Spectral Analysis of Convolutional Coded DPIM for Indoor Optical Wireless Communications

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***Abstract*—Power spectral density (PSD) of the convolutional coded digital pulse interval modulation (CC-DPIM) is derived analytically based on the stationarization of variable-length word sequence (VLWS). To verify the theoretical model developed, CC-DPIM is simulated in Matlab and its spectrum is compared with the predicted results showing a close match. As in DPIM scheme, CC-DPIM spectrum also contains a distinct slot (clock) component which can be recovered at the receiver for both slot and symbol synchronization.**

I. Introduction

A number of modulation techniques have been proposed and studied for use in indoor as well as outdoor optical wireless communication links to meet the key requirements of power and bandwidth efficiencies. Equally powerful characteristics when selecting a modulation scheme in particular pulse modulation schemes is the power spectral density (PSD). Spectral evaluation of a data sequence relates to the statistical and stationary properties of the transmitted waveform. For a fixed length encoded messages such as the on-off keying (OOK) and the pulse position modulation (PPM), and the variable length digital pulse interval modulation (DPIM) the PSDs have been well studied and reported in the literature [1-6]. In [7] CC-DPIM scheme capable of detecting and correcting errors proposed for the diffuse indoor optical wireless links have been investigated. However, to the best of our knowledge no work has been reported on the spectral characteristics of the CC-DPIM scheme. In this paper for the first time, we theoretically investigate the PSD of the proposed CC-DPIM and verified the approach adopted by means of Matlab simulation.

Methods of evaluating the PSD of a signal include the periodogram technique to compute the spectral characteristics of dual header pulse interval modulation (DHPIM) as outlined in [8]. The autocorrelation function of a cyclostationary process for the DPPM scheme has been calculated to estimate the spectrum of the transmitted pulse stream in [9]. The method of stationarization is also evaluated in [10] and the theory of variable length random processes (or VLWS) has been developed in [11, 12] where stationarization of the vectors with random lengths is achieved by considering the vector length probabilities.

The rest of this paper is organized as follows: Section II outlines the basic concept of DPIM and CC-DPIM and shows their corresponding symbol structures. Section III outlines the theoretical approached adapted to evaluate the spectral property of CC-DPIM. In section IV spectral plots and discusses are given, whereas in section V the concluding remarks are presented.

II. Symbol Structure

DPIM is a modulation technique in which each block of $\log_2 L$ data bits $\{d_m\}$ is mapped to one of $L = 2^M$ possible symbols $\{D_m\}$, each different in length with the m^{th} symbol having the structure of $D_m = [D_m^0, D_m^1, \ldots D_m^{L_m-1}]$. Each symbol of variable length L_m begins with a pulse of one slot duration (or "1", i.e. $D_m^0 = 1$) followed by a number of empty slots (or "0", i.e. $D_m^k = 0,\ 0 < k < L_m$) corresponding to the decimal value of the block of data bits being encoded. The mapping of data bits into 4-DPIM using no guard slot (GS) is depicted in Table I.

The average symbol length of the coded data $\Gamma^{-1} = \sum_{\lambda} \lambda\, \mathrm{P}[L_m = \lambda]$, where P[.] is the probability function and $\lambda \in \mathrm{I} = \{L_0, L_1, \ldots, L_{L-1}\}$.

For L-DPIM with no GS with identically independent distribution (i.i.d), $\mathrm{P}[L_0 = \lambda] = L^{-1}$, and $\Gamma^{-1} = \frac{1}{L}\sum_{1}^{L} \lambda = \frac{1}{L}\frac{L(L+1)}{2} = \frac{L+1}{2} = L_{avg}$. The slot duration defined in terms of data bit duration is $T_s = T \log_2 L / L_{avg}$.

