# ***Problems of Interoperability in Information Systems***

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# Four Challenges

- Enterprise Interoperability
- Knowledge-oriented Collaboration
- Web Technologies
- Interoperability Service Utility

Need dynamic connections

# What is Underlying Logic?

- Not set theory
  - OK for closed local systems
  - But falls foul of Gödel as higher-order operations needed
  - Neither complete nor decidable outside FOPC
  - CWA is not realistic
  - But experimental verification is valuable
- Not pure category theory
  - Axiomatic
  - So also falls foul of Gödel

# Process Logic

- Strong candidate
- Long pedigree
  - Heraclites
  - Whitehead
  - Category theory
  - Cartesian closed categories

# Uses of Category Theory

- Cartesian closed categories (CCC, naturality)
- Systems theory with Heyting logic (open systems)
- Topos (SoS)
- Monad (transaction logic, process)
- Adjointness (relationships)
- 2-categories (vertical + horizontal composition)
- Higher-order logic in CCC
  - Without axioms and reliance on number
  - Gödel free in connecting systems in our view
- For good practice, avoid categorification

# Twin-track Approach

- Two subsystems
- 1. Data Structures and Rules
  - 3-level architecture
  - In terms of mappings A →B → C →D
    - With dual D → C → B → A
- 2. Behaviour
  - 3-level architecture
  - In terms of cycles F: A→ B; G: B→ A
    - GF 3 times
    - FG 3 times

# Example of Adjointness

$$
\begin{tikzcd}
L \arrow[r, shift left=2, "F"] & \mathbf{R} \arrow[l, shift left=2, "G"]
\end{tikzcd}
$$

- If conditions hold, then we can write F ⊣ *G*
- The adjunction is represented by a 4-tuple:
  - <F,G,η, ε>
- η  and ε are unit and counit respectively
- L, R are categories; F, G are functors

# Data Structures and Rules

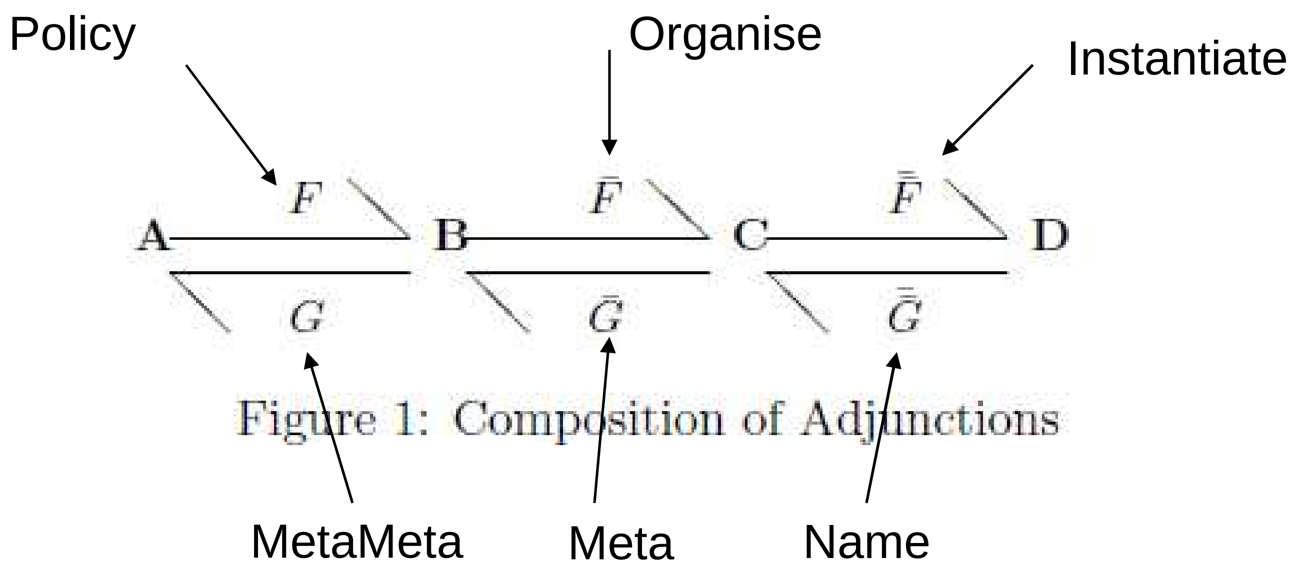


Figure 1: Composition of Adjunctions

A is category for Concepts
B is category for Constructs
C is category for Schema
D is category for Data

