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Citation: Fernando, Michael, Elsdon, Michael, Busawon, Krishna and Smith, Dave (2010) A novel simplified mathematical model for antennas used in medical imaging applications. Journal of Physics Conference Series, 255 (1). ISSN 1742-6596

Published by: IOP Publishing

URL: <http://dx.doi.org/10.1088/1742-6596/255/1/012001>
<<http://dx.doi.org/10.1088/1742-6596/255/1/012001>>

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A Novel Simplified Mathematical Model for Antennas used in Medical Imaging Applications

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Abstract

In this paper a new technique has been proposed to model the current across a monopole antenna and thereby the radiation fields of the antenna can be calculated. Generally, the method of Moments (MOM) techniques is used for this purpose whereby the integral equations are discretised to find the fields of an antenna. The proposed model requires only the knowledge of three parameters (Initial Current I_0 , Damping coefficient α and the radial parameter τ) and hence considerably reduces the computational time and space as its results does not depend on the number of functions involved. The new technique has also been developed to take account of the conductivity property of the surrounding medium. Hence this technique can be used in field prediction for antennas employed in medical imaging application. Initial results obtained from the new technique shows good correlation in comparison with MOM technique.

1. Introduction

One of the fundamental problems in predicting the electromagnetic field generated by a semi rigid co-axial wire "monopole antenna" at a particular point in space has been addressed in this paper. This requires, in particular, the knowledge of the current density across the wire. Conventionally Method of Moments (MOM) techniques are used for this purpose whereby integral equations are discretised to predict the current across the wire [1, 2]. Although this technique is versatile it requires large amount of computation ($N \times N$) as the accuracy of the numerical result directly proportional to the number of basis functions N involved [3]. In addition, the MOM techniques cannot be easily employed for application involving medium other than free space such as medical imaging application which require the use of coupling medium [4, 5]. Because of the conductive nature of the coupling medium the current prediction along the wire using MOM technique becomes very laborious. More specifically $(N \times N)^2$ computations are required.

In this paper a new technique has been proposed to model the current across a monopole antenna. The proposed model requires only the knowledge of three parameters for the prediction of the current along the wire; hence, reducing the computational time drastically as its results does not depend on the number of function involved. Also the new model will be useful to predict the antenna fields and impedance in different coupling media easily. Consequently this technique can be used in field prediction for antennas employed in medical imaging application.

This paper is organised as follows: The next section details the method of moment's procedures to calculate the current across the monopole antenna involving the pocklington integral equations. Section 3 describes the new mathematical model. In Section 4 the results of the integral equation method is compared with the results obtained through our new mathematical model. Finally the conclusions on the model obtained are presented.

2. Method of Moment technique

In this section the so-called Method of Moment (MOM) technique is recalled. Consider an equation of the form:

$$Lf = g \quad (1)$$

where L is a continuous linear operator such as the integral operators or the differential operators, function f is the unknown to be determined and g represents a known excitation. If a unique solution exists, it is given by $f = L^{-1}g$, where L^{-1} is the inverse operator. L^{-1} cannot be determined analytically and thereby numerical solution is sorted [1]. MOM is one such numerical solution in which the continuous equation is discretised into matrices. The specific discretisation places a limit on the accuracy of a numerical result for a fixed number of basis functions and determines whether or not the numerical result will converge to the exact solution as the number of basis function increased. The solution to the above equation (1) can be given as

$$f = \sum_{n=1}^N \alpha_n \beta_n \quad (2)$$

Where the functions B_n are known basis functions defined on the domain of L and the scalars α_n are unknown coefficients to be determined. In what follows the MOM technique will be employed to calculate the current across the monopole.

2.1. Current along the wire using MOM technique

Consider a straight wire of length l and of radius a , the wire is placed along the z axis as shown in Figure 1.

