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## **AN EXPERIMENTAL STRATEGY FOR FRACTIONATING $3^3$ AND $3^4$ FACTORIAL EXPERIMENTS**

**Abstract:** *In the design of statistical experiments, situations may arise when resource constraints hinder the use of factorial designs for process improvement. This paper explores how 9, 18 and 27-run orthogonal arrays compare against each other and against a proposed experimental plan referred to as a ‘Segmented Fractional Plan’ when used to fractionate  $3^3$  and  $3^4$  factorial experiments. Based on the analysis of 8 responses from 6 factorial experiments, it was observed that to identify the process setting that produces the desired product quality, with a reduced number of experimental runs, the segmented fractional plan can perform as well or better than some orthogonal arrays thus, providing an option for fractionating  $3^3$  and  $3^4$  factorial experiments.*

**Keywords:** *Three-level fractional factorial designs, three-level orthogonal arrays, design of experiments, process improvement, quality improvement.*

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### **1. Introduction**

Process quality improvement has been the focus of many industries due to the competitive advantage a process with high quality can provide (Montgomery & Woodwall, 2008). Experimentation plays a major role in improving the quality of industrial processes. Engineers engage in experimentation for various reasons. These include (Dean et al., 2017; Rodrigues & Lemma, 2015):

- To determine which factors have the most influence on the process output;
- To determine the settings of the influential factors that optimise the process output;
- To determine the settings of the influential factors that minimises the variability in the process output;

- To determine the settings of the influential factors that minimise the effect of uncontrollable factors on the process output.

In improving the quality of a process when several factors are to be investigated, factorial experiments are recommended over other experimental strategies as they can be used to estimate all factor effects (Montgomery, 2013). However, improving the performance of a process using factorial designs may not be possible when resources are limited due to the number of experimental runs needed to conduct some factorial designs (Montgomery, 2013; Wu & Hamada, 2000).

Orthogonal arrays (OAs) and fractional factorial designs (also OAs) are commonly used to fractionate factorial designs (Montgomery, 2013; Wu & Hamada, 2000). A good OA design can not only reduce run size and cost, but also provide precise

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estimation of factorial effects of interest (Tang & Xu, 2014). See (Tutar et al., 2014; Vankanti & Ganta, 2014) for examples on the use of OAs for process improvement.

In addition, OAs are used for screening experiments in which the objective is to identify the most important factor effects from a list of many potential ones (Nguyen & Pham, 2016; Xu et al., 2004). Based on the knowledge gained from the screening experiment, the process can be optimised (Xu et al., 2014). See (Tagliaferri et al., 2013) for an example on the use of OAs for factor effect screening.

The focus of this paper is on the fractionation of 3-level factorial designs for 3 and 4 factors ( $3^3$  and  $3^4$  factorial designs). Although 3-level factorial designs are not recommended when less expensive second order designs can be used to investigate a process, 3-level factorial designs are useful in situations where the process settings are discrete in form (Montgomery, 2013; Wu & Hamada, 2000). In addition, practical situations may arise where discrete and continuous factors are mixed at three levels. There is no standard approach to analysing experiments in such situations with Box-Behnken designs requiring the factors to be continuous (NIST/SEMATECH e-handbook of statistical methods 2013) and, more recent 3-level designs (Definitive Screening Designs) designed to accommodate discrete factors at 2-levels (Jones & Nachtsheim, 2013).

When the aim of the experiment is to identify factor settings that improve the quality of the process output, one way to analyse such experiments is to treat the continuous factors as discrete. This research focuses on cases where:

- 1) the goal of the experiment is to identify factor settings that improve the quality of the process output,
- 2) continuous factors are mixed with discrete factors at three levels and the experimenter chooses to treat the continuous factors as discrete.

A 9-run fractional factorial design is used to fractionate a  $3^3$  factorial design while 9 and 27-run fractional factorial designs are used to fractionate a  $3^4$  factorial design (Montgomery, 2013; Wu & Hamada, 2000). Xu et al (2004) developed 3-level 18-run OAs for screening important factors from a large number of potential factors and also detecting interactions among a subset of active factors when 3 to 7 factors are to be studied. To distinguish factorial designs from fractional factorial designs, factorial designs are referred to as full factorial designs and the 9 and 27-run fractional factorial designs are referred to as OAs to simplify discussions on these designs and the 18-run OA. This paper analyses eight responses from three  $3^3$  full factorial experiments and three  $3^4$  full factorial experiments using the 9-run OA, the 18-run OAs of Xu et al., the 27-run OA and a proposed experimental plan referred to as a 'Segmented Fractional Plan' (SFP). The OAs and the SFP are analysed based on their ability to use their respective experimental runs to identify the optimal process setting obtained from the  $3^3$  and  $3^4$  full factorial experiments.

## 2. Research Methodology

### 2.1. The Segmented Fractional Plan

In the design of experiments, experiments with more than one replicate are recommended. However, situations may arise where due to a lack of resources, a minimal number of experimental runs is sought to improve the process (Montgomery, 2013; Wu & Hamada, 2000). The SFP is proposed for cases where a single replicate of the experiment is preferred due to the minimal availability of resources. The SFP uses a full factorial experiment ( $2^3$  or  $2^4$  full factorial experiment) at the high and low factor settings to identify the most important factor in the system and to also determine the optimal process setting when curvature resulting from the medium settings of the factors is not detected.

