# Investigation of Punctured LDPC Codes and Time-Diversity on Free-Space Optical Links

Martin Czaputa*, Tomaž Javornik †, Erich Leitgeb*, Gorazd Kandus †, Zabih Ghassemlooy ‡
*Institute of Broadband Communications, Graz University of Technology, Austria
† Department of Communication Systems, Jožef Stefan Institute, Ljubljana, Slovenia
‡ Optical Communications Research Group, NCLab Northumbria University, Newcastle upon Tyne, UK
martin_c@sbox.tugraz.at, tomaz.javornik@ijs.si, erich.leitgeb@TUGraz.at
gorazd.kandus@ijs.si, z.ghassemlooy@northumbria.ac.uk

*Abstract*— **In this paper, we analyze the behavior of DVB-S2 un-punctured/punctured low-density parity-check (LDPC) coded on-off-keying (OOK) under atmospheric turbulence conditions by utilizing time diversity. A performance characterization between these schemes is evaluated, where punctured LDPC code provides a penalty of around 0.1 to 0.2 dB against unpunctured LDPC codes but still provides a coding gain of several dB against uncoded OOK. The combination of channel coding and a bit interleaver results in performance improvements in turbulence conditions. For example, such a system can achieve a coding gain of 16.7 dB in moderate turbulence conditions compared to uncoded OOK.**

## I. Introduction

Free space optical (FSO) systems have the capability to satisfy future demands on high speed data links. Its advantages range from high transmission security, high data throughput, license free and ease installation to high immunity on interference. It can be used in a variety of applications e.g. linking LAN-to-LAN networks, extending metropolitan networks (MAN) and is considered to solve the "last mile" problem. However, an optical wave propagating through the air experiences atmospheric attenuations. One major impairment is atmospheric turbulence induced fading, also called scintillation [1], where strong turbulence could cause link attenuations in the order of 30 dB/km. Many different methods have been investigated to counter these signal fluctuations including spatial diversity [2], [3], aperture averaging [1], adaptive coded modulation [4] and rateless coding [5]. The optical signal intensity variations on an FSO channel are very slow compared to the data rates typically used in FSO. Error control coding can increase the reliability. The additional usage of interleavers can further provide improvements by reducing the temporal correlation of the sent information. The interleaver length must be designed in such a way that it is much longer than the coherence time of the fading process in order to decode properly. The combination of channel coding and additional interleaving improves the link quality during atmospheric turbulence [6]. For channel coding, LDPC codes have recently become very popular due its strong error correction capability. Also, a variety of LDPC codes is defined in the new digital video broadcast satellite standard (DVB-S2) [7]. The advantage of DVB-S2 LDPC codes includes efficient encoding and decoding algorithms which are already implemented in hardware. Together with puncturing techniques, various code rates can be supported and system complexity is kept low at the same time.

In this work, we investigate the encoding abilities of punctured DVB-S2 LDPC together with channel interleavers to exploit time diversity. We first compare punctured/unpunctered LDPC coded OOK in weak turbulence conditions. Further an interleaver is introduced and results for weak and moderate turbulence follow. The balance of this paper is organized as follows. In Sec. 1 the channel model is presented and a brief discussion about LDPC codes and puncturing techniques are given in Sec. III. The simulation model is presented in Sec. IV while different system simulations are evaluated in Sec. V and results are concluded in Sec. VI.

## II. Channel model

General, a FSO link consists of a laser diode (LD) at the transmitter and a photo diode (PD) at the receiver. The information, in form of a electrical signal, is modulated by the LD to an optical intensity signal $x$ and is sent through the atmosphere. At the receiver, the optical beam is converted back into an electrical signal. This conversion is determined by a constant factor, defined as the responsitivity $R$. Consequently, this is termed an intensity modulation and direct detection (IM/DD) system. Furthermore, this FSO design is constrained by two factors. First, eye safety regulations allow only a limited transmitting average optical power $\mathbb{E}\{x\} \leq P$ and since the transmit signal is an optical intensity signal, $x \geq 0$. Under these constrains a common used discrete channel model [8] forms to

$$y = Rhx + n, \tag{1}$$

where $y$ is the received electrical signal, $x$ is the on-off-keying (OOK) modulated optical intensity signal taking values $x \in \{x_0, x_1\}$, $h$ is the channel state and $n$ is an independent noise process. The noise is assumed to be additive white Gaussian with zero-mean and a variance of $\sigma_n^2$. Without loss of generality, we set $R$ to unity, since it only scales the intensity signal. The channel state $h$ models the random fluctuations of the atmospheric channel. As the prime interest in this work is focused on atmospheric turbulence fading, the channel state equals

