Northumbria Research Link

A Spatial Signature of Sprawl:
Or the Proportion and Distribution of Linear Network Circuits

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Abstract
This paper sets out to investigate whether the frequency distribution of the linear network circuits within a graph-based representation of a road transportation system can be helpful in identifying sprawl and, in particular, whether a *'spatial signature of sprawl'* can be determined. This paper is based upon an earlier study on Peachtree City, Georgia and in particular of its dual transportation system (roads and golf cart paths). In order to fully understand the effect that the dual transportation system has upon Peachtree City, the frequency distribution of its circuits are compared to three, supposed, 'suburban' areas and three, supposed, 'urban' districts. The conclusion of this paper is that there is, unquestionably, a measurable continuum between 'suburbia' and 'urbanity' and that this is reflected in the frequency, length and distribution of the graph network circuits. The main section of this paper is concerned with the presentation and discussion of alternative algorithms for calculating these circuits. This section is followed by an introduction of a selection of methods for interpreting the resultant data. Finally, with respect to Peachtree City, this paper concludes that the effect of the dual transportation system is to make it more 'urban' than it would otherwise be, although it remains a distinctly suburban environment.

1. Introduction
This paper focuses upon a set of methodological techniques originally developed as part of a larger research project (Conroy Dalton and Dalton, 2005) conducted into understanding the spatial transportation network/s of Peachtree City. Peachtree City[2] is a commuter satellite-city to the South East of Atlanta, Georgia, USA. It is accessed via Interstate 85, a major travel corridor through the Deep South connecting

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[1] During the data-gathering and analysis stages of this paper, Dr. Ruth Dalton was an Assistant Professor in the College of Architecture at the Georgia Institute of Technology (Atlanta) and Nick Dalton was an employee of GRTA, the Georgia Regional Transportation Authority.

[2] Community website http://www.peachtreecityweb.com/

Petersburg, Virginia in the North with Montgomery, Alabama to the South and passing through key Southern cities such as Charlotte and Atlanta en route. The city is accessed directly from Highways 74 and 54 which intersect approximately 1km to the west of Lake Peachtree, an artificial lake, which forms the heart of the city. The city was a planned community built entirely by private developers; it was chartered on March 9, 1959. Its area covers approximately 15,500 acres with a current population of 31,580 (2000 census). Estimates for 2003 indicate a population of 33,010 with projected growth calculations suggesting an ultimate population limit of 45,000-50,000. The location of Peachtree City is shown in Figure 1 below.
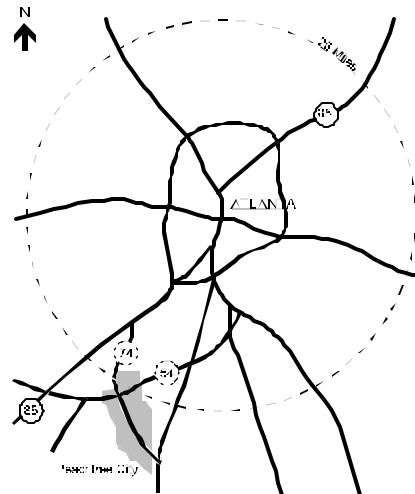

Figure 1. Location of Peachtree City

What makes Peachtree City a particularly interesting topic for study and what distinguishes it from the ubiquitous, suburban sprawl[3] that characterizes much of recent development in North America is that Peachtree City boasts a network of leisure 'paths' or trails forming a network of 80-90 miles. The original research project set out to determine why the path system of Peachtree City was so successful and whether there were fundamental spatial, configurational properties which underpinned its achievement. This research has been expanded in this paper.

2. Space Syntax Analysis

The basic analyses used to investigate the dual-transportation network of the city were based upon a set of theories and analytic techniques (essentially linear spatial-network graph analyses) known as *space syntax* (Hillier and Hanson, 1984; Hillier, 1998). Using these methods, a linear-spatial network (broadly analogous to road center-lines in GIS data (Dalton, Peponis & Conroy Dalton, 2003; Turner, 2005)), known as the *axial line* map, is derived; axial lines essentially represent the least number of changes of direction required when navigating the system. Next, a graph representation, or

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[3] According to http://www.planningweb.com the Vermont Forum on Sprawl defines sprawl as "dispersed development outside of compact urban and village centers along highways and in rural countryside." It is interesting to note, however, that of the many definitions of sprawl almost none of them propose morphological measures as indicators.

meta-graph of the network is generated, in which each line is represented as a node in the graph and each line-intersection corresponds to an edge in the graph. The majority of the subsequent graph-techniques described in this paper were performed on the *meta-graph* rather than the spatial, axial line graph.