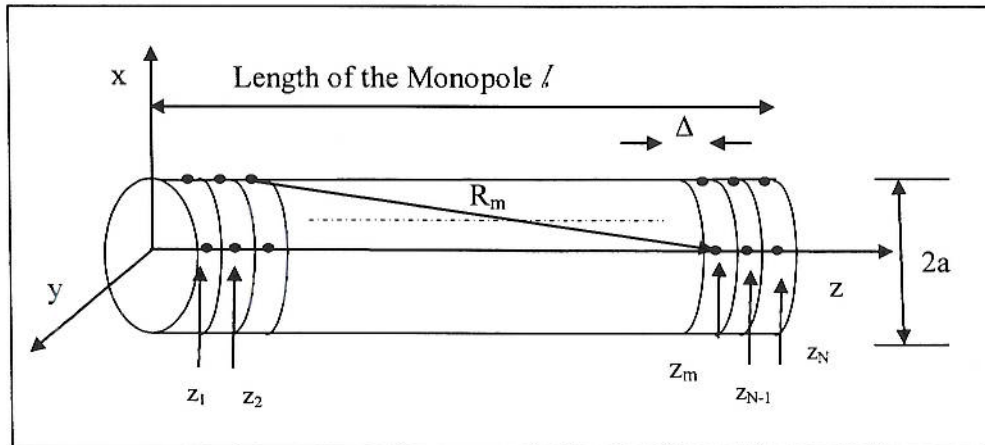


Figure 1: Segmented Wire.

A linear charge distribution $\rho(r')$ creates an electric potential $V(r)$ across the wire[6] which is given by

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{R} dl' \quad (3)$$

Where $r(x', y', z')$ denotes the source coordinates, $r(x, y, z)$ denotes the observation coordinates dl' is the path of integration and the R is the distance from any point on the source to the observation point, which is given by

$$R(r - r') = |r - r'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2} \quad (4)$$

The unknown charge distribution $\rho(z')$ can be approximated by an expansion of N known terms with constant, but unknown coefficients, that is,

$$\rho(z') = \sum_{n=1}^N \alpha_n g_n(z') \quad (5)$$

This expansion for the unknown charge distribution can be substituted in equation (3) and after interchanging the summation and integration the electric potential can be written as

$$4\pi\epsilon_0 = \sum_{n=1}^N a_n \int_0^l \frac{g_n(z')}{\sqrt{(z-z')^2 + a^2}} dz' \quad (6)$$

The wire is now divided into N uniform segments, each of length $\Delta = l/N$ the $g_n(z')$ function is referred as basis functions and is given by

$$g_n(z') = \begin{cases} 0 & z' < (n-1)\Delta \\ 1 & (n-1)\Delta \leq z' \leq n\Delta \\ 0 & n\Delta < z' \end{cases} \quad (7)$$

Replacing z in equation (6) by a fixed point such as z_m on the surface of the wire and choosing N observation points $z_1 \dots z_N$ on the surface of the wire each at the centre of each Δ length element to form N linearly independent equations to obtain the solutions for $a_1 \dots a_N$ equation (6) can be expressed in matrix notation as

$$[I_n] = [Z_{mn}]^{-1} [V_m] \quad (8)$$

Where

$$[V_m] = \begin{pmatrix} 4\pi\epsilon_0 \\ \vdots \\ 4\pi\epsilon_0 \end{pmatrix}, [I_n] = \begin{pmatrix} a_1 \\ \vdots \\ a_1 \end{pmatrix}$$

And

$$Z_{mn} = \int_0^l \frac{g_n(z')}{\sqrt{(z_m - z')^2 + a^2}} dz' = \int_{(n-1)\Delta}^{n\Delta} \frac{1}{\sqrt{(z_m - z')^2 + a^2}} dz'$$

Equation (8) gives the current across the wire of length l and radius a using the MOM technique. In the next subsection the pocklington integrodifferential equation will be employed to calculate the current of the semi-rigid cable.

2.2. Semi-rigid cable current using Pocklington Integrodifferential equation

In the case of the monopole antenna constructed by stripping the outer conductor of the semi rigid coaxial cable (Figure 2) and exposing the centre conductor, the wire exposed will experience an incident field across the surface caused by the coaxial aperture of the outer conductor.

