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Optimization Methods on Electricity Generation and Transmission Expansion Planning Problem

Mahdi Noorizadegan, Alireza Shokri

Abstract

As a powerful analytical method, optimisation has been used for energy planning problems for a long time. Recent developments in computational capabilities have made it possible to include complex assumptions such as integration of natural gas and power networks, and uncertainty of various parameters with reasonable details in one energy planning problem. It is important to study the integrated natural gas and power networks to reduce the impact of variation in power generation of renewable sources. Moreover, advantages of renewable energy resources have been encouraging many countries to assign a large share of their energy portfolio to these resources. Due to uncertainty, finding a reliable and secure combination of technologies including thermal and renewable sources is significantly complicated. Approaches such as chance constrained programming and robust optimisation have been used to handle reliability and uncertainty of renewable resources and demand. Non-linearity of natural gas network leads to complex energy planning problems. In order to provide a practical pathway to carry out an energy planning problem, we categorise and discuss important topics in energy planning under three main subjects: problem settings and model, uncertainty and solution methods. We suggest a relatively comprehensive optimisation model which includes key features of an integrated power generation and transmission expansion plan and natural gas network. Then, to deal with uncertainty of net load and equipment failure, we suggest robust optimisation and a cutting plane-based method. Finally, we review solution methods used to solve similar problems.

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1. Introduction

Electricity is considered as the heart of modern economies and is predicted to have a significant increase in its share in the global energy mix i.e., twice the rate of primary energy demand (IEA, 2019). Solar and wind will have the highest growth rates among other electricity resources from 2018 to 2040 (IEA, 2019). According the same report, in a sustainable development scenario where electricity plays a larger role in energy demand, renewable resources will account for two thirds of global electricity demand. Therefore, given the energy mix transition towards electricity (particularly renewable sources), energy planning studies mainly focus on power Generation Expansion Planning (GEP). In general, GEP seeks an optimal investment of power generation units over a planning horizon to meet predicted/projected energy consumption (load) subject to a variety of constraints and considerations. Moreover, transmission facilities play important economic and technical roles in GEP as their installation cost and technical constraints could have substantial impacts on generation expansion decisions. Therefore, many studies combine these two problems and reviewed an integrated generation and transmission expansion planning problem (GTGP). Whilst there are different versions of GEP, the main decision variables include investment schedule for generation units. While transmission expansion planning is included in the problem, location of new power generation units and decisions for transmitting power from generating locations to demand points/areas are also decided. However, decision variables are not limited to only these generation units and transmission lines/corridors. Depending on the setting and assumptions, a problem may include many other types of decision variables such as decommission decisions, power generated by units, phase angles of voltages and currents, etc. For instance, Micheliet al. (2020) consider decommissioning variables. Direct Current (DC) load flow is an approximation of Alternating Current (AC) load flow and has been considered in

some studies (Caunhye and Cardin 2018) while it has been ignored in many GTEP related studies. This involves the computation voltage angle which depends on geographical properties and technical characteristics of transmission lines. Some studies (Coester et al. 2018) incorporated less technical details and instead focused more on economic analysis and environmental aspects of GTEP and GEP.

In recent years, technological and economic advancements in renewable sources as well as environmental requirements have directed the focus of energy planning problems towards GTEP with high renewable energy penetration. Despite their advantages, renewable sources impose considerable complexities to power supply. Although cost of renewable sources has substantially declined (e.g., 70% for solar from 2010 to 2018), it seems that renewables still cannot effectively compete with thermal technologies as their cost has also reduced (Fu et al. 2018; and Feldman and Margolis 2019). Therefore, governments designed attractive incentive schemes to encourage companies for investing in renewable sources. Levin et al. (2019) studied incentives mechanisms under four categories: 1) investment support, 2) generation support, 3) quantity targets, and 4) carbon policies. There are various studies for further investigation of incentive mechanisms (Alolo et al. 2020; and Newbery 2016). Because of their uncertain power generation, integrating renewable sources into existing power systems which mainly consist of thermal units, is complex and requires sophisticated planning and scheduling. Moreover, technical restrictions such as rampage constraints of thermal units limit the utilisation of renewable sources. For instance, Duck curve is a critical concept to address the impact of power generation by solar units on power systems (Denholm et al. 2015). Capacity factors of renewable sources are another important component that can have a considerable impact on economic and technical analysis in GTEP. Capacity factors are usually estimated for an entire year. In fact, this type of complexity and limitation makes renewable sources more expensive. The wide use of renewable sources imposes another complication to GEP. The consumption of natural gas by gas-fired units is significantly affected by the uncertainty of power generated by renewable sources. In other words, the production of gas turbines needs to be adjusted with respect to power generation changes of renewable sources to satisfy demand. As a result, the uncertainty of renewables is transferred to the gas network. In order to maintain gas pressure at a safe level in a gas network for other usage (e.g., residential and industrial sectors), gas and electricity storage devices and gas compressors need to be installed. Integrated gas and power networks have been studied in operational level (Fallahi and Maghouli 2020a). However, recently there has been an interest for this integration in planning problem (Conejo et al. 2020). Such problems are in general non-convex non-linear mixed integer problems (Esmaili et al. 2020). The source of non-linearity is the gas flow equation known as Weymouth equation.