11th International Conference on Telecommunications - ConTEL 2011
ISBN: 978-3-85125-161-6, June 15-17, 2011, Graz, Austria

$$h = h_s, \tag{2}$$

where $h_s$ is random distributed regarding scintillation. In FSO, scintillation arises through the varying refractive index due to inhomogenous temperature and pressure fluctuations along the propagation path. As a consequence of the randomly behaving amplitude and phase of the signal, fading occurs at the receiver. The coherence time of such fades is in the order of 1-100 ms [1]. The extensive study of atmospheric turbulence on optical intensity signals has led to various statistical models. One of the recent proposed schemes models the intensity fluctuations as a Gamma-Gamma distribution [9]. The pdf of $h_s$ is given as

$$f_{h_s} = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} h_s^{(\alpha\beta)/2-1} K_{\alpha-\beta}\left(2(\alpha\beta h_s)\right), \tag{3}$$

where $K_{\alpha-\beta}(\cdot)$ is the modified Bessel function of the second kind and $\alpha$ and $\beta$ are related to large- and small-scale eddies. Considering a large link distance (often 1 km), we can assume a plane optical wave propagation. For such a case the pdf parameters follow as

$$\alpha = \left[\exp\left(\frac{0.49\sigma_R^2}{(1+1.11\sigma_R^{12/5})^{7/6}}\right) - 1\right]^{-1}, \tag{4}$$

$$\beta = \left[\exp\left(\frac{0.51\sigma_R^2}{(1+0.69\sigma_R^{12/5})^{5/6}}\right) - 1\right]^{-1}, \tag{5}$$

where $\sigma_R^2$ is the Rytov variance defined as $\sigma_R^2 = 1.23\,k^{7/6}C_n^2 z^{11/6}$, where $k = 2\pi/\lambda$, $\lambda$ is the optical wavelength and $C_n$ is the refractive index structure parameter.

## III. LDPC and Punctured LDPC codes

### A. DVB-S2 LDPC Codes

The LDPC codes invented by Gallager [10], and re-discovered in recent times, are proven to provide very high coding gains. Usually the LDPC codeword length is very long, so that the encoding and decoding speed was way to slow in the past. Improved hardware and decoding algorithms has made it possible to design reasonably time efficient LDPC codes. LDPC codes have been proposed in the DVB-S2 standard (Digital Video Broadcast - Satellite) [7] as an inner code for forward error correction. The maximum decoder speed for the DVB-S2 LDPC codes was recently improved up to 1 Gbps [11].

Generally, LDPC codes belong to a class of binary systematic codes and are defined by a sparse parity check matrix $H$. A codeword consists of $n$ bits where $k$ is the number of information bits and $n-k$ the number of parity bits. Particularly, the DVB-S2 standard specifies two possible code lengths where here the code with $n = 64800$ bits is considered. To decode these LDPC codes, the sum product algorithm is utilized based on belief propagation. This algorithm uses hard or soft-information of the received symbols to iteratively update the reliability information of the received bits [7]. If all parity check parity sums are satisfied or a specified limit of iterations is reached, hard decision is utilized to finally determine the decoded bits.

### B. Puncturing

Typical, puncturing is used to generate codes with different code rates based on one standard low rate encoder [12]. Using this technique is often simpler and more cost effective rather to invest in various encoders with different code rates. For this study, the DVB-S2 LDPC code is used with coding rate $r_m = 1/2$ as a mother code to generate the punctured LDPC (termed LDPC-P) codes utilized in simulations. The puncturing process is realized as follows: information bits are first encoded through the LDPC encoder. Then, with a code rate of $r_m = 1/2$, 32400 information bits are encoded, hence, 32400 redundancy bits are generated. By puncturing redundancy bits according to a code rate $r$, the codeword size reduces to $n_p$ bits. After receiving the OOK modulated intensity signal and demodulation, the received codeword is depunctured and finally decoded.

Puncturing can be done randomly or deterministically [12]. In this work only random puncturing is considered since it has a good performance compared to other schemes. This technique randomly chooses the to be punctured bits on a block by block basis.

