The Natural Metaphysics of Computing Anticipatory Systems

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Outline

- Systems theory
  - Pivotal Role of Adjointness
  - Rosen’s influence
  - Free and Open Systems
- Composition of Systems for Complexity
  - Godement
  - Cube, Adjunctions
- Anticipation as Structural Ordering

Purpose

- To attempt to show that the natural relationship between category theory and systems provides the basis for a metaphysical approach to anticipation

Systems Theory

- Important for Information Systems
- Challenging Areas
  - pandemics
  - prediction of earthquakes
  - world finance (credit crunch)
  - world energy management policy
  - climate change
- Globalisation
- Freeness and Openness needed

Features of Dynamic Systems

- Natural entities
  - easier to recognise than to define
- Second-order Cybernetics
  - observer is part of the system
  - distinguish between
    - modelling components/components of system itself
- General Information Theory (Klir)
  - handling uncertainty
- Theory of Categories (Rosen)

System Theory

- Basic concepts
  - internal connectivity of components
    - Plato (government institution)
    - Aristotle (literary composition)
    - von Bertalanffy
      - theory of categories (vernacular)
      - to be replaced by an exact system of logico-mathematical laws.

Complexity of System

- System is a model of a whole entity
  - hierarchical structure
  - emergent properties
  - communication
  - control (Checkland)
- Complexity -- openness and freeness
  - self-organisation
  - anticipation (Dubois, Klir)
  - global interoperability

Key Elements in the Definition of a System

system	natural relationship	locality
closed	*intra*-connectivity	local
open	*inter*-connectivity	local
self-organised	*intra*-activity	non-local
free	*inter*-activity	non-local