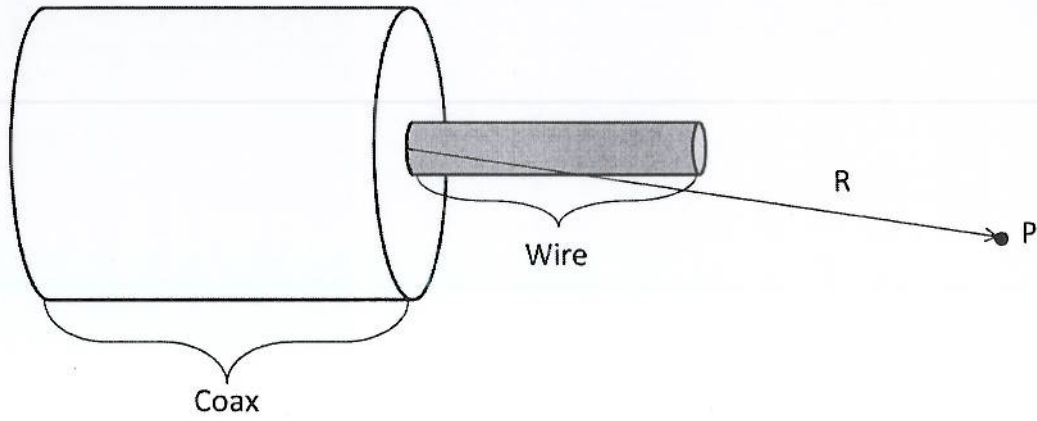


Figure 2: Semi Rigid Coax Monopole Antenna

Figure (3) shows the coaxial apertures and its equivalent magnetic frill given by equivalence principle which states "two sources producing the same field within a region of space are said to be equivalent within that region". There by the coaxial aperture is replaced by an equivalent magnetic frill to calculate the fields from coaxial apertures. The Magnetic frill consists of a circumferentially directed magnetic current density that exists over an annular aperture with inner radius a , which is usually chosen to be the radius of the wire, and an outer radius b . Figure 4 shows the segmented wire with the magnetic frill.

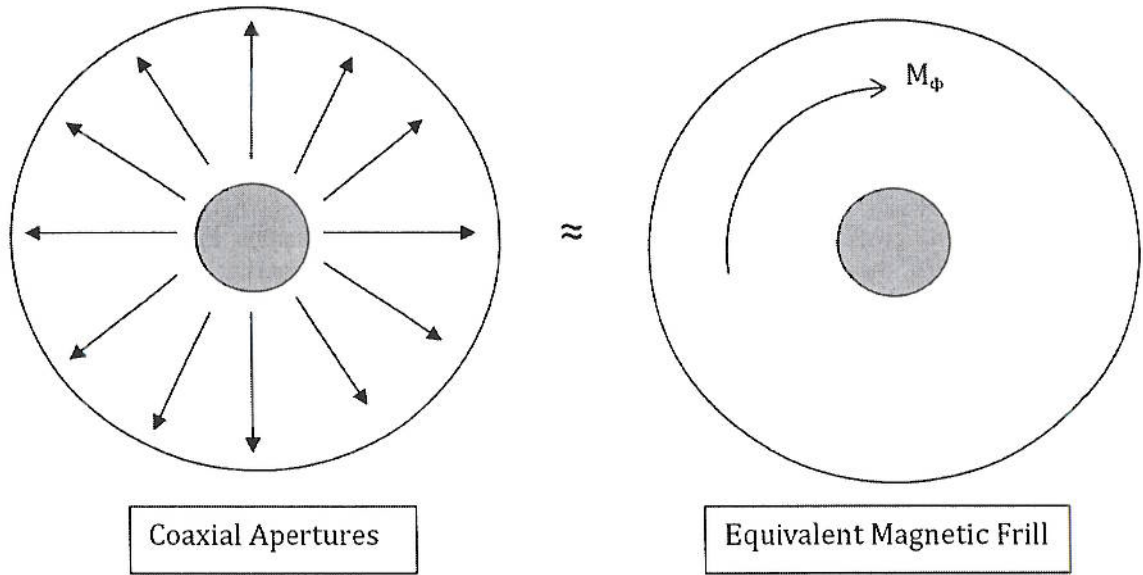


Figure 3 Equivalence Principle

Pocklington integrodifferential equation[7] (11) can be used to determine the equivalent filamentary line source current of a thin wire, by knowing the incident field on the surface of the wire[8].

$$\int_0^1 I_z(z') \frac{e^{-jkR}}{4\pi R^5} [(1 + jkR)(2R^2 - 3a^2) + (kaR)^2] dz' = -j\omega\epsilon E_z^i(\rho = a) \quad (11)$$