This book chapter provides a relatively comprehensive overview on GTEP and suggests an optimisation modelling framework for GTEP that includes important features. The rest of this chapter is organised as follow. In section 2, we focus on the mathematical modelling of a general GTEP where various types of objective functions and constraints are discussed. In section 3, we discuss two types of

uncertainties: demand and power generation of renewable resources, and equipment failure. We suggest Interval Optimisation to deal with demand and generation of renewable resources and a cutting plane-based method for equipment failure. The both approaches are conservative and consider the worst possible situations. In Section 4, we briefly review the solution methods and suggest a simulation-based optimisation framework for solving practical GTEP problems. Finally, we provide a summary of this chapter in section 5.

Notation - In this chapter, we use a simple notation for simplicity. Bold face characters and symbols indicate vector i.e., $\mathbf{x}_t^D := [x_{ikt}^D]_{ik}$. Subscripts denote indices while superscripts denote the type of variables (e.g., D stands for decommissioned equipment). The main sets, indices, decision variables and parameters are defined as per below. The rest of variables and parameters are defined where needed.

Sets, indices, superscripts

I : the set of locations (nodes),

t : index for time period (year),

T_t : a subset within time period t ,

i, j : location indices,

h : index for hours of a day,

k : index for technology type (either power generation unit or transmission line),

l : index for fuel type,

G : used to denote the natural gas,

st : used to denote storage devices,

dis : used to denote discharging storage devices,

ch : used to denote charging storage devices,

D : used to denote decommissioned equipment,

N : used to denote new equipment.

Decision variables

$\mathbf{x}_t^N := [x_{ikt}^N]_{ik}$: binary variables representing whether at location i , a unit of technology k at time period t is installed,

$\mathbf{x}_t^D := [x_{ikt}^D]_{ik}$: binary variables representing whether a unit of technology k located at location i at time period t is decommissioned,

$\mathbf{y}_t^N := [y_{ijkt}^N]_{ijk}$: binary variables representing whether between locations i and j , a transmission line of technology k at time period t is installed,

$\mathbf{p}_{ht} := [p_{ikht}]_{ik}$: continuous variables representing power generation at location i with technology k at hour h in time period t ,

$\mathbf{f}_{ht} := [f_{ijkht}]_{ijk}$: continuous variables representing power flow between locations i and j using transmission line technology k at hour h in time period t ,

$\mathbf{I}_{ht} = [I_{ikht}]$: continuous variables representing inventory of storage at location i with technology k at hour h in time period t ,

θ_{iht} : continuous variables representing the phase angle at location i at hour h in time period t ,

$\mathbf{dis}_{ht} := [dis_{ikht}]_{ik}$: continuous variables representing discharge of storage at location i with technology k at hour h in time period t ,

$\mathbf{ch}_{ht} := [ch_{ikht}]$: continuous variables representing discharge of storage at location i with technology k at hour h in time period t ,

π_{iht} : continuous variables representing gas pressure at location i at hour h in time period t ,

$\mathbf{g}_{ht} := [g_{ijht}]_{ij}$: continuous variables representing natural flow between locations i and j at hour h in time period t ,

M, M^G : large enough numbers.

Parameters

$\mathbf{Q} := [Q_k]_k$: maximum capacity of power generation unit of technology k ,

$\mathbf{Q}^{\min} := [Q_k^{\min}]_k$: minimum level of power generation unit of technology k ,

$\mathbf{F} := [F_k]_k$: maximum capacity of power flow for line of technology k ,

$\mathbf{r} := [r_k]_k$: increasing ramp rate for power generation unit of technology k ,

L_{lT_t} : maximum available fuel of type k in time period T_t

$\alpha_h := [\alpha_{kh}]_k$: capacity factor for hourly power generation unit of technology k ,

$\hat{\mathbf{x}} := [\hat{x}_{ik}]_{ik}$: indicator for existing unit at location i , a unit of technology k in the beginning of the planning horizon,

$\hat{\mathbf{y}} := [\hat{y}_{ijk}]_{ijk}$: indicator for existing transmission line between locations i and j of technology k in the beginning of the planning horizon,

$\pi_i^{\min}, \pi_i^{\max}$: minimum and maximum permitted gas pressure at location i .

Note that $\mathbf{x}_t = \hat{\mathbf{x}} + \mathbf{x}_t^N$ and $\mathbf{y}_t = \hat{\mathbf{y}} + \mathbf{y}_t^N$. We also eliminate the transpose sign in the notation.

2. Mathematical model

Key components of GEPR include but not limited to objective function, environmental impacts of sources of power generation, reliability, resiliency, uncertainty, operational restriction and consideration, and impact of power generated by gas network. GTEP decides on facilities (generation units and transmission lines) with effective lives of more than 30 years. Therefore, similar to long term planning problems, some technical details such as daily operational details are replaced with their approximations or ignored. However, this could affect the electricity mix leading to higher costs or in some severe cases infeasible situations in reality. On the other hand, building and solving a GTEP with

many details are formidable tasks. Therefore, a reasonable trade-off between operational details and the problem complexity and computational challenges is usually sought.

2.1. Objective function

Whilst majority of studies consider cost-based models, some (Lohmann and Rebennack, 2017; and Allahdadi Mehrabadi et al. 2020) study a welfare or profit-based objective functions. Electricity markets will have to be simulated to detect the electricity price to model a maximisation of GTEP model. Simulating electricity markets with reasonable details is a complex topic. However, some studies (Coester et al. 2018) applied a rather simple methods such as Merit Order Curve. In such models, regulators also play an important role in setting electricity markets. We refer to Cramton et al. (2013) for further discussion on energy and capacity markets. The main components of cost for GTEP include investment, decommission, operation, and fixed maintenance costs.