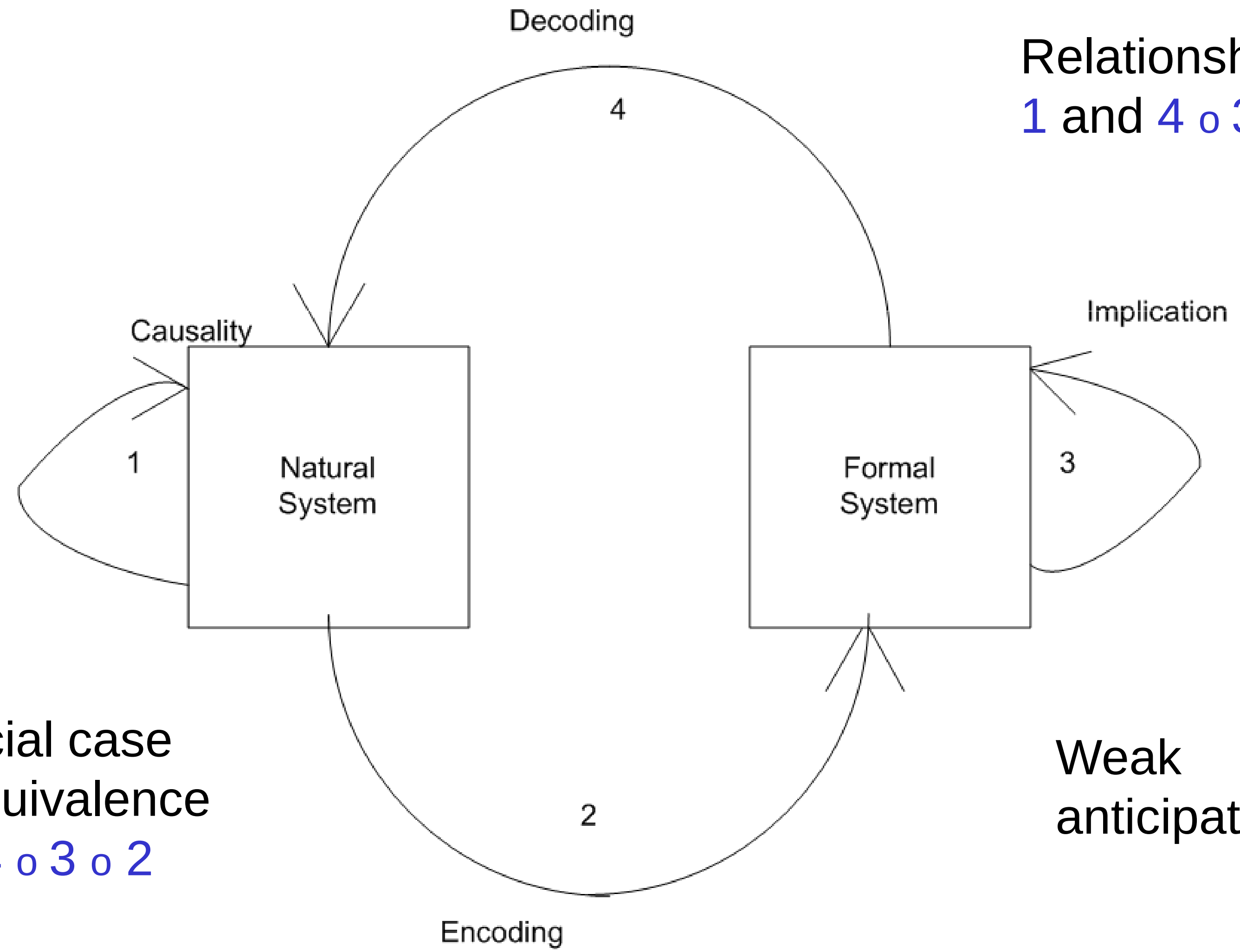
Concept of Openness

- Open
  - defined inductively on open interval -- difficult to formalise
- Dedekind cut
  - section of pre-defined field -- local
- Topology
  - $\mathfrak{I}$-open
    - system is open to its environment
    - intuitionistic logic
  - Limited by reliance on set theory

Category of Systems

- To make formal
  - intraconnectivity
  - interconnectivity
  - intra-activity
  - Interactivity
- Theory is realisable -- constructive
- Work on process -- Whitehead