By adding a centre point run to the full factorial design, a test for curvature is conducted using a statistical test and, a main effects and centre point plot (m-c plot). Curvature as referred to herein signifies that the medium setting is the best setting of one or more factors. The statistical test for curvature uses centre point runs to check for the possibility of a curvilinear relationship between the factors being studied and the response of interest (Montgomery, 2013). As a curvilinear relationship may mean that the medium setting of a factor produces a better result than its high and low settings, the statistical test is employed to test for curvature. In a single replicate experiment, the statistical test can be used by removing the least significant interaction effect from the response prediction model. This may compromise the goodness of the statistical test when the effect is not small enough (Montgomery, 2013). Hence, it is used in conjunction with an m-c plot to improve the detection of curvature. Using both tests, if the response at the centre point is worse than the mean response from the full factorial experiment, it is assumed curvature is unlikely. The m-c plot shows the position of the response of the centre point run relative to the mean response from the full factorial experiment and the mean response of the high and low setting of each factor. Across the 3-level full factorial design space, if the mean response at the high and low setting of a factor or a combination of factors is worse than the mean response of its medium setting, it may be reflected in the response of the centre point run as the response resulting from the main effects of these factors at their medium setting as well as their interactions with the medium settings of other factors may better the response associated with their optimum settings (best average response between the factor settings) across the full factorial design at their high and low settings. In using the m-c plot, when the centre point run produces a better result than the response associated with the optimal setting of at least one factor across the  $2^3$  or  $2^4$  full factorial design space, it is

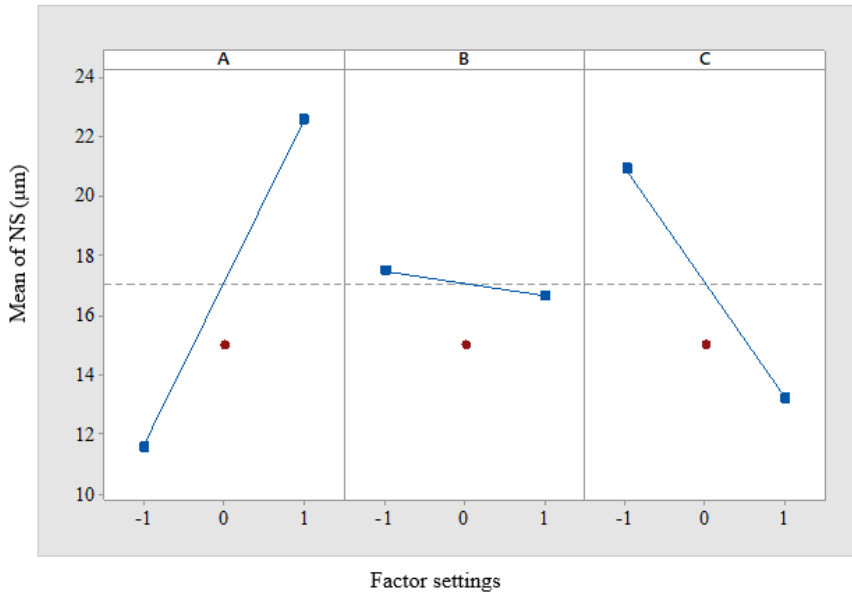
assumed that curvature may be present. When this is not the case, it is assumed curvature is unlikely.

Figure 1 which is based on a  $3^3$  full factorial experiment (Bhavsar et al., 2005) used to investigate the influence of polymer concentration (factor A), amount of nanoparticles (factor B) and stirring speed (factor C) on the size of nanoparticles produced (NS), is used to demonstrate how the m-c plot is used. Figure 1a is an m-c plot from the experiment and figure 1b is a main effects plot from the  $3^3$  full factorial of the same experiment. The low, medium and high factor settings are represented by the numbers -1, 0 and 1 respectively. This is the same for all other experiments described in this paper. For a minimal size of nanoparticles, the m-c plot showed that the response of the centre point run ( $15\mu\text{m}$ ) was better than the response associated with the optimal setting of factor B ( $16.65\mu\text{m}$ ) across the  $2^3$  full factorial design space. Based on the interpretation of the m-c plot, this may suggest the presence of curvature. Comparing the m-c plot to the main effects plot from the  $3^3$  full factorial experiment, it can be seen that curvature exists as the optimal setting of factor B is its medium setting.

