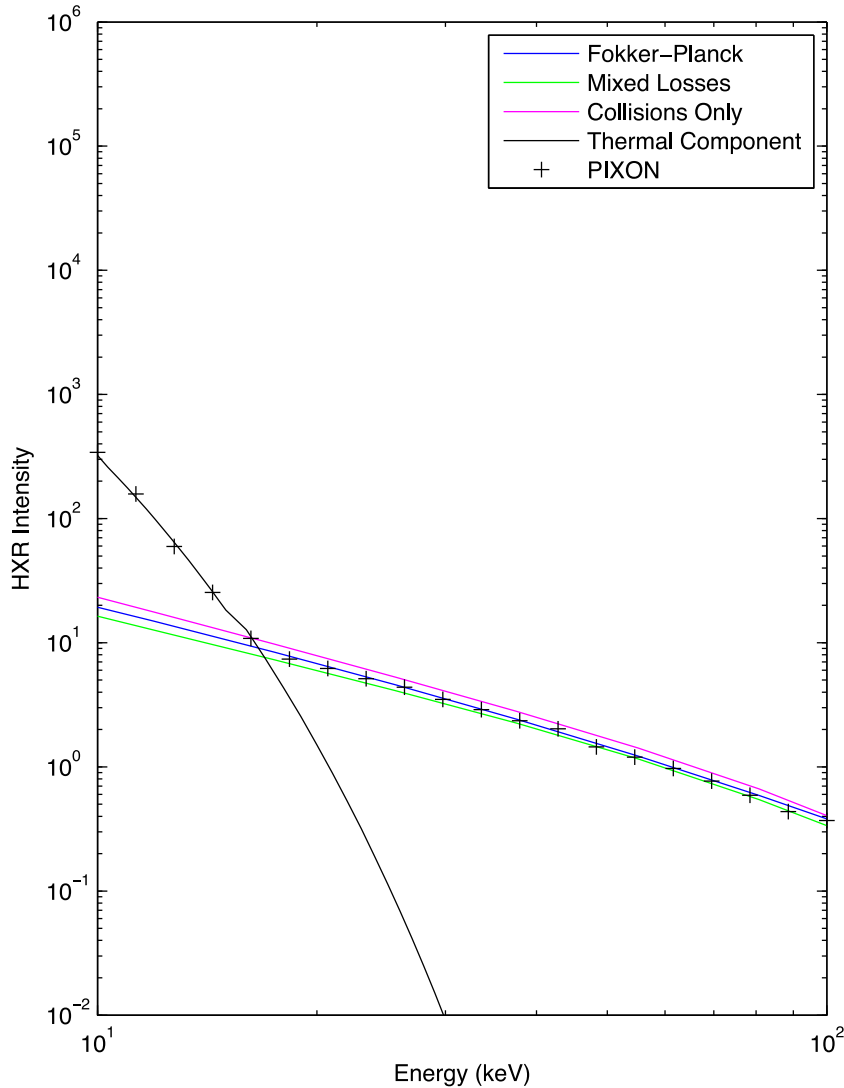
#### 3.2.1 Methodology

Using the proposed iterative method for analytical calculation of DDs for precipitating beam electrons with MEL (Section 2.1), one can apply a spectral fitting of the observed HXR emission to those calculated using the MEL distribution functions. This procedure was applied to the main events (emission at the flare footpoints) of nine flares (GOES class C, M and X) summarized in Table 5. The HXR photon energy spectra were obtained for the main HXR bursts in the energy bands from 10 to 100 keV and plotted in Fig. 5.

The images are reconstructed using the high-precision PIXON algorithm (Hurford et al. 2002) from the RHESSI branch of SolarSoft (Lin et al. 2002). The integration time was chosen according to a flare class, with 8 s for X-class, 12 s for M-class, and 20 s for C-class flares. 20 PIXON images were reconstructed for each flare, using the energy bins with logarithmic spacing, ranging from 10 to 100 keV following the methodology described by Hurford et al.

**Table 5.** Variations of spectral indices  $\gamma$  of the HXR emission produced by beam electrons with the initial energy fluxes  $F_0$  and ratios of precipitating/returning electrons with the best fit to the RHESSI observations.