One of the early hypotheses of the original study was that the cart paths in Peachtree City were serving to reduce sprawl (a sort of anti-sprawl mechanism). However, in order to investigate this, a morphological measure of sprawl was required, which was not hitherto available. There are currently many indicators of sprawl but the majority of them are economic or land-use related rather than *intrinsically* spatial or morphological. However, an early observation of the significant reduction of the number of dead-ends in the system, caused by the merging of the cart path and road networks, suggested that the amount of 'ringiness[4]' (Hillier and Hanson, 1984, p. 104) in the system might serve to be a good indication of sprawl. In order to investigate the role of 'ringiness' or the *proportion* of circuits[5] in axial maps and their relationship to sprawl, a method needed to be developed to count the quantity and length (the number of *edges* in the circuit) of the circuits in the meta-graph.

3. Comparative Suburban and Urban Areas

For the purposes of determining whether there was a pattern inherent in the frequency distribution of the circuits, it was decided to compare Peachtree City (with and without the cart paths) to a number of other areas, which could broadly be put into the category of 'suburban' or 'urban' (see column three of Table 1). The majority of these are North America examples, however, there are two UK examples included in the sample. The other maps used for comparison are shown in Table 1, along with a brief description of the location/extents of the map. Given that Peachtree City (containing only the road network) can be held initially to be 'suburban', our test is to determine the effect of incorporating the cart path network into the analysis. For this reason, Peachtree City - Dual (the dual transportation network) is left undefined, in terms of its 'suburban' or 'urban' category; its final status is still to be determined. The Peachtree City (Dual) map can, therefore, be compared to three other 'suburban' areas and three 'urban' districts.

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[4] The definition and equation for *axial ringiness* is given on page 104 in the Social Logic of Space. Axial 'ringiness' is defined as being (2L-5)/I, where L is the number of axial lines and I is the number of islands or rings (or circuits in graph theoretic terms).

[5] In graph theory rings are known as *circuits*; a *circuit* is a path which starts and ends at the same node (and has a step depth greater than 1, otherwise it would be a loop).

Table 1 Maps used for the Comparative Analysis

Location	Description	S or U?
Atlanta	An area covering all of Downtown and Midtown Atlanta, Georgia.	U
Borehamwood	A former village, approx. 11 miles to the north of London, now part of London's outer suburbs.	S
Crabapple, Georgia	A rapidly developing suburb, on the extreme northern edge of Atlanta, near Alpharetta.	S
Manhattan	Manhattan Island, New York.	U
Peachtree City (Roads only)	Peachtree City, Georgia – map of the road network only.	S
Peachtree City (Dual)	Peachtree City, Georgia – map of the dual transportation network, roads and cart paths.	?
Soho, London	An inner district of Central London.	U

4. Algorithms for Calculating Circuits

Clearly, if axial-line graphs were planar graphs (as road center-line networks are) then calculating the unique shortest circuits would be an easy task, as this would simply be the *order* (or number of nodes) of the *dual* of the network graph. Equally, it could be calculated using Euler's Polyhedron Formula.

Equation 1. Euler's Polyhedron Formula

$$(n - m + f = 2)$$

Where n = number of vertices, m = number of edges and f = number faces/circuits

Note that the number 2 in Equation 1 can be substituted for number 1 if the 'external face' is to be discounted. It is well known that many operations that are straightforward for planar graphs become extremely complex when dealing with non-planar ones; to solve the problem of identifying axial map circuits, a number of different algorithms were devised, implemented and assessed.

The first method presented, is a variant of Euler's Polyhedron Formula. Essentially, it uses the axial lines' connectivity values, C^6, in order to approximate the number of edges and nodes in the resultant planar graph, were the axial map to be *'segmented'*. Please note, this *approximates* the act of segmentation, rather than actually performing it (which is computationally expensive).

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[6] If the line is represented as a node in the *meta-graph*, then the connectivity of the line (the number of other lines it intersects) is simply the degree of the node.

