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SPECIAL SECTION ON RECENT ADVANCES IN FAULT DIAGNOSIS AND FAULT-TOLERANT CONTROL OF AEROSPACE ENGINEERING SYSTEMS

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Robust Fault Tolerant Control for Discrete-Time Dynamic Systems With Applications to Aero Engineering Systems

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ABSTRACT Unexpected faults in actuators and sensors may degrade the reliability and safety of aero engineering systems. Therefore, there is motivation to develop integrated fault tolerant control techniques with applications to aero engineering systems. In this paper, discrete-time dynamic systems, in the presence of simultaneous actuator/sensor faults, partially decoupled unknown input disturbances, and sensor noises, are investigated. A jointly state/fault estimator is formulated by integrating an unknown input observer, augmented system approach, and optimization algorithm. Unknown input disturbances can be either decoupled by an unknown input observer, or attenuated by a linear matrix inequality optimization, enabling the estimation error to be input-to-state stable. Estimator-based signal compensation is then implemented to mitigate adverse effects from the unanticipated actuator and sensor faults. A pre-designed controller, which maintains normal system behaviors under a fault-free scenario, is allowed to work along with the presented fault tolerant mechanism of the signal compensations. The fault-tolerant closed-loop system can be ensured to mitigate the effects from the faults, guarantee the input-to-state stability, and satisfy the required robustness performance. The proposed fault estimation and fault tolerant control methods are developed for both discrete-time linear and discrete-time Lipschitz nonlinear systems. Finally, the proposed techniques are applied to a jet engine system and a flight control system for simulation validation.

INDEX TERMS Discrete-time systems, aero engineering systems, fault estimation, partically decoupled unknown inputs, signal compensation.

I. INTRODUCTION

Working under challenging operating conditions, real engineering systems are unavoidably subjected to abnormal/faulty behaviors, which degrade the functionality of the systems. Consequently, there is a high demand to develop advanced fault diagnosis and fault tolerant control strategies for enhancing the system reliability and safety. A variety of fault diagnosis and fault tolerant control techniques were developed during last decades (see [1]–[4]). Among industrial plants, aero systems are extremely safety-critical; hence, the level of fault diagnosis and tolerant control is required to be even higher. Therefore, fruitful results in terms of advanced fault diagnosis and tolerant control for aero engineering systems were documented, which can be found in the review work [5], [6].

Fault estimation/reconstruction is a multi-mission fault diagnosis technique, which can provide rich information of faults and allow successful design of fault tolerant controller to mitigate the influences from unexpected faults. Specifically, advanced observer methods, such as sliding mode observer [7], [8], and augmented observers [9]–[11], were applied to design state/fault estimator-based fault tolerant controller for aerospace systems corrupted with actuator/sensor faults. Actuator/sensor fault reconstruction for aircraft was proposed in [12]. Fault estimations for discrete-time systems were developed in [13]–[15]. Among various fault estimation techniques, augmented system approach possesses advantages in achieving simultaneous states and faults estimation, where unmeasurable system states can be estimated as a byproduct.

Once the faults can be estimated, signal compensation is known as an effective fault tolerant method, which can work with a pre-designed controller, e.g., see the pioneering work [9], [10]. It takes advantages to remove/mitigate the influences from the actuator/sensor faults so that the system can work well even when a fault occurs (e.g., see [9], [10], [15], [16]). The application of signal compensation to aero engine systems can be found in [12] and [13]. As signal compensators achieve fault tolerance by providing compensated signals to actuators and sensors, successful implementation of signal compensation depends on effective fault estimation. Unknown inputs including modelling errors, uncertainties, and extra perturbations, etc., are unavoidable in engineering plants, but may decrease the sensitivity of fault reconstruction. Unknown input observer (UIO) [17] is then motivated to be applied for decoupling the influences from the unknown inputs when carrying out fault estimation [18], [19].

As the complexity and diversity of unknown input disturbances increase, traditional UIO techniques cannot decouple all unknown inputs. UIO associated with optimization scheme was hence developed for continuous-time systems subjected to partially decoupled unknown inputs in our previous work [20], where unknown inputs that cannot be decoupled by UIO were attenuated by using linear matrix inequality (LMI). Therefore, a robust estimator-based signal compensation approach was designed for a continuous-time wind turbine system corrupted by faults and partially decoupled unknown inputs in [21]. However, the aforementioned works [20], [21] were based on continuous-time systems only, which cannot be applicable to discrete-time dynamic systems. In practice, real-time implementation of monitoring and control need to use digital signals, therefore, discrete-time diagnosis and tolerant control techniques need to be explored. As a result, it is well motivated to design robust UIO-based fault tolerant control techniques for discrete-time dynamic systems subjected to faults and partially decoupled unknown inputs, which can be applied to aero engineering systems. Due to the nature of discrete-time dynamics, some well-known techniques for continuous systems such as high-gain observer techniques etc., [9], [10], have been found difficult to apply for discrete-time systems.

In this paper, an integrated robust fault estimator-based fault tolerant control approach is addressed for discrete-time systems in presence of simultaneous faults and partially decoupled unknown inputs. Specifically, robust fault estimation can be obtained by integrating augmented system approach, UIO, and LMI techniques, such that the estimation error dynamic is input-to-state stable. Signal compensation method is then developed to achieve tolerance against actuator and sensor faults, and maintain the stability of the closed-loop system. The robust fault estimator-based fault tolerant control approaches are presented for both linear and Lipschitz nonlinear discrete-time systems. The novelties and contributions of this work include: 1) Simultaneous discrete-time state/fault estimation techniques with robustness against partially decoupled unknown inputs are developed, with the aid of input-to-stability theory. The input disturbances are assumed not to be completely decoupled, which can meet more general practical engineering conditions. 2) Robust fault estimation-based signal compensation for fault tolerant control is addressed without replacing the pre-existing controller, which makes the tolerant control strategies simple to apply and capable of avoiding performance fluctuations due to controllers switching. 3) Input-to-state stability theory is used for the stability proof of the estimation error dynamics and tolerant closed-loop control system, which is shown to be an effective tool for handling discrete-time estimation and control issues. 4) Case studies on two aero engineering systems are used to demonstrate the effectiveness.

The rest of the paper is organized as follows. In Section II, the UIO-based fault estimation approach is addressed for discrete-time linear dynamic systems subjected to both faults and partially decoupled unknown input uncertainties. Estimator-based signal compensation tolerant technique is developed in Section III. UIO-based fault estimation and fault tolerant control approaches for discrete-time Lipschitz nonlinear systems are presented in Section VI. The developed integrated fault tolerant control strategies are demonstrated using two aero systems for case studies in Section V. The paper ends with the conclusion in Section VI.

Notation: The notations in this paper are standard. The superscript "T" represents the transpose of matrices or vectors. $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ stand for the n-dimentional Euclidean space and the set of $n \times m$ real matrices, respectively. $\mathcal{R}_+$ and $\mathcal{J}_+$ represent the set of nonnegative reals and nonnegative integers, respectively. $X < 0$ indicates the symmetric matrix X is negative definite, while the notation $X > Y$ means that $X - Y$ is positive definite. I_n denotes the identity matrix with the dimension of $n \times n$, while 0 is a scalar zero or a zero matrix with appropriate zero entries. For a complex number z, $|z|$ denotes the module of z; while for a vector x, $|x|$ refers to the Euclidean norm of the vector. $|x|_2 = \left(\sum_{k=0}^{\infty} x^T(k)x(k)\right)^{1/2}$, and $|A| = \sqrt{\lambda_{max}\left(A^TA\right)}$ for a real matrix A. $\forall$ means for all. Denotes $\|v\| = \sup\{|v(k)| : k \in \mathcal{J}_+\} \leq \infty$, which indicates $\|v\|$ is the standard l_∞ norm when v is bounded. For brevity, $\begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} \overset{\Delta}{\Longleftrightarrow} \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix}$.

II. UIO-BASED FAULT ESTIMATION FOR DISCRETE-TIME LINEAR DYNAMIC SYSTEMS

Consider a discrete-time plant subjected to actuator faults, sensor faults, and unknown disturbances in the form of

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_f f(k) + B_d d(k) \\ y(k) = Cx(k) + D_f f(k) + D_d d_s(k) \end{cases} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ represents state vector with initial value of $x_0 \in \mathcal{R}^n$; $u(k) \in \mathcal{R}^m$ and $y(k) \in \mathcal{R}^p$ stand for control input vector and measurement output vector, respectively; $d(k) \in \mathcal{R}^{l_d}$ is a bounded unknown input vector caused by

either disturbances or modelling errors; $d_s(k) \in \mathcal{R}^{l_s}$ is the measurement noise; $f(k) \in \mathcal{R}^{l_f}$ is the fault vector involving actuator faults and sensor faults; $k \in \mathcal{J}_+$ is the discrete-time instant. A, B, C, B_d, B_f, D_d and D_f are known constant coefficient matrices with appropriate dimensions. In addition, $B_d = [B_{d1}\ B_{d2}]$, $d(k) = [d_1^T(k)\ d_2^T(k)]^T$, $d_1(k) \in \mathcal{R}^{l_{d1}}$ and $d_2(k) \in \mathcal{R}^{l_{d2}}$, where $d_1(k)$ rather than $d_2(k)$ is assumed to be decoupled, and B_{d1} is of full column rank.

Define

$$\Delta f(k) = f(k+1) - f(k) \tag{2}$$

and assume $\Delta f(k)$ is bounded.

Denote

$$\begin{aligned} \bar{n} &= n + l_f \\ \bar{x}(k) &= \begin{bmatrix} x^T(k) & f^T(k) \end{bmatrix}^T \in \mathcal{R}^{\bar{n}}, \\ \bar{d}(k) &= \begin{bmatrix} d^T(k) & \Delta f^T(k) \end{bmatrix}^T \in \mathcal{R}^{l_d + l_f}, \end{aligned}$$

Therefore, system (1) can be represented by an augmented system as follows:

$$\begin{cases} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) + \bar{B}_d\bar{d}(k) \\ y(k) = \bar{C}\bar{x}(k) + D_d d_s(k) \end{cases} \tag{3}$$

and the corresponding system coefficients are:

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & B_f \\ 0_{l_f \times n} & I_{l_f} \end{bmatrix} \in \mathcal{R}^{\bar{n}\times\bar{n}}, \quad \bar{B} = \begin{bmatrix} B \\ 0_{l_f \times m} \end{bmatrix} \in \mathcal{R}^{\bar{n}\times m}, \\ \bar{B}_d &= \begin{bmatrix} B_d & 0_{n\times l_f} \\ 0_{l_f \times l_d} & I_{l_f} \end{bmatrix} \in \mathcal{R}^{\bar{n}\times(l_d+l_f)}, \\ \bar{C} &= \begin{bmatrix} C & D_f \end{bmatrix} \in \mathcal{R}^{p\times\bar{n}}. \end{aligned}$$

Denote $\bar{d}_2(k) = \begin{bmatrix} d_2^T(k)\ \Delta f^T(k) \end{bmatrix}^T \in \mathcal{R}^{l_{d2}+l_f}$, and $\bar{d}(k) = [d_1^T(k)\ \bar{d}_2^T(k)]^T$, where $d_1(k)$ can be decoupled whereas $\bar{d}_2(k)$ cannot be decoupled. Moreover, we let $\bar{B}_d = [\bar{B}_{d1}\ \bar{B}_{d2}]$, where $\bar{B}_{d1} = \begin{bmatrix} B_{d1} \\ 0_{l_f\times l_{d1}} \end{bmatrix} \in \mathcal{R}^{\bar{n}\times l_{d1}}$ and $\bar{B}_{d2} = \begin{bmatrix} B_{d2} & 0_{n\times l_f} \\ 0_{l_f\times l_{d2}} & I_{l_f} \end{bmatrix} \in \mathcal{R}^{\bar{n}\times(l_{d2}+l_f)}$.