Adjunctions compose naturally
F-|G is one of 6 adjunctions (if they hold)

# Principles

- Have pairs of abstractions
- Each level is defined by level above
- Adjunctions permit relationships less than equivalence between the levels
- Having more than three levels of abstraction does not achieve greater precision
- Can be viewed as multi-level type subsystem

# Six Possible Adjunctions

$F \dashv G$ $\qquad \overline{F} \dashv \overline{G}$ $\qquad \overline{\overline{F}} \dashv \overline{\overline{G}}$

$\overline{F}F \dashv G\overline{G}$ $\qquad \overline{\overline{F}}\,\overline{F} \dashv \overline{G}\,\overline{\overline{G}}$

$\overline{\overline{F}}\,\overline{F}F \dashv G\overline{G}\,\overline{\overline{G}}$

# Adjunctions in More Detail
# Simple Pairs

We can define these in more detail with their units and counits of adjunction as follows:

$$< F, G, \eta_a, \epsilon_b >: \mathbf{A} \longrightarrow \mathbf{B} \tag{1}$$

$\eta_a$ is the unit of adjunction $1_a \longrightarrow GFa$ and $\epsilon_b$ is the counit of adjunction $FGb \longrightarrow 1_b$

$$< \bar{F}, \bar{G}, \bar{\eta}_b, \bar{\epsilon}_c >: \mathbf{B} \longrightarrow \mathbf{C} \tag{2}$$

$\bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{F}b$ and $\bar{\epsilon}_c$ is the counit of adjunction $\bar{F}\bar{G}c \longrightarrow 1_c$

$$< \bar{\bar{F}}, \bar{\bar{G}}, \bar{\bar{\eta}}_c, \bar{\bar{\epsilon}}_d >: \mathbf{C} \longrightarrow \mathbf{D} \tag{3}$$

$\bar{\bar{\eta}}_c$ is the unit of adjunction $1_c \longrightarrow \bar{\bar{G}}\bar{\bar{F}}c$ and $\bar{\bar{\epsilon}}_d$ is the counit of adjunction $\bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$

# Adjunctions in More Detail Doubles

$$< \bar{F}F, G\bar{G}, G\bar{\eta}_a F \bullet \eta_a, \bar{\epsilon}_c \bullet \bar{F}\epsilon_c\bar{G} >: \mathbf{A} \longrightarrow \mathbf{C} \tag{4}$$

$G\bar{\eta}_a F \bullet \eta_a$ is the unit of adjunction $1_a \longrightarrow G\bar{G}\bar{F}Fa$ and $\bar{\epsilon}_c \bullet \bar{F}\epsilon_c\bar{G}$ is the counit of adjunction $\bar{F}FG\bar{G}c \longrightarrow 1_c$

The unit of adjunction is a composition of $\eta_a : 1_a \longrightarrow GFa$ with $G\bar{\eta}_a F : GFa \longrightarrow G\bar{G}\bar{F}Fa$

The counit of adjunction is a composition of $\bar{F}\epsilon_c\bar{G} : \bar{F}FG\bar{G}c \longrightarrow \bar{F}\bar{G}c$ with $\bar{\epsilon}_c : \bar{F}\bar{G}c \longrightarrow 1_c$

We have retained the symbol $\bullet$ indicating vertical composition as distinct from normal horizontal composition indicated by the symbol $\circ$ [13].

$$< \bar{\bar{F}}\bar{F}, \bar{G}\bar{\bar{G}}, \bar{G}\bar{\bar{\eta}}_b\bar{F} \bullet \bar{\eta}_b, \bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} >: \mathrm{B} \longrightarrow \mathrm{D} \tag{5}$$

$\bar{G}\bar{\bar{\eta}}_b\bar{F} \bullet \bar{\eta}_b$ is the unit of adjunction $1_b \longrightarrow \bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}B$ and $\bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}}$ is the counit of adjunction $\bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow 1_d$

The unit of adjunction is a composition of $\bar{\eta}_b : 1_b \longrightarrow \bar{G}\bar{F}b$ with $\bar{G}\bar{\bar{\eta}}_b\bar{F} : \bar{G}\bar{F}b \longrightarrow \bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}b$

The counit of adjunction is a composition of $\bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} : \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{\bar{G}}d$ with $\bar{\bar{\epsilon}}_d : \bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$.