Where for observations along the centre of the wire ($\rho=0$)

$$R = \sqrt{a^2 + (z - z')^2}$$

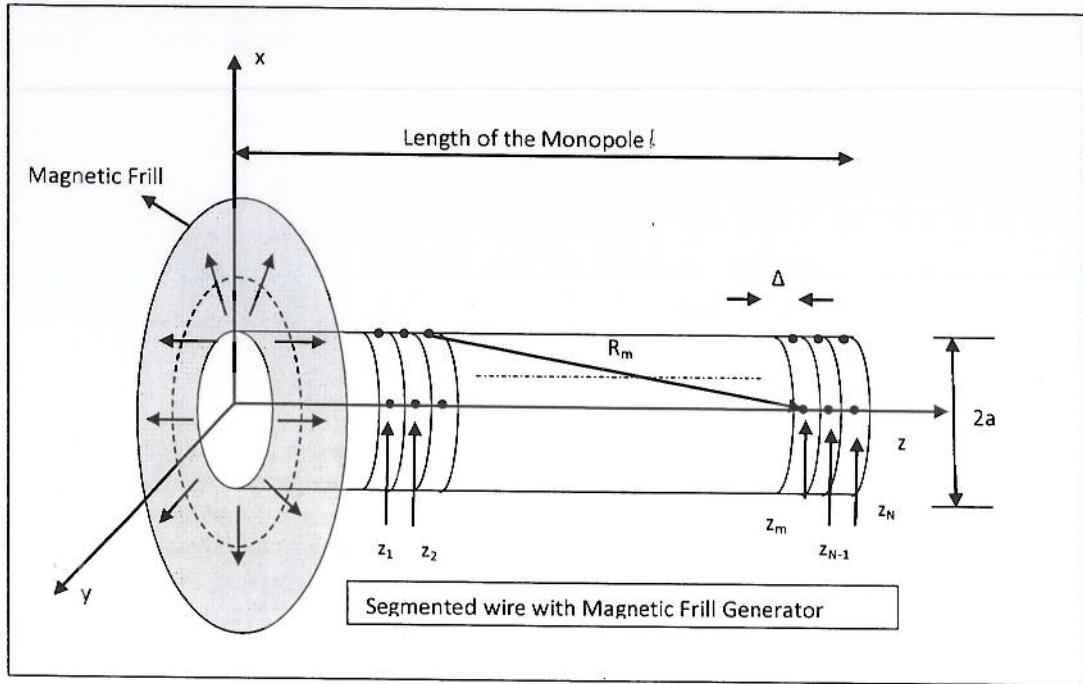


Figure 4: Segmented Wire with Magnetic Frill

Over the annular aperture of the magnetic frill generator, the electric field is represented by the TEM mode field distribution of a coaxial transmission line is given by

$$E_f = \hat{a}_\rho \frac{V_i}{\ln(b/a)} \quad a \leq \rho' \leq b$$

Therefore the corresponding equivalent magnetic current density M_f for the magnetic frill generator used to represent the aperture is equal to

$$M_f = -2\hat{n} \times E_f = -2\hat{a}_z \times \hat{a}_\rho E_\rho = -\hat{a}_\phi \frac{V_i}{\rho' \ln(b/a)} \quad a \leq \rho' \leq b \quad (12)$$

The field generated by the magnetic frill generator of (12) on the surface of the wire along the axis ($\rho=0$) is given by [9]

$$E_z^i(\rho=0, 0 \leq z \leq l) = -\frac{V_i}{2 \ln(b/a)} \left[\frac{e^{-jkR_1}}{R_1} - \frac{e^{-jkR_2}}{R_2} \right] \quad (13)$$

Where

$$R_1 = \sqrt{z^2 + a^2} \quad R_2 = \sqrt{z^2 + b^2}$$

Knowing the incident field, the current across the wire can be calculated using (13). Using MOM the current $I_z(z')$ can be given by

$$I_z(z') = \sum_{n=1}^N \alpha_n g_n(z')$$

Replacing $I_z(z)$ in equation (11)

$$\int_0^1 \sum_{n=1}^N \alpha_n g_n(z') \frac{e^{-jkR}}{4\pi R^5} [(1+jkR)(2R^2 - 3a^2) + (kaR)^2] dz' = -j\omega \epsilon E_z^i \quad (14)$$