It is clear that $\bar{x}(k)$ contains the original state vector $x(k)$ and the concerned fault vector $f(k)$. As a result, these two components can be estimated simultaneously by designing an observer for the augmented system (3).

To attenuate the influences from the unknown inputs, a UIO can be constructed for system (3) as follows:

$$\begin{cases} \bar{z}(k+1) = R\bar{z}(k) + T\bar{B}u(k) + (L_1 + L_2)\,y(k) \\ \hat{\bar{x}}(k) = \bar{z}(k) + Hy(k) \end{cases} \tag{4}$$

where $\bar{z}(k) \in \mathcal{R}^{\bar{n}}$ is the state vector of dynamic system (4) and $\hat{\bar{x}}(k) \in \mathcal{R}^{\bar{n}}$ represents the estimation of $\bar{x}(k) \in \mathcal{R}^{\bar{n}}$, while $R \in \mathcal{R}^{\bar{n}\times\bar{n}}$, $L_1 \in \mathcal{R}^{\bar{n}\times p}$, $L_2 \in \mathcal{R}^{\bar{n}\times p}$, $T \in \mathcal{R}^{\bar{n}\times\bar{n}}$ and $H \in \mathcal{R}^{\bar{n}\times p}$ are the gain matrices to be designed.

Defining the estimation error as

$$e(k) = \bar{x}(k) - \hat{\bar{x}}(k) \tag{5}$$

it can be calculated that

$$\begin{aligned} e(k+1) &= \bar{x}(k+1) - \hat{\bar{x}}(k+1) \\ &= \left(I_{\bar{n}} - H\bar{C}\right)\bar{x}(k+1) - \bar{z}(k+1) \\ &\quad - HD_d d_s(k+1) \end{aligned} \tag{6}$$

Using (3)-(6), we have

$$\begin{aligned} &e(k+1) \\ &= \left[\left(I_{\bar{n}} - H\bar{C}\right)\bar{A} - L_1\bar{C}\right]\bar{x}(k) - R\hat{\bar{x}}(k) \\ &\quad + \left[\left(I_{\bar{n}} - H\bar{C}\right) - T\right]\bar{B}u(k) + \left(I_{\bar{n}} - H\bar{C}\right)\bar{B}_{d1}d_1(k) \\ &\quad + \left(I_{\bar{n}} - H\bar{C}\right)\bar{B}_{d2}\bar{d}_2(k) - L_1 D_d d_s(k) \\ &\quad + (RH - L_2)y(k) - HD_d d_s(k+1) \end{aligned} \tag{7}$$

Estimation error dynamic (7) can be reduced to

$$\begin{aligned} e(k+1) &= Re(k) + T\bar{B}_{d2}\bar{d}_2(k) - L_1 D_d d_s(k) \\ &\quad - HD_d d_s(k+1) \end{aligned} \tag{8}$$

if the following conditions hold

$$\left(I_{\bar{n}} - H\bar{C}\right)\bar{B}_{d1} = 0 \tag{9}$$

$$R = \bar{A} - H\bar{C}\bar{A} - L_1\bar{C} \tag{10}$$

$$T = I_{\bar{n}} - H\bar{C} \tag{11}$$

$$L_2 = RH \tag{12}$$

In order to make conditions (9)-(12) hold, we have the following Lemma:

Lemma 1: The sufficient and necessary conditions for the existence of the UIO (4) for the system (3) are:

(i) $\operatorname{rank}(CB_{d1}) = \operatorname{rank}(B_{d1})$;

(ii). $\operatorname{rank}\begin{bmatrix} A - I_n & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix} = n + l_{d1} + l_f$;

(iii). $\operatorname{rank}\begin{bmatrix} A - zI_n & B_{d1} \\ C & 0 \end{bmatrix} = n + l_{d1}$, $\forall z$, with $|z| \geq 1$ and $z \neq 1$.

Proof: See Appendix.

A special solution of (9) is

$$H = \bar{B}_{d1}[(\bar{C}\bar{B}_{d1})^T(\bar{C}\bar{B}_{d1})]^{-1}(\bar{C}\bar{B}_{d1})^T \tag{13}$$

From (8), one can see $d_1(k)$ has been decoupled, by deriving H from (13) to satisfy condition (9), but $\bar{d}_2(k)$ still exists. Therefore, the observer design is transformed to seek the observer gains to ensure the estimation error $e(k)$ stable and attenuate the influences of $\bar{d}_2(k)$ on estimation error $e(k)$.

The following definitions and lemmas about input-to-state stability are introduced.

Definition 1 [22]: A function $\gamma : \mathcal{R}_+ \longrightarrow \mathcal{R}_+$ is said to be a $\mathcal{K}$-function if it is continuous, strictly increasing, and satisfy $\gamma(0) = 0$. γ is a $\mathcal{K}_\infty$ function if it is a $\mathcal{K}$-function, and also $\gamma(s) \to \infty$ as $s \to \infty$.

Definition 2 [22]: A function $\beta : \mathcal{R}_+ \times \mathcal{R}_+ \longrightarrow \mathcal{R}_+$ is said to be a $\mathcal{KL}$-function if for each fixed $t \geq 0$, the function $\beta(s,t)$ is a $\mathcal{K}$-function, and for each fixed $s \geq 0$, the function is decreasing, and $\beta(s,t) \longrightarrow 0$ as $t \longrightarrow \infty$.

Consider the following discrete-time dynamic system

$$x(k+1) = h(x(k), v(k)) \tag{14}$$

where $x(k) \in \mathcal{R}^n$ is system state and $v(k) \in \mathcal{R}^v$ is input. For system (14), we have the following definition.

Definition 3 [23]: System (14) is said to be input-to-state stable, if there exist functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that for any initial condition $x(k_0) = x_0$, $v(k) \in l_\infty^v$, and $k \in \mathcal{J}_+$ one has

$$|x(k, x_0, v)| \leq \beta(|x_0|, k) + \gamma(\|v\|) \tag{15}$$

From *Definition 3*, we can know input-to-state stability takes the influences of inputs on stability into consideration, and reflects that bounded inputs result in bounded system states. It can indicate the robustness of a system.

Lemma 2 [23]–[25]: Let $V : \mathcal{R}^n \to \mathcal{R}_+$ be a continuous function. If there exist $\mathcal{K}_\infty$ functions ψ_1 and ψ_2, such that

$$\psi_1(|x|) \leq V(x) \leq \psi_2(|x|), \quad \forall x \in \mathcal{R}^n \tag{16}$$

and if there exist a $\mathcal{K}_\infty$ function ψ_3 and a $\mathcal{K}$ function ψ_4, such that

$$V(h(x, v)) - V(x) \leq -\psi_3(|x|) + \psi_4(|v|) \quad \forall x \in \mathcal{R}^n, \quad \forall v \in \mathcal{R}^v \tag{17}$$

then system (14) is input-to-state stable.

Based on the definitions and lemmas above, it is time to design robust observer (4) so that the error dynamic (8) is input-to-state stable.

Theorem 1: For system (3), there exists a robust UIO in the form of (4) such that the error dynamic system (8) is input-to-state stable, if there exist a positive definite matrix P and matrix Y, and the positive scalars α, γ_{d2}, γ_{ds} and γ_{ds1} such that inequality (18) holds, which is shown at the bottom of this page.

Furthermore, one can calculate $L_1 = P^{-1}Y$.

Proof: Take the following Lyapunov function candidate for error dynamic system (8):

$$V(e(k)) = e^T(k)Pe(k) \tag{19}$$

It is clear that

$$\lambda_{min}(P)|e(k)|^2 \leq V(e(k)) \leq \lambda_{max}(P)|e(k)|^2 \tag{20}$$

indicating that $V(e(k))$ satisfies condition (16) in *Lemma 2*, with $\psi_1(|e(k)|) = \lambda_{min}(P)|e(k)|^2$, and $\psi_2(|e(k)|) = \lambda_{max}(P)|e(k)|^2$.

Define $\eta(k) = e(k+1) - e(k)$. From (8), one can derive

$$\eta(k) = (R - I_{\bar{n}})e(k) + T\bar{B}_{d2}\bar{d}_2(k) - L_1 D_d d_s(k) - HD_d d_s(k+1) \tag{21}$$

By using (19) and (21), one can have

$$\begin{aligned}
&\Delta V(e(k)) \\
&= V(e(k+1)) - V(e(k)) \\
&= e^T(k+1)Pe(k+1) - e^T(k)Pe(k) \\
&= [e(k+1) - e(k)]^T P[e(k+1) - e(k)] \\
&\quad + 2e^T(k)Pe(k+1) - 2e^T(k)Pe(k) \\
&= [e(k+1) - e(k)]^T P[e(k+1) - e(k)] \\
&\quad + 2e^T(k)P[e(k+1) - e(k)] \\
&= -[e(k+1) - e(k)]^T P[e(k+1) - e(k)] \\
&\quad + 2e^T(k)P[e(k+1) - e(k)] \\
&\quad + 2[e(k+1) - e(k)]^T P[e(k+1) - e(k)] \\
&= -\eta^T(k)P\eta(k) + 2e^T(k)P\eta(k) + 2\eta^T(k)P\eta(k) \\
&= -\eta^T(k)P\eta(k) + 2e^T(k)P(R - I_{\bar{n}})e(k) \\
&\quad + 2e^T(k)PT\bar{B}_{d2}\bar{d}_2(k) - 2e^T(k)PL_1 D_d d_s(k) \\
&\quad - 2e^T(k)PHD_d d_s(k+1) \\
&\quad + 2\eta^T(k)P(R - I_{\bar{n}})e(k) + 2\eta^T(k)PT\bar{B}_{d2}\bar{d}_2(k) \\
&\quad - 2\eta^T(k)PL_1 D_d d_s(k) - 2\eta^T(k)PHD_d d_s(k+1)
\end{aligned} \tag{22}$$

Adding and subtracting

$$\begin{aligned}
&-\gamma_{d2}^2\bar{d}_2^T(k)\bar{d}_2(k) - \gamma_{ds}^2 d_s^T(k)d_s(k) \\
&-\gamma_{ds1}^2 d_s^T(k+1)d_s(k+1) + \alpha V(e(k))
\end{aligned}$$

to the right side of (22), one has

$$\begin{aligned}
&\Delta V(e(k)) \\
&= \begin{bmatrix} \eta^T(k) & e^T(k) & \bar{d}_2^T(k) & d_s^T(k) & d_s^T(k+1) \end{bmatrix} \\
&\quad \times \Theta \begin{bmatrix} \eta(k) \\ e(k) \\ \bar{d}_2(k) \\ d_s(k) \\ d_s(k+1) \end{bmatrix} \\
&\quad + \gamma_{d2}^2\bar{d}_2^T(k)\bar{d}_2(k) + \gamma_{ds}^2 d_s^T(k)d_s(k) \\
&\quad + \gamma_{ds1}^2 d_s^T(k+1)d_s(k+1) - \alpha V(e(k))
\end{aligned} \tag{23}$$

where Θ is shown in the first equation at the top of the next page.