Investment cost - The investment for power system equipment is capital intensive and usually involves long-term financial arrangements. Uncertainty of demand and power generation by renewable sources complicates the risk assessment for investors. Hence, some studies (Simo et al. 2015; and de Oliveira et al. 2017) formulate GTEP as a dynamic program. GTEP can be considered as a facility location problem with fixed cost. This approach essentially requires a long-term planning horizon (to include the full effective lifecycle of all equipment) in order to make a right balance between operational and investment costs. However, due to complications such as disparity of lifecycle of different equipment, it is not always possible. Instead of total fixed investment costs, an equivalent annual cost for each equipment is computed. In this situation, additional constraints are required to ensure of availability of a selected facility for its entire lifetime. The investment cost of facility is computed towards the end of planning horizon (Caunhye and Cardin, 2018; and Ding et al., 2018).

Let $\mathcal{F}_t^{inv}(\mathbf{x}_t^N, \mathbf{y}_t^N)$ denote the investment cost function at time period t .

Decommission and upgrade cost - The main reasons to retire a unit are its high maintenance and operational cost and its high rate of failure and unreliability. In practice, even units may be used beyond their nominal effective lifetime when they are properly maintained and looked after. Decommissioning some units such as nuclear units incurs cost while decommissioning units such as gas turbines may lead to profit as they have salvage values. In some cases, old units may not be decommissioned and may be used to cover limited peak loads as an alternative for installing new units for this purpose. The decision to keep old units or to decommission them should be made through GTEP models. Moreover, units can be upgraded in an extra cost. In some cases, upgrading old units can be more cost effective than investing in new ones. For instance, gas turbines may be converted into combined cycle units or an overhaul may significantly increase the efficiency of steam units. Upgrading a unit usually involves considerable modelling complexities. Related constraints need to be defined based on the type of upgrade. A simple solution is to define two new variables one for investment in the unit upgrade and

one for decommissioning the old unit. We use $\mathcal{F}_t^{D,U}(\mathbf{x}_t, \mathbf{y}_t)$ to denote the cost function for decommissioning and upgrading power generation units and transmission line at time period t . It is worth mentioning that some units such as distributed generators have shorter effective lifecycle and may be installed and also decommissioned within the planning horizon.

Operation cost - Alongside the investment cost, operation costs (i.e., variable cost) comprise the main part of electricity cost. The operation cost includes fuel cost, water supply, pollution and emission cost, start-up cost. All components of operation cost may depend on the age of units. Thermal units can usually work with more than one type of fuel, which increases the availability of units. However, the efficiency, and emission produced by units depend on the type of fuel. For instance, due to restriction of natural gas network in winter as the result of higher level of consumption, other fuels such as Mazut are used in power plants. As such periods are usually short, modelling other fuels can have a considerable impact on the electricity mix. The reason is that when the natural gas limitation is enforced, a single-fuel GTEP model would change the mix e.g., installing sufficient fuel-efficient units to ensure the feasibility. Once fuel-efficient units are installed, the power generation plan and consequently the operation cost would change. However, due to modelling and computational complexities, alternative fuels are generally ignored in GTEP. Temperature and altitude also affect power generation by almost 10 percent (Sen et al., 2018). Whilst it is not difficult to incorporate temperature and altitude in GTEP models, their impacts are usually neglected. It is worth mentioning that the fuel consumption function is not linear; but, a linear approximation is usually studied for more simplicity. Start-up cost is often considered in GTEP models. Modelling start-up requires constraints that link power generation in different hours. Such constraints are complicating constraints and increase the computational complexity.

Although water supply is crucial for thermal units, it is not included in GTEP studies as it is considered water is available everywhere. However, this is not a correct assumption. Water supply at certain dry locations can be quite expensive or in some areas impossible. We carried out a simple experiment to investigate the impact of water supply cost and restrictions. We noticed that water supply in areas with particular restrictions could play an important role in determining the electricity mix.

Environmental consideration is among the most influential factors to shift electricity mix towards renewable sources. The emission cost is now a key part of operation costs, and is mainly considered for NO_x, SO₂, CO, SPM, CO₂, CH₄ and N₂O (Li and Taeihagh, 2020). It is worth mentioning that the penalty for each one is different. In addition to penalising, the production of some pollutants may be restricted. The emission cost depends on several factors such as distance between power plants and cities, population of cities, type of emission and pollution and type of fuel. Moreover, some environmental consideration may forbid installing power plants in special areas. This is important when the decision for transmission lines/corridors is included.

Let $\mathcal{F}_t^{opr}(\mathbf{x}_t, \mathbf{y}_t)$ denote the operation cost function at time period t .

Fixed and maintenance cost - There is usually a schedule for power plants and unit maintenance, which depends on hours that each unit produces electricity in each year. Some types of maintenance activities are short, but some are longer. The cost for each type is therefore different. But it is common to consider a fixed cost for annual maintenance for each unit depending on the technology of units. A fixed cost is also considered for each unit, which does not depend on its performance. We use $\mathcal{F}_t^{fix}(\mathbf{x}_t, \mathbf{y}_t)$ to denote the fixed and maintenance cost function at time period t .