Early Adjointness from Rosen



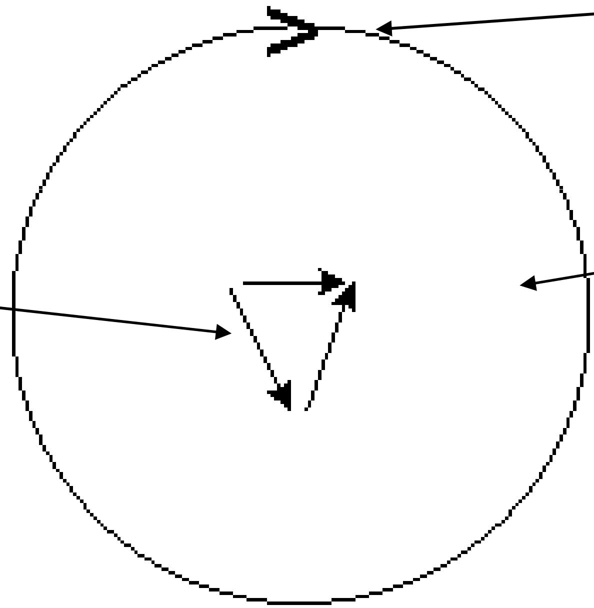
Relationship between
1 and 4 o 3 o 2

Special case
of equivalence
1 = 4 o 3 o 2

Weak
anticipation

Identity Functor as Intension of Category-System

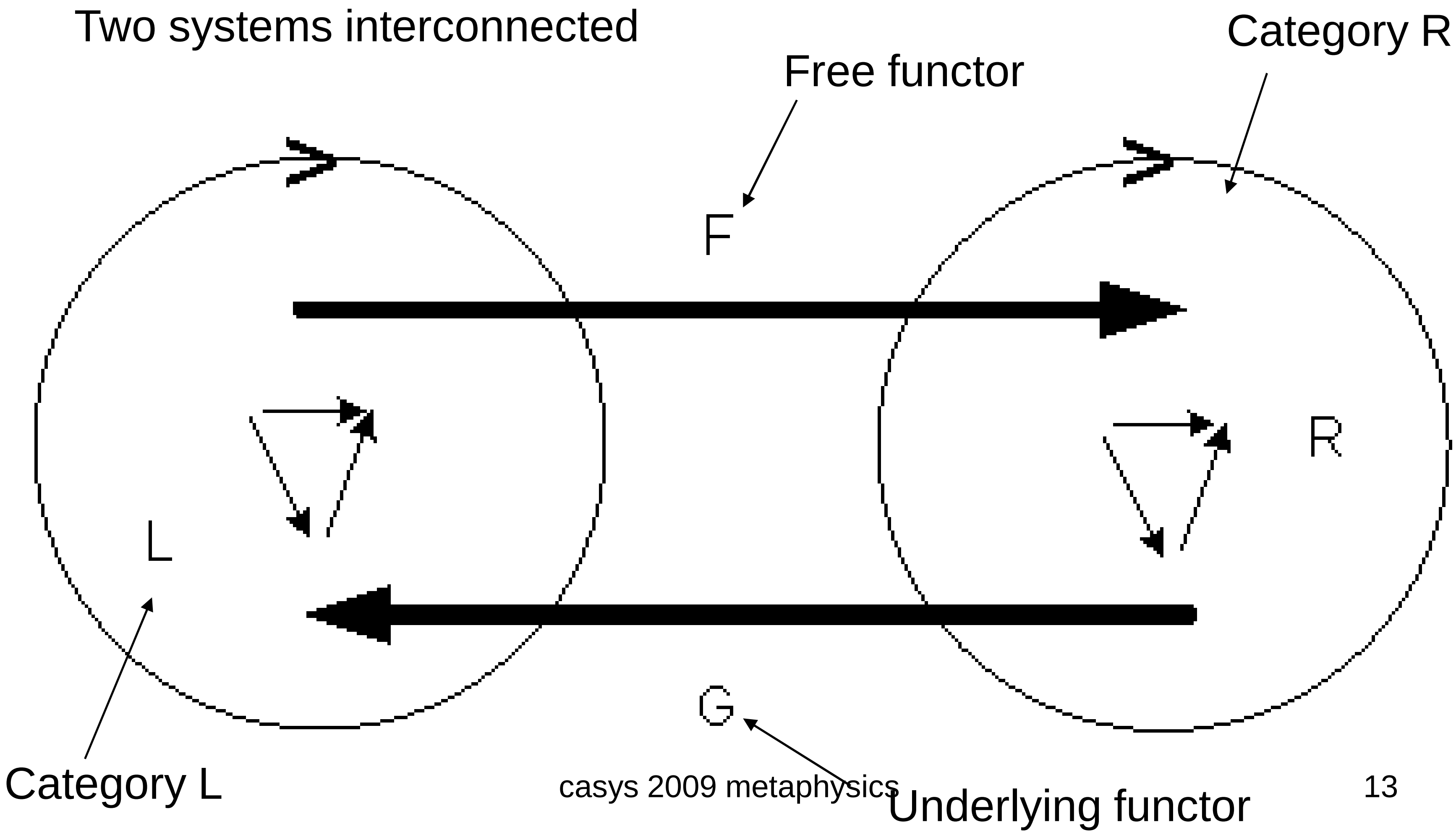
Cartesian closed category



Intraconnectivity

System is one large arrow (process)
Identity functor is intension
All internal arrows are extension

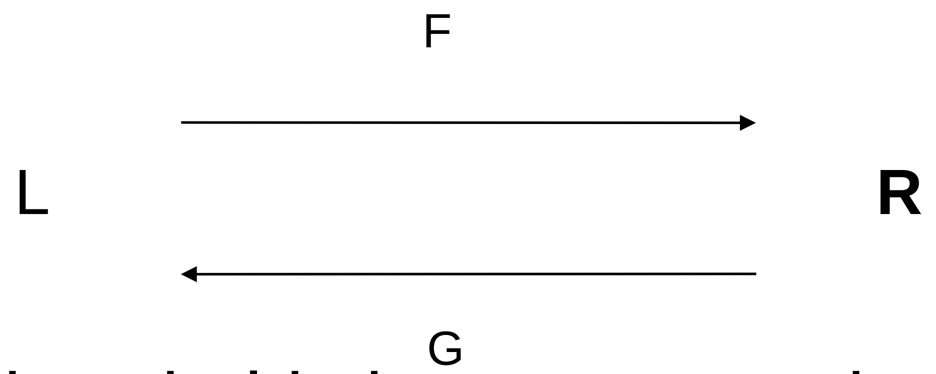
Interconnectivity between two Identity Functors leading to Interactivity between Category-Systems.