## IV. System model

In our simulations, we consider OOK as the modulation scheme. This conventionally modulation is frequently utilized in commercial systems because of its ease of implementation and its robustness against various channel impairments. We set the symbol levels to $x_0 = 0$, $x_1 = 1$ corresponding to bit zero and bit one. As introduced in Sec. III soft-information in form of log-likelihoods ratios are required for the LDPC decoder. For an AWGN channel, the conditional pdf of the received signal $y$ is

$$P(y|x) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(y-hx)^2}{2\sigma_n^2}\right) \tag{6}$$

and the log-likelihood for a OOK symbol, assuming equiprobable input bits, forms to

$$\mathrm{LLR}(y) = \ln\frac{P(y|x_1)}{P(y|x_0)} = \frac{2hy - h^2}{2\sigma_n^2}. \tag{7}$$

The conditional pdf in (6) is dependent on the channel state $h$. We assume perfect channel knowledge for simulation, hence, $h$ is known at the receiver for each time interval. This is reasonable since the channel can be accurately estimated [5] due to a long fading interval ($10^{-3}$ s) compared to a short symbol duration ($T_s = 10^{-9}$ s). In case of hard decision, the received symbol is set to $\hat{x} = x_1$ if LLR $> 0$ and $\hat{x} = x_0$ otherwise. In Fig. 1 the block diagram for the simulation setup is shown. At the transmitter, 32400 bit data blocks are LDPC encoded with coding rate of $r_m = 1/2$, then the codeword is punctured to achieve a required code rate of $r$ and is stored in a buffer. A pseudo random interleaver, of $n_i$ bits long, mixes the to be sent bits randomly. After transmitting the OOK modulated signal through


ConTEL 2011, ISBN: 978-3-85125-161-6

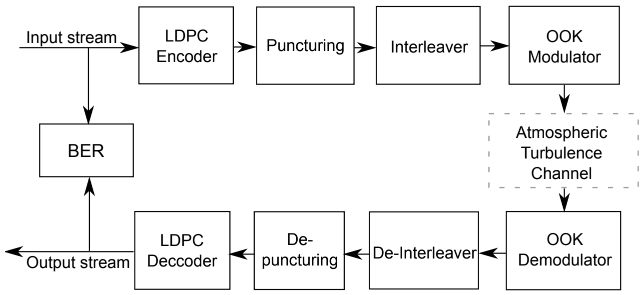


Figure 1. Communication block diagram for simulation.

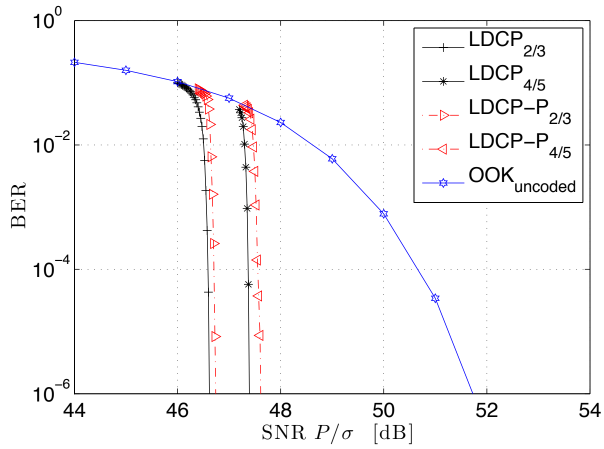


Figure 2. BER peformance comparison of punctured/unpunctured DVB-S2 LDPC codes.

the atmosphere, the optical intensity signal is received and demodulated. Finally, the received data stream gets deinterleaved, depunctured and finally decoded. In order to calculate the bit error rate (BER), the sent and received data bits are compared. Moreover, we define the optical signal-to-noise ratio (SNR) as $\mathrm{SNR} = P/\sigma$.

## V. Simulation Setup and System Performance Evaluation

Monte Carlo simulations are performed to evaluate the BER rates of various selected coding techniques and the performance improvement compared to uncoded OOK in turbulence regimes. A strong lower code rate of $r = 2/3$ is investigated in further simulations. Simulations are stopping after $5 \cdot 10^{-8}$ transmitted bits for high signal-to-noise ratios (SNR) and for low SNR conditions a number of around $10^{-5}$ bits are used. The LDPC decoder was limited to have a maximum of 30 decoding cycles.