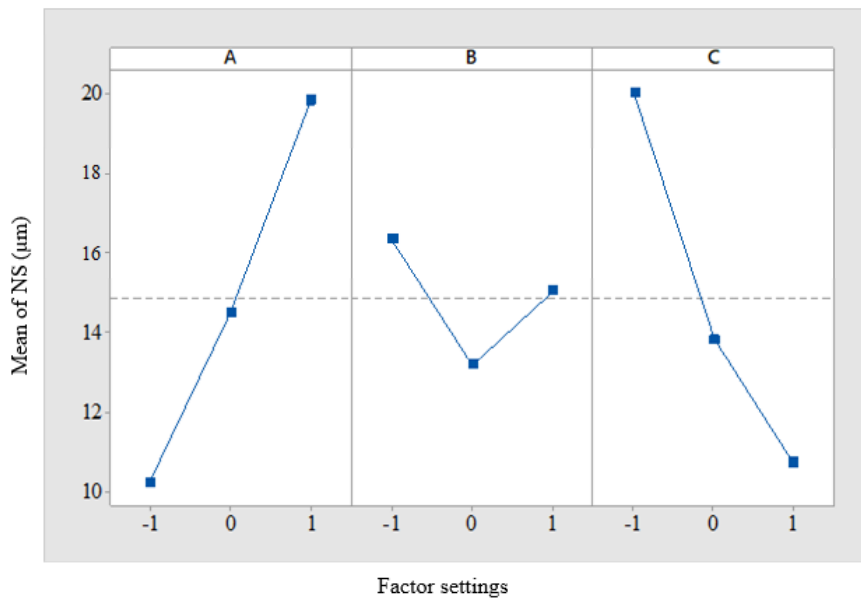
If either the statistical test or the m-c plot, or both, suggest the possibility of curvature, the medium setting of the factors should be explored. If both tests suggest curvature is unlikely, the full factorial experiment can be analysed to identify the optimal process setting. To identify the optimal process setting when the tests for curvature suggest curvature may be present, an experimental run is conducted by changing the most important factor to its medium setting while keeping other factors at their best setting from the full factorial experiment. Retaining the setting of the most important factor that produced the best response, a second 2-level full factorial experiment of the less important factors at their best setting from the first full factorial experiment and their medium setting is performed. The optimal factor setting corresponds to the factor settings that produce

the best response across all experiments conducted. In explaining why the most important factor was changed to its medium setting and why a full factorial of the less

important factors was used, a synergistic and anti-synergistic interaction are defined as follows.



**Figure 1.** (a) M-C plot for the NS experiment, ■ represents the runs at the factorial points, ● represents the centre point run



**Figure 1.** (b) Main Effects plot for the NS experiment

A synergistic interaction is an interaction which provides an additional improvement to the system response when main effects are positively exploited compared to a model of main effects only. On the other hand, an anti-synergistic interaction worsens the system response when main effects are positively exploited compared to a model of main effects only. Positive exploitation of main effects mean that the main effects are set at levels that improve the system response while negative exploitation of main effects mean that the main effects are set at levels that worsen the system response (Frey & Jugulum 2006). The following example from a 2<sup>3</sup> full factorial experiment investigating the effects of temperature (factor A), initial pH of solution (factor B) and the ionic strength of dispersion (factor C) on the maximum adsorption of an anionic dye (Brilliant Yellow) onto sepiolite (Bingol et al., 2010) is used to illustrate how synergistic and anti-synergistic interactions work.

The regression equation based on the maximum dye adsorption (Q<sub>e</sub>) was:

$$Q_e = 1.7458 - 0.1433A - 0.3400B + 0.0808C + 0.0675AB - 0.0450AC + 0.2117BC + 0.0125ABC \quad (1)$$

Positively exploiting the main effects, the following statements hold:

- a. For a main effects model only, Q<sub>e</sub> = 2.3099mg/g
- b. For a model of main effects with AB, AC and ABC synergistic interactions, Q<sub>e</sub> = 2.4349mg/g. The synergistic interaction improved the response of the main effects model.
- c. For a model of main effects with BC anti-synergistic interaction, Q<sub>e</sub> = 2.0982mg/g. The anti-synergistic interaction worsened the response of the main effects model.

In using the SFP, the most important factor is selected to be changed to its medium setting instead of other factors due to the following reasons: Firstly, by varying the setting of the most important factor first in a 2-level full

factorial experiment, there is a reduced chance that the interaction effects which act opposite to the direction of exploitation of its main effect will overcome its main effect as well as the interaction effects acting in the direction of exploitation of its main effects compared to when other factors are changed first (Frey & Jugulum, 2006). This is demonstrated using an experiment (Seki et al., 2006) conducted to investigate the influence of adsorbent type (factor A), pH of solution (factor B) and temperature (factor C) on the adsorption of boron from aqueous solution (Y). The regression equation from the designed experiment is as follows:

$$Y = 0.4840 + 0.0790A - 0.0206B - 0.0666C + 0.0071AB + 0.0421AC - 0.0161BC + 0.0146ABC \quad (2)$$

From the regression equation, it can be seen that factor A is the most important followed by factors C and B respectively.

When the aim of the experiment is to increase the amount of boron adsorbed, the following statements hold:

- a) The optimal process settings are A=+, B=- and C=-.
- b) Positively exploiting the main effects of factor A, B and C, the main and interaction effect model produced a response of Y=0.5995 mgL<sup>-1</sup>. This response corresponds to the optimal process setting.
- c) Positively exploiting the effect of factor B (B=-) while keeping factor A at its less optimal setting from its main effect analysis (A=-) and keeping factor C at its optimal setting from its main effect analysis (C=-), the main and interaction effects model produced a response of Y = 0.5107mgL<sup>-1</sup>.
- d) Negatively exploiting the effect of factor B (B=+) while keeping factors A and C at the same settings from the previous step (A=-, C=-), the main and interaction effects model produced a response of Y =

0.5167mgL<sup>-1</sup>. The main effect of factor B is not reflected as the response has improved compared to when factor B was positively exploited. Even though the main effect of factor B and the ABC synergistic interaction had a larger value than the AB, AC and BC anti-synergistic interactions at the optimal process setting (A=+, B=-, C=-), which is a representation of the positive exploitation of all the factors, at the process setting (A=-, B=+, C=-), the interaction effects which acted opposite to the direction of exploitation of the main effect of factor B (AC, BC, ABC) overcame the main effect of factor B as well as the interaction effects which acted in the direction of exploitation of the main effect of factor B (AB).