Features of Adjointness F --| G

- Free functor (F) provides openness
- Underlying functor (G) enforces rules
- Natural so one (unique) solution
- Special case
  - GF(L) is the same as L AND
  - FG(R) is the same as R
  - Equivalence relation
- Adjointness in general is a relationship less strict than equivalence
  - 1_L <= GF if and only if FG <= 1_R

Example of Adjointness



- If conditions hold, then we can write $F \dashv G$
- The adjunction is represented by a 4-tuple:
  - <F,G,η, ε>
- η and ε are unit and counit respectively
  - $\eta : L \rightarrow GFL$; $\varepsilon : FGR \rightarrow R$
  - Measure displacement in mapping on one cycle
- L, R are categories; F, G are functors

Category-Systems

- Makes formal
  - intraconnectivity identity functor
  - interconnectivity functors
  - intra-activity self-organisation (L and R are indistinguishable)
  - interactivity adjointness
- Right-hand category-system R
  - free and open category system
  - freedom from free functor F
  - determination by underlying functor G

Anticipation

- Not simply F
  - While this takes process one step forward
    - On its own it lacks context
- Not simply G
  - This appears to take the process backwards
- It’s F --| G, that is F in the context of G
  - The forward step as limited by G

Composition of Systems for Handling Complexity

- With category-systems
  - Composition is natural
  - Godement calculus
    - Compose all arrows at whatever level they are defined
    - Categories, Functors, Natural transformations, Adjoints
  - Obtain expressions showing equality of paths

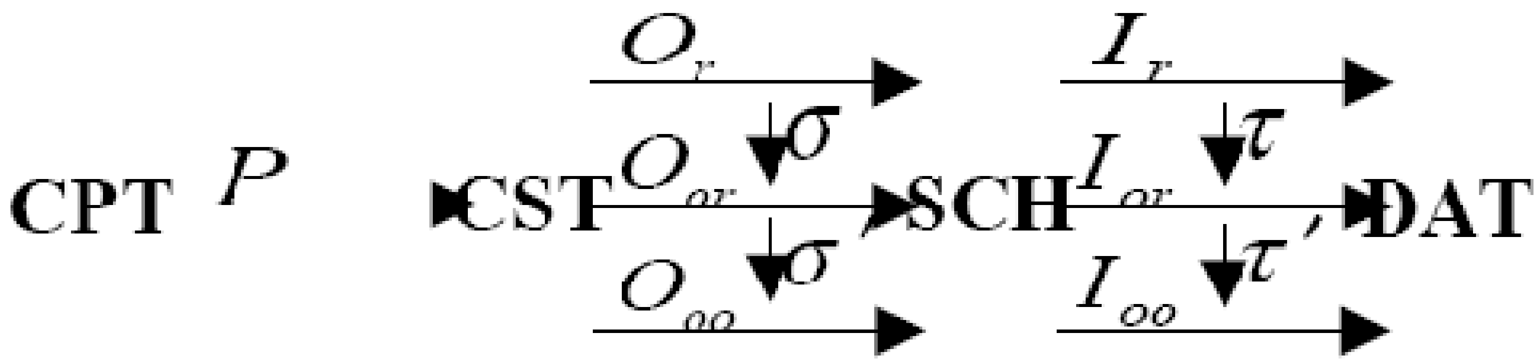
The Cube – Composing All Arrows

- Higher dimensional arrows can deconstruct higher dimensional spaces into simple one dimensional paths
- Power of category theory:
  - Abstract (compact) notation can also be represented in an equivalent more detailed notation.
  - Both are robust
  - One more suited to description, the other to implementation.