In case of no scintillation, the BER rates of punctured LDPC codes and LDPC codes with equal rate are illustrated in Fig. 2. The BER rate of the code with rate $r = 4/5$ is here also simulated to show the penalty in coding gain due to puncturing. With a code rate of $r = 4/5$ the LDPC code has a coding gain of 3.62 dB at $\mathrm{BER} = 10^{-5}$. The punctured code $r = 4/5$ behaves only 0.21 dB weaker than the unpunctured one. As the punctured code rate gets closer to the mother code rate, the performance difference of punctured and unpunctured code decreases. For $r = 2/3$ a coding gain of 4.65 dB for unpuncured and 4.53 for punctured LDPC codes is achieved.

Now, we consider the additional influence of scintillation where we choose $\sigma_R = 0.2$ in weak and $\sigma_R = 1$ in moderate atmospheric turbulence conditions. Assume a fixed coherence time of $\tau = 1$ ms and a data rate of 1 Gbps with a symbol duration of $T_s = 10^{-9}$ then the fading $h$ varies over blocks of $N_s = \tau/T_s = 10^6$ symbols. In the moderate turbulence scenario the coherence time was measured to be on average around $\tau = 3.6$ ms ($N_s = \tau/T_s = 3.6 \cdot 10^6$) [5]. To reduce computation time, also the moderate turbulence case was simulated with $\tau = 1$ ms. The results stay valid as long as the degree of time diversity is kept at a specified level. This means that if the block fading interval increases, also the interleaver size must be scaled up.

During weak turbulence (see Fig. 3), the performance of uncoded OOK decreases. The SNR to achieve, for example, $\mathrm{BER} = 10^{-5}$ is for uncoded OOK 52.3 dB. In further discussions, coding gain is always compared at $\mathrm{BER} = 10^{-5}$ if not differently stated. The additional usage of forward error correction (FEC) without time diversity (no interleaver) shows interesting improvements. The good results are gained from the long codeword length and the usage of soft-decoding of these LDPC codes. Also in Fig. 3, the BER curves using time diversity are demonstrated. First, the interleaver size is chosen to be around $N_{\mathrm{intl}} = 2N_s = 2 \cdot 10^6$ bits where $N_{\mathrm{intl}}$ is the number of bits randomly mixed by the interleaver. This number of bits correspondents to signal latency of 2 ms. We name this signal delay, time diversity (TD). The number of $N_{\mathrm{intl}}$ may change from the individual used code since the interleaver size is set to a multiple of the codeword length. As a result of the interleaver, the block faded symbols are spread so the LDPC encoder faces smaller burst or individual bit errors. In case of no coding, the extra diversity of time has no benefit. However, with channel coding only, a performance in coding gain is observed of around 3.29 dB compared to uncoded OOK. Furthermore, the additional usage of time diversity with latency of 2 ms (TD = 2) has an improved coding of 4.26 dB and 5 dB for TD = 4 , respectively.

In case of moderate atmospheric turbulence, the simulation results with the same parameters as in the weak turbulence case are presented in Fig. 4. With the interleaver (TD = 2) and a coding rate of $r = 2/3$ an improvement of 7.72 dB is obtained. Note that the link latency would enhance with a correlation time of $\tau = 3.6$ ms to 7.2 ms with TD = 2. By increasing the interleaver depth (TD = 4), the system gains performance by spreading the blocks of high attenuated symbols. In this case, a coding gain of 16.74 dB is achieved. These results confirm the usefulness of time diversity to improve FSO communication during scintillation effects. But notice that this modified system also implicates a higher system complexity and an increased link latency.

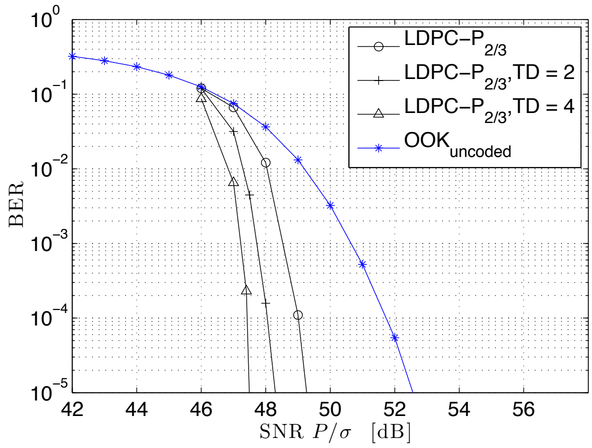


Figure 3. BER under weak turbulent conditions, $\sigma_R = 0.2$.