From (10), it is clear that $PR = PT\bar{A} - Y\bar{C}$, where $Y = PL_1$. Therefore, Θ can be rewritten as follows, which is shown in the second equation at the top of the next page.

LMI (18) indicates $\Theta < 0$, leading to

$$\begin{aligned}
\Delta V(e(k)) < &-\alpha V(e(k)) + \gamma_{d2}^2\bar{d}_2^T(k)\bar{d}_2(k) \\
&+ \gamma_{ds}^2 d_s^T(k)d_s(k) + \gamma_{ds1}^2 d_s^T(k+1)d_s(k+1)
\end{aligned} \tag{24}$$

$$\begin{bmatrix}
-P & PT\bar{A} - Y\bar{C} - P & PT\bar{B}_{d2} & -YD_d & -PHD_d \\
* & 2PT\bar{A} - 2Y\bar{C} + (\alpha - 2)P & PT\bar{B}_{d2} & -YD_d & -PHD_d \\
* & * & -\gamma_{d2}^2 I_{(l_{d2}+l_f)} & 0 & 0 \\
* & * & * & -\gamma_{ds}^2 I_{l_{ds}} & 0 \\
* & * & * & * & -\gamma_{ds1}^2 I_{l_{ds}}
\end{bmatrix} < 0 \tag{18}$$

$$\Theta = \begin{bmatrix} -P & P(R-I_{\bar{n}}) & PT\bar{B}_{d2} & -PL_1D_d & -PHD_d \\ * & 2P(R-I_{\bar{n}})+\alpha P & PT\bar{B}_{d2} & -PL_1D_d & -PHD_d \\ * & * & -\gamma_{d2}^2 I_{(l_{d2}+l_f)} & 0 & 0 \\ * & * & * & -\gamma_{ds}^2 I_{l_{ds}} & 0 \\ * & * & * & * & -\gamma_{ds1}^2 I_{l_{ds}} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} -P & PT\bar{A}-Y\bar{C}-P & PT\bar{B}_{d2} & -YD_d & -PHD_d \\ * & 2PT\bar{A}-2Y\bar{C}+(\alpha-2)P & PT\bar{B}_{d2} & -YD_d & -PHD_d \\ * & * & -\gamma_{d2}^2 I_{(l_{d2}+l_f)} & 0 & 0 \\ * & * & * & -\gamma_{ds}^2 I_{l_{ds}} & 0 \\ * & * & * & * & -\gamma_{ds1}^2 I_{l_{ds}} \end{bmatrix}$$

Since $-\alpha V(e(k)) \leq -\alpha\lambda_{min}(P)|e(k)|^2$, and form (24) one can have

$$\begin{aligned} &\Delta V(e(k)) \\ &\quad < -\alpha\lambda_{min}(P)|e(k)|^2 + \gamma_{d2}^2\bar{d}_2^T(k)\bar{d}_2(k) \\ &\qquad + \gamma_{ds}^2 d_s^T(k)d_s(k) + \gamma_{ds1}^2 d_s^T(k+1)d_s(k+1) \end{aligned} \tag{25}$$

which means condition (17) in *Lemma 2* is met. Specifically, $\psi_3(|e(k)|) = \alpha\lambda_{min}(P)|e(k)|^2$ and $\psi_4(|v(k)|) = \max\{\gamma_{d2}^2, \gamma_{ds}^2, \gamma_{ds1}^2\}|v(k)|^2$, in which $v(k) = \left[\bar{d}_2^T(k)\ d_s^T(k)\ d_s^T(k+1)\right]^T$. As a result, the error dynamic system (8) is input-to-state stable. This completes the proof.

Now the design procedure of the fault estimator can be summarized as follows:

Procedure 1 (UIO-Based State/Fault Estimation):

(1) Construct the augmented system in the form of (3) for the discrete-time model (1).
(2) Solve H from Equation (13).
(3) Solve the LMI (18) to obtain the matrices P and Y, and calculate the gain $L_1 = P^{-1}Y$.
(4) Calculate the gain matrices R, T and L_2 following the formulae (10)-(12), respectively.
(5) Generate the augmented estimate $\hat{\bar{x}}(k)$ by implementing UIO (4), leading to the simultaneous estimates of the state and fault as $\hat{x}(k) = \left[I_n\ 0_{n\times l_f}\right]\hat{\bar{x}}(k)$ and $\hat{f}(k) = \left[0_{l_f\times n}\ I_{l_f}\right]\hat{\bar{x}}(k)$, respectively.

III. FAULT TOLERANT CONTROL FOR DISCRETE-TIME LINEAR SYSTEM

Assume there is a pre-existing dynamic output feedback controller, designed for normal operating conditions (*i. e.*, fault free scenario), in the form of

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c y(k) \\ u(k) = C_c x_c(k) \end{cases} \tag{26}$$

where $x_c(k) \in \mathcal{R}^{n_c}$ is the state of the dynamic controller, A_c, B_c and C_c are control gains with appropriate dimensions, whose designs are beyond this study.

Based on the estimation of $\hat{\bar{x}}$, the original system state and fault vector can be reconstructed as

$$\hat{x}(k) = \left[I_n \quad 0_{n\times l_f}\right]\hat{\bar{x}}(k) \tag{27}$$

and

$$\hat{f}(k) = \left[0_{l_f\times n} \quad I_{l_f}\right]\hat{\bar{x}}(k) \tag{28}$$

Suppose

$$\text{rank}\left[B \quad B_f\right] = \text{rank}B \tag{29}$$

and the compensated signal for the actuator is designed as $u_f = K_f\hat{f}$, where

$$K_f = B^+B_f \tag{30}$$

Therefore, we have

$$B_f - BK_f = 0 \tag{31}$$

Using $-D_f\hat{f}(k)$ to compensate the measurement output, we have

$$\begin{aligned} y_c(k) &= y(k) - D_f J_2\hat{\bar{x}}(k) \\ &= Cx(k) + D_f f(k) - D_f\hat{f}(k) + D_d d_s(k) \\ &= Cx(k) + D_f J_2 e(k) + D_d d_s(k) \end{aligned} \tag{32}$$

where $J_2 = \left[0_{l_f\times n}\ I_{l_f}\right]$.

Subtracting u_f from the actuator input, and using the compensated measurement output y_c to replace the actual measurement y, the controller with signal compensation can thus be updated as follows:

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c y_c(k) \\ u(k) = C_c x_c(k) - K_f J_2\hat{\bar{x}}(k) \end{cases} \tag{33}$$

Substituting controller (33) to system (1), the resulting closed-loop system can be established

$$\begin{cases}\tilde{x}(k+1)=\tilde{A}\tilde{x}(k)+\tilde{B}_d\tilde{d}(k)+B_e e(k)\\ y_c(k)=\tilde{C}\tilde{x}(k)+D_f J_2 e(k)+J_d\tilde{d}(k)\end{cases} \tag{34}$$

where

$$\tilde{x}(k)=\begin{bmatrix}x^T(k) & x_c^T(k)\end{bmatrix}^T,\quad \tilde{A}=\begin{bmatrix}A & BC_c\\ B_cC & A_c\end{bmatrix},$$
$$\tilde{B}_d=\begin{bmatrix}B_d & 0\\ 0 & B_cD_d\end{bmatrix},\quad \tilde{C}=[\,C\quad 0_{p\times n_c}\,],$$
$$\tilde{d}(k)=\begin{bmatrix}d^T(k) & d_s^T(k)\end{bmatrix}^T,\quad B_e=\begin{bmatrix}B_fJ_2\\ B_cD_fJ_2\end{bmatrix},$$

and $J_d=[\,0_{p\times l_d}\ D_d\,]$.

One can presume that under controller (26), the closed-loop system is input-to-state stable in fault-free scenario, with the following robustness performance

$$|y|_2^2\le\gamma_p\left|\tilde{d}\right|_2^2 \tag{35}$$

where γ_p is a positive scalar.

Since system (1) is input-to-state stable under controller (26) for fault-free case, there is a Lyapunov function V_c which satisfies condition (16) and (17) of *Lemma 2*. Without loss of generality, one can presume the Lyapunov function can be found as $V_c(\tilde{x}(k))=\tilde{x}^T(k)\tilde{P}\tilde{x}(k)$, such that

$$\psi_{c1}(|\tilde{x}(k)|)\le V_c(\tilde{x}(k))\le\psi_{c2}(|\tilde{x}(k)|) \tag{36}$$

and

$$\Delta V_c(\tilde{x}(k))<-\alpha_c|\tilde{x}(k)|^2+\gamma_c\left|\tilde{d}(k)\right|^2 \tag{37}$$

where $\tilde{P}$ is a positive-definite matrix with appropriate dimension, $\psi_{c1},\psi_{c2}\in\mathcal{K}_\infty$, α_c and γ_c are positive scalars.

Now it is ready to discuss the stability and robustness of the dynamic system (34) in faulty case after signal compensation.

Theorem 2: If there is a pre-existing controller in the form of (26) to ensure the closed-loop system of the plant (1) to be input-to-state stable under fault-free case and satisfy the robust performance index (35), the tolerant controller (32)-(33) can drive the trajectories of the closed-loop system of the plant (1) to be input-to-state stable when a fault occurs, and satisfy the following robust performance index:

$$|y_c|_2^2\le\gamma_0\left|\tilde{d}\right|_2^2+\gamma_{0e}|e|_2^2 \tag{38}$$

where γ_0 and γ_{0e} are positive scalars representing the robust performance indices.