Equation 2. An Approximation to Euler's Polyhedron Formula for Axial Maps

$$f = 1 + \sum (C - 1) - \frac{\left(\sum (C)\right)}{2}$$

Where f = the number of circuits and C = the axial line connectivity values

However, this can only be an approximation to the true number of circuits, as it is based upon the assumption that most lines intersect with only one other axial line at any single point in space (an extremely good heuristic, but unfortunately not universally true). Equally, it identifies a high proportion of *'trivial circuits'*, circuits typically caused by the manner and convention by which axial maps are drawn.

The second method, we termed *step-depth sequencing,* and is based on existing algorithms. One node in the graph is randomly selected to be the origin, and all the 'step-depths' from that node are calculated (i.e. all lines that are one step (in the graph) away from the origin node are labeled '1', all lines two steps away are labeled '2' etc. until all lines/nodes in the graph have been identified and tagged). Then, all paths originating from the origin-node are examined with respect to the *sequence of 'step-depths'* along a path; if the step-depths cease to increment, then this is indicative of the presence of a circuit. The circuit-path is then followed until it reaches the origin-node and its length (the number of edges in the circuit) is noted. Each axial line is 'tagged' with a number representing the *size* (number of edges) of the smallest *sub-graph/circuit* of which it constitutes part. If a line does not form part of a circuit, then this number is zero. This process is repeated for all nodes in the graph. The frequency counts of these 'circuit-lengths' are used to produce a measure of the number (and the lengths) of circuits in a system. This second algorithm is relatively efficient, but is still an approximation to the absolute number of circuits. This is because we are interested in finding the *least number of unique circuits*, and this method can occasionally identify circuits which contain a number of smaller circuits and hence should be eliminated from the final count. Equally, *'trivial circuits'* are still identified, however, with this method, they can be discarded, as all circuits of length 3 (the majority of trivial circuits are length 3) can be excluded from the final tally.

The final algorithm is a further refinement of the *step-depth sequencing* method, we have termed it *step-depth sequencing with sets*. Every time a circuit is identified, the unique IDs of the lines constituting the circuit are stored as a set, S_1, S_2, S_3 to S_n which are then canonicalized. Duplicate sets are removed, as are union sets, see example below.

Equation 3. Set Culling Test Statement for all Union Sets

$$\text{If } S_3 = \left(S_1 \cup S_2\right) \text{ then } S_3 \rightarrow \{\ \}$$

This method calculates the *minimum number of unique circuits* in the graph and permits the exclusion of *'trivial circuits'*, however, it is computationally more

expensive than the previous two algorithms. To test the efficacy of the *step-depth sequencing* algorithms, we implemented all three methods for each of the seven non-planar axial maps (Atlanta, Borehamwood, Crabapple, Manhattan, Peachtree City (x2) and Soho, London) and compared the results/process time to the equivalent (if applicable) algorithms for planar graphs. The planar graphs were produced by passing the same seven maps through a 'segmentation' algorithm, which effectively cuts an axial line at all its points of intersection. Next it produces an alternative meta-graph in which line-segments are translated into nodes in the graph and where the endpoints of two segments are coincident, those nodes (the segments) are connected by an edge in the graph.

The test algorithms implemented for the purposes of this paper are summarized in Table 2 along with a statement of their computational expense. Before performing any actions on the segmented map (planar graph), it was first necessary to run the segmentation algorithm and the expense of running this has been added to the algorithmic expense. In Table 2, K = a constant and C = the connectivity of the axial lines.

Table 2. Algorithms for Calculating Linear Circuits

	Axial Map (Non-planar graph)	Segment Map (Planar Graph)
Method 1	Approximation to Euler's Polyhedron Formula(Equation 2)	Euler's Polyhedron Formula (Equation 1)
Method 2	Step-depth sequencing [$n^2 log(n)$]	Step-depth sequencing (plus segmentation) [$C n^2 log(Cn)$]
Method 3	Step-depth sequencing with sets [$K n^2 log(n)$]	'with sets' operation proved unnecessary [$C n^2 log(Cn)$]