2.2. Constraints and technical conditions

We classify the constraints and technical conditions of a GTEP model into three groups: 1) investment related constraints, 2) capacity constraints, and 3) technical constraints. These constraints are related to power generation units, transmission lines, and gas network. A key factor in formulating a GTEP is time interval which is usually hourly based intervals. But depending on the problem, 24 hours in a day could be split into 6 intervals. This will significantly reduce the number of variables and constraints. In the following constraints, we consider hourly interval and present a brief mathematical model for a general GTEP.

Investment related constraints - As mentioned in the objective function description, when the investment decisions are annually modelled, additional constraints are required to ensure that once a unit is installed, it will be available for the rest of the planning horizon. Analogously, we need to make sure once a unit is decommissioned, it will be no longer available for production.

$$\mathbf{x}_t^N \leq \mathbf{x}_{t+1}^N \quad (1)$$

$$\mathbf{y}_t^N \leq \mathbf{y}_{t+1}^N \quad (2)$$

$$\mathbf{x}_t^D \geq \mathbf{x}_{t+1}^D \quad (3)$$

$$\mathbf{y}_t^D \geq \mathbf{y}_{t+1}^D \quad (4)$$

Capacity constraints - These constraints enforce capacity constraints for new and existing power generation units and transmission lines.

$$\sum_{l \in \mathcal{L}} \mathbf{p}_{htl} \leq \alpha_h (\mathbf{Q}\mathbf{x}_t + \mathbf{Q}(1 - \mathbf{x}_t^D)) \quad (5)$$

$$|\mathbf{f}_{ht}| \leq \mathbf{F}\mathbf{y}_t + \mathbf{F}(1 - \mathbf{y}_t^D) \quad (6)$$

$$\sum_{l \in \mathcal{L}} \boldsymbol{\eta} \mathbf{p}_{htl} \leq L_{lT_t} \quad (7)$$

where $\boldsymbol{\eta}$ is the vector of fuel consumption rate for 1MWh corresponding to units in vector \mathbf{p}_{htl} . When hydropower units are included in GTEP, additional constraints for their power generation should be considered such as intakes and reservoir levels. Since other entities (agricultural related organizations) are involved, hydropower units may not be always available in particular during pick times.

Technical constraints - In an accurate model, all technical constraints in a Security Unit Commitment (SUC) problem will have to be considered. However, as GTEP is a long-term planning problem, only important conditions are studied. Given their complexities, effective approximations for some constraints are developed and used in GTEP. Here, we consider ramp rate constraints, start-up related constraints, and DC power flow requirements. In order to formulate ramp rate constraints, additional binary variables are needed for each unit. However, it may not be vital to include such details for a GTEP. We suggest using the following ramp rate constraints:

$$\mathbf{p}_{ht} - \mathbf{p}_{h+1,t} \leq \min\{\mathbf{r}, \alpha_h(\mathbf{Q}\mathbf{x}_t + \mathbf{Q}(1 - \mathbf{x}_t^D) - \mathbf{p}_{ht})\} \quad (8)$$

The above inequality only enforces increasing ramp rate limits. A similar inequality can be used for modelling decreasing ramp rate; but it is not crucial to add the latter to the model. Equivalent ramp rate functions can be used to further simplify the ramp rate constraints. Lohmann and Rebennack (2017) proposed an efficient way of modelling unit start-up. They divided the power generation of each unit to two parts: \mathbf{p}_{ht}^L is a vector of variable for power generation of units up to their \mathbf{Q}^{\min} , and \mathbf{p}_{ht}^U is another vector of variables for power generation between \mathbf{Q}^{\min} and \mathbf{Q} . Then, the difference between $\mathbf{p}_{h-1,t}^L$ and \mathbf{p}_{ht}^L approximates the start-up variable \mathbf{u}_{ht} . The below inequalities compute the start-up variables

$$\mathbf{Q}^{\min}\mathbf{p}_{ht}^L + (\mathbf{Q} - \mathbf{Q}^{\min})\mathbf{p}_{ht}^U = \mathbf{p}_{ht} \quad (9)$$

$$\mathbf{p}_{ht}^L + \mathbf{p}_{h-1,t}^L \leq \mathbf{u}_{ht} \quad (10)$$

$$\mathbf{p}_{ht}^U \leq \mathbf{p}_{ht}^L \quad (11)$$

The last inequality ensures that the load below minimum generation always exceeds the load above minimum generation. It is trivial that without this inequality, the start-up variable may be zero. AC load flow equations involve non-linear and complex terms. Even in SUC problems which needs to be accurate, they are approximated by DC load flow equations. In a long-term planning, a good approximation may be sufficient. Therefore, we only enforce the key equations for load flow as follows.

$$-M(1 - \mathbf{y}_{ht}) \leq \mathbf{f}_{ht} - \mathbf{S}\hat{\boldsymbol{\theta}}_{ht} \leq M(1 - \mathbf{y}_{ht}) \quad (12)$$

where \mathbf{S} is the matrix of reactance of lines and $\hat{\boldsymbol{\theta}}_{ht}$ is the vector of difference of phase angles at two ends of each line. We misuse the notation in the above inequality for the notation brevity and simplicity. But it is worth mentioning that it should be constructed so that we have $f_{ijht} = s_{ij}(\theta_{iht} - \theta_{jht})$ if a line is installed or exists. We have observed that when the above inequality is removed, the solution of GTEP significantly changes and may not be feasible for a real situation.