- e) However, positively exploiting the effect of factor A across all combinations of factor settings of factors B and C improves the system response compared to when factor A is negatively exploited. This is shown in table 1.

**Table 1.** Experimental data set

B	C	Boron adsorption (mgL <sup>-1</sup> ) at A = -	Boron adsorption (mgL <sup>-1</sup> ) at A = +
-	-	0.5107	0.5995
+	-	0.5167	0.5755
-	+	0.3547	0.5535
+	+	0.2379	0.5235

The second reason the most important factor was chosen to be changed first to its medium setting compared to other factors was due to the hierarchical ordering principle for factorial effects which states that lower order effects are more likely to be important than higher order effects. In other words, main effects are more likely to be important than two factor interaction effects, two factor interaction effects are more likely to be important than three factor interaction effects, etc. Focusing on the main effect of the most

important factor as it is the most likely to obey the principle, this signifies that the main effect of the most important factor is more likely to be larger than any interaction effect (Wu & Hamada, 2000). Thus, by changing first, the most important factor in a process to its medium setting, there is a reduced chance that the interaction effects which act opposite to the direction of exploitation of its main effect will overcome its main effect as well as the interaction effects acting in the direction of exploitation of its main effects compared to when other factors are changed first. When the most important factor across the 3-level full factorial experiment is different from that obtained from the full factorial involving the medium settings of the factors and their best setting from the full factorial experiment at their high and low settings, the likelihood of the SFP identifying the optimal process setting is reduced.

Responses due to anti-synergistic interactions can only be confirmed by using a full factorial design as it explores all possible combinations of factor settings. Where the optimal process settings obtained from the main effect analysis of a full factorial experiment do not correspond to the optimal process settings across the full factorial design matrix, it is as a result of anti-synergistic interactions present in the system (Frey & Jugulum, 2006). The first full factorial employed by the SFP will identify the optimal process setting resulting from anti-synergistic interactions within the design space of the full factorial experiment at the high and low factor settings and the second full factorial of the less important factors at the best setting of the most important factor will identify an optimal process setting which results from anti-synergistic interactions at the best setting of the most important factor and the settings of the less important factors being studied. For 3<sup>3</sup> experiments, the SFP will require 9, 12 or 13 experimental runs, and for 3<sup>4</sup> experiments, 17, 24 or 25 experimental runs. This is dependent on the presence of curvature in the system. A flow chart of the SFP is shown in figure 2 (See Appendix).

## 2.2. Experimental data set

Six full factorial experiments (Bocchini et al., 2002; Bhavsar et al., 2006; Vitanov et al., 2010; Reddy & Rao, 2005; Erkan et al., 2013; Ozcelik et al., 2005) identified from literature on designed experiments were used to

compare the performance of the OAs and the SFP. The response investigated and the factors studied are given in table 2. Experiments 1, 2, and 3 are  $3^3$  full factorial experiments and experiments 4, 5 and 6 are  $3^4$  full factorial experiments.

**Table 2.** Experimental data set

Experiment number	Response, units	Factors
Experiment 1	Xylanase production, U/ml	Xylan (A), pH (B) and cultivation time (C)
Experiment 2a (maximum response value)	Size of nanoparticles-in-microsphere, $\mu\text{m}$	Polymer concentration (A), amount of nanoparticles (B) and stirring speed (C)
Experiment 2b (minimum response value)	Size of nanoparticles-in-microsphere, $\mu\text{m}$	Polymer concentration (A), amount of nanoparticles (B) and stirring speed (C)
Experiment 3	Coating bond strength of micro friction surfacing process, N	Rotational speed (A), traverse rate of the substrate (B) and feed rate of the mechtrode (C)
Experiment 4	Surface roughness of medium carbon steel, $\mu\text{m}$	Speed (A), feed (B), radial rake angle (C) and nose radius (D)
Experiment 5	Damage factor in the end milling of glass fibre reinforced plastic composites, mm	Number of flutes (A), cutting speed (B), depth of cut (C) and feed rate (D)
Experiment 6a	Surface roughness values of Inconel 718 superalloy across the feed, $\mu\text{m}$	Cutting speed (A), feed (B), axial depth of cut (C) and radial depth of cut (D)
Experiment 6b	Surface roughness values of Inconel 718 superalloy transverse to the feed, $\mu\text{m}$	Cutting speed (A), feed (B), axial depth of cut (C) and radial depth of cut (D)

The response in experiment 2 was analysed based on its minimum and maximum response values as both responses were desirable, depending on the aim of the experiment. Also, two responses were analysed in experiment 6 namely: the surface roughness values across the feed and the surface roughness values transverse to the feed. Thus, eight responses from the six full factorial experiments were analysed. The responses from the experiments are coded herein as follows; Experiment 1 (XA), Experiment 2a (NS) [maximum response value], Experiment 2b (NS) [minimum response value], Experiment 3 (ST), Experiment 4 (SR), Experiment 5 (DF), Experiment 6a (SAF), and Experiment 6b (STF).