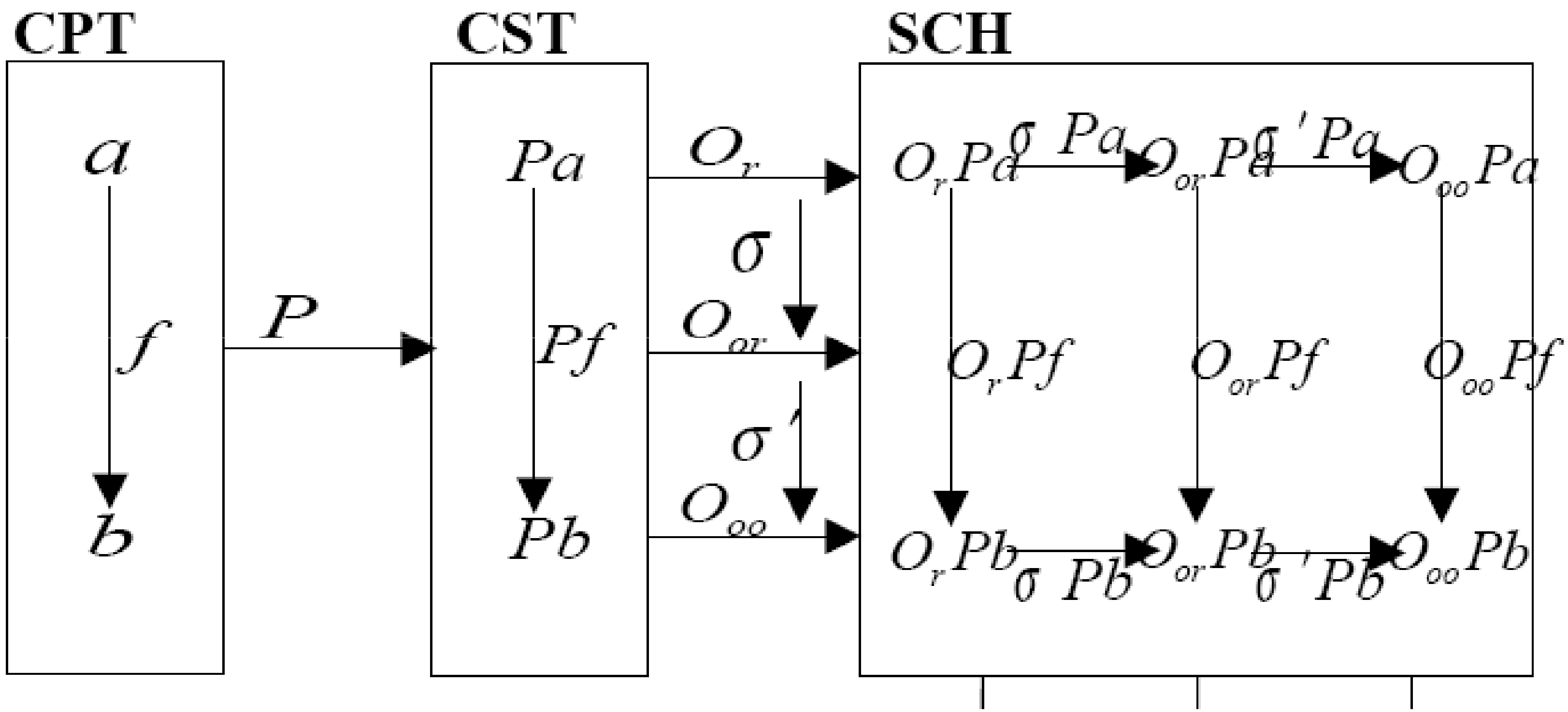
Abstract Notation

Comparison of four systems (categories):
CPT, **CST**, **SCH**, **DAT**
by functors P, O, I