Flare date	Peak time UT	Class	Flux $F_0$ , $\text{erg cm}^{-2} \text{s}^{-1}$	MEL $\gamma$	FP $\gamma$	p/r ratio per cent
01 Oct 2012	05:03:26	C1.8	$5.2 \times 10^8$	3.2	3.2	95/05
17 Jan 2014	08:29:30	C2.2	$6.1 \times 10^8$	3.4	3.4	95/05
17 Jan 2014	19:20:22	C7.1	$10.1 \times 10^8$	3.6	3.7	90/10
13 Jan 2014	21:51:02	M1.3	$9.7 \times 10^{10}$	4.1	4.1	75/25
19 Dec 2013	23:18:26	M3.5	$12.8 \times 10^{10}$	4.2	4.3	70/30
08 Jan 2014	03:46:10	M3.6	$13.0 \times 10^{10}$	4.3	4.4	70/30
25 Oct 2013	07:59:34	X1.7	$11.6 \times 10^{11}$	5.3	5.3	55/45
23 Oct 2012	03:17:22	X1.8	$12.0 \times 10^{11}$	5.1	5.2	53/47
25 Oct 2013	15:01:14	X2.1	$12.3 \times 10^{11}$	5.2	5.4	50/50

**Figure 5.** Examples of the HXR photon energy spectra obtained for the main HXR bursts in the energy bands from 10 to 100 keV for a flare of class M (see for more plots the supplementary Fig. 5) for the flares from the list summarized in Table 5. The fitting is done using (a) thermal component (black line); (b) the updated CE solutions for Coulomb collisions (magenta); (c) the CE solutions for MEL (green) for the relevant ratios of returning and precipitating electron abundances and (d) FP solutions for the same beam parameters (blue line). The parameters are compared to the HXR intensity of the flares estimated by PIXON algorithm (crosses) (see the text for details).

(2002). The best image quality was obtained using the detectors 2 to 7 (front segments only). Then, the thermal component is fitted to the observed photon flux, using fitting routines in the OSPEX application (Hurford et al. 2002; Lin et al. 2002, 2003).

Finally, the HXR parts of the photon spectra derived from RHESSI payload are fitted with the HXR emission obtained from CEs for the MEL (Paper I), the original CE for pure collisions and from the numerical FPA (Siversky & Zharkova 2009).

### 3.2.2 Fitting the simulated HXR spectra to observations

The results obtained through the spectral fitting using the standard RHESSI methods and the method described in the previous section are presented as a sample in Fig. 5 and the supplementary Fig. 5 for nine flares with the flare parameters summarized in Table 5. The observed HXR intensity is represented by crosses, with the black line defining the thermal component. The same initial energy fluxes are used for electron DDs derived from the original CE (with pure collisional losses), from the two CEs with MELs using the iterative method described in this paper (see Section 2.1) and for FPA using the same MELs.

From Fig. 5, one can see a clear increase in the overall HXR intensity with the increasing flare index likely produced by electron beams with a higher initial energy flux. For C class flares the HXR emission is observed in the band of 16–18 keV, for M class flares in the band of 18–20 keV, and for X-class flare in 27–30 keV. For more powerful flares (or beams) the HXR energy spectra become double power laws. The break in the HXR photon spectra shifts towards higher energies with an increase in the flare class (or an increase in the injected beam initial energy flux).

The investigation of spectral fitting with the different solutions revealed that the increase in the initial energy flux of beam electrons would increase their resulting HXR intensity. The spectral index used for the MEL model would differ by only 0.1 from the one used with FPA. Only in a single case the difference would become 0.2, for the most powerful flare of 2013 October 25 (GOES class X2.1) (Table 5), which coincides with the conclusions from the previous paper (Paper I).

The spectral indices of the HXR emission derived from the MEL and FP solutions are presented in the fifth and sixth columns of Table 5 with the ratios of precipitating/returning electrons best fitting the observations shown in the last column. In general, fitting of the observed HXR emission by that simulated for the electrons beams with MELs can be summarized as follows.

- (i) For medium (class M) and powerful (class X) flares the proposed joint MEL solutions combining pure collisions and ohmic losses show a closest fit to the observed curves of HXR photon spectra for a particular fraction of returning and precipitating electrons shown in the last column of Table 5.
- (ii) For more powerful flares of class X the impact of ohmic losses becomes more pronounced resulting in higher fraction of returning electrons, and thus, in noticeable spectral flattening of HXR intensity at lower energies.
- (iii) Spectral indices of the simulated HXR emission derived with the MEL approach are rather close to those derived with FP approach for similar MEL.

Hence, the MEL approach can explain in very simple terms the spectral flattening towards lower energies by the increased ratio of returning electrons compared to the precipitating ones as pointed out by ZG06 and Siversky & Zharkova (2009). In addition, the proposed MEL approach allows us to derive a preliminary fraction

of returning electrons in the whole population in beam electrons, which provides a good starting point for the further FP fitting, if required.

Of course, some discrepancies appearing between MEL and FP approaches can occur due to the fact that a full pitch angle scattering is not considered in the semi-analytical MEL approach that can reduce the derived spectral indices compared to those found from FP solutions. Also in either approaches we do not consider magnetic mirroring in converging magnetic field affecting electrons with higher energies that can also slightly reduce the spectral index of resulting HXR emission at higher energies (Zharkova et al. 2011). However, in the four cases out of nine flares considered, the spectral indices derived with the MEL approach are practically identical to FPA, with a difference of less than 0.1 (see Table 5).

