The discharge of the pipelines supplying public fountains in Roman Pompeii

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ABSTRACT

The public fountains of the ancient Roman town of Pompeii were supplied through pipelines descending from the nearby water towers. The water discharge was determined by the height of the water towers as well as the hydraulic features of the pipes, including the average diameter and the internal surface roughness. In the present study the possible connections between each fountain and the nearby water towers are assessed. Based on available elevation data and measurements taken in various survey campaigns, a range is defined for the pipeline hydraulic slope. The discharge of the pipelines is estimated through a graphical method, which can conveniently be applied by scholars of non-technical background; values of the diameter between those of the ancient Roman *digitus* and *denaria*, and values of the hydraulic absolute roughness k_s between 0.1 and 0.5 mm are considered. For 47 connections of fountains to water towers, the discharge is calculated to be between 0.1 and 2.5 l/s. The original height of the towers 12 and 13, not reconstructed after the 62-63 CE earthquake, is estimated as close to 6 m. The values of discharge in the pipelines generally confirm the figures obtained in 2021 by Monteleone *et al.*, in a study on the discharge of the Pompeian fountains overflow channels.

Keywords: Pompeii water supply; Roman fountains supply; *lacus*; Roman lead pipelines; Pompeii fountains

Declaration of interest: The authors declare that there is no special personal/financial interest from any of the people/institutions related to the production of this paper, that could affect the objectivity of the statements, methods and results presented.

Introduction

The aim of this paper is to use an engineering approach to amplify the considerations provided by archaeologists on the public *lacus* fountains of the Roman city of Pompeii, verifying what would have worked hydraulically, and what would not. The paper is structured as follows. The background sections briefly consider previous work on the discharge of water from Roman fountains and summarize the authors' previous work on the Pompeian *lacus*. The methodological sections of our paper cover the causes of head loss in ancient lead pipes, presenting experimental work recently completed. We introduce a graphical method for identifying the discharge of a pipeline, connecting two points of known elevation, which can be used by professionals of non-technical background. We provide an application of sensitivity analysis, considering the uncertainties in the input data which translate in a range of possible results for the discharge. After applying the graphic method to various possible connections, we compare the results obtained on the fountain supply with the overflow figures calculated in our 2021 work. Finally, we draw a range of conclusions which we trust will be useful for scholars researching on water use and water demand in Pompeii, as well as in other ancient Roman towns.

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Previous quantitative studies on Roman fountains

The distribution of water in the ancient Roman towns was accomplished through a variety of structures, interconnected in a network. Construction materials and complexity varied, adapting to the hydrological features of the areas and the morphology of the surface terrain.

The quantities of water supplied to the fountains within the urban areas have been estimated for some medium size and large size monumental fountains, supplied by channels and terracotta pipelines. For example, Ortloff and Crouch (2001) calculated for the Fountain House in Ephesus a supply of 38 l/s; Tuttahs (2007) estimated the flowrate supplied to the II century *nymphaeum* in Miletus as 53 l/s; Pisani-Sartorio *et al.* (2011) provided the value of 200-300 l/s for the supply of the *Nymphaeum Alexandri* in Rome; Vekemans and Haut (2017) estimated the maximum discharge of the *Nymphaeum* and *Euripus* channel at Perge as 196 l/s. The high values for the discharge were justified by the volume of the open basins, ranging between 20 and 300 m^3[2].

Till present, there are no published studies on the operation of the smaller size *lacus*, consisting of a stone basin supplied by a pipeline, with a volume usually smaller than 7-8 m^3[3]. Agusta-Boularot (2008) and Schmölder-Veit (2009) described in detail the remains of fountains found in many Roman towns. In some cases, pipes have been found near the fountains, for example in Herculaneum (Fig. 1a and b) and Lyon (Fig. 1c); however, estimates for the pipeline discharges were not provided.