After piecewise integration the current across the n segments is given by

$$[I_n] = [Z_{mn}]^{-1} [V_m] \quad (15)$$

where voltage at m^{th} segment is given by

$$V_m = E_z^i \Delta$$

And Z_{mn} is given by

$$[Z_{mn}] = \int_{n\Delta}^{(n+1)\Delta} \frac{e^{-jkR(z_m, z'_n)}}{4\pi R^5(z_m, z'_n)} \left\{ \left(1 + jkR(z_m, z'_n)\right) \left(2R(z_m, z'_n)^2 - 3a^2\right) + (kaR(z_m, z'_n))^2 \right\}$$

Knowing the values for Z_{mn} and V_m across the segment, the current across the wire I_n can be calculated using equation (15). This equation calculates the current across the wire of length l and radius a using the MOM technique. However this technique is a time consuming process which requires $N \times N$ computations to predict the current. Also as the surrounding medium becomes conductive the calculation becomes complex thereby increasing the computational time to $(N \times N)^2$. In the next section the mathematical model for the current distribution developed in the paper is presented.

3. The Proposed Mathematical Model

The extent of the limitation of the traditional MOM technique discussed above is directly proportional to the number of basis functions involved. In antenna field calculations this limitation is related to the segmentation of the wire. As the segment length Δ decreases, the number of segments N increases. As N increases, the accuracy of the current distribution along the wire increases, however this also increases the complex nature of the matrix equation involved, and thereby increases the computational time. This limitation of MOM technique increases as we address the issue of field calculation of antenna in conducting medium. In conducting medium the dielectric property of the medium is complex [10] and it takes into account the loss tangent of the medium. Thereby the propagation constant k will be of complex nature,

$$k = k' + jk''$$

and this will in turn increase the computational time from $N \times N$ to $(N \times N)^2$. In order to address the limitations of the MOM technique for current distribution calculation, a new mathematical model is proposed. The new model aims to reduce the computational time and the tedious nature of the MOM equations explained in the previous section. The expression for the new model is given below in equation 16.

$$I(z) = I_0 e^{-\alpha z} \sin(k(l-z)) + f(z, \tau) \quad (16)$$

Equation 16 consists of two parts; the first part,

$$I_0 e^{-\alpha z} \sin(k(l-z))$$

accounts for the damping in the current distribution curve of Figure 5. This characterises the effect of the surrounding medium of the wire. The current distribution curve in Figure 5 is of the wire of length $\lambda/2$ in free space. In this case the damping coefficient α is zero and its value changes as the surrounding medium changes. This is very effective for applications involving coupling medium with complex dielectric properties, such as medical imaging applications.

This part also provides the overall shape of the current distribution curve in Figure 5. This part of the equation is similar to that of the current distribution expression given in [1]. The final part of the expression is generally given by,

$$f(z, \tau) = \begin{cases} d_0 + \frac{\tau}{4} \sin(2k(l-z)) , & \text{for } l = (2w+1) \frac{\lambda}{4} \\ d_0 + 2\tau \sin(2k(l-z)) , & \text{for } l = 2w(\lambda/4) \end{cases}$$

Where d_0 is the dc component and w is a positive integer. This part accounts for the variation due to the radii of the wire; it acts like the dc term in the expression. It also provides the delay element in the current distribution curve in Figure 5.