With a code rate of ½ and a constraint length of 3, the convolutional coded DPIM with 2GSs is also shown in Table I, where each symbol start with a unique pattern of {11 10 11} followed by a number of zeros representing the input data as outlined in [7]. For CC-DPIM with $L = 2^M$ symbols, the symbol length is $\lambda \in \mathrm{I} = \{6, 8, \ldots, 2(L+2)\}$ and the average symbol length with an i.i.d is $L + 5$. Hence, $\Gamma = (L+5)^{-1}$.

Table I illustrates the symbol structure for the DPIM and CC-DPIM schemes.

Table I.
DPIM and CC-DPIM symbol structures for 2-bit input data

M-bit Input data, $\{d_m\}$	4-DPIM, D_m of length L_m		
	NGS	2GS	CC-DPIM (2GS)
00	1	1 0 0	**11 10 11**
01	1 0	1 0 0 0	**11 10 11** 00
10	1 0 0	1 0 0 0 0	**11 10 11** 00 00
11	1 0 0 0	1 0 0 0 0 0	**11 10 11** 00 00 00

III. POWER SPECTRAL DENSITY

A DPIM pulse train may be expressed as [12]:

$$x_c(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_s) \tag{1}$$

which is cyclostationary, where $p(t)$ is the rectangular pulse shape, T_s is the slot duration and $a_n \in \{0,1\}$ for all n is a set of random variables that represent the presence or absence of a pulse in the n^{th} time slot.

$x_c(t)$ can be stationarized with the introduction of a continuous variable φ to give $x_s(t) = x_c(t + \varphi)$, where φ is equally distributed over $[0, T_s]$ and is independent of a_n. The distribution of stationarization depends on the length probabilities given as:

$$p(k) = L_{avg}{}^{-1} \sum_{\lambda > k} \mathrm{P}[L_0 = \lambda]. \tag{2}$$

The general expression for the spectral distribution expressed by the spectral density is given as: [11, 12]

$$R_{vs}(f) = \frac{1}{T}|P(f)|^2 \left[R_c(fT) + \sum_{m=-\infty}^{+\infty} \Delta F_c(f_m T)\delta(f - f_m) \right] \tag{3}$$

where T is the input period of the $\{a_n\}$, $P(f)$ is the Fourier transform of $p(t)$, and $|P(f)|^2 = T^2\, Sinc^2(fT)$.

The continuous spectrum of $\{a_n\}$ is defined as:

$$R_c(u) = \Gamma\left(C(z) + \Lambda\Gamma|A(z)|^2 + 2\Re[A(z)B(z)^*]\right) \qquad z = e^{i2\pi u} \tag{4}$$

and the discrete part of the spectrum (spectral lines) is given by:

$$\Delta F_c(u_m) = |\Gamma A(z_m)|^2,\ z_m = e^{i2\pi u_m}, u_m = m/\Lambda \tag{5}$$

and by considering equations (20) from [12], we can obtain for CC-DPIM:

$$A(z) = 1 + z + z^2 + z^4 + z^5$$
$$B(z) = \sum_{\lambda} V_\lambda(z) m'_\lambda \left[X(z^\Lambda) z^{-\lambda} - \Gamma\Lambda U_\lambda(z^\Lambda)\right] \tag{6}$$
$$C(z) = (1 + z + z^2 + z^4 + z^5)(1 + z^{-1} + z^{-2} + z^{-4} + z^{-5})$$

Where

$$X(z) = \Gamma\Lambda h(z)/g(z),$$
$$h(z) = \sum_{\lambda} p_\lambda \sum_{k=0}^{\lambda/\Lambda - 1} (\lambda/\Lambda - k) z^{-k}, \tag{7}$$
$$g(z) = \sum_{\lambda} p_\lambda \sum_{k=0}^{\lambda/\Lambda} z^{-k},$$
$$U_\lambda(z) = \sum_{k=0}^{\lambda/\Lambda} z^{-k}.$$
$$p_\lambda = \mathrm{P}[L_m = \lambda],$$
$$V_\lambda(z) = [1, z, \ldots, z^{\lambda-1}],$$

and Λ is the greatest common divisor.