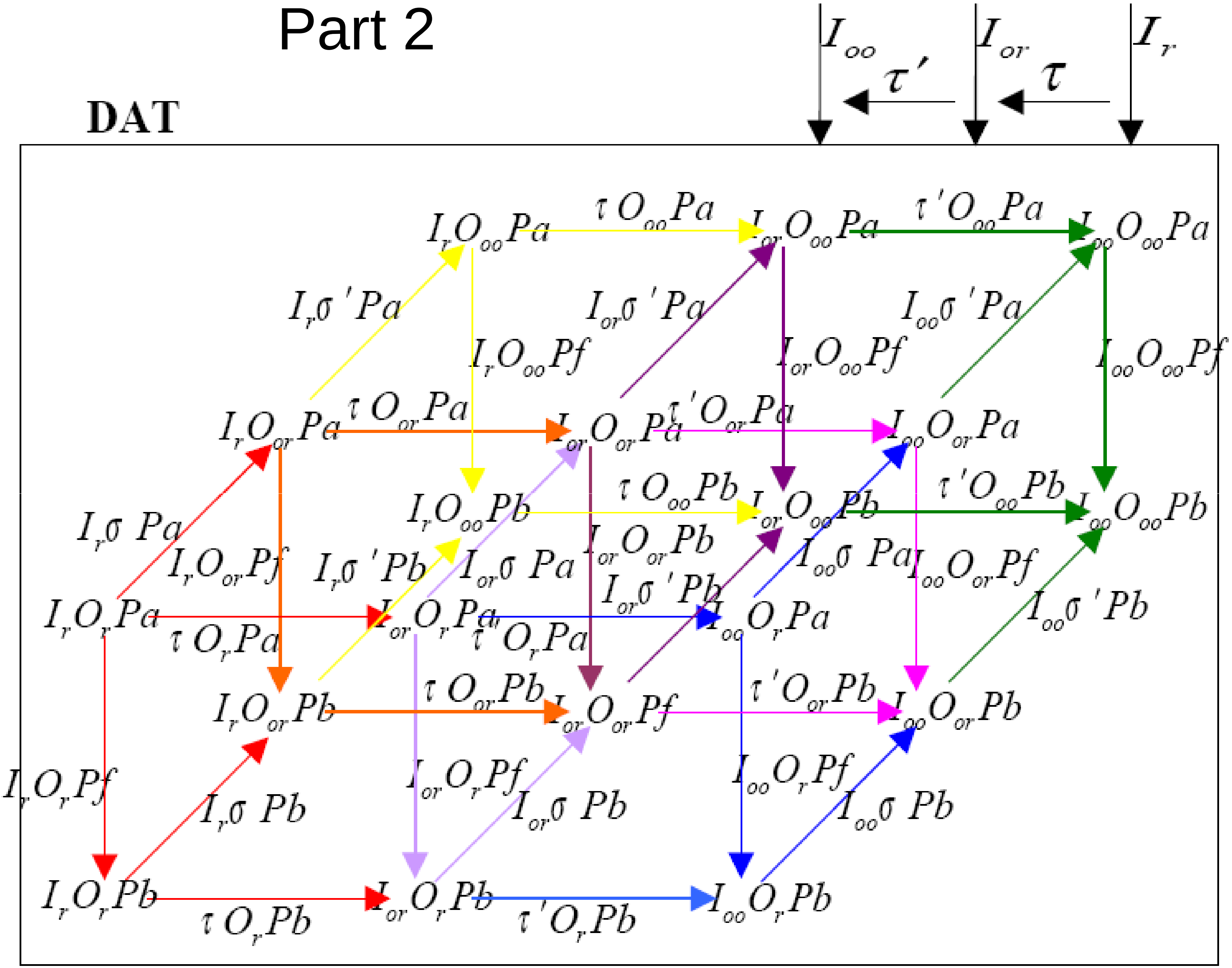
O, I have variants
σ, τ compare variations (natural transformations)