Proof: Choose Lyapunov function as

$$\begin{aligned}\tilde{V}(\tilde{x}_e(k))&=V_c(\tilde{x}(k))+\xi V(e(k))\\&=\tilde{x}^T(k)\tilde{P}\tilde{x}(k)+\xi e^T(k)Pe(k)\end{aligned} \tag{39}$$

where ξ is a positive scalar, $\tilde{x}_e(k)=\left[\tilde{x}^T(k)\ e^T(k)\right]^T$.

Therefore, $\tilde{V}(\tilde{x}_e(k))$ satisfies condition (16) in *Lemma 1* with $\tilde{\psi}_1(|\tilde{x}_e(k)|)=\min\{\lambda_{min}(\tilde{P}),\xi\lambda_{min}(P)\}|\tilde{x}_e(k)|^2$, and $\tilde{\psi}_2(|\tilde{x}_e(k)|)=\max\{\lambda_{max}(\tilde{P}),\xi\lambda_{max}(P)\}|\tilde{x}_e(k)|^2$.

From (25), one can have

$$\Delta V(e(k))<-\alpha_e|e(k)|^2+\gamma_e|v(k)|^2 \tag{40}$$

where $\alpha_e=\alpha\lambda_{min}(P)$, and $\gamma_e=\max\{\gamma_{d2}^2,\gamma_{ds}^2,\gamma_{ds1}^2\}$.

It should be noticed that, condition (37) holds when system (1) is free of faults. Nevertheless, considering faulty cases, the closed-loop system of (1) is subjected to the estimation error as shown in (34). As a result, $\Delta V_c(\tilde{x}(k))$ in (37) is also corrupted with estimation error $e(k)$ under faulty case.

According to (34), (37), (39) and (40), we have

$$\begin{aligned}\Delta\tilde{V}<&-\alpha_c|\tilde{x}(k)|^2+\gamma_c\left|\tilde{d}(k)\right|^2+e^T(k)B_e^T\tilde{P}B_e e(k)\\&+2\tilde{x}^T(k)\tilde{A}^T\tilde{P}B_e e(k)+2e^T(k)B_e^T\tilde{P}\tilde{B}_d\tilde{d}(k)\\&-\xi\alpha_e|e(k)|^2+\xi\gamma_e|v(k)|^2\\\le&-\alpha_c|\tilde{x}(k)|^2+\gamma_c\left|\tilde{d}(k)\right|^2+\xi_e|e(k)|^2\\&+\xi_x|\tilde{x}(k)||e(k)|+2\xi_d|e(k)|\left|\tilde{d}(k)\right|\\&-\xi\alpha_e|e(k)|^2+\xi\gamma_e|v(k)|^2\\\le&-\alpha_c|\tilde{x}(k)|^2-(-\xi_e+\xi\alpha_e-\xi_d)|e(k)|^2\\&+\xi_x|\tilde{x}(k)||e(k)|+(\gamma_c+\xi_d)\left|\tilde{d}(k)\right|^2\\&+\xi\gamma_e|v(k)|^2\end{aligned} \tag{41}$$

where ξ_e, ξ_x, and ξ_d are positive scalars such that $\xi_e=\left|B_e^T\tilde{P}B_e\right|$, $\xi_x=2\left|\tilde{A}^T\tilde{P}B_e\right|$, $\xi_d=\left|B_e^T\tilde{P}\tilde{B}_d\right|$.

Selecting

$$\xi\ge\frac{\xi_x^2+(\xi_d+\xi_e)\alpha_c}{\alpha_c\alpha_e} \tag{42}$$

and from (41) and (42), one has

$$\begin{aligned}\Delta\tilde{V}\le&-\frac{\alpha_c}{2}|\tilde{x}(k)|^2-\frac{-\xi_e+\xi\alpha_e-\xi_d}{2}|e(k)|^2\\&+\xi\gamma_e|v(k)|^2+(\gamma_c+\xi_d)\left|\tilde{d}(k)\right|^2\\\le&-\alpha_{xe}|\tilde{x}_e(k)|^2+\beta_{xe}|\tilde{v}(k)|^2\end{aligned} \tag{43}$$

in which $\alpha_{xe}=\min\{\frac{\alpha_c}{2},\frac{-\xi_e+\xi\alpha_e-\xi_d}{2}\}$, $\tilde{v}(k)=\left[v^T(k)\ \tilde{d}^T(k)\right]^T$, and $\beta_{xe}=\max\{\xi\gamma_e,\gamma_c+\xi_d\}$.

Therefore, the input-to-state stability of the close-loop system (34) has been proved.

Now it is time to discuss the robustness performance of the closed-loop system (34).

Considering (34) for fault-free case and using (35), one has

$$\begin{aligned}&\sum_{k=0}^N y^T(k)y(k)\\&=\sum_{k=0}^N[\tilde{x}^T(k)\tilde{C}^T\tilde{C}\tilde{x}(k)+\tilde{d}^T(k)J_d^TJ_d\tilde{d}(k)\\&\quad+2\tilde{x}^T(k)\tilde{C}^TJ_d\tilde{d}(k)]\\&\le\gamma_p\sum_{k=0}^N\tilde{d}^T(k)\tilde{d}(k)\end{aligned} \tag{44}$$

For system (34) under faulty scenario, and using (44), one can derive

$$\begin{aligned}
&\sum_{k=0}^{N} y_c^T(k)\, y_c(k) \\
&= \sum_{k=0}^{N} [\tilde{x}^T(k)\, \tilde{C}^T \tilde{C} \tilde{x}(k) + \tilde{d}(k)^T J_d^T J_d \tilde{d}(k) \\
&\quad + 2\tilde{x}^T(k)\, \tilde{C}^T J_d \tilde{d}(k) + e^T(k)\, J_2^T D_f^T D_f J_2 e(k) \\
&\quad + 2\tilde{x}^T(k)\, \tilde{C}^T D_f J_2 e(k) + 2e^T(k)\, J_2^T D_f^T J_d \tilde{d}(k)] \\
&\le \sum_{k=0}^{N} [(\gamma_p + \left|J_2^T D_f^T J_d\right|) \left|\tilde{d}(k)\right|^2 + (\left|J_2^T D_f^T D_f J_2\right| \\
&\quad + \left|\tilde{C}^T D_f J_2\right| + \left|J_2^T D_f^T J_d\right|) |e(k)|^2 + \left|\tilde{C}^T D_f J_2\right| |\tilde{x}(k)|^2]
\end{aligned} \tag{45}$$

Adding and subtracting $\sum_{k=0}^{N} \xi \Delta V_c(\tilde{x}(k))$ to (45), and using (37), one can have

$$\begin{aligned}
&\sum_{k=0}^{N} y_c^T(k)\, y_c(k) \\
&\le \sum_{k=0}^{N} [(\gamma_p + \left|J_2^T D_f^T J_d\right| + \xi\gamma_c) \left|\tilde{d}(k)\right|^2 \\
&\quad + (\left|J_2^T D_f^T D_f J_2\right| + \left|\tilde{C}^T D_f J_2\right| + \left|J_2^T D_f^T J_d\right|) |e(k)|^2 \\
&\quad + (\left|\tilde{C}^T D_f J_2\right| - \xi\alpha_c) |\tilde{x}(k)|^2 - \xi \Delta V_c(\tilde{x}(k))] \\
&\le \sum_{k=0}^{N} [\gamma_0 \left|\tilde{d}(k)\right|^2 + \gamma_{0e} |e(k)|^2 \\
&\quad + (\left|\tilde{C}^T D_f J_2\right| - \xi\alpha_c) |\tilde{x}(k)|^2] - \sum_{k=0}^{N} \xi \Delta V_c(\tilde{x}(k))
\end{aligned} \tag{46}$$

where $\gamma_0 = \gamma_p + \left|J_2^T D_f^T J_d\right| + \xi\gamma_c$ and $\gamma_{0e} = \left|J_2^T D_f^T D_f J_2\right| + \left|\tilde{C}^T D_f J_2\right| + \left|J_2^T D_f^T J_d\right|$.

Selecting

$$\xi \ge max \left\{ \frac{\xi_x^2 + (\xi_d + \xi_e)\alpha_c}{\alpha_c \alpha_e}, \frac{\left|\tilde{C}^T D_f J_2\right|}{\alpha_c} \right\} \tag{47}$$

using (46), we have

$$\begin{aligned}
\sum_{k=0}^{N} y_c^T(k)\, y_c(k) \le \sum_{k=0}^{N} [\gamma_0 \left|\tilde{d}(k)\right|^2 + \gamma_{0e} |e(k)|^2] \\
- \sum_{k=0}^{N} \xi \Delta V_c(\tilde{x}(k))
\end{aligned} \tag{48}$$

Under zero initial conditions, one has

$$\sum_{k=0}^{N} \xi \Delta V_c(\tilde{x}(k)) = \xi V_c(\tilde{x}(N)) \ge 0 \tag{49}$$

As a result, the inequality (48) can be further reduced to

$$\sum_{k=0}^{N} y_c^T(k)\, y_c(k) \le \sum_{k=0}^{N} [\gamma_0 \left|\tilde{d}(k)\right|^2 + \gamma_{0e} |e(k)|^2] \tag{50}$$

which indicates the robustness performance (38) holds for the closed-loop system (34). This completes the proof.

Now, we can conclude the design procedure of the robust fault estimator-based fault tolerant control strategies.

Procedure 2 (Tolerant Control With Signal Compensation):

(1) The estimate of the augmented state vector $\hat{\tilde{x}}$ is produced from the robust estimation algorithm described in *Procedure 1*.

(2) Based on a pre-existing controller (26), implement the tolerant controller in the form (32)-(33) with signal compensation for system (1).

Remark 1: If the pre-existing controller is a static output feedback controller in the form of

$$u(k) = Ky(k) \tag{51}$$

so that the resulting closed-loop system of (1) under fault-free condition is input-to-state stable and satisfies the robust performance index in the form of (35), the tolerant controller becomes

$$u(k) = Ky_c(k) - K_f J_2 \hat{\tilde{x}}(k) \tag{52}$$

where $y_c(k)$ is defined as in (32) and K_f is given by (30). Under faulty scenario, the closed-loop system becomes:

$$\begin{cases} x(k+1) = A_k x(k) + B_{kd}\tilde{d}(k) + B_{ke} e(k) \\ y_c(k) = Cx(k) + D_f J_2 e(k) + J_d \tilde{d}(k) \end{cases} \tag{53}$$

where $A_k = A + BKC$, $B_{kd} = \left[B_d \;\; BKD_d\right]$, $B_{ke} = BKD_f J_2 + B_f J_2$, and the other symbols are defined as before. Define the storage function as

$$\begin{aligned}
V_g(x_e(k)) &= V_c(x(k)) + \xi V(e(k)) \\
&= x^T(k) Q x(k) + \xi e^T(k) P e(k)
\end{aligned} \tag{54}$$

where P and Q are both positive definite matrices, ξ is a positive scalar, $x_e(k) = \left[x^T(k)\; e^T(k)\right]^T$. By using the same proof manner of Theorem 2, one can derive the result straightforward: the tolerant controller (52) can drive the trajectories of the closed-loop system of the plant (1) to be input-to-state stable when a fault occurs, and satisfy a robust performance index in the form of (38).