It should be clear from Table 2 that performing the step-depth sequencing method (either with or without sets) is less expensive to perform on the non-planar graphs, *if the cost of segmentation* is taken into account. However, some interesting discoveries were made in performing these tests. First, it was discovered that there was no need to perform the step-depth sequencing method *with sets* for the set of planar graphs; the step-depth sequencing method alone, produced perfect results and these results were subsequently used as the 'gold-standard' for all the non-planar graph tests in this paper. This saving was translated into a cost saving, as no additional computational stages were required. Unfortunately, despite the fact that the step-depth sequencing with sets was efficient (for the non-planar graphs), we discovered some unique cases where circuits were omitted and hence it has to be concluded that this method is, *in some particular cases*, unreliable. This is not to say, however, that it could not be used as a useful 'rule of thumb' algorithm. Once again, these tests reinforce the salutary lesson that operations that are easy to perform on planar graphs can become extremely difficult when applied to non-planar ones. The results of one circuit-counting trial can be seen in Figure 2 below, in this case, for Borehamwood.

Figure 2. Borehamwood Circuits (on axial, non-planar graph)

5. Overall Results

First, let us consider the set of seven axial maps, three of which were of 'suburban' areas and three of which were of 'urban' areas (and one to be determined). If we plot the histogram of the frequencies of circuits of incremental lengths for all the maps, the resultant data can be seen in Figure 3 below.
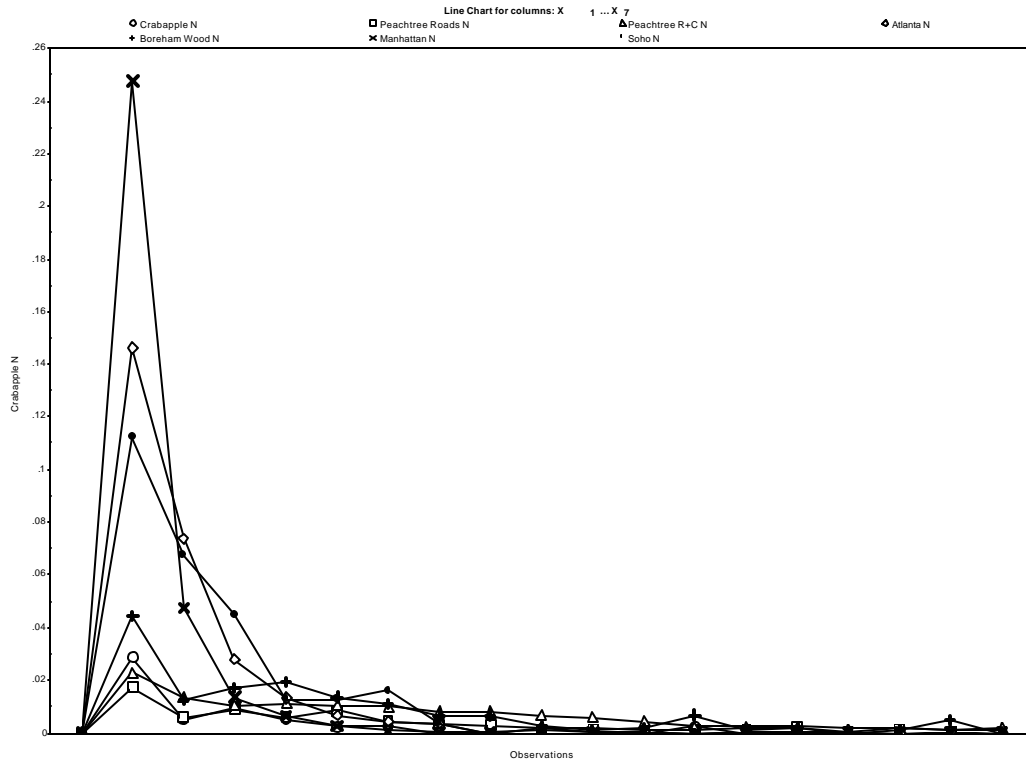