The mean vector $m_\lambda = \mathrm{E}[D_m; L_m = \lambda] = p_\lambda \beta_\lambda$, and β_λ is the set of words with a length λ defined as:

$$\beta_\lambda = \left[\beta'_{\lambda 0}, \beta'_{\lambda 1}, \ldots \beta'_{\lambda N_\lambda - 1}\right], \tag{8}$$

where $\beta_{\lambda n} = [\beta^0_{\lambda n}, \beta^1_{\lambda n}, \ldots, \beta^{\lambda-1}_{\lambda n}]$, $n \in \{0, 1, \ldots, N_\lambda - 1\}$, where N_λ is the number of words with length $\lambda \in \mathrm{I}$, where for CC-DPIM and DPIM, N_λ = 1. Since the symbol length is even then $\Lambda = 2$.

IV. RESULTS AND DISCUSSION

Figure 1(a) shows the predicted PSD of the 8-CC-DPIM using (3-7), assuming that pulse shape $p(t)$ is rectangular with 100% duty cycle. The spectrum displays a sinc function profile with a distinctive clock (slot) frequency and its harmonics at odd multiples, and nulls at even multiples of the slot frequency. The nulls at normalized frequencies $(fT)_0 = \pm 1, \pm 2, \ldots$ are poles on the unit circle, followed by two symmetrically close poles on both sides at $fT = (fT)_0 \pm 0.15$. The spectral density (4) is a rational function of z and can be factored to two casual and stable functions of z and z^{-1} as $R_c(z) = H(z)H(z^{-1})$. To build $H(z)$ one can derive poles and zeros of the PSD and extract those inside unit circle ($|z_i| < 1$). Having this information we can build filter $H(z)$ which can be implemented as an Auto Regressive Moving Average (ARMA) filter.

The presence of the slot component suggest that a simple phase locked loop could be employed to recover the clock signal form the incoming CC-DPIM symbol stream for slot synchronization at the receiver. Unlike PPM, the PSD does not fall to zero at DC region similar to the DPIM. Thus, implying that it too would also be susceptible to the effect of baseline wander due to high-pass filtering of the ambient light at the receiver [13-15]. To confirm the theoretical analysis, the proposed scheme was simulated in Matlab and its estimated PSD is depicted in Figure 1(b), showing a close match to the predicted PSD.