The  $3^3$  and  $3^4$  full factorial experiments were selected to represent the following:

- a) Interactions of vaying strength;
- b) Cases where the optimal process setting was influenced by synergistic and anti-synergistic interactions;
- c) Experiments with and without curvature.

The methodology used for classifying the strength of interactions in this paper was adopted from Frey et al (2003). It is based on the contribution of the interactions to the total sum of squares of the system. The interaction strength is calculated by dividing the sum of squares due to interaction effects by the sum of squares due to the total factor effects (main and interaction effects). Based on this ratio,



the interaction strengths are classified. To facilitate the grouping of the experiments used in this paper based on their interaction strengths, three classes of interactions are used. These are given in table 3 as follows:

**Table 3.** Classification of Interaction strength

Class of Interaction	Strength of interactions
Mild	0 to 0.1
Moderate	0.1 to 0.25
Strong	Above 0.25

Experiments with various interaction strengths were chosen to increase the chances of interactions negatively affecting the ability of the OAs and the SFP to identify the optimal process setting and, to increase the chances of interactions compromising the goodness of the statistical test. The 3<sup>3</sup> and 3<sup>4</sup> full factorial experiments analysed in this study are each representative of the three classes of interactions.

In analysing the experimental data set, it was assumed that the errors associated with the experimental runs were at their lowest. Thus, it is assumed the main and interaction effects obtained from the analysis of the experimental data set represent their best possible estimate.

The experiments were analysed using Minitab statistical software. The 3<sup>3</sup> full factorial experiments were analysed using the 9-run OA, the 18-run OA and the SFP while the 3<sup>4</sup> full factorial experiments were analysed using the 18-run OA, the 27-run OA and the SFP. Main effects in the 9-run OA are analysed by assuming that two factor interactions and higher are negligible (Wu & Hamada, 2000). Hence, the results of the 9-run OA are based on a main effects analysis as interaction effects cannot be analysed using this array. Although the 18-run OA was designed to be used to test for interactions when the factor settings are continuous, the results of the 18-run OA are based on main effects analysis as the factor settings are analysed in a discrete manner in this study.

The results of the 27-run OA are based on a main and two factor interaction effect analysis. In the 27-run OA, main effects and some two factor interaction effects are aliased with three factor interaction effects and, some two factor interaction effects are aliased with other two factor interaction effects. The two factor interaction effects can be estimated by assuming that three factor interaction effects and higher are negligible (Wu & Hamada, 2000). Based on this assumption, the main effects and all two factor interaction effects were analysed.

For 3<sup>4</sup> full factorial experiments the 18-run OA was analysed instead of the 9-run OA as the additional number of experimental runs in the 18-run OA provides more experimental data and as such will increase the chances of correctly estimating the main effects.

### 3. Results and Discussions

#### 3.1. Testing for curvature using the SFP

In testing for curvature using the SFP, the m-c plot and the statistical test for curvature produced the same result in experiments 1, 4 and 5. In experiments 2a and 6a, no tests for curvature were conducted as the response from the centre point run was worse than the mean response from the full factorial experiment at the high and low settings of the factors. However, in experiments 2b, 3 and 6b, the advantage of combining the m-c plot with the statistical test is observed. These experiments are discussed as follows:

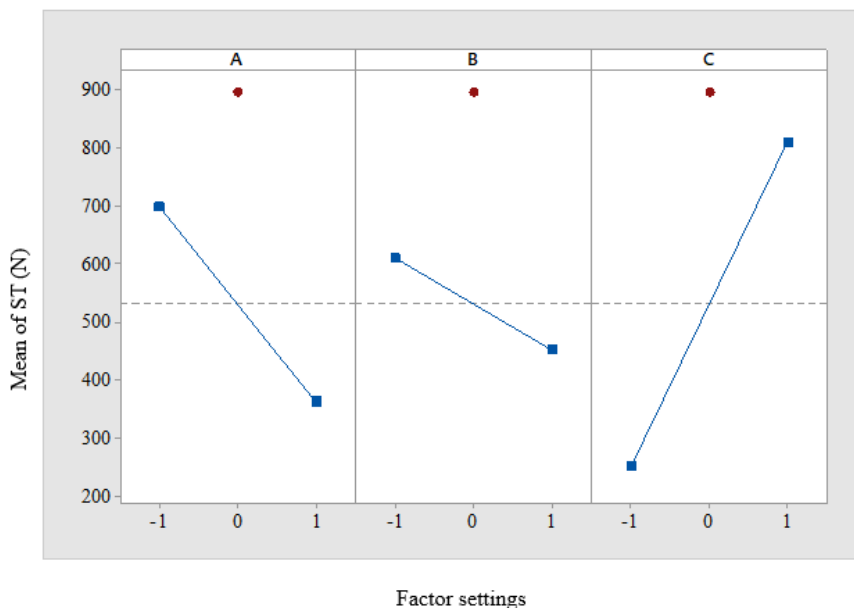
Experiment 2b (NS): In this experiment, the statistical test for curvature generated a p-value of 0.361 which implied that curvature was unlikely. However, the m-c plot showed that the response at the centre point was better than the response associated with the optimal setting of factor B (amount of nanoparticles) across the 2<sup>3</sup> full factorial design space. As this was the same experiment described in figure 1, it can be seen from figure 1 that a main effect analysis of the 3<sup>3</sup> full factorial experiment confirmed the presence of

curvature as the optimal setting of factor B (amount of nanoparticles) was its medium setting. This shows the advantage of combining the statistical test for curvature with the m-c plot.