Figure 3. Frequency and Lengths of Network Circuits

In Figure 3, the x-axis represents the number of lines/edges constituting the circuit. It should be noted that the first data point for all lines, represents circuits of 'size 3', which have been set to zero (these are, for the most part, *trivial circuits*). The y-axis represents the count of the total of number of circuits of each size. However, the y-axis has been 'normalized'; because the maps varied considerably, in terms of the total number of lines in each map, it was deemed necessary to divide the 'total count' by the number of 'edges' in each map in order to be able to compare across the different systems. Now, let us look at the differences between the supposed 'urban' and 'suburban' systems (Figures 4 and 5).
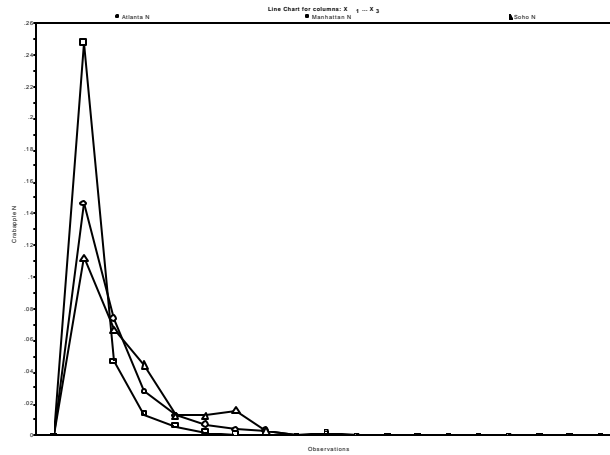
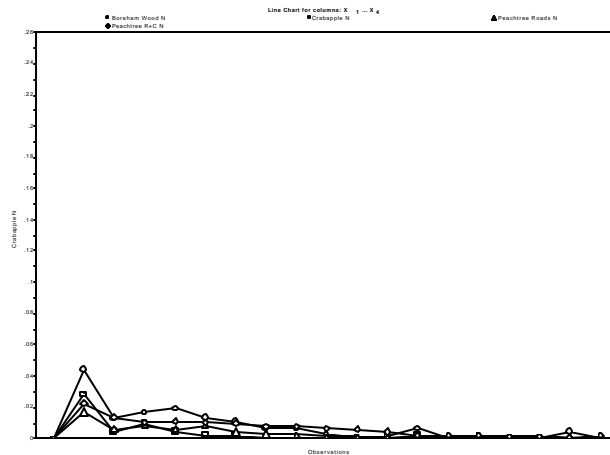Figure 4. Frequency and Lengths of Network Circuits for Urban Systems

Figure 5. Frequency and Lengths of Network Circuits for Suburban Systems

It can clearly be seen from the graphs in Figures 4 and 5 that there is a marked difference between the urban and suburban areas (the y-axis has the same scale and range on both figures). In the urban areas, there tends to be an strong early peak, around circuit lengths 4 and 5. This pattern of frequency drops off sharply, but smoothly. In the suburban areas, the peak is still evident, but is far, far shallower and the whole graph is much flatter; in the suburban areas there is a greater distribution of longer circuits. Imagine, for a moment, an archetypal, 'gridded' downtown area; for a perfect grid, all circuits will be of size/length 4 (a grid-square) with few, if any, circuits of larger lengths. In contrast, in the three suburban areas, circuit lengths of 40, 50, 60 (and higher) are being recorded. It should be noted that Figure 3 illustrates the maps of Atlanta (Central), Manhattan and Soho (London), and Figure 4 is of the maps of Borehamwood, Crabapple and Peachtree City.

In terms of descriptive statistics, a number of measures of the above graphs appear to be useful indicators of suburbia/urbanity. In particular, a small sample - mean appears to represent suburban areas, as do small standard errors, standard deviations and standard variances. Urban areas tend to be characterized by higher peaks, which are easily measurable and suburban areas identified by the comparative lengths of their longest circuits. Given the smaller 'count' values, in suburban areas (since there is a greater range of circuit lengths), the sum of the frequency counts, appears to be a useful indicator, with small sum counts corresponding to suburban areas. Finally, axial 'ringiness' is once again, an extremely useful benchmark, which is ironic as it has been an established (if seldom used) measure for more than thirty years.

Finally, it was hypothesized that the frequency distribution of the 'urban' areas might more closely approximate a Poisson distribution (as compared to the much flatter distribution of the suburban areas). If so, then the application of a 'goodness of fit' test, such as the x^2-*test* could be an extremely good indicator of the range between urban and suburban. Unfortunately, for the purposes of this paper, we were unable to test this hypothesis and must relegate this to 'future work'.