Storage Constraints - Storages substantially complicate the problem; because it includes binding constraints that connect power generation of different hours (similar to rampage constraints). Therefore, although they have become vital in power systems with high renewable penetration, many studies still do not explicitly formulate them (Chen et al., 2019). They are very important for technical purposes

such as helping to cover load rampage, stability of power network, and variation of renewables' power generation. Storages are especially useful when the difference of the maximum and minimum electricity loads is very high. In this case, there will be enough idle units to charge storages during off-peak times to be used in pick times. Storages conventionally are batteries and pumped-storage hydroelectricity. Recently, Power-to-Gas (PtG) systems are used to produce gas during off-pick times, in order to be used to generate power when required (Ban et al. 2017; and Fallahi and Maghouli 2020b).

$$\mathbf{I}_{h+1,t} = \mathbf{I}_{ht} - \gamma^{dis} \mathbf{dis}_{ht} + \gamma^{ch} \mathbf{ch}_{ht} \quad (13)$$

$$\mathbf{I}_{ht} \leq \mathbf{Q}^{st} (\mathbf{Q} \mathbf{x}_t^{st} + \mathbf{Q} (1 - \mathbf{x}_t^{D,st})) \quad (14)$$

$$\mathbf{dis}_{ht} \leq \boldsymbol{\iota}^{dis} (\mathbf{Q} \mathbf{x}_t^{st} + \mathbf{Q} (1 - \mathbf{x}_t^{D,st})) \quad (15)$$

$$\mathbf{ch}_{ht} \leq \boldsymbol{\iota}^{ch} (\mathbf{Q} \mathbf{x}_t^{st} + \mathbf{Q} (1 - \mathbf{x}_t^{D,st})) \quad (16)$$

where γ^{ch} and γ^{dis} are charging and discharging efficiency vectors, respectively. Also, $\boldsymbol{\iota}^{ch}$ and $\boldsymbol{\iota}^{dis}$ are charge and discharge rate vectors, respectively. Equations (13) state the storage balance equation between two hours. Constraints (14-16) enforce inventory, discharging and charging restrictions based on the existing, installed and decommissioned capacity.

Gas Network - Natural gas is the main fuel used thermal units. The variation of power generation by renewable sources changes the natural gas consumption of thermal units and consequently the gas pressure in gas network. This could affect the natural gas consumption of residential and industrial sectors. Therefore, it is important to include the gas equation into GTEP to manage the impact of power system with high renewable penetration on gas network. Below, we suggest a simplified variation of gas network modelling. We use a non-vector notation for clarity.

$$-M^G (1 - \bar{y}_{ijt}^G) \leq g_{ijht} |g_{ijht}| - \phi_{ij} (\pi_{jht}^2 - \pi_{iht}^2) \leq M^G (1 - \bar{y}_{ijt}^G) \quad (17)$$

$$\pi_{iht} \leq \pi_{jht} \leq \Gamma \pi_{iht} \quad (18)$$

$$\pi_{iht}^{\min} \bar{y}_{ijt}^G \leq \pi_{iht} \leq \pi_{iht}^{\max} \bar{y}_{ijt}^G \quad (19)$$

$$g_{iht}^{\min} \bar{y}_{ijt}^G \leq g_{iht} \leq g_{iht}^{\max} \bar{y}_{ijt}^G \quad (20)$$

$$p_i^{G,\min} \leq p_{iht}^G \leq p_i^{G,\max} \quad (21)$$

where ϕ_{ij} is the parameter of natural gas pipeline, Γ is the compression ratio and $\bar{y}_{ijt}^G = 1$ if there exists a pipeline between node i and j , and otherwise, $\bar{y}_{ijt}^G = y_{ijt}^G$ (i.e., a decision variable). Constraints (17) state relation between gas flow and gas pressure for new and existing pipelines. Constraints (18) model the impact of compressors on the gas pressure. Constraints (19 and 20) respectively enforce the pressure and flow restrictions on new and existing pipelines. Constraints (21) ensure gas production restrictions on gas production nodes. Adding the above set of inequalities to GTEP results in a non-linear program. There are various methods such as Newton method and decomposition-based methods to deal with the nonlinear terms. The reader is referred to Ding et al. (2018) and Fallahi and Maghouli (2020b) for

further topics on non-linear gas network related terms. Note that in these inequalities, we assume that the gas flow direction is known in each pipeline. We also neglected modelling line-pack and installing new compressors.

Balance Equations - Natural gas and power networks have to be separately balanced at each node

$$\mathbf{p}_{ht} + \mathbf{dis}_{ht} - \mathbf{ch}_{ht} + \mathbf{f}_{ht}^{in} - \mathbf{f}_{ht}^{out} = \mathbf{d}_{ht} \quad (22)$$

$$\mathbf{p}_{ht}^G + \mathbf{g}_{ht}^{in} - \mathbf{g}_{ht}^{out} = \mathbf{d}_{ht}^G + \boldsymbol{\eta}\mathbf{p}_{htl} \quad (23)$$

where l is the fuel index for natural gas, \mathbf{f}_{ht}^{out} is a vector with element f_{iht}^{out} presented as $\mathbf{f}_{ht}^{out} = [f_{iht}^{out}]_i$, $f_{iht}^{out} = \sum_{j \in I} f_{ijht}$. Similarly, we have $\mathbf{f}_{ht}^{in} = [f_{iht}^{in}]_i$, $f_{iht}^{in} = \sum_{j \in I} f_{jih}$, $\mathbf{g}_{ht}^{out} = [g_{iht}^{out}]_i$, $g_{iht}^{out} = \sum_{j \in I} g_{ijht}$, $\mathbf{g}_{ht}^{in} = [g_{iht}^{in}]_i$, and $g_{iht}^{in} = \sum_{j \in I} g_{jih}$. The first balance equation ensures that electricity load at each node is satisfied. The second balance equation connects the natural gas network to the power network. The last term in this equation ($\boldsymbol{\eta}\mathbf{p}_{htl}$) is the consumption of natural gas by power generating units.