IV. ESTIMATOR-BASED FAULT TOLERANT CONTROL FOR DISCRETE-TIME LIPSCHITZ NONLINEAR SYSTEMS

In Sections II and III, robust simultaneous state and fault estimator and estimator-based tolerant controller are developed for discrete-time linear systems. It is noted that some aero engineering systems are Lipschitz nonlinear systems, therefore, it is of interest to extend the results obtained in the Sections II and III to Lipchitz nonlinear dynamic systems.

A. STATE AND FAULT SIMULTANOUES UIO-BASED ESTIMATOR FOR LIPSCHITZ NONLINEAR DISCRETE-TIME DYNAMIC SYSTEMS

A discrete-time plant with Lipschitz nonlinear constraints is described as follows:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_f f(k) \\ \qquad\qquad + B_d d(k) + \Phi(x(k), u(k)) \\ y(k) = Cx(k) + D_f f(k) + D_d d_s(k) \end{cases} \tag{55}$$

where $\Phi(x(k), u(k))$ is a Lipschitz nonlinear function vector, i.e., $\forall x(k), \hat{x}(k) \in \mathcal{R}^n$, and $u(k) \in \mathcal{R}^m$, there is a constant $\theta > 0$, such that

$$\left|\Phi(x(k), u(k)) - \Phi\left(\hat{x}(k), \quad u(k)\right)\right| \leq \theta \left|x(k) - \hat{x}(k)\right| \tag{56}$$

and the other symbols are the same as defined in (1).

Defining an augmented state vector to be $\bar{x}(k) = \left[x^T(k) \; f^T(k)\right]^T \in \mathcal{R}^{\bar{n}}$, one can obtain an equivalent augmented system as follows:

$$\begin{cases} \bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}u(k) + \bar{B}_d\bar{d}(k) + \bar{\Phi}(x(k), u(k)) \\ y(k) = \bar{C}\bar{x}(k) + D_d d_s(k) \end{cases} \tag{57}$$

where $\bar{\Phi}(x(k), u(k)) = \left[\Phi^{\mathrm{T}}(x(k), u(k)) \; 0_{1\times lf}\right]^T \in \mathcal{R}^{\bar{n}}$, and the other symbols are defined the same as those in (3).

The nonlinear UIO is then designed for the augmented system (57) as follows:

$$\begin{cases} \bar{z}(k+1) = R\bar{z}(k) + T\bar{B}u(k) + T\bar{\Phi}\left(\hat{x}(k), u(k)\right) \\ \qquad\qquad + (L_1 + L_2)\, y(k) \\ \hat{\bar{x}}(k) = \bar{z}(k) + Hy(k) \end{cases} \tag{58}$$

where R, T, L_1, L_2 and H are the observer gains to be designed by satisfying (9)-(12).

The estimation error is the same as defined as (5). From (5), (57) and (58), one can obtain the estimation error equation as

$$\begin{aligned} e(k+1) = {} & Re(k) + T\bar{B}_{d2}\bar{d}_2(k) + T\tilde{\Phi}(k) \\ & - L_1 D_d d_s(k) - HD_d d_s(k+1) \end{aligned} \tag{59}$$

where $\tilde{\Phi}(k) = \bar{\Phi}(x(k), u(k)) - \bar{\Phi}\left(\hat{x}(k), u(k)\right)$, H is obtained by (13), T is calculated by (11); $R = \bar{A} - H\bar{C}\bar{A} - L_1\bar{C} = T\bar{A} - L_1\bar{C}$, in which L_1 is to be designed in the following theorem.

Theorem 3: For system (57), there exists a robust UIO in the form of (58) such that the error dynamic system (59) is input-to-state stable, if there exist a positive definite matrix P and matrix Y, and the positive scalars α, γ_{d2}, γ_{ds}, γ_{ds1} and γ_θ, such that inequality (60) holds, which is shown at the bottom of this page.

Furthermore, one can calculate $L_1 = P^{-1}Y$.

Proof: Choosing the Lyapunov function in the form of (19), which satisfies (16) in *Lemma 2* according to (20).

Define $\eta(k) = e(k+1) - e(k)$. From (59), we can have

$$\begin{aligned} \eta(k) = {} & (R - I_{\bar{n}})\, e(k) + T\bar{B}_{d2}\bar{d}_2(k) - L_1 D_d d_s(k) \\ & - HD_d d_s(k+1) + T\tilde{\Phi}(k) \end{aligned} \tag{61}$$

According to (19) and (61) and using the similar manner to derive (22), we have

$$\begin{aligned} & \Delta V(e(k)) \\ & = -\eta^T(k) P\eta(k) + 2e^T(k) P\eta(k) + 2\eta^T(k) P\eta(k) \\ & = -\eta^T(k) P\eta(k) + [2e^T(k) P(R - I_{\bar{n}})e(k) \\ & \quad + 2e^T(k) PT\bar{B}_{d2}\bar{d}_2(k) + 2e^T(k) PT\tilde{\Phi}(k) \\ & \quad - 2e^T(k) PL_1 D_d d_s(k) - 2e^T(k) PHD_d d_s(k+1) \\ & \quad + 2\eta^T(k) P(R - I_{\bar{n}})\, e(k) + 2\eta^T(k) PT\bar{B}_{d2}\bar{d}_2(k) \\ & \quad + 2\eta^T(k) PT\tilde{\Phi}(k) - 2\eta^T(k) PL_1 D_d d_s(k) \\ & \quad - 2\eta^T(k) PHD_d d_s(k+1) \end{aligned} \tag{62}$$

Adding and subtracting

$$\begin{aligned} & -\gamma_\theta \tilde{\Phi}^T(k)\tilde{\Phi}(k) - \gamma_{d2}^2 \bar{d}_2^T(k)\bar{d}_2(k) - \gamma_{ds}^2 d_s^T(k) d_s(k) \\ & \qquad - \gamma_{ds1}^2 d_s^T(k+1) d_s(k+1) + \alpha V(e(k)) \end{aligned}$$

to the right side of (62), one has

$$\begin{aligned} & \Delta V(e(k)) \\ & \leq \left[\eta^T(k) \quad e^T(k) \quad \bar{d}_2^T(k) \quad d_s^T(k) \quad d_s^T(k+1) \quad \tilde{\Phi}^T(k)\right] \\ & \quad \times \Omega \begin{bmatrix} \eta(k) \\ e(k) \\ \bar{d}_2(k) \\ d_s(k) \\ d_s(k+1) \\ \tilde{\Phi}(k) \end{bmatrix} \\ & \quad + \gamma_{d2}^2 \bar{d}_2^T(k)\bar{d}_2(k) + \gamma_{ds}^2 d_s^T(k) d_s(k) \\ & \quad + \gamma_{ds1}^2 d_s^T(k+1) d_s(k+1) - \alpha V(e(k)) \end{aligned} \tag{63}$$

where Ω is shown at the top of the next page.

From the LMI (60), it is evident that

$$\Omega < 0 \tag{64}$$

indicating

$$\begin{aligned} \Delta V(e(k)) < {} & -\alpha V(e(k)) + \gamma_{d2}^2 \bar{d}_2^T(k)\bar{d}_2(k) \\ & + \gamma_{ds}^2 d_s^T(k) d_s(k) + \gamma_{ds1}^2 d_s^T(k+1) d_s(k+1) \\ \leq {} & -\alpha\lambda_{min}(P)\left|e(k)\right|^2 + \gamma_{d2}^2 \bar{d}_2^T(k)\bar{d}_2(k) \\ & + \gamma_{ds}^2 d_s^T(k) d_s(k) + \gamma_{ds1}^2 d_s^T(k+1) d_s(k+1) \end{aligned} \tag{65}$$

which means $V(e(k))$ satisfies condition (17) in *Lemma 2*, with $\psi_3(|e(k)|) = \alpha\lambda_{min}(P)|e(k)|^2$ and $\psi_4(|v(k)|) = \max\{\gamma_{d2}^2, \gamma_{ds}^2, \gamma_{ds1}^2\}|v(k)|^2$, in which $v(k) = \left[\bar{d}_2^T(k) \; d_s^T(k) \; d_s^T(k+1)\right]^T$. As a result, the error dynamic system (55) is input-to-state stable. This completes the proof.

$$\begin{bmatrix} -P & PT\bar{A} - Y\bar{C} - P & PT\bar{B}_{d2} & -YD_d & -PHD_d & PT \\ * & 2PT\bar{A} - 2Y\bar{C} + (\alpha - 2)P + \gamma_\theta\theta^2 & PT\bar{B}_{d2} & -YD_d & -PHD_d & PT \\ * & * & -\gamma_{d2}^2 I_{(l_{d2}+l_f)} & 0 & 0 & 0 \\ * & * & * & -\gamma_{ds}^2 I_{l_{ds}} & 0 & 0 \\ * & * & * & * & -\gamma_{ds1}^2 I_{l_{ds}} & 0 \\ * & * & * & * & * & -\gamma_\theta I_{\bar{n}} \end{bmatrix} < 0 \tag{60}$$

$$\Omega = \begin{bmatrix} -P & PT\bar{A} - Y\bar{C} - P & PT\bar{B}_{d2} & -YD_d & -PHD_d & PT \\ * & 2PT\bar{A} - 2Y\bar{C} + (\alpha - 2)P + \gamma_\theta \theta^2 & PT\bar{B}_{d2} & -YD_d & -PHD_d & PT \\ * & * & -\gamma_{d2}^2 I_{(l_{d2}+l_f)} & 0 & 0 & 0 \\ * & * & * & -\gamma_{ds}^2 I_{l_{ds}} & 0 & 0 \\ * & * & * & * & -\gamma_{ds1}^2 I_{l_{ds}} & 0 \\ * & * & * & * & * & -\gamma_\theta I_{\bar{n}} \end{bmatrix}.$$

B. FAULT ESTIMATOR BASED TOLERANT CONTROL FOR LIPSCHITZ NONLINEAR DISCRETE-TIME DYNAMIC SYSTEMS

Assume there is a pre-existing nonlinear dynamic output feedback controller, designed for normal operating conditions (*i. e.*, fault-free scenario), in form of (26). The fault-tolerant controller (32)-(33) by using signal compensation is then employed for the plant (55). As a result, the resulting closed-loop system is obtained as follows:

$$\begin{cases} \tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}_d\tilde{d}(k) + \overline{\overline{\Phi}}(k) + B_e e(k) \\ y_c(k) = \tilde{C}\tilde{x}(k) + D_f J_2 e(k) + J_d \tilde{d}(k) \end{cases} \tag{66}$$

where $\overline{\overline{\Phi}}(k) = \left[\Phi^T((x(k), u(k))\ 0_{n_c}\right]^T$, and the other symbols are the same as defined in (34).