Detailed notation – The Cube – Part 1



Part 2


DAT
I_oo
τ'
I_or
τ
I_r
I_r O_oo Pa
τ O_oo Pa
I_or O_oo Pa
τ' O_oo Pa
I_oo O_oo Pa
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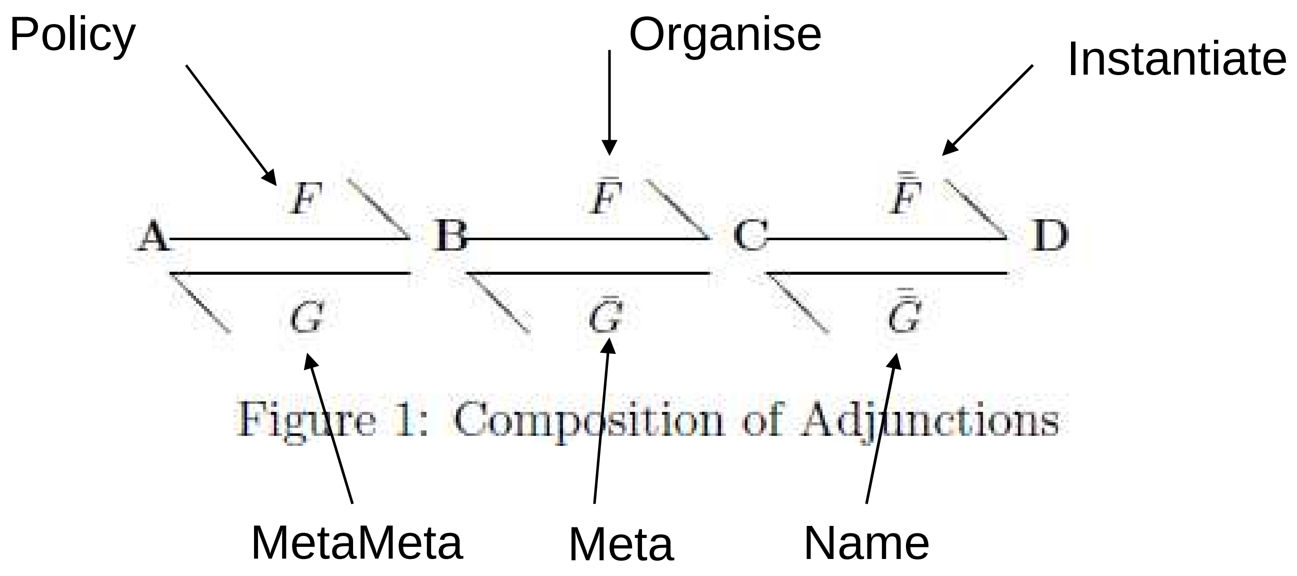
Composing Adjunctions



A is category for Concepts
B is category for Constructs
C is category for Schema
D is category for Data

Example for data structures

Adjunctions compose naturally
F-|G is one of 6 adjunctions (if they hold)

Six Possible Adjunctions

Simple

$F \dashv G$ $\quad \overline{F} \dashv \overline{G}$ $\quad \overline{\overline{F}} \dashv \overline{\overline{G}}$

Pairs

$\overline{F}F \dashv G\overline{G}$ $\quad \overline{\overline{F}}\,\overline{F} \dashv \overline{G}\,\overline{\overline{G}}$

Triples

$\overline{\overline{F}}\,\overline{F}F \dashv G\overline{G}\,\overline{\overline{G}}$

Simple Adjunctions Not composed

We can define these in more detail with their units and counits of adjunction as follows:

$$< F, G, \eta_a, \epsilon_b >: \mathbf{A} \longrightarrow \mathbf{B} \tag{1}$$

η_a is the unit of adjunction $1_a \longrightarrow GFa$ and ϵ_b is the counit of adjunction $FGb \longrightarrow 1_b$

$$< \bar{F}, \bar{G}, \bar{\eta}_b, \bar{\epsilon}_c >: \mathbf{B} \longrightarrow \mathbf{C} \tag{2}$$

$\bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{F}b$ and $\bar{\epsilon}_c$ is the counit of adjunction $\bar{F}\bar{G}c \longrightarrow 1_c$

$$< \bar{\bar{F}}, \bar{\bar{G}}, \bar{\bar{\eta}}_c, \bar{\bar{\epsilon}}_d >: \mathbf{C} \longrightarrow \mathbf{D} \tag{3}$$