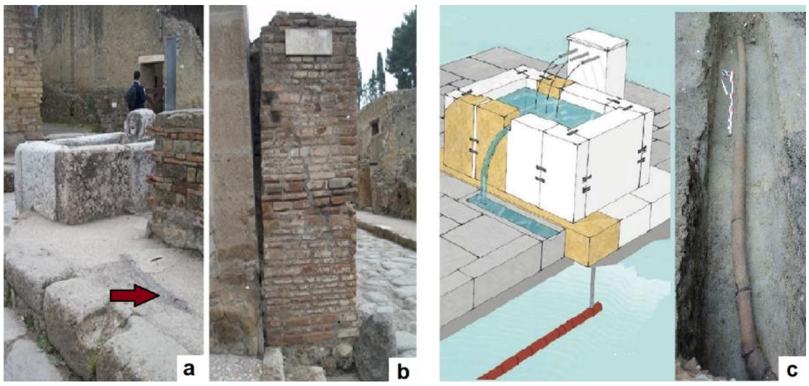

Fig. 1. Examples of the simple *lacus* fountains found with remains of the supply pipelines nearby: (a) Neptune fountain in Herculaneum; the arrow indicates the lead pipeline; and (b) the water tower from which it departed (photos by the authors); (c) reconstruction of the fountain found in Lyon Place d'Albon, including the possible connection to the terracotta pipeline, shown in the picture to the right (Ramona, 2019).

Public fountains and water towers in Pompeii

Pompeii is one of the western Roman towns where the simple *lacus* are abundant and well preserved. The public fountains identified in 1983 by Eschebach and Schafer were 42, of which 35 are in the form of a *lacus*; an additional basin was uncovered during the recent excavations in Regio V[4]. The fountain basins are composed of stone slabs assembled to contain an average volume of 0.80 m^3. The supply pipelines are not preserved: the vertical elements placed to protect the pipes, show the grooves that contained them. The pipelines terminated at the orifice still visible in the spout stone, placed on the basin rim. Fig. 2 b, c and d show the typical Pompeian *lacus*.

[2] One of the largest known *lacus* is the Tritons' *Nymphaeum* in Hierapolis, for which Campagna e Scardozzi (2013) gave dimensions of 57.15 x 4.70 m, corresponding to a volume of around 269 m^3.
[3] For example, from the dimensions given by Ramona (2019, Fig. 23), the volume of the *lacus* in Lyon could have ranged between 4.8 and 6.2 m^3.
[4] The pictures were published in the official blog of Pompeii's archaeological park (www.facebook.com/pompeiisoprintendenza/photos/a.1523717371268809/2245875905719615); the description of the structures has not yet been published at the time of writing.

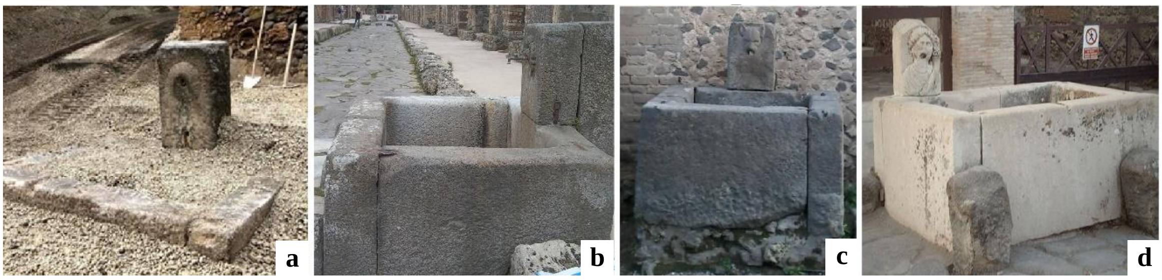

Fig. 2. Public *lacus* fountains in Pompeii: (a) the recently excavated fountain in Vicolo dei Balconi of Regio V (https://www.facebook.com/pompeiisoprintendenza/photos/pcb.2245877522386120/2245875905719615); (b) fountain in via di Nola; (c) fountain in Vico dei Soprastanti; (d) the so called 'Fontana di Cerere' in via dell'Abbondanza (photos by the authors).

The water towers have been known as 14 in number, until the 2020 discovery of a new tower in Regio V, close to the mentioned fountain.

Fig. 3 shows, on an aerial map, the location of the 43 public fountains, identified with the Eschebach and Schafer catalogue number, and the 15 water towers. The blue boxes indicate the *lacus*, while the pale-yellow boxes indicate other types of public fountains[5]. The water towers are shown with orange boxes; the two towers (12 and 13) damaged in the 62-63 CE[6] earthquake and not restored to their original height, are in yellow. The green colour (numbers 15 and 44) identifies the recently excavated fountain and water tower of Regio V.