Theorem 4: If there is a pre-existing controller in the form of (26) to ensure the closed-loop system of the plant (55) to be input-to-state stable under fault-free situation and satisfy the robust performance index (35), the tolerant controller (32)-(33) can drive the trajectories of the closed-loop system of the plant (55) to be stable under faulty scenarios and satisfy a robust performance index in the form of (38).

Proof: The proof is similar to *Theorem 2*, which is omitted.

Now, we can conclude the procedure to design robust fault estimation and fault tolerant control strategies for Lipschitz nonlinear systems

Procedure 3 (UIO-Based State/Fault Estimation for Lipschitz Nonlinear Systems):

(1) Construct the augmented system in the form of (57) for the discrete-time Lipschitz nonlinear plant (55).
(2) Solve H from Equation (13), and calculate T from (11).
(3) Solve the LMI (60) to obtain the matrices P and Y, and calculate the gain $L_1 = P^{-1}Y$.
(4) Calculate the gain matrices R from (10).
(5) Generate the augmented estimate $\hat{\bar{x}}(k)$ by implementing UIO (58), leading to the simultaneous estimates of the state and fault as $\hat{x}(k) = \left[I_n\ 0_{n\times l_f}\right]\hat{\bar{x}}(k)$ and $\hat{f}(k) = \left[0_{l_f\times n}\ I_{l_f}\right]\hat{\bar{x}}(k)$, respectively.
(6) Based on a pre-existing controller (27), implement the tolerant controller in the form of (32)-(33).

Remark 2: If the pre-existing controller is a static output feedback controller in the form of

$$u(k) = Ky(k) \tag{67}$$

and a tolerant controller is described by

$$u(k) = Ky_c(k) - K_f J_2 \hat{\bar{x}}(k) \tag{68}$$

one can obtain the same result straightforward as in Theorem 4, that is, the tolerant controller (68) can ensure the closed-loop system of (55) to be input-to-stable stable even when a fault occurs, and satisfy a robust performance index in the form of (38).

V. CASE STUDY: AERO ENGINEERING SYSTEMS

In this section, the proposed fault estimator-based fault tolerant strategies are demonstrated by two aero engineering systems, i.e. a linear discrete-time jet engine system, and a discrete-time flight control system with Lipschitz nonlinear components.

A. JET ENGINE SYSTEM

A gas turbine engine is modeled as a linearized 17-order system at some operating point, and the state variables include pressure, air and gas mass flow rates, shaft speeds, absolute temperatures and static pressure. The control inputs are the fuel flow rate and exhaust nozzle area. For practical reasons and convenience of design, the 17-order model can be reduced to a 5-order jet engine model in the form of (1), and the system matrices are given by [1] as follows:

$$A = \begin{bmatrix} -0.981 & 7.532 & -0.598 & 0.486 & -0.698 \\ 0.284 & -0.083 & 0.078 & -0.062 & 0.093 \\ -6.859 & 28.916 & -2.056 & 1.608 & -2.261 \\ 1.224 & -5.661 & 0.402 & -0.319 & 0.414 \\ 13.266 & -53.405 & 4.739 & -3.771 & 5.367 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.000139 & 0.000195 \\ 0.000067 & -0.000005 \\ 0.003188 & 0.000601 \\ 0.007840 & -0.000273 \\ 0.003123 & -0.001516 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.003 & 0.001 & -0.0005 \\ 0.002 & 0.003 & -0.0015 \\ -0.001 & -0.002 & 0.001 \\ 0.005 & 0.004 & 0.002 \\ 0.004 & -0.001 & 0.0005 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D_d = \begin{bmatrix} 0 & 0.01 \\ 0 & 0.03 \\ 0 & 0.02 \\ 0 & 0.04 \\ 0 & -0.01 \end{bmatrix} \tag{69}$$

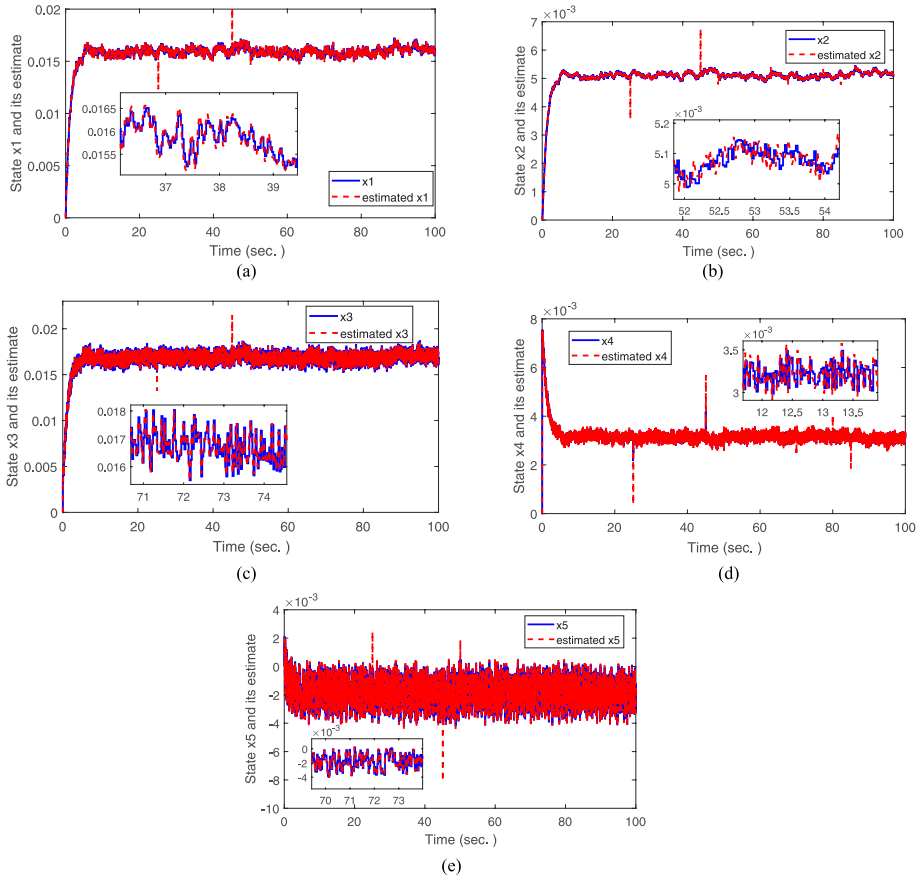

FIGURE 1. **(a) System state x_1 and its estimate. (b) System state x_2 and its estimate. (c) System state x_3 and its estimate. (d) System state x_4 and its estimate. (e) System state x_5 and its estimate.**

and the sampling period $T_s = 0.026$ s. The total running time is 100 seconds. The unknown inputs are given as random signals with range from -10^{-2} to 10^{-2}. Actuator fault vector is $f_a = [f_{a1}\ f_{a2}]^T$, where f_{a1} is 10% loss of actuation effectiveness from 25 second to 45 second, and f_{a2} is $-0.5 + 0.1\sin(kT_s)$ from 50 second to 65 second. In this case, the distribution matrix of the actuator fault vector is $B_{fa} = B$. Sensor fault $f_s = [f_{s1}\ f_{s2}]^T$ occurs in the first two outputs, where f_{s1} is 15% loss of effectiveness from 70 second to 80 second, and f_{s2} is a stuck fault from 85 second. Then $D_{fs} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T$. Consequently, the fault vector considered is $f = [f_a^T\ f_s^T]^T$ with $B_f = [B_{fa}\ 0_{5\times 2}]$ and $D_f = [0_{5\times 2}\ D_{fs}]$.

H can be solved from (13). Selecting $\gamma_{d2} = 0.01$, $\gamma_{ds} = 0.08$, $\gamma_{ds1} = 0.06$,and $\alpha = 0.05$, and solving the LMI (18), the observer gain L_1 can be calculated. Therefore R and L_2 can be obtained following the formulae (10) and (12), respectively. As a result, the obtained gains of the UIO in the form of (4), that is, H, T, L_1, R and L_2, are shown in (70) at the top of the next page.

There is a pre-designed feedback controller

$$u(k) = Ky(k) \tag{71}$$

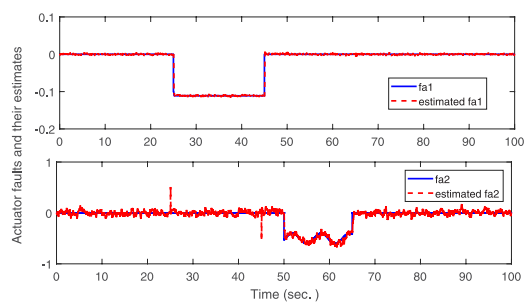

FIGURE 2. **Actuator faults and their estimates.**

where the control gain is given by [13], as follows

$$K = \begin{bmatrix} -0.0346 & 0.1076 & -0.0120 & 0.0096 & -0.0135 \\ 0.0376 & -0.1703 & 0.0139 & -0.0111 & 0.0156 \end{bmatrix}.$$

In this case, the tolerant controller is in the form of

$$u(k) = Ky_c(k) - K_f J_2 \hat{\bar{x}}(k) \tag{72}$$

where $K_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, and $J_2 = [0_{4\times 5}\ I_4]$.