However, the proposed MEL approach to the calculation of electron beam densities and their resulting HXR emission allows any user to reduce by 30–40 times the computation time (a few minutes) required for interpretation of the RHESSI spectra, compared to hours required to calculate distribution functions and HXR emission for a flare using FP approach. This IDL procedure can be easily implemented in the RHESSI software for the benefits of all the RHESSI users.

## 4 CONCLUSIONS

In this paper we consider the simultaneous analytical solutions of continuity equations for electron beam precipitation (a) in collisional losses and (b) in ohmic losses, or MELs, by applying the iterative method to calculate the resulting DDs at given precipitation depth. This approach allows us to provide quick estimations of the contribution of self-induced electric field in the electron beam precipitation and to derive their resulting HXR emission produced by both precipitating and returning electrons forming the electric circuit in solar flares (ZG06; Siversky & Zharkova 2009).

The HXR intensities are compared for the following solutions: the updated CE for Coulomb collisions (Paper I), the updated CE for mixed Coulomb collisions and ohmic energy losses (presented here) and the numerical FP solution for Coulomb collisions, ohmic losses and pitch angle scattering (Siversky & Zharkova 2009). In general, HXR bremsstrahlung emission produced by precipitating beam electrons is directed mostly towards the photosphere. Thus, it is considered here to be fully reflected back to the corona (a full albedo effect) (Massone et al. 2004), while the emission of returning electrons is produced directly towards the observer in the top of the atmosphere.

In general, the MEL solutions for DDs and HXR emission produced by both precipitating and returning electrons can be summarized as follows.

- (i) DDs of precipitating electrons derived from the MEL semi-analytical solutions become much more flattened at lower energies below 10–30 keV compared to the purely collisional case. This flattening becomes stronger with an increasing precipitation depth turning into a positive slope at greater precipitation depths in the chromosphere, producing, thus, a DD distribution with maximum, which shifts towards higher energies with the increase in a column depth.
- (ii) On the other hand, the resulting differential densities of combined precipitating and returning electrons are higher at every precipitation depth than those for a pure collisional case. This increases the resulting differential density of beam electrons at a given precipitation depth, which contributes to the HXR bremsstrahlung emission produced towards the top of a flaring atmosphere.

(iii) An overall increase in the differential density of precipitating plus returning electrons is more significant for powerful electron beams, as they induce a stronger electric field, and a larger number of returning beam electrons. Hence, the inclusion of MELs into the semi-analytical approach proposed here allows us to estimate the effects of self-induced electric field on electron beam precipitation into flaring atmosphere without running computer-expensive numerical FP simulations.

(iv) A fraction of precipitating and returning electrons is shown to be strongly dependent on the initial energy flux, spectral indices and pitch angle dispersion of electron beam at injection. Weak electron beams are found to have the highest fraction of precipitating electrons being slightly lower for softer electron beams and higher for harder ones. While electron beams with moderate and strong initial energy fluxes there are more electrons accelerated backwards the corona, resulting in a lower fraction of precipitating electrons.

(v) Moreover, the fraction of returning electrons is found to be dependent on a pitch angle dispersion of the beam at its injection on the top boundary. For mildly directed beams (with a pitch angle dispersion of 0.2) an increased number of returning electrons (43–57 per cent) is found at the very top precipitation depths, compared to a much smaller number (<20 per cent) derived for the narrower beams with a smaller pitch angle dispersion of 0.02.

(vi) Simultaneous consideration of both collisional and ohmic energy losses combining the HXR emission produced by precipitating and returning electrons reveals a noticeable flattening of their HXR intensity spectra at lower energies (<80 keV). This flattening is stronger for beams with higher initial energy fluxes, or for more powerful flares.

(vii) Spectral fitting of the HXR emission observed by RHESSI for nine flares providing the closest fit of the simulated curves to the observed HXR photon spectra helps one to evaluate a fraction of returning electrons versus precipitating ones in these flares shown in Table 5.

(viii) In addition, spectral fitting of the HXR emission found from the MEL approach is very close to the observed HXR photon spectra and with those derived from FP approach within 0.1–0.2.

(ix) As discussed in Introduction, the number of returning electrons can be even further enhanced by a magnetic mirroring, not considered in the current model, while dominating at lower atmospheric depths where the magnetic convergence and magnitude are the highest. This enhancement can noticeably modify the resulting HXR and MW emission for flares with visibly converging magnetic loops that needs to be considered in the future interpretation.

Therefore, the proposed MEL approach can be utilized at the initial investigation stage of spectral fitting providing an opportunity to account simultaneously for both collisional and ohmic losses without lengthy numerical FP simulations. The IDL code implementing the proposed MEL approach can be incorporated into the RHESSI Solar Software for a quicker fit at the initial data processing stage.

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