# Adjunctions in More Detail
# Triples

$$< \bar{\bar{F}}\bar{F}F, G\bar{G}\bar{\bar{G}}, G\bar{G}\bar{\bar{\eta}}_a\bar{F}F \bullet G\bar{\eta}_a F \bullet \eta_a, \bar{\bar{\epsilon}}_d \bullet \bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} \bullet \bar{\bar{F}}\bar{F}\epsilon_d\bar{G}\bar{\bar{G}} >: \mathbf{A} \longrightarrow \mathbf{D} \tag{6}$$

The unit of adjunction is a composition of:

$\eta_a : 1_a \longrightarrow GFa$ with $G\bar{\eta}_a F : GFa \longrightarrow G\bar{G}\bar{F}Fa$ with $G\bar{G}\bar{\bar{\eta}}_a\bar{F}F : G\bar{G}\bar{F}Fa \longrightarrow G\bar{G}\bar{\bar{G}}\bar{\bar{F}}\bar{F}Fa$

The counit of adjunction is a composition of:

$\bar{\bar{F}}\bar{F}\epsilon_d\bar{G}\bar{\bar{G}} : \bar{\bar{F}}\bar{F}FG\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d$ with $\bar{\bar{F}}\bar{\epsilon}_d\bar{\bar{G}} : \bar{\bar{F}}\bar{F}\bar{G}\bar{\bar{G}}d \longrightarrow \bar{\bar{F}}\bar{\bar{G}}d$ with $\bar{\bar{\epsilon}}_d : \bar{\bar{F}}\bar{\bar{G}}d \longrightarrow 1_d$

# Desired Properties

- If all adjunctions hold
  - Have clearly-defined multi-level type subsystem
- Can relate one subsystem to another by
  - Natural transformation
    - Maps between functors
- Provides interoperability between subsystems for
  - Data structures and rules

# Natural Transformation

$$\begin{tikzcd} L \arrow[r, shift left=3, "F"] \arrow[r, shift right=3, "F'"'] \arrow[r, phantom, "\Downarrow\alpha"] & \mathbf{R} \end{tikzcd}$$

α is natural transformation comparing F and F′

# Behaviour/Anticipation Monad/Comonad

- Define subsystem
  - Handle transactions
    - ACID properties
    - Atomicity, Consistency, Isolation, Durability
  - Have 3 cycles
    - 1. make changes
    - 2. review changes
    - 3. holistic check that all is well
  - Example with Bank ATM:
    - 1. debit account
    - 2. check funds available
    - 3. holistic check that all changes recorded safely

# Monad

- Construction for transactions is the Monad
- Monad is a triple <T, η, μ>
  - T is an endofunctor (functor with same source and target)
    - e.g. GF : A → B → A
  - η is unit of adjunction e.g. $1_L \rightarrow GF(L)$
    - Compares initial value for object L with value for L after one cycle
  - μ is multiplication $T^2 \rightarrow T$
    - comparing result from 2nd cycle with 1st
    - e.g. GFGF → GF
- Full details of definition involve $T^3$ (GFGFGF)

# Comonad

- Monad gives left-hand-perspective (L)
- Comonad gives right-hand perspective (R)
- Comonad is a triple <S, ε, δ>
  - S is FG
    - e.g. B → A → B
  - ε is counit of adjunction e.g. FG(R) → $1_R$
  - δ is comultiplication T → $T^2$
    - Anticipation – looking forward