Experiment 3 (ST): The statistical test for curvature generated a p-value of 0.214 which implied that curvature was unlikely. Checking for curvature with the m-c plot (figure 3), it showed that the response at the centre point was better than the response associated with the optimal setting of all three

factors across the  $2^3$  full factorial design space.

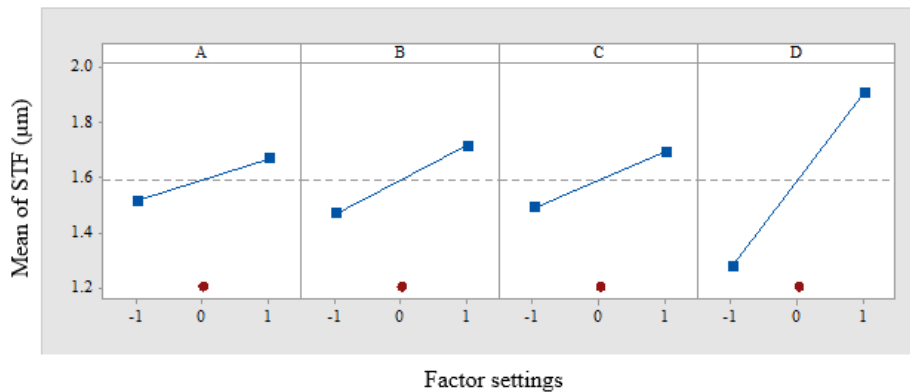
Hence, a suggestion for the experimenter to explore the design space associated with the medium settings of all the factors. A main effect analysis of the  $3^3$  full factorial experiment showed that the optimal setting of factor A was its medium setting as the optimal process setting was A = medium, B = low, C = high. This again, shows the advantage of combining the statistical test for curvature with the m-c plot.



**Figure 3.** M-C plot for ST experiment, ■ represents the runs at the factorial points, ● represents the centre point run

Experiment 6b (STF): The statistical test for curvature generated a p-value of 0.106. This indicated curvature was unlikely. On the other hand, the m-c plot (figure 4) showed that the response at the centre point was better than the response associated with the optimal setting of all four factors across the  $2^4$  full factorial design space. Thus, indicating that curvature may be present. A main effect analysis of the  $3^4$  full factorial experiment revealed that the optimal process setting included the medium setting of the cutting

speed (factor A) and the axial depth of cut (factor C) which signified the presence of curvature. On this occasion, the m-c plot proved to be useful as it was able to identify the curvature in the system. Experiments 2, 3 and 6b demonstrate the advantage of combining the statistical test for curvature with the m-c plot when testing for curvature. Even though the statistical test suggested curvature was unlikely in these experiments, the use of the m-c plot improved the chance of identifying curvature in the system.



**Figure 4.** M-C plot for STF experiment, ■ represents the runs at the factorial points, ● represents the centre point run

### 3.2. Comparing the performance of the OAs and SFP

Table 4 compares the results of the OAs, the SFP and the 3-level full factorial design for all the experiments investigated, as well as the number of experimental runs needed for the experimental plans. In table 4, L/B (larger-the-better) signifies that a larger response value was desired and S/B (smaller-the-better) signifies that a smaller response value was desired.

#### 3<sup>3</sup> full factorial experiments (experiments 1, 2a, 2b and 3):

In these experiments, the 18-run OA performed as well as or better than the 9-run OA. In experiment 3, the 9-run OA produced a result of 1026N and the 18-run OA produced a result of 882N. From the point of view of array efficiency, the 18-run OA produced a better result as its result was the same as that obtained from the main effect analysis of the 3<sup>3</sup> full factorial experiment. For the SFP, it performed as well as or better than the 18-run OA with a reduced number of experimental runs across all 3<sup>3</sup> full factorial experiments.

#### 3<sup>4</sup> full factorial experiments (experiments 4, 5, 6a and 6b):

In experiment 4 and 5, the 18-run OA performed as well as the 27-run OA. In

experiment 6a, the main effect analysis of the 3<sup>4</sup> full factorial experiment produced a result of 0.280µm and the best response across the 3<sup>4</sup> full factorial design space was 0.245µm. In this experiment, the 18-run OA produced a result of 0.315µm while the 27-run OA produced a result of 0.245µm. In this case, the result of 0.245µm produced by the 27-run OA is by chance as it was not obtained from an analysis of the interaction effects. In experiment 6b, the best response across the 3<sup>4</sup> full factorial design space was 0.480µm. In this experiment, the 18-run OA produced a result of 0.520µm which corresponded to the result obtained from a main effect analysis of the 3<sup>4</sup> full factorial experiment. On the other hand, the 27-run-OA produced a result of 1.083µm which was worse than that of the 18-run OA. For the SFP, with the exception of experiment 6a, it performed as well as or better than the 27-run OA with a reduced number of experimental runs and; in experiments 5 and 6a it performed better than the 18-run OA with a reduced number of experimental runs.