Table 3. Ranked Order of Frequency Distribution Results

Location	Axial Ringiness	Sample Mean
Crabapple, Georgia	14.98 (0.07)	0.0007
Peachtree City (Roads only)	9.76 (0.10)	0.0010
Peachtree City (Dual)	5.68 (0.18)	0.0017
Borehamwood	3.36 (0.30)	0.0020
Soho, London	1.28 (0.78)	0.0035
Atlanta	1.03 (0.97)	0.0036
Manhattan	0.51 (1.95)	0.0042

Let us briefly look at two measures, smallest mean and axial ringiness [7] (the reciprocal values for axial ringiness are given in brackets). The values for the seven areas are shown in Table 3 above and have been ranked in order of 'most suburban' to 'most urban'. One interpretation of these results is that that although, at first glance, the maps appear to be forming two clusters (suburban and urban) in reality there is more of a range or continuum between either extreme. In the case of the seven examples, the most extreme case of suburbia (or indeed, sprawl) is Crabapple, Georgia. This seems to fit with a common-sense appraisal of the maps; Crabapple is characterized by the rapid development of sub-divisions containing a high proportion of dead-ends creating a dendritic structure of streets. Crabapple certainly represents the archetypal American suburb. At the other end of the spectrum is, hardly surprisingly, Manhattan, famous for its gridded street-network. Staying at the urban end of the spectrum, Atlanta (Central) is closer to Manhattan than is Soho, London. Again, if you are

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[7] In The Social Logic of Space (Hiller and Hanson, 1984), the values for axial ringiness are given as the reciprocal of the equation (although not explicitly so). There is some justification to this, as more densely urban areas seem, intuitively, more 'ringy' and suburban areas more 'tree-like'. This discrepancy needs to be clarified, and if the reciprocal is to be used for future work, then the accepted equation should be altered accordingly.

familiar with either location, this seems a fair ranking. More or less in the middle of the range is Borehamwood, the London outer suburb, which is almost a small town rather than a suburb. This leaves Peachtree City. Interestingly, Peachtree City, with only the roads, is closer to, but not so extreme as, Crabapple. Through the use of this spectrum, Peachtree City would have to be classified as an essentially suburban development. However, once you add the cart paths to the city, its ranking alters and it moves closer to the middle of the spectrum than to the suburban edge of it. In other words, the effect of the cart path system is to make Peachtree City more urban and less suburban than it would otherwise be. In answer to the question, how should Peachtree City with the cart paths be classified? It is clear that it should either be classified as suburban or grouped with Borehamwood as *semi-urban*. In the next and final section we shall continue by examining the Peachtree City results in greater detail.

6. Peachtree City Results

This paper was prompted by a previous study into the dual transportation system of Peachtree City. It has already been suggested that the combination of the golf cart path system and the road system seem to be altering the manner in which Peachtree City functions; indeed it could be said to be changing the very nature of the city (perhaps into something less 'suburban') or even acting as a sort of 'anti-sprawl device as suggested in a previous paper (Dalton and Dalton, 2005). But what effect is it having and how might be analyze it or quantify it? First, let us look at some measures in more detail. These measures are shown in Table 4 below.

Table 4. Values of Measures of the Road and Road/Cart Systems
with their Proportional Differences

	Road System Only	Roads & Cart Paths Combined	% Change
Mean axial line connectivity	2.41	2.70	112 %
Mean axial line integration	0.43	0.48	112 %
Number of dead ends	431	337	78 %
Number of circuits	460	1160	252 %
Mean length of circuits	11.56	9.79	85 %
Axial ringiness	0.102	0.176	172 %

As well as the vast reduction in the number of dead ends in the system, after adding the cart paths to the analysis, the most significant change produced by combining the cart paths and the roads is the increase in the number of circuits in the dual system. By including the cart paths in the analysis it can be shown that there are more than double (252%) the number of circuits in the resultant axial map. Furthermore, it can be shown that the mean length of the paths forming the circuits falls to 85% (i.e. there are more circuits and they are shorter). If the distributions of the circuit lengths are plotted as two histograms (for roads only and the combined system), a striking pattern of differences between the road system and the integrated cart-and-road system can be discerned. Figure 6 overleaf shows the pair of histograms.