2.3. Final deterministic model

The summary of this section is a deterministic optimisation model as presented below:

$$\begin{aligned} \min \sum_{t \in T} \mathcal{F}_t^{inv}(\mathbf{x}_t^N, \mathbf{y}_t^N) + \mathcal{F}_t^{fix}(\mathbf{x}_t, \mathbf{y}_t) + \mathcal{F}_t^{D,U}(\mathbf{x}_t, \mathbf{y}_t) + \mathcal{F}_t^{opr}(\mathbf{x}_t, \mathbf{y}_t) \\ \text{s.t., (1-23)} \end{aligned}$$

The above model can be used as a base model for the next stage, which is to consider uncertain parameters. The above problem has a diagonal structure based on t . In other words, there is no constraint coupling variables for different t . The objective function is also separable based on t . As it will be explained in the solution method section, decomposition methods can be applied to such a structure. In some studies (Moradi Sepahvand and Amraee, 2020), reserve and spinning reserve are included in GTEP models. In security-constrained unit commitment problems, reserve and spinning reserve are considered to respond unforeseen events such as demand variations and equipment failure. A simple and practical way of computing reserve and spinning reserve is to consider a certain fraction of load (Moradi Sepahvand and Amraee, 2020). As reserve and spinning reserve are mainly operational decisions, we do not independently address them in this model. In the next section, we study uncertainty in GTEP which are due to two events: net load variation and equipment.

3. Uncertainty

Electricity demand and power generation by renewable sources are two key sources of uncertainty in GTEP. Power unit and transmission line failures are also uncertain events in a power system. These

two types of uncertainty are usually dealt with differently. We briefly review main approaches for both types of uncertainties in this section.

3.1. Uncertain electricity demand and power generation by renewable sources

Stochastic programming (Ding et al. 2018) and robust optimisation (Jabr 2013) are the common approaches used to deal with power generation of renewable sources and load uncertainties. The application of stochastic programming involves scenario generation for possible electricity load and power generation by renewable sources for the planning horizon. For a GTEP problem, the planning horizon is generally more than 15 years. Based on prediction/projection methods, a discrete set of possible realisations of each uncertain parameter is generated. Therefore, the number of scenarios for hourly electricity load and power generation by renewable sources will be exponential. As the first step to reduce the number of scenarios, only selective days are considered for modelling (e.g., few days per month, or even per season for each year). Another way of reducing number of scenarios is to merge 24 hours of a day into fewer time blocks. Then, scenario reduction approaches are applied to eliminate less likely scenarios. However, solving a large multi-stage stochastic problem specially for practical cases is still very challenging.

Alternatively, robust optimisation takes a less complex but more conservative approach and plans for the worst cases. The worst cases can be formed prior to the start of solution procedures. Multivariate statistical analysis based methods such as “flying-brick” have been developed to deal with variable requirements of the look-ahead generation capacity, ramping capability, and ramp duration for unit commitment problems. For more details see Pourahmadi et al. (2020). We focus on Interval Optimisation approach developed for unit commitment problems by Wu et al. (2012). They used the concept of net load (NL) which is equal to total demand minus wind generation output minus solar output generation. Net load is commonly used because wind and solar generation, and demand have some similar characteristics such as they are non-dispatchable, they depend on the weather condition, and they deviate from forecasts (Makarov et al. 2010). Therefore, the electricity balance equation is modified by the concept of net load. The key idea is to make sure that the installed electricity mix is capable of responding to extreme situations which are illustrated in Figure 1. The worst situations are as follow: 1) power generation units including thermal and hydropower units are able to increase their generation to satisfy the net load from hour h where the net load is in its lowest level to hour $h + 1$ where the net load is in its highest level. 2) power generation units can deal with duck curve from mid-day towards night peak.

To this end, we need to define three power generation variable \mathbf{p}_{ht}^P , \mathbf{p}_{ht}^E , \mathbf{p}_{ht}^O , power generation for pessimistic, expected and optimistic net loads. Then, the rampage constraints need to be imposed for all combinations of these new variables (e.g., $\mathbf{p}_{ht}^P - \mathbf{p}_{h-1,t}^O \leq \min\{\mathbf{r}, \alpha_h(\mathbf{Q}\mathbf{x}_t + \mathbf{Q}(1 - \mathbf{x}_t^D) - \mathbf{p}_{ht}^O)\}$).

When the ramp rate constraints are revised as explained, it can be ensured that the duck curve is also addressed.