Figures 1(a)-1(e) show five system states and their estimates, while Figures 2 and 3 exhibit the faults and their

$$H = \begin{bmatrix} 0.1636 & 0.1091 & -0.0545 & 0.2727 & 0.2182 \\ 0.1091 & 0.0727 & -0.0364 & 0.1818 & 0.1455 \\ -0.0545 & -0.0364 & 0.0182 & -0.0909 & -0.0727 \\ 0.2727 & 0.1818 & -0.0909 & 0.4545 & 0.3636 \\ 0.2182 & 0.1455 & -0.0727 & 0.3636 & 0.2909 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0.8364 & -0.1091 & 0.0545 & -0.2727 & -0.2182 & 0 & 0 & -0.1636 & -0.1091 \\ -0.1091 & 0.9273 & 0.0364 & -0.1818 & -0.1455 & 0 & 0 & -0.1091 & -0.0727 \\ 0.0545 & 0.0364 & 0.9818 & 0.0909 & 0.0727 & 0 & 0 & 0.0545 & 0.0364 \\ -0.2727 & -0.1818 & 0.0909 & 0.5455 & -0.3636 & 0 & 0 & -0.2727 & -0.1818 \\ -0.2182 & -0.1455 & 0.0727 & -0.3636 & 0.7091 & 0 & 0 & -0.2182 & -0.1455 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -1.1697 & -0.7800 & 12.119 & 8.9338 & -10.140 \\ -0.5135 & 0.3424 & -5.1466 & 3.7253 & -4.1978 \\ -0.9721 & -0.6482 & -12.619 & 9.1329 & -10.086 \\ -0.9573 & -0.6383 & -9.2758 & 6.5616 & -7.4296 \\ 1.2829 & 0.8555 & 19.7218 & -13.563 & 15.8453 \\ 0.7847 & 0.5270 & 62.495 & -22.365 & 46.383 \\ 20.052 & 13.3865 & 1614.5 & 417.499 & -854.99 \\ 1.1034 & 0.0691 & 0.1857 & -0.7713 & 0.6238 \\ -0.0457 & 0.9694 & 0.1857 & -0.6803 & 0.3114 \end{bmatrix}$$

$$R = \begin{bmatrix} -3.2841 & 21.862 & 10.354 & -7.5231 & 8.1382 & -0.0025 & 0.0006 & 1.0061 & 0.6709 \\ -1.5177 & 9.2926 & 4.4470 & -3.1708 & 3.4220 & -0.0017 & 0.0003 & 0.4045 & 0.2697 \\ -4.7293 & 25.048 & 10.951 & -7.8331 & 8.2596 & 0.0041 & 0.0005 & 1.0266 & 0.6846 \\ -3.6067 & 17.5601 & 7.7338 & -5.3394 & 5.6717 & 0.0034 & 0.0004 & 0.6846 & 0.4565 \\ 7.3527 & -36.194 & -16.538 & 11.025 & -12.22 & -0.0004 & -0.0010 & -1.5011 & -1.001 \\ -0.7847 & -0.5270 & -62.495 & 22.365 & -46.383 & 1 & 0 & -0.7847 & -0.5270 \\ -20.052 & -13.3865 & -1614.5 & -417.499 & 854.99 & 0 & 1 & -20.052 & -13.387 \\ -1.1034 & -0.0691 & -0.1857 & 0.7713 & -0.6238 & 0 & 0 & -0.1034 & -0.0691 \\ 0.0457 & -0.9694 & -0.1857 & 0.6803 & -0.3114 & 0 & 0 & 0.0457 & 0.0306 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 1.0066 & 0.6711 & -0.3355 & 1.6777 & 1.3421 \\ 0.4047 & 0.2698 & -0.1349 & 0.6745 & 0.5396 \\ 1.0270 & 0.6847 & -0.3423 & 1.7117 & 1.3694 \\ 0.6849 & 0.4566 & -0.2283 & 1.1415 & 0.9132 \\ -1.5017 & -1.0011 & 0.5006 & -2.5028 & -2.0022 \\ -0.7975 & -0.5317 & 0.2658 & -1.3292 & -1.0633 \\ -20.1260 & -13.4173 & 6.7087 & -33.5434 & -26.8347 \\ -0.1037 & -0.0691 & 0.0346 & -0.1729 & -0.1383 \\ 0.0456 & 0.0304 & -0.0152 & 0.0760 & 0.0608 \end{bmatrix} \tag{70}$$

estimates. One can see both the system states and the monitored actuator and sensor faults are estimated excellently. The influences of the unknown inputs are decoupled/attenuated successfully.

Fig. 4 (a)-(e) exhibit five system outputs under three scenarios for comparisons: healthy outputs in fault-free cases, faulty cases without fault tolerant control (FTC), and faulty cases after FTC. From Figure 4, one can see that the faults have made the outputs significantly distorted compared with the healthy system outputs. However, after signal compensation (e.g., FTC), the system outputs are recovered successfully which are consistent with the healthy system outputs.

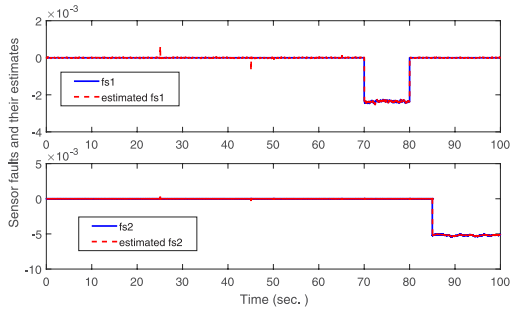

FIGURE 3. **Sensor faults and their estimates.**

As a result, the proposed fault estimation and fault tolerant control techniques are effective.

B. FLIGHT CONTROL SYSTEM

In this example, the methods developed for discrete-time Lipchitz nonlinear plants are verified by a nonlinear flight control system.

The model of a simplified longitudinal flight control system can be described by a discrete-time Lipschitz nonlinear system in the form of (55), where $x(k) = \left[\eta_y(k)\ \omega_z(k)\ \delta_z(k)\right]^T$ with initial condition $x_0 = \left[1\ 0.5\ 2\right]^T$, η_y is the normal velocity, ω_z is the pitch rate, and δ_z is the pitch angle. $u(k)$ is the elevator control signal, taking value at $u(k) = 10$. The sampling period is $T_s = 0.01$ s. Lipschitz nonlinear component is $\Phi(x(k), u(k)) = \left[0\ 0.005\sin(x_1(k))\ 0\right]^T$. The system parameters are given as follows [26]:

$$A = \begin{bmatrix} 0.9944 & -0.1203 & -0.4302 \\ 0.0017 & 0.9902 & -0.0747 \\ 0 & 0.8187 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4252 \\ -0.0082 \\ 0.1813 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B_{d1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

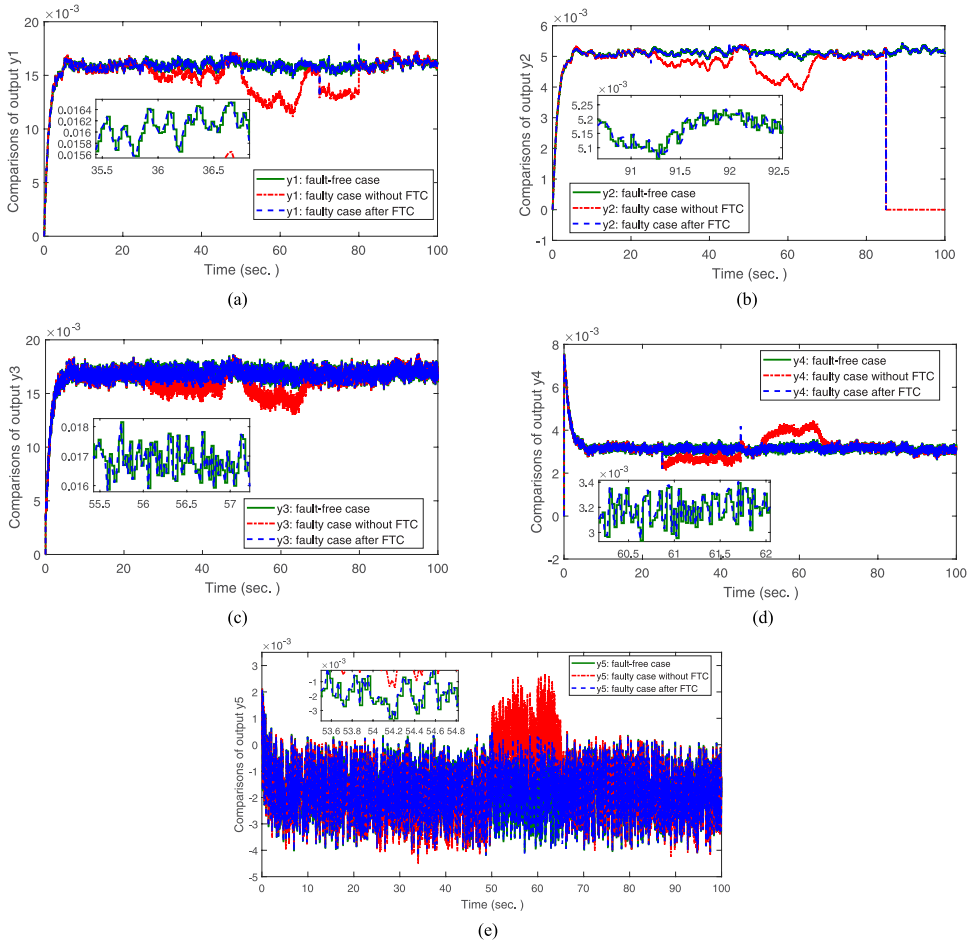

FIGURE 4. **(a) Comparisons of output y_1: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control. (b) Comparisons of output y_2: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control. (c) Comparisons of output y_3: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control. (d) Comparisons of output y_4: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control. (e) Comparisons of output y_5: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control.**

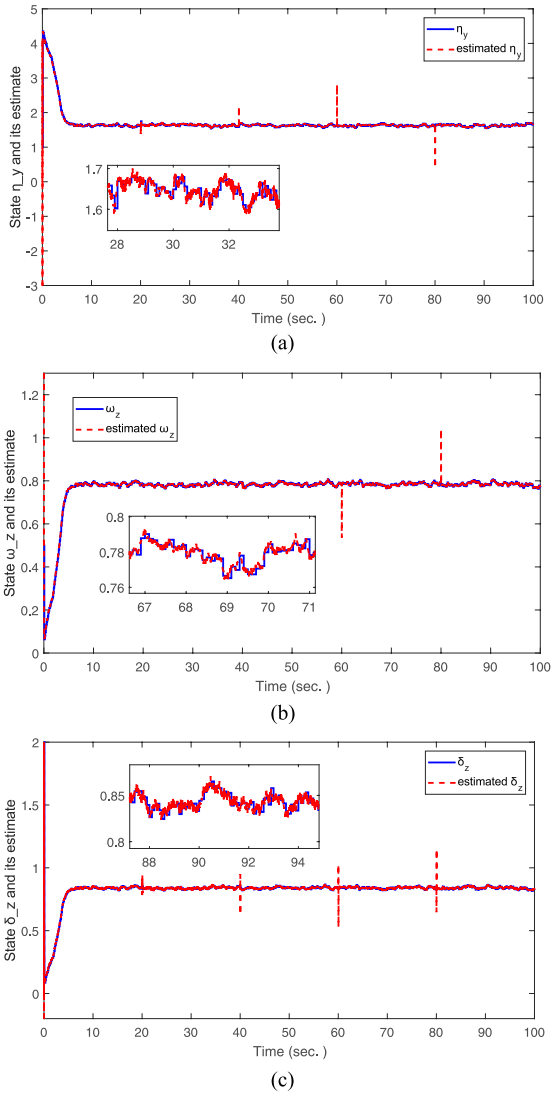

FIGURE 5. (a) Velocity η_y and its estimate. (b) Pitch rate ω_z and its estimate. (c) Pitch angle δ_z and its estimate.