With the exception of experiment 6a, the SFP performed as well as or better than the 9, 18 and 27-run OAs across all experiments (3<sup>3</sup> and 3<sup>4</sup> full factorial experiments). Compared to the OAs, an advantage of performing a full factorial experiment at the high and low factor settings is the identification of the optimal factor setting due to anti-synergistic

interactions within this design space. For instance, in the damage factor experiment (experiment 5), using 17 experimental runs, the SFP identified the optimal process setting resulting from anti-synergistic interactions which produced a result of 1.1383 $\mu$ m as curvature was not detected in the system. On the other hand, both the 18 and 27-run OA produced the same result of 1.1401 $\mu$ m with 18 and 27 experimental runs respectively. Because the OAs are not full factorial designs, the experimenter cannot identify for certain responses due to anti-synergistic

interactions. Also, the interaction tests of the 27-run OA may or may not identify them. Cases may exist when the optimal process setting produced by the main and interaction effect analysis of the OAs is present in their design matrix. In such a case, a comparison can be made between the optimal process settings identified from the main and interaction effect analysis to the optimal process setting across the OA design matrix. The better response can then be selected based on the comparison.

**Table 4.** Summary of the results from the OAs, the SFP and the 3-level full factorial design

<b>Exp. 1 (XA). Exp. plans</b>	<b>9-run OA</b>	<b>18-run OA</b>	<b>SFP</b>	<b>Full factorial</b>
Response (units in U/ml) (L/B)	<b>15.81</b>	<b>22.4</b>	<b>22.45</b>	<b>22.45</b>
Run size	<b>9</b>	<b>18</b>	<b>13</b>	<b>27</b>
Exp. 2a (NS). Exp. plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>27</b>	<b>27</b>	<b>31.60</b>	<b>31.60</b>
Run size	<b>9</b>	<b>18</b>	<b>9</b>	<b>27</b>
Exp. 2b (NS). Exp. plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>7.51</b>	<b>7.51</b>	<b>6.80</b>	<b>6.80</b>
Run size	<b>9</b>	<b>18</b>	<b>13</b>	<b>27</b>
Exp. 3 (ST). Exp. plans	9-run OA	18-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>1026</b>	<b>882</b>	<b>1249</b>	<b>1249</b>
Run size	<b>9</b>	<b>18</b>	<b>13</b>	<b>27</b>
Exp. 4 (SR). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>0.460</b>	<b>0.460</b>	<b>0.460</b>	<b>0.460</b>
Run size	<b>18</b>	<b>27</b>	<b>17</b>	<b>81</b>
Exp. 5 (DF). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>1.1401</b>	<b>1.1401</b>	<b>1.1383</b>	<b>1.1383</b>
Run size	<b>18</b>	<b>27</b>	<b>17</b>	<b>81</b>
Exp. 6a (SAF). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>0.315</b>	<b>0.245</b>	<b>0.270</b>	<b>0.245</b>
Run size	<b>18</b>	<b>27</b>	<b>17</b>	<b>81</b>
Exp. 6b (STF). Exp. plans	18-run OA	27-run OA	SFP	Full factorial
Response (units in U/ml) (L/B)	<b>0.520</b>	<b>1.083</b>	<b>0.520</b>	<b>0.480</b>
Run size	<b>18</b>	<b>27</b>	<b>25</b>	<b>81</b>

A second advantage of the SFP over the 18 and 27-run OA is that in minimising the chances of exploring insignificant design spaces by means of the tests for curvature, the process performance can be improved by identifying anti-synergistic interactions with nine less experimental runs than the 18-run OA (for 3<sup>3</sup> full factorial experiments), one

less experimental run than the 18-run OA (for 3<sup>4</sup> full factorial experiments) and ten less experimental runs than the 27-run OA (for 3<sup>4</sup> full factorial experiments). Furthermore, the SFP will require less experimental runs than running two full factorial experiments at the high and low factor settings and, the medium and the best settings from the full factorial at

the high and low factor setting. For  $3^3$  experiments, this strategy requires fewer experimental runs than the 18-run OA as it uses 15. Comparing the performance of this strategy to the OAs and the SFP (for  $3^3$  experiments) showed it produced the same result as the SFP.

In using the SFP, the most important factor across the first 2-level full factorial experiment and the full factorial involving the medium settings of the factors and their best setting from the first full factorial experiment may not be the same. In such a case, changing the most important factor to its medium setting at the best settings of other factors from the first full factorial experiment may produce sub optimal results as the response at the medium setting of the most important factor may be affected by anti-synergistic interactions. To minimise this, the factor settings should be evenly spaced out when possible. This can reduce the chances of choosing factor settings that do not reflect the true importance of the factors. Furthermore, if the most important factor obtained from the full factorial experiment at the high and low factor settings varies from that obtained from the 3-level full factorial experiment, the performance of the SFP may be affected by the choice of the most important factor.