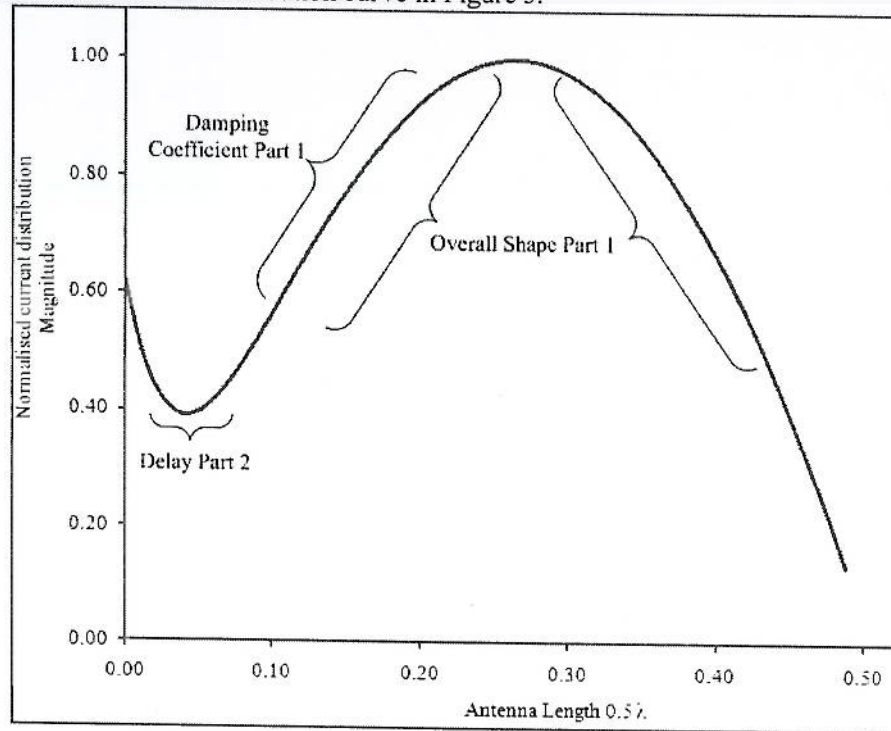


Figure 5: current distribution curve of the semi rigid coaxial wire of length $\lambda/2$.

This new mathematical model decreases the computational time as it depends on only three parameters; Initial current I_0 , damping coefficient α and radial parameter τ . Initial current I_0 is the current at the first segment of the wire, damping coefficient α characterises the conductivity of the surrounding medium. It is this parameter of the expression which makes this model suitable for the predicting the current distribution of the wire in different surrounding medium other than free space. And finally, τ is a parameter related to the radius of the wire.

4. Initial Results and Model Validation

The ability to predict the current across the monopole antenna using this new mathematical model has been analysed by predicting the current distribution of two different lengths of wire. Length of quarter wavelength ($\lambda/4$) and three quarter wavelength ($3\lambda/4$) monopoles are considered. The predicted current distributions of the above lengths of wire using conventional technique were available to compare with the results from the new mathematical model. Antenna radius a is given as 0.005λ and the outer radius b is related to the inner radius by $b = 2.3a$, and the wire is considered along the z axis. For the conventional method the number of basis function is kept at 80, i.e. the wire along the z axis is discretised to 80 segments. The initial current parameter I_0 of the new model is measured for different lengths of the antenna. Using these initial current values the other two parameters damping coefficient α and radial parameter τ are calculated using optimisation techniques.

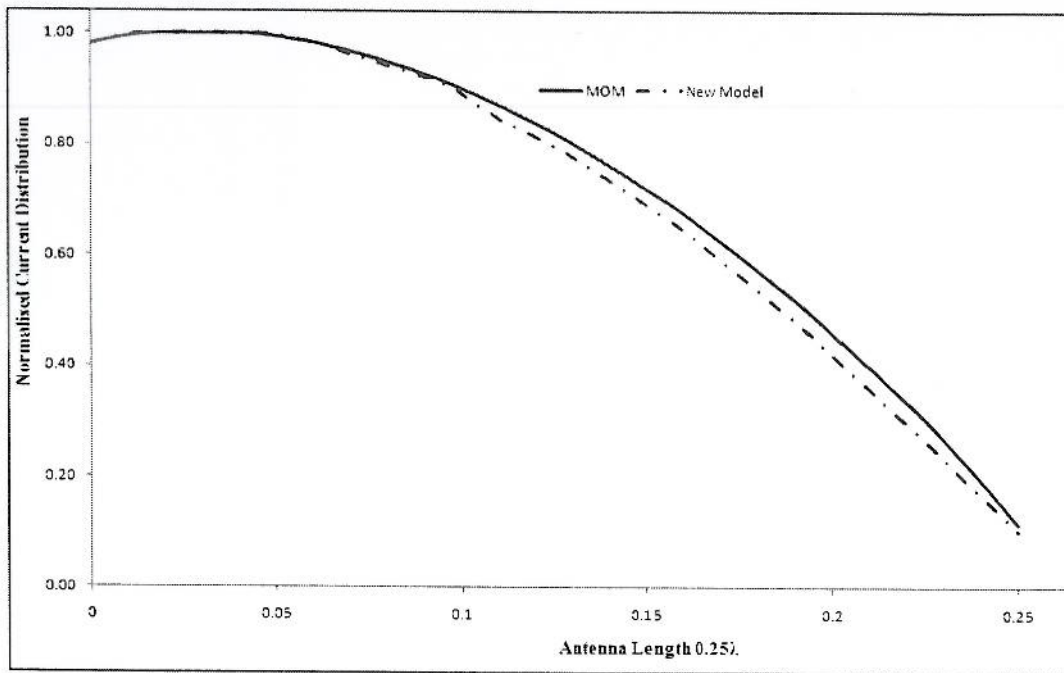


Figure 6: current distribution curve of the semi rigid coaxial wire of length $\lambda/4$.