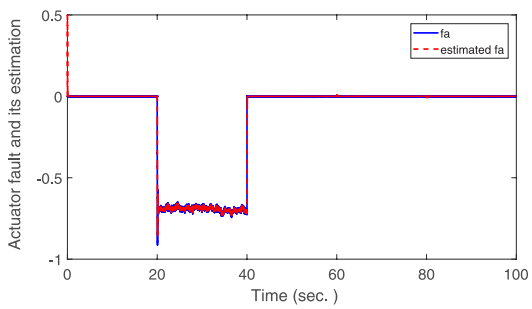

FIGURE 6. Actuator fault and its estimate.

$$B_{d2} = \begin{bmatrix} 0.1 & -0.05 \\ 0.3 & -0.15 \\ -0.4 & 0.2 \end{bmatrix}, \quad D_d = \begin{bmatrix} 0.01 \\ -0.02 \\ 0.04 \end{bmatrix}. \tag{73}$$

The unknown input vector $d_1(k) = \begin{bmatrix} \Delta a_{21} & \Delta a_{22} & \Delta a_{23} \end{bmatrix} x(k)$ represents the parameter perturbations in matrix A, i.e.

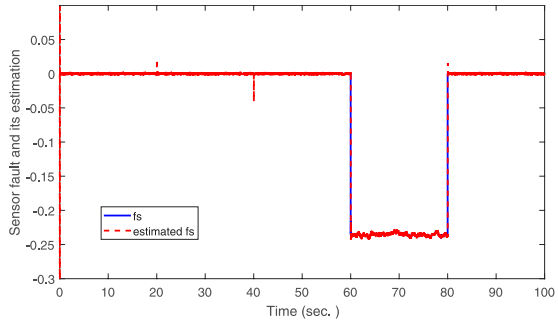

FIGURE 7. Sensor fault and its estimate.

$\Delta a_{2j} = 0.1 a_{2j}, j = 1, 2, 3$. Unknown input vector $d_2(k)$ represents the extra disturbances, with value from -0.01 to 0.01 randomly. d_s is the measurement noise vector, taking values from -0.001 to 0.001. The faults under consideration are 50% loss of the actuation effectiveness f_a from 20s to 40s, and 30% loss of effectiveness f_s in the second sensor from 60s to 80s. In this case, $B_{fa} = B$, and $D_{fs} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$. Consequently, the fault vector considered is $f = \begin{bmatrix} f_a & f_s \end{bmatrix}^T$ with $B_f = [\, B_{fa} \;\; 0_{3\times 1} \,]$ and $D_f = [\, 0_{3\times 1} \;\; D_{fs} \,]$.

There is a pre-designed feedback controller

$$u(k) = Ky(k) \tag{74}$$

where $K = \begin{bmatrix} -2.1710 & -9.0038 & 2.0115 \end{bmatrix}$.

Then the fault estimation and fault tolerant control strategies designed in Section IV can be implemented to the flight control system. H can be solved from (13). Selecting $\gamma_{d2} = 0.05$, $\gamma_{ds} = 0.4$, $\gamma_{ds1} = 0.03$, $\gamma_\theta = 0.05$, and $\alpha = 0.01$, and solving the LMI (60), the observer gain L_1 can be calculated. Therefore R and L_2 can be obtained following the formulae (10) and (12), respectively. As a result, the obtained gains of the UIO in the form of (58) are obtained as follows:

$$H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 1.8430 & -0.0280 & -0.4031 \\ -0.3533 & 0.5063 & 1.0176 \\ 0.0858 & 1.0168 & 0.8098 \\ 1.9738 & 0.5029 & 0.3314 \\ 0.3532 & -0.5205 & -1.0176 \end{bmatrix}$$

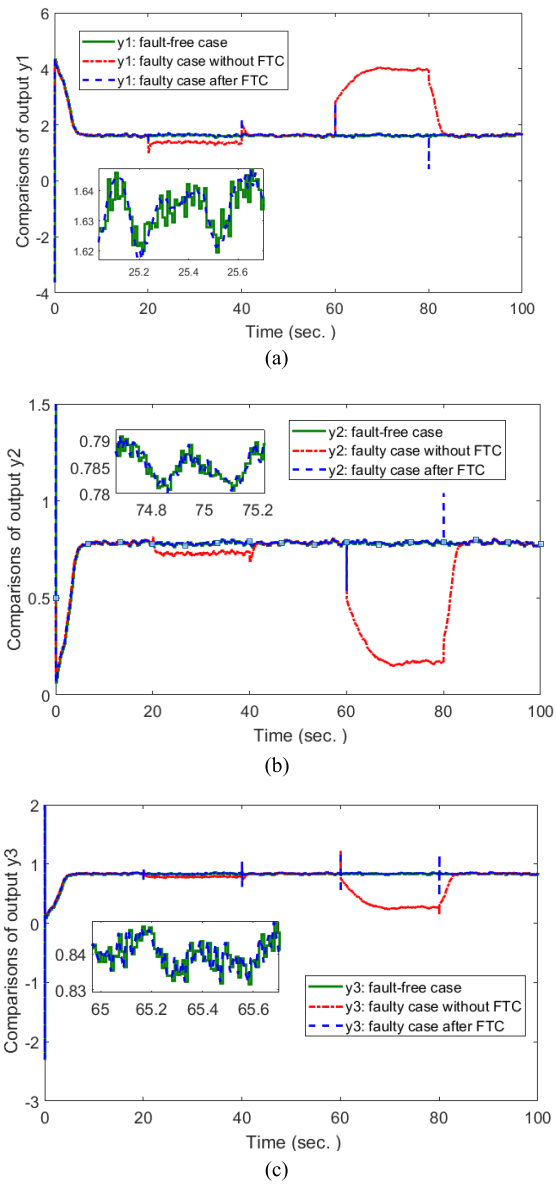

FIGURE 8. **(a) Comparison of output y_1: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control. (b) Comparison of output y_2: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control. (c) Comparison of output y_3: fault-free output, output subjected to faults without tolerant control, and output subjected to faults after tolerant control.**

$$R = \begin{bmatrix} -0.8486 & -0.0923 & -0.0271 & 0.4252 & 0.0280 \\ 0.3533 & -0.5063 & -1.0176 & 0 & -1.5063 \\ -0.0858 & -0.1981 & -0.8098 & 0.1813 & -1.0168 \\ -1.9738 & -0.5029 & -0.3314 & 1 & -0.5029 \\ -0.3532 & 0.5205 & 1.0176 & 0 & 1.5205 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 0 & -0.0923 & 0 \\ 0 & -0.5063 & 0 \\ 0 & -0.1981 & 0 \\ 0 & -0.5029 & 0 \\ 0 & 0.5205 & 0 \end{bmatrix} \tag{75}$$

The tolerant controller should be in the form of

$$u(k) = Ky_c(k) - K_f J_2 \hat{\bar{x}}(k) \tag{76}$$

where $K_f = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $J_2 = \begin{bmatrix} 0_{2\times 3} & I_2 \end{bmatrix}$, and K is defined immediately after (74).

The estimation performances of full system states, i.e. velocity, pitch rate, and pitch angle, are shown in Figures 5(a)-(5c). The actuator and sensor faults and their estimates are depicted by Figures 6 and 7. One can see both the system states and faults are estimated satisfactorily.

The curves displayed in Figures 8(a)-(c) show the comparisons of the three system outputs under three scenarios: healthy system outputs, faulty system outputs without FTC, and faulty system outputs after FTC. One can see the faulty system output performances are significantly degraded if no measures are taken. However, it is encouraging to see the effects from the faults are successfully mitigated/removed by using the proposed tolerant control strategy. As a result, the developed integrated fault tolerant technique is effective.

VI. CONCLUSION

In this study, an integrated fault tolerant control technique has been developed for discrete-time dynamic systems with applications to aero engine system and flight control system. Augmented approach, UIO and LMI have been integrated to construct a simultaneous state/fault estimator with robustness against partially decoupled unknown inputs and measurement noises. Estimator-based signal compensation, associated with a pre-designed controller, has been then developed to attenuate the effects of both actuator and sensor faults. As a result, the system outputs after fault tolerant control can track the healthy outputs satisfactorily. The stabilization of the fault-tolerant control system is addressed in the sense of the input-to-state stability. In the future, it is encouraging to develop fault tolerant control mechanisms for aero engineering systems with higher nonlinearities and stochastic dynamics.

APPENDIX

PROOF OF LEMMA 1

According to [17] and [27], the sufficient and necessary conditions for the existence of the UIO (4) for the system (3) are:

$$\text{rank}\left(\bar{C}\bar{B}_{d1}\right) = \text{rank}(\bar{B}_{d1}) \tag{A1}$$

$(\bar{C}, \bar{A}_1)$ is a detectable pair, where

$$\bar{A}_1 = \left(I_{\bar{n}} - H\bar{C}\right)\bar{A}. \tag{A2}$$

It is noticed that

$$\bar{C}\bar{B}_{d1} = \begin{bmatrix} C & D_f \end{bmatrix} \begin{bmatrix} B_{d1} \\ 0_{l_f \times l_{d1}} \end{bmatrix} = CB_{d1} \tag{A3}$$

and

$$\text{rank}(\bar{B}_{d1}) = \text{rank}(B_{d1}) \tag{A4}$$

Therefore one can observe that condition (i) in *Lemma 1*, that is, rank $(CB_{d1}) = \text{rank}(B_{d1})$, is equivalent to (A1).

If (A1) holds, (A2) is equivalent to that the transmission zeros from the unknown inputs to the measurements must be stable [17], [27], [28], *i.e.,*

$$rank\begin{bmatrix} zI_{\bar{n}}-\bar{A} & -\bar{B}_{d1} \\ \bar{C} & 0 \end{bmatrix} = \bar{n}+l_{d1}, \quad \forall z \text{ with } |z|\geq 1 \tag{A5}$$

It is noticed that

$$\begin{aligned} &\text{rank}\begin{bmatrix} zI_{\bar{n}}-\bar{A} & -\bar{B}_{d1} \\ \bar{C} & 0 \end{bmatrix} \\ &= \text{rank}\begin{bmatrix} zI_n-A & -B_f & -B_{d1} \\ 0_{l_f\times n} & zI_{l_f}-I_{l_f} & 0_{l_f\times l_{d1}} \\ C & D_f & 0_{p\times l_{d1}} \end{bmatrix} \\ &= \begin{cases} \text{rank}\begin{bmatrix} A-I_n & B_f & B_{d1} \\ C & D_f & 0 \end{bmatrix}, & z=1 \\ \text{rank}\begin{bmatrix} A-zI_n & B_{d1} \\ C & 0 \end{bmatrix}+l_f, & z\neq 1 \end{cases} \end{aligned} \tag{A6}$$

Therefore, it is clear that the conditions (ii) and (iii) in *Lemma 1*, is equivalent to the condition (A5), which is equivalent to the condition (A2). This completes the proof.

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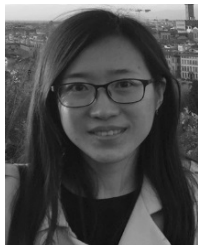

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