Fig. 3. Aerial map of Pompeii's archaeological site, showing the location and numbering for the fountains and the water towers.

Fountain basin overflow data

The dimensions of the spout orifices were surveyed by the authors, so that an estimate for the diameters of the original supply pipes was provided, assuming a pipe thickness of 5 mm (Monteleone

[5] Those with other layouts are the fountain within the arch at the NE corner of the *forum* (25), the fountain within the Arch of Caligula (15), the no longer visible fountain in the Palestra (36), the *labrum* in the Triangular Forum area (35) and the three street fountains a that have a spout stone but not a front basin (11, 30 and 38).

[6] The exact date for the earthquakes is Pompeii remains uncertain; Keenan-Jones (2015) noted that the 64 CE earthquake in Naples could have produced effects in Pompeii, and the interval 62-64 CE should be considered to identify changes in the Vesuvian area's hydrology.

et al., 2021). The average internal pipe diameter is compared in Fig. 4 with the standard diameters of the Roman pipes in the range *digitus* to *denaria* described by Frontinus[7].

It can be seen how the supply pipe diameter varies between 1.5 and 5 cm; however, 15 out of 28 values are close to the *senaria* and *settenaria* sizes, which are regularly found along the Pompeian streets, also supplying private properties. 26 out of 28 values indicate diameters in the range *senaria* to *denaria*. Nappo (2002) found various stretches of pipes directed towards the fountains, but never preserved up to the basin; he was tempted to conclude that all pipes supplying fountains were *denariae*; however, as will be demonstrated later in this paper, the difference of level existing between the top of the towers and the spout orifice axis required in some instances the use of small diameter pipes, such as the *digitus*, if the assumptions presented on the surface roughness of the ancient pipes are correct.

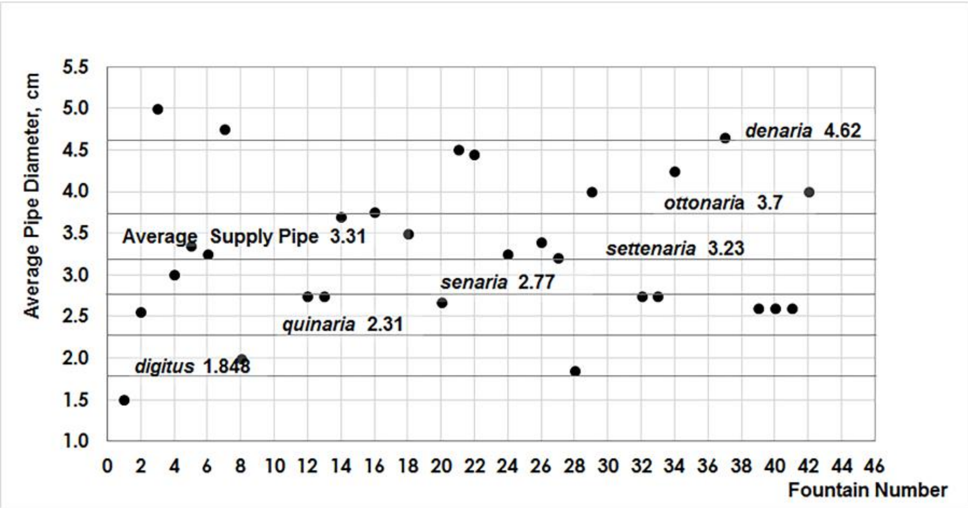

Fig. 4. Diameters of the supply pipes for the Pompeian *lacus*, estimated from the orifice size, identified in the chart as dots; the horizontal grey lines identify the standard Roman pipe diameters from *digitus* to *denaria* (Monteleone *et al.*, 2021).

In 2021 Monteleone *et al.* described the geometry of the overflow channels carved on the top rim of the stone slabs; the discharge of the channel was calculated for various water levels at the channel inlet cross section (1 cm, half the cross-section height and full cross-section height), as reported in Table 1.

Table 1. Values for the overflow channel discharge (Q), calculated for three water levels at the channel inlet: 1 cm above the channel bed, half the channel cross section height, and full inlet cross section (Monteleone *et al.*, 2021).

Fountain	Q (1 cm) (l/s)	Q (H/2) (l/s)	Q (H) (l/s)
1	0.04	0.41	1.73
2	0.08	0.24	0.76
3	0.12	0.66	2.04*
4	0.1	0.14	0.45
5	0.03	0.44	2.08*

[7] *De Aquis* 37-63, in Rodgers (2004). In this study the values summarised by Nir-El (2017) are taken as a reference.