Bar Chart for column: X   1   Peachtree City (Roads)

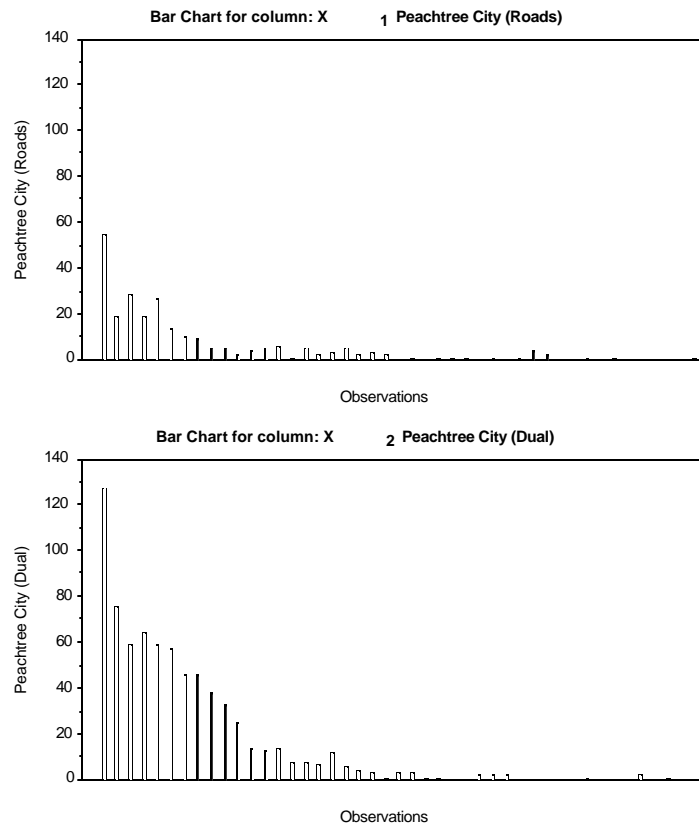**Bar Chart for column: X   2   Peachtree City (Dual)**

Figure 6. Histograms Showing the Distribution of Circuit Lengths for Peachtree City
For Cars (Top) and for Cars and Cart Paths (Bottom)

There is also a greater increase in the number of shorter circuits. Prior to the inclusion of the cart path system, the axial analysis of the roads included a number of extremely long circuits (i.e. of circuit length 79[8]). After the insertion of the cart path system, the maximum circuit depth fell to 66. The outcome of these analyses begins to suggest that such a set of measures, as described in this paper, of the *proportion and distribution of circuit lengths* could provide one morphological definition of the difference between suburbia and urbanity or even sprawl. The paper suggests that without the cart path system, Peachtree City would consist of nothing more than aggregations of typical suburban developments with one or two primary road-entrances accessed from arterial roads and containing a high ratio of cul-de-sacs.

7. Conclusions

First, on the basis of the Peachtree City data and its comparison to other, selected, environments, both suburban and urban, this paper suggests that the differences between developments are clearly discernable and even quantifiable. It goes on to suggest that there is a continuum between suburbia and urbanity and that any

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[8] These are *segments* not *axial lines*. In the previous paper (Dalton and Dalton, 2005) circuit lengths were expressed in terms of axial lines only. This is noted for comparative reasons.

settlement may be analyzed and placed along that spectrum. It further suggests that there is a clear 'profile' to classically 'suburban' and to densely 'urban' areas and that this might be particularly beneficial for the development of a characteristic *spatial signature for sprawl,* an issue which is an acknowledged, growing problem in many countries. With particular respect to Peachtree City, this paper concludes that Peachtree City is essentially a suburban development. However, through the beneficial inclusion of the cart path network, the city has become more urbanized than it would otherwise be. This is achieved by reducing the length of circuits in the transportation network whilst increasing the overall number of circuits.

Second, a number of methods for the generation of circuits were presented in this paper along with initial analyses of the efficacy and expense of the associated algorithms. On the basis of the tentative results in this paper, it is clear that future research needs to be undertaken to further develop these algorithms (or indeed create new methods), with the goal of establishing a reliable yet computationally acceptable solution. Having worked on the methods illustrated in this paper, the authors are clear that there are already opportunities for optimizing them in terms of their performance. Finally, more empirical data needs to be analyzed in order to fully establish the characteristics of sprawl's spatial signature. Our goal for future research is to create a database of settlements, analyzed with respect to the frequency distributions of their circuits in order to expand the spectrum presented in this paper and ultimately aid the development of such definitions.

8. Acknowledgements

9. References

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