3.2 Uncertain equipment failure

Security and resiliency of a power system are usually defined by uncertain equipment failures which can be due to technical failure, natural disasters, or sabotage. One solution to deal with unforeseen equipment failures is to allocate a sufficient level of spinning and non-spinning reserves, which is usually a topic for daily operation (Morales et al. 2009). Another approach is to set the N-k security criterion. That means if for any reasons, k equipment (mainly lines) fails at the same time, there will be no power cut in the power system. This criterion can be imposed locally with different values for k. As the number of equipment is high in a power system, contingencies are limited to a pre-defined set of contingency scenarios (Qiming Chen and McCalley 2005). Then, binary variables or indicators and a set of related constraints are used to model the N-k security criterion. This idea is applied within bi-level programming, multi-stage robust optimisation and multi-stage stochastic programming (Wu et al. 2016). There are also some probabilistic versions of N-k security criterion (Sundar et al., 2018). But due to the complexity of probabilistic constraints, this approach is not popular for GTEP.

As GTEP problems expand existing electricity networks, it may not be necessary to define a binary variable for each line for the N-k security criterion. In other words, it is very likely that there are already other routes to a demand bus if one line fails. Studying the topology of the network could be very useful. Therefore, instead of initially defining binary variables for each line, cutting plane methods can be used to ensure the N-k security criterion with much less computational complexity. In a cutting plane method, the N-k security criterion is first relaxed and the problem is solved. Then, using a separation algorithm, it is checked to find a violation of the N-k security criterion for each demand bus. If found, a cutting plane is constructed to enforce the security criterion for that bus. In general, separation algorithms are usually quite fast and it is relatively simple to identify violated constraints which were relaxed (Nemhauser and Wolsey, 1988). For a GTEP, graph-based problems such as maximum flow problem and shortest path problem could be used in designing separation algorithms. Therefore, it is expected to achieve a better computational efficiency in particular for real problems, as significantly less binary variables are required in the model.

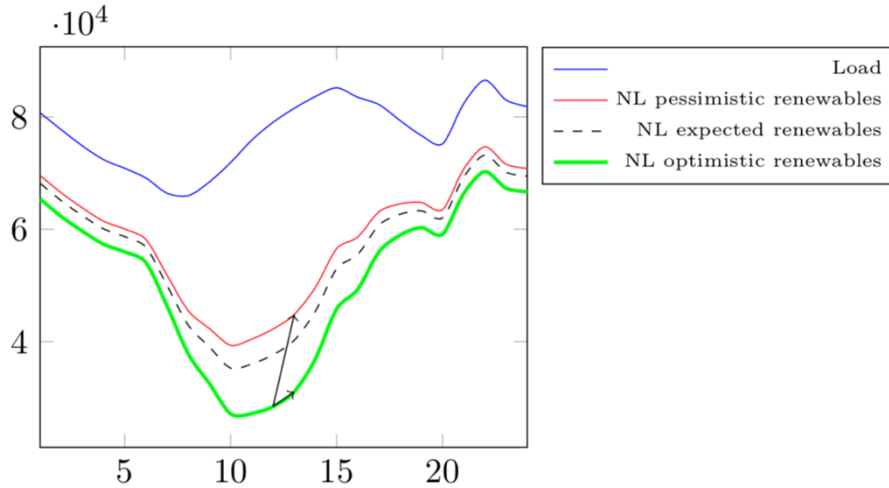


Figure 1: Net load uncertainty intervals for a sample daily load.

4. Solution method

There is a longitudinal study on GTEP in which the majority of them use Benders' decomposition-based methods to solve their problems (such as Lohmann and Rebennack (2016) and Wu et al. (2016)). Therefore, this section reviews some principles of Benders' decomposition and few important tips for implementing Benders' decomposition particularly useful for solving practical problems.

Decision variables in a GTEP problem are naturally divided into strategic decisions and operational decisions. This division paves the way for applying Benders' decomposition where the investment and operational decisions are made in the master problem and subproblems, respectively. In particular, Benders' decomposition is applied to two or multi-stage stochastic programming or robust optimisation variations of GTEP. Some studies (Lohmann and Rebennack, 2017) have further explored the structure of their problem and proposed nested Benders decomposition reformulations. As the operational problems are independent some time intervals, they can be solved separately. Constraints such as available fuel and maximum amount of pollution produced by units are usually defined seasonally or annually. These constraints link operational variables within a season or a year. In these cases, seasonal or annual operational problems can be independently solved. Such further breakdowns can help to reduce the computational efforts.

Connecting the master problem and the subproblems is done using optimality and feasibility cuts. Optimality cuts approximate the impact of the master problem decisions on the cost of the subproblem (Conejo et al., 2006). It is worth mentioning that if there are binary or integer decision variables in the subproblem, standard optimality which are developed for pure linear continuous subproblems cannot be used. The reason is that optimality cuts are constructed based on the dual form of the subproblem. The linear programming duality theorem does not hold for an integer program (Nemhauser and Wolsey, 1988). This is a common mistake in studies about GTEP problems. Further details can be found in

studies about the concept of value function (Guzelsoy and Ralphs 2006; and Trapp et al. 2013). Feasibility cuts are driven when a solution of the master problem leads to an infeasible subproblem. When a subproblem is infeasible for a master problem solution, feasibility cuts driven for that solution only remove that solution from the search space. Nevertheless, it is possible that next solutions of the master problem still lead to infeasible subproblems. Significant computational efforts thus would be spent on finding master problem solutions with feasible subproblems. It will be computationally beneficial to avoid feasibility cuts, if possible. Investment decisions e.g., power generation unit installation, are made in the master problem. For a minimisation problem, in the first iteration, no new investment is made to keep the master problem cost at its minimum level. This will lead to some infeasible subproblem. To avoid dealing with feasibility cuts and their drawbacks, valid inequalities that reflect the subproblem's feasible region of subproblems can be derived and added to the master problem prior to the solution process. For power generation expansion decisions, a constraint can be formed to enforce the sufficiency of the accumulative capacity to satisfy a selective peak load. Similar valid inequalities can be formed for transmission lines. Moreover, storage devices are complicating components of GTEP problems as they are only available when they are charged. Because the investment cost of storage devices is usually lower than other technologies, the solution of the master problem may include too many storage devices at the first iteration. As a result, subproblems may be infeasible. To overcome this issue, some valid inequalities approximating storage constraints (charge and discharge processes) are useful in the master problem.