A disadvantage of the SFP compared to the OAs is that the optimal factor setting due to main effects in the full factorial experiment at the high and low factor settings may differ from those obtained from a 3-level full factorial experiment due to interactions. This

is more likely to affect small main effects. This disadvantage also applies to the full factorial experiments involving the high and low factor settings and, the medium and the best settings from the full factorial at the high and low factor setting. In such situations, when curvature is present and anti-synergistic interactions do not determine the optimal process setting, the OAs may outperform the SFP.

#### 4. Conclusion

In summary, the SFP provides an option for economic experimentation without neglecting the influence of interactions on the system response. Although the SFP has its disadvantages, the SFP can be useful in situations where resources are scarce and process optimisation with minimal amount of resources is the primary objective.

Future research will focus on identifying ways to better quantify the performance of the SFP. One way to do this is by characterising the relative probabilities of interactions which act opposite to and in the direction of main effects when the main effects are positively and negatively exploited. This way, the performance of the SFP can be quantified when the most important factor in the 3-level full factorial experiment and the full factorial involving the medium settings of the factors and their best setting from the first full factorial experiment are the same or otherwise.

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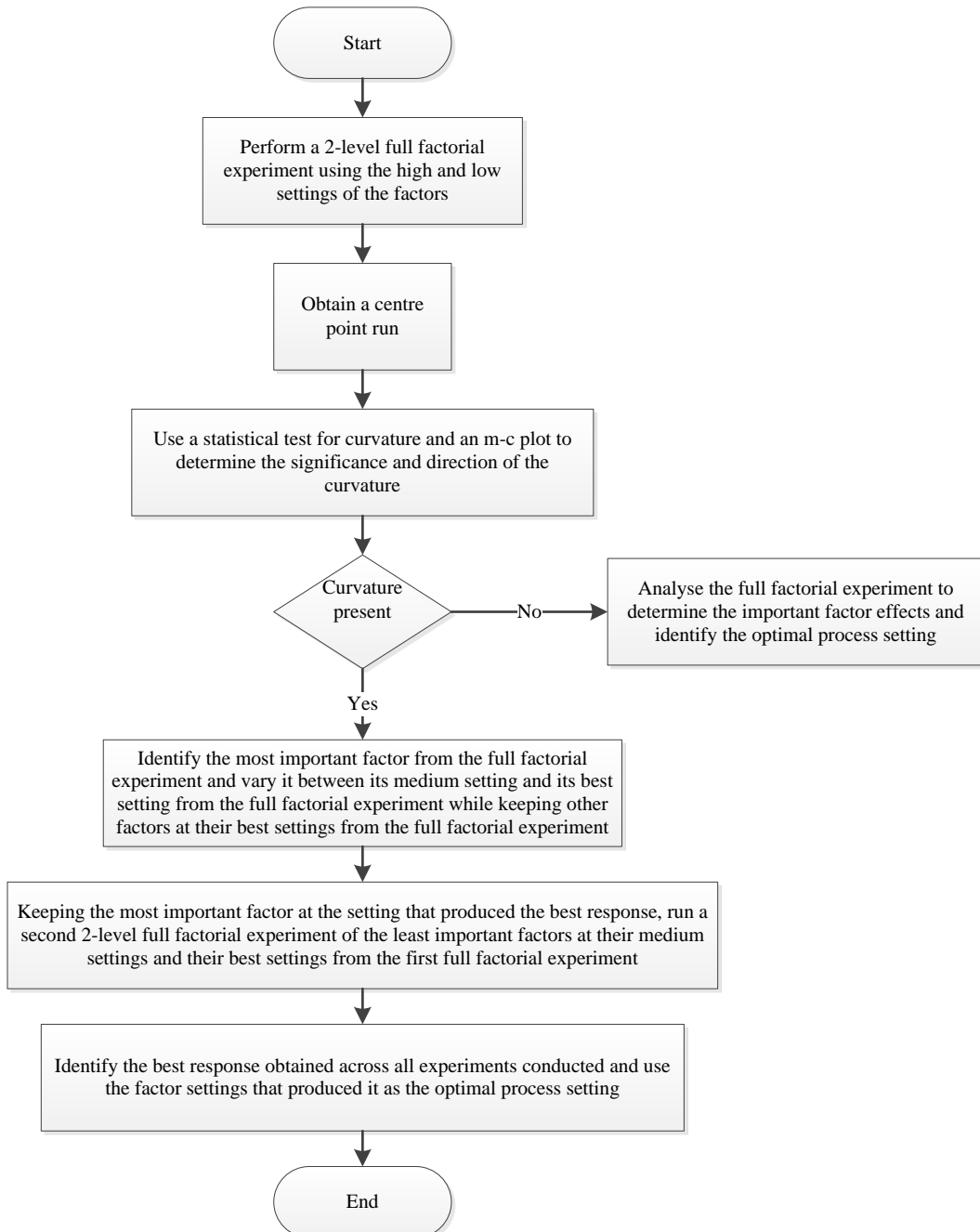
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**Appendix:**



**Figure 2.** A flow chart of the segmented fractional plan