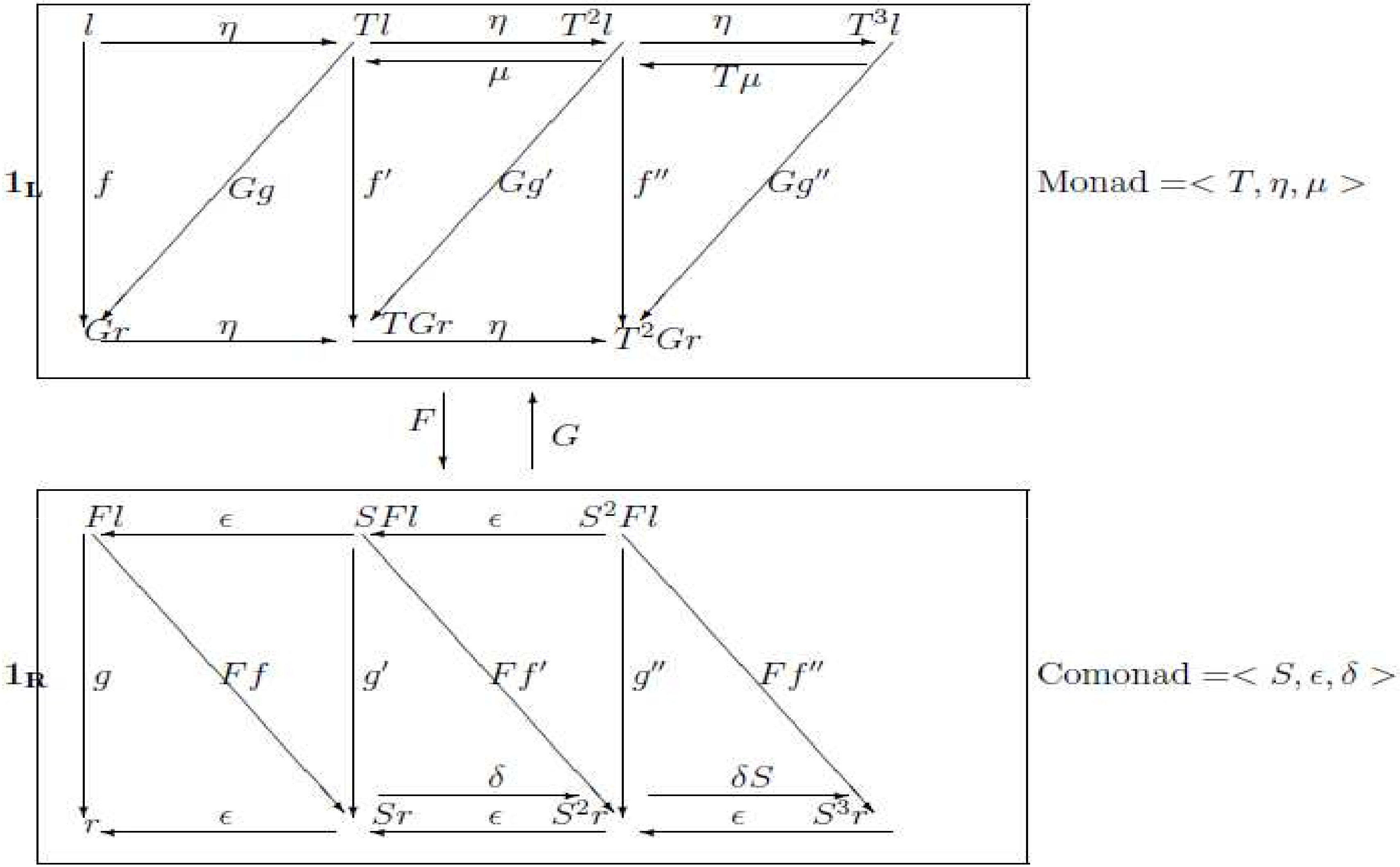


Figure 2: After three cycles $GFGFGF$ from left-hand category and three cycles $FGFGFG$ from right-hand category: $\eta$ and $\delta$ map onto other than $\perp$, $\top$ maps onto other than $\epsilon$ and $\mu$

# System Viewpoint for Interoperability

- Have a system formed from 2 subsystems
  - For data structures/rules
    - 3 levels of mapping as functors between categories
    - Each mapping represents a level-pair of abstractions
  - For behaviour
    - 3 cycles as a monad/comonad structure
- Interoperability
  - Comparing one system with another by natural transformations or higher-order categories
- Recent work on Security by PhD student Dimitris Sisiaridis with category theory produces the system unification

# Possible Way Forward

- Not for everybody to learn category theory!
- Development of tool
  - Assist with interoperability
  - Based on process category theory
  - Graphical
  - Haskell is a candidate
    - Facilities include monads