$\bar{\bar{\eta}}_c$ is the unit of adjunction $1_c \longrightarrow \bar{\bar{G}}\bar{\bar{F}}c$ and $\bar{\bar{\epsilon}}_d$ is the counit of adjunction $\bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$

Composition of Adjunctions Pairs

$$< \bar{F}F, G\bar{G}, G\bar{\eta}_a F \bullet \eta_a, \bar{\epsilon}_c \bullet \bar{F}\epsilon_c\bar{G} >: \mathbf{A} \longrightarrow \mathbf{C} \quad (4)$$

$G\bar{\eta}_a F \bullet \eta_a$ is the unit of adjunction $1_a \longrightarrow G\bar{G}\bar{F}Fa$ and $\bar{\epsilon}_c \bullet \bar{F}\epsilon_c\bar{G}$ is the counit of adjunction $\bar{F}FG\bar{G}c \longrightarrow 1_c$

The unit of adjunction is a composition of $\eta_a : 1_a \longrightarrow GFa$ with $G\bar{\eta}_a F : GFa \longrightarrow G\bar{G}\bar{F}Fa$

The counit of adjunction is a composition of $\bar{F}\epsilon_c\bar{G} : \bar{F}FG\bar{G}c \longrightarrow \bar{F}\bar{G}c$ with $\bar{\epsilon}_c : \bar{F}\bar{G}c \longrightarrow 1_c$

We have retained the symbol $\bullet$ indicating vertical composition as distinct from normal horizontal composition indicated by the symbol $\circ$ [13].

$\bar{G}\bar{\bar{\eta}}_b\bar{F} \bullet \bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}B$ and $\bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}}$ is the counit of adjunction $\bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow 1_d$

The unit of adjunction is a composition of $\bar{\eta}_b : 1_b \longrightarrow \bar{G}\bar{F}b$ with $\bar{G}\bar{\bar{\eta}}_b\bar{F} : \bar{G}\bar{F}b \longrightarrow \bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}b$

The counit of adjunction is a composition of $\bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} : \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{\bar{G}}d$ with $\bar{\bar{\epsilon}}_d : \bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$.

Composition of Adjunctions Triples

$$< \bar{\bar{F}}\bar{F}F, G\bar{G}\bar{\bar{G}}, G\bar{G}\bar{\bar{\eta}}_a\bar{F}F \bullet G\bar{\eta}_a F \bullet \eta_a, \bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} \bullet \bar{\bar{F}}\bar{F}\epsilon_d\bar{G}\bar{\bar{G}} >: \mathbf{A} \longrightarrow \mathbf{D} \qquad (6)$$

The unit of adjunction is a composition of:

$\eta_a : 1_a \longrightarrow GFa$ with $G\bar{\eta}_a F : GFa \longrightarrow G\bar{G}\bar{F}Fa$ with $G\bar{G}\bar{\bar{\eta}}_a\bar{F}F : G\bar{G}\bar{F}Fa \longrightarrow G\bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}Fa$

The counit of adjunction is a composition of:

$\bar{\bar{F}}\bar{F}\epsilon_d\bar{G}\bar{\bar{G}} : \bar{\bar{F}}\bar{F}FG\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d$ with $\bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} : \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{\bar{G}}d$ with $\bar{\bar{\epsilon}}_d : \bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$

General Solution

- In general, adjointness gives a logical ordering:
  - iff the operation of an environment C on a subobject A has a solution subobject B then
    - A implies B in the environment of C.
  - This can be represented as the adjunction

$$C\times A\rightarrow B \dashv C\rightarrow B^{A}$$

Metaphysical Ordering

- This adjunction is the natural metaphysical ordering which constitutes anticipation
- Thus causation (left adjoint) and Heyting inference (right adjoint) are both stationary forms of the predicate of anticipatory systems
  - these dominate the two mainstream applications of AI and databases
- In AI the left adjoint is a relevance connection in context and the corresponding right adjoint is cognition
- For data warehousing, data mining, the semantic web, etc, a query in context is left adjoint and the resultant retrieval right adjoint

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