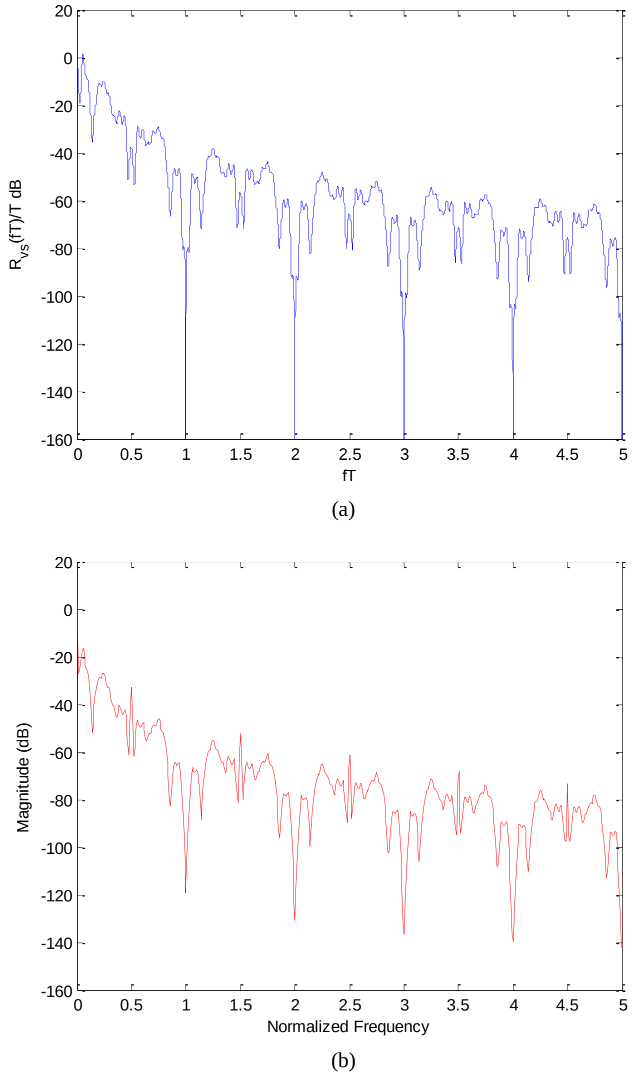

Figure 1. PSD of 8-CC-DPIM with 100% pulse duty cycle against the normalized frequency: (a) predicted, and (b) simulated

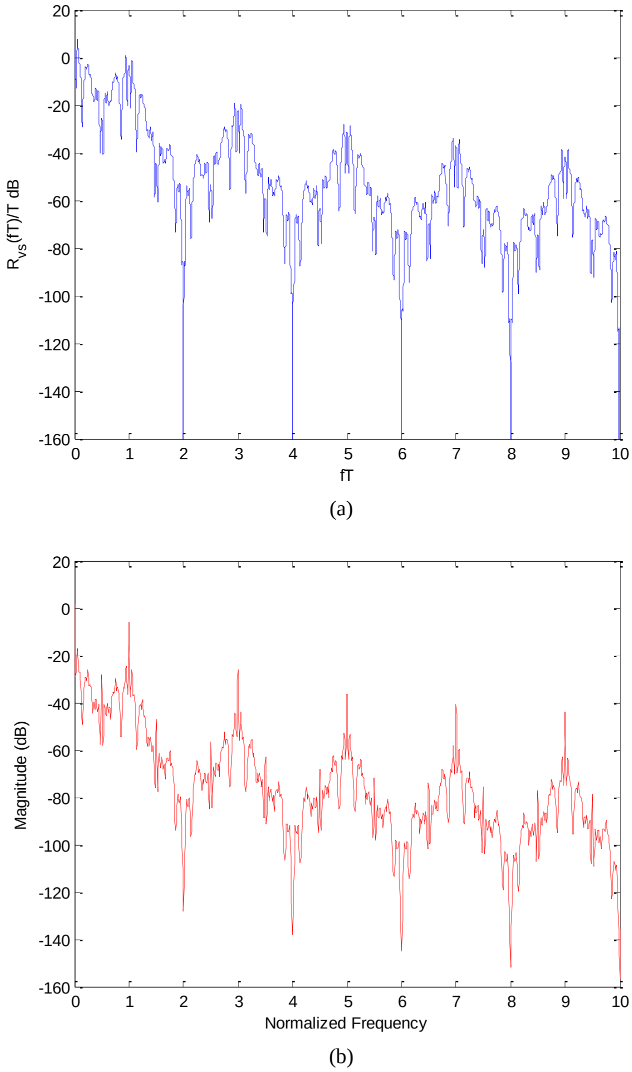

Figure 2. PSD of 8-CC-DPIM with 50% pulse duty cycle against the normalized frequency: (a) predicted, and (b) simulated

Figure 2 shows the PSD plots for the CC-DPIM with the header pulses having 50% duty cycle. The profile is the same as in Fig. 1 except for the first zero crossing being shifted to the right. In both figures, the slight difference in the profiles is due to the fact that in the theoretical analysis both the discrete and continuous components of the spectrum are separated while in the simulation these components are combined.

Figure 3 compares the spectrum of 8-CC-DPIM and 8-DPIM. For the normalized frequency $u = 0$, we can compute $A(z) = 5$, $C(z) = 25$ and $B(z) = -1.8047$. By incorporating these values into (3) the DC component $R_{vs}(0)/T_0 = 0.8307$ is obtained as shown in Fig. 3. The DC component is 14.4219 times as high as the 8-DPIM DC value of 0.0576. Note the PPM scheme has no DC component, a desirable characteristics in optical wireless communications.