Given the significant number of variables and constraints, it may take several days to solve a practical problem even with very powerful machines. The feasible region can be reduced before the solution process by applying methods such as Merit Order or even simple economic analysis to only include power generation technologies which are likely to be a part of the optimal solution. In addition, decomposition methods such as Dantzig Wolfe decomposition and column generation methods have proved to be very efficient for optimisation problems with binary variables (Singh et al. 2009). Nevertheless, they have not received much attention for reformulating and solving GTEP problems (Flores-Quiroz et al. 2016).

A practical GTEP with fair number of details, which addresses concerns for independent system operators may not be solvable with a reasonable time. Key aspects of a power system such as frequency response control, power system inertia, various types of losses (particularly important for distributed generators) are usually ignored. As mentioned, gas network and storages are also mainly neglected in GTEP problems. Simulation-based optimisation is a practical way of addressing all important components, factors and constraints of GTEP and at the same time solving the resulting problem. The application of simulation-based optimisation to GTEP is an independent topic with many technical details. Here, we emphasise on its benefit for GTEP and outline some general steps. For more detail, we refer the reader to Rodgers et al. (2018).

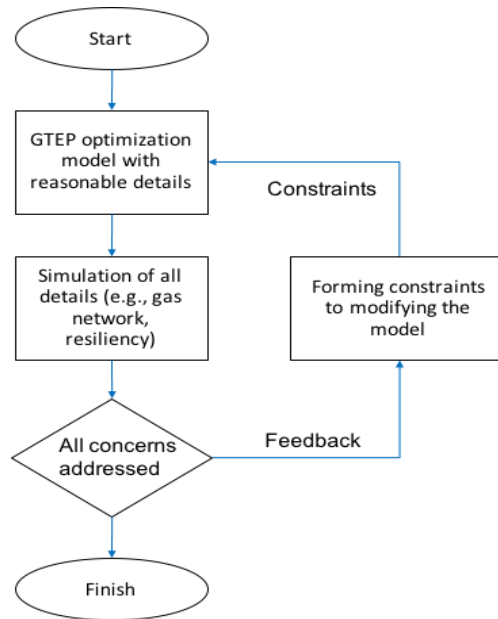


Figure 2: The diagram for a simulation-based optimisation method

As it is illustrated in Figure 2, we can start with a GTEP optimisation model including only key constraints. The GTEP optimisation model box may contain several sub-boxes in relation to modelling and solution methods (e.g., cutting planes and decomposition methods). In particular, with uncertainty assumptions, it is usually the case that the problem is decomposed into a master problem and several sub-problems. In the initial optimisation model, complicating components such as the gas network (i.e., constraints (17-21) and (23)) and N-k security criterion may be ignored. Once the optimisation model is solved, its solution can be used for a Monte-Carlo simulation model with more details (including the gas network and N-k security criterion) compared with the initial optimisation model. The aforementioned complicating components do not directly affect the objective function i.e., there is no term for these components in the objective function. It is worth noting that the focus here is to solve a GTEP in which the impact of the gas network is also considered, and we do not intend to optimise the gas network as well. Therefore, these components affect the feasibility of the problem. If the solution satisfies all requirements, then the optimal solution is generated. It is worth mentioning that the notion of optimal solution here may be challenged. Otherwise, constraints that enforce the violated requirements are built and added to the optimisation problem. Procedures to generate feedbacking constraints that impose the conditions of the eliminated components are difficult to develop and highly depend on the assumptions made on these eliminated components. However, there are two advantages of using a simulation-based optimisation in solving GTEP. First, constraints associated with these components are complex and maybe non-linear and non-convex if gas network is included. Even if these constraints have no impact on the optimal solution, they make it very difficult for solvers to find even a feasible solution. Thus, it would be good to eliminate them in the initial problem. Second, if these constraints change the optimal solution, the initial GTEP without these constraints will produce good working solutions for the complex components. That will help us to define feedback constraints

around these solutions and apply neighbourhood search methods, instead of searching the entire feasible region of GTEP.

5. Summary

We discussed the key features of a power generation and transmission expansion planning problem as an optimisation problem. We provided a general framework for modelling an integrated GTEP and gas network with reasonable features. An approximation of gas network was used to manage the effect of natural gas consumed by gas-fired units which can be considerably affected by uncertainty of power generation by renewable sources. We applied a conservative approach (Interval Optimization) to deal with uncertainty of electricity load and power generation by renewable sources. This approach is particularly useful as 1) the resulting model is significantly smaller than models produced by other approaches, and 2) it ensures that in the worst cases and duck curve, net load will be safely managed. We also discussed equipment failures as another source of uncertainty and suggested a cutting plane-based method for N-k security criterion. Finally, solution methods were reviewed. Benders' decomposition is by far the most commonly used method. To incorporate more technical details of GTEP for practical purposes, we suggested a simulation-based optimisation framework.

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