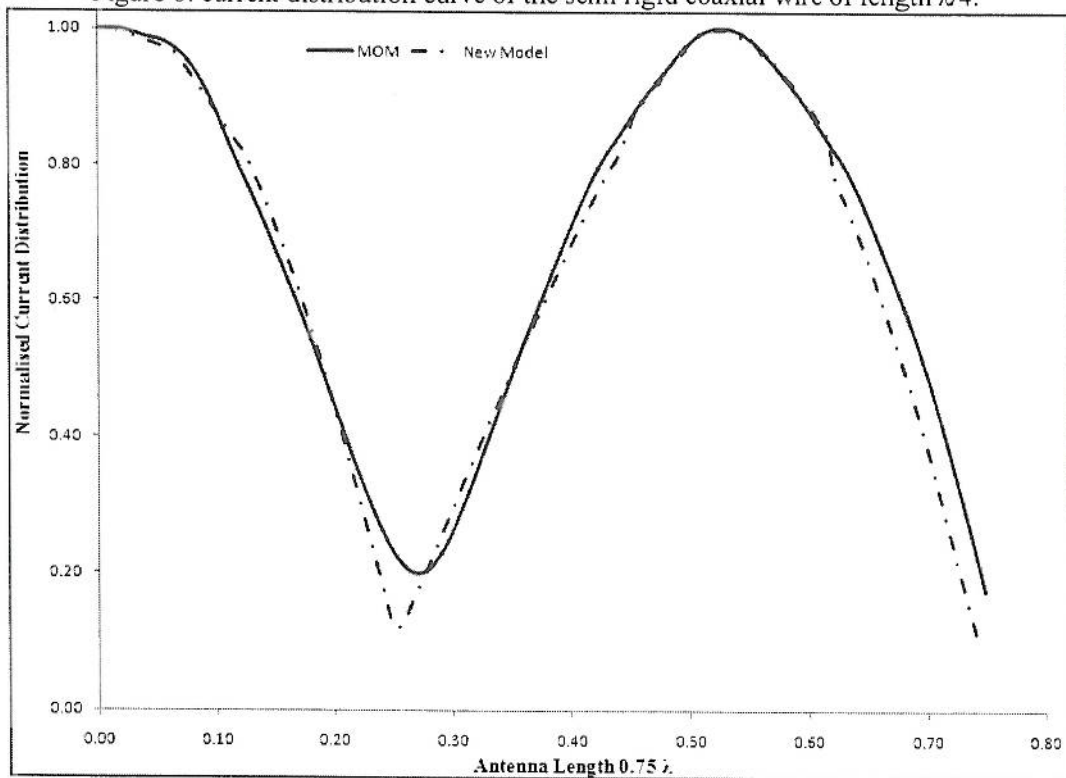


Figure 7: current distribution curve of the semi rigid coaxial wire of length $3\lambda/4$.

Figure 6 & 7 shows the result of both MOM and the new model. As it shown there is a good correlation between the two techniques for the two different lengths of the wire. From the current values across the wire, the vector potential A at a distance P (refer to Figure 2) can be calculated using the expression

$$A(x, y, z) = \frac{\mu}{4\pi} \int I_c(x', y', z') \frac{e^{-jkR}}{R} dl'$$

Once the vector potential is known the electric field E at the point P due to the wire can be calculated. Thus the electric field of a monopole antenna can be calculated using our new model, because of the less complex nature of the new model and its flexibility to predict the current distribution in media other than free space (damping coefficient α), this can be extended to surroundings involving coupling medium.

5. Conclusion

This paper presents a new model to predict the current across the monopole antenna thereby useful in predicting the electromagnetic field at a particular point in space. The new model is computationally less tedious and laborious than the traditional MOM method. The initial results have shown a good correlation between the two models and thereby the new model can be used in monopole field prediction in coupling medium environment. Further work will investigate the validity of the new model in media such as saline solution, vegetable oil and other coupling liquids used for medical applications. It will also refine the second part of the new model.

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