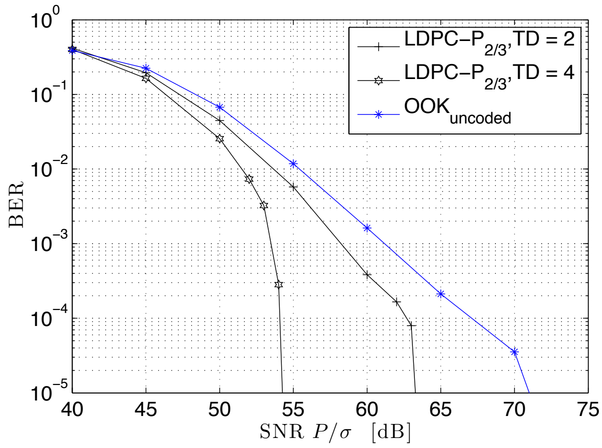


Figure 4. BER under moderate turbulent conditions, $\sigma_R = 1$.

## VI. Conclusion

In the presence of scintillation, FSO systems encounter communication issues due to signal attenuating fading intervals. In this work, we utilized additional channel coding to mitigate the impairments of atmospheric turbulence on FSO links. Effective error correcting LDPC codes from the DVB-S2 standard were investigated with additional puncturing techniques. The advantage of puncturing is the integration of only one LDPC encoder with a specified low mother code rate. Different code rates can be achieved by puncturing and de-puncturing redundancy bits of the original LDPC codeword. Hence, this lowers the system complexity if a variety of channel codes with different code rates is required. In addition, an interleaver is considered to make use of time diversity. One punctured LDPC code was evaluated in terms of bit error rates and provided performance improvements. For example, the code with rate of 2/3 and TD = 4 achieved a coding gain of 16.74 dB at $\mathrm{BER} = 10^{-5}$ under moderate turbulence conditions. Note that the decision of the interleaver size is always a tradeoff between link latency and system improvement. If the number of interleaved bits is increased, the error correction may improve but the latency increases simultaneously.

## References


[1] L. C. Andrews and R. L. Phillips, *Laser Beam Propagation Through Random Media*. WA: SPIE, 1998.

[2] E. Lee and V. Chan, “Part 1: Optical communication over the clear turbulent atmospheric channel using diversity,” *Selected Areas in Communications, IEEE Journal on*, vol. 22, pp. 1896 – 1906, nov. 2004.

[3] W. Popoola, Z. Ghassemlooy, J. Allen, E. Leitgeb, and S. Gao, “Free-space optical communication employing subcarrier modulation and spatial diversity in atmospheric turbulence channel,” *Optoelectronics, IET*, vol. 2, pp. 16 –23, feb. 2008.

[4] I. B. Djordjevic, “Adaptive modulation and coding for free-space optical channels,” *Optical Communications and Networking, IEEE/OSA Journal of*, vol. 2, pp. 221 –229, May 2010.

[5] J. Anguita, M. Neifeld, B. Hildner, and B. Vasic, “Rateless coding on experimental temporally correlated fso channels,” *Lightwave Technology, Journal of*, vol. 28, pp. 990 –1002, April 2010.

[6] F. Xu, A. Khalighi, P. Caussé, and S. Bourennane, “Channel coding and time-diversity for optical wireless links,” *Opt. Express*, vol. 17, pp. 872–887, Jan 2009.

[7] ETSI, “Digital video broadcasting (DVB); second generation framing structure, channel coding and modulation systems for broadcasting, interactive services, news gathering and other broadband satellite applications.” EN 302 307, V1. 1. 1, Jun. 2004.

[8] S. Hranilovic, *Wireless Optical Communication Systems*. New York: Springer, 2005.

[9] M. A. Al-Habash, L. C. Andrews, and R. L. Phillips, “Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media,” *Optical Engineering*, vol. 40, no. 8, pp. 1554–1562, 2001.

[10] R. Gallager, “Low-density parity-check codes,” *Information Theory, IRE Transactions on*, vol. 8, pp. 21 –28, Jan. 1962.

[11] S.-M. Kim, C.-S. Park, and S.-Y. Hwang, “A novel partially parallel architecture for high-throughput ldpc decoder for dvb-s2,” *Consumer Electronics, IEEE Transactions on*, vol. 56, pp. 820 – 825, May 2010.

[12] M. Smolnikar, T. Javornik, M. Mohorcic, S. Papaharalabos, and P. Mathiopoulos, “Rate-compatible punctured dvb-s2 ldpc codes for dvb-sh applications,” in *Satellite and Space Communications, 2009. IWSSC 2009. International Workshop on*, pp. 13 –17, 2009.



ConTEL 2011, ISBN: 978-3-85125-161-6