In most scenarios, the data sequence which generates the pulse stream is either wide sense stationary (WSS) or wide sense cyclostationary (WSCS). In both cases the timing jitter induced by the noise while propagating through the channel will introduce frequency offset, thus affecting the continuous components of the spectrum. This will result in a more complex clock recovery scheme at the receiving end.

The discrete components (delta functions (5)) corresponding to the mean of the sequence, are due to the data asymmetry. This can be reduced or eliminated if the mean is zero. For bipolar input data $a_n \in \{1,-1\}$ the discrete components are no longer present in spectrum.

V. Conclusion

Power spectral density of the convolutional coded digital pulse interval modulation was derived analytically based on the stationarization of variable-length word sequence. The predicted and simulated spectrums were compared showing a close match. The spectrum contains a slot (clock) component which could be extracted at the receiver for slot and symbol synchronizations, but it also has a DC component which is higher than that of DPIM and PPM.

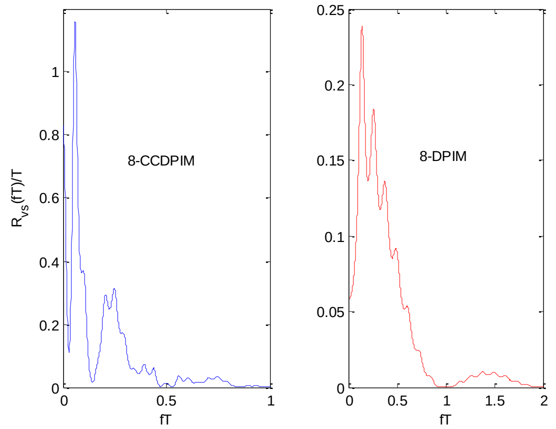

Figure 3. Spectrum comparison between DPIM and CC-DPIM.

APPENDIX

DERIVATION OF (6)

The probability of two words $\{D_m\}$ has the distance of k digits in the symbol sequence $\{a_n\}$ can be introduced by the cumulative length probabilities which satisfy the following difference equation:

$$y(k) = \sum_{\lambda} p_{\lambda} y(k-\lambda), \quad k > 0 \tag{9}$$

the sequence $y(k\Lambda)$ converges to $y(\infty) = \Lambda\Gamma$ and the sequence $x(k) = y(k\Lambda) - y(\infty)$ is absolutely summable with *z-Transform*:

$$X(z) = \sum_{k=0}^{\infty} [y(k\Lambda) - y(\infty)] z^{-k} = \Gamma\Lambda \frac{h(z)}{g(z)} \tag{10}$$

The continuous spectrum of the sequence $\{a_n\}$ is evaluated by the equation (4) where:

$$\begin{aligned} A(z) &= \sum_{\lambda} V_{\lambda}(z) m'_{\lambda} \\ B(z) &= \sum_{\lambda} V_{\lambda}(z) m'_{\lambda} \left[X(z^{\wedge}) z^{-\lambda} - \Gamma\Lambda U_{\lambda}(z^{\wedge}) \right] \\ C(z) &= \sum_{\lambda} V_{\lambda}(z) \beta'_{\lambda} diag[p_{\lambda}] \beta_{\lambda} V'_{\lambda}(z^{-1}) \end{aligned} \tag{11}$$

where ‘ is the transpose and *diag* is the diagonal matrix.

For CC-DPIM:

$\beta_{\lambda} = [1110110\ldots0]$, $\Gamma = \dfrac{1}{L+5}$, $\Lambda = 2$

$p_{\lambda} = \dfrac{1}{L}$ (All symbols are equally likely)

$m_{\lambda} = \mathrm{E}[D_m; L_m = \lambda] = p_{\lambda}\beta_{\lambda} = \dfrac{1}{L}\beta_{\lambda}$

$V_{\lambda}(z) = [1, z, \ldots, z^{\lambda-1}]$

Hence, by using these into (11) we can obtain (6).

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