7	0.11	0.44	1.35
8	0.11	0.26	0.82
9	0.09	0.46	1.55
12	0.09	0.21	0.64
13	0.13	0.95	2.92*
14	0.11	0.62	1.85
16	0.04	0.33	1.22
17	0.06	0.38	1.33
18	0.05	0.19	0.76
20	0.03	0.12	0.55
21	0.08	0.30	1.03
22	0.06	0.67	2.35
23	0.15	0.49	1.43
24	0.20	0.96	2.91*
26	0.08	0.33	1.15
27	0.06	0.64	2.79*
28	0.10	0.43	1.39
29	0.06	0.15	0.49
32	0.04	0.16	0.75
33	0.06	0.36	1.14
34	0.13	0.33	0.98
37	0.12	0.80	2.50*
39	0.07	0.22	0.68
40	0.09	0.50	1.76
41	0.07	0.72	2.42*
42	0.08	0.35	1.18
Average	0.09	0.43	1.45

*These values remain improbable, since at this discharge the water jet would have surpassed the basin width.

The pipelines connecting water towers to fountains

Nappo (2002) recorded the diameter and depth below the footway for some sections of pipelines directed towards fountains 8, 9 and 28; with the help of a metal detector, Jansen (2002) could trace the pipeline connecting water tower 9 to fountain 27 in Vico della Maschera. The two scholars agreed on the fact that in Pompeii the pipes serving the fountains, as well as those serving the private houses, were not part of a branched pipe network, but each end user was assigned its unique supply pipe, directly attached to the tank at the top of a water tower[8].

Only for fountain 27 in Vico della Maschera is the supply pipe visible as a short stretch emerging from the footway (Fig. 5a, b and c). Considering that the cross section external minimum and maximum dimensions are 3.4 and 5.4 cm, and that the minimum and maximum dimensions of the spout orifice are 3.5 and 4.9 cm, the visible pipe might have continued with the same cross section up to the spout. Small fragments of the pipe are also attached to the orifice surface[9]. Fig. 5 also shows the spout stone of the newly excavated fountain in Regio V (Fig. 5d), in which a small diameter pipe is inserted in the spout orifice. It was not possible to access this fountain to verify if the small diameter pipe is connected to a larger diameter pipe at the level of the fountain base.

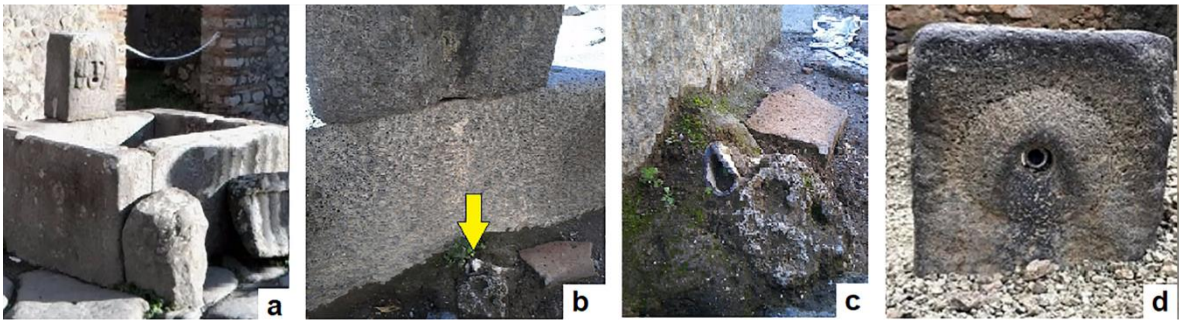

Fig. 5. Remains of supply pipes: (a) Fountain 27 in Vico della Maschera, (b) remains of the supply pipe at the back and (c) details of the pipe cross section; (d) detail of the spout of the newly excavated fountain in Regio V showing the small pipe inserted in the spout. Photos a, b and c taken by the authors; picture d from the Pompeii archaeological park webpage (www.facebook.com/pompeiisoprintendenza/photos/a.1523717371268809/2245875812386291).

Traces of small diameter pipelines, descending on the side of the tower, are visible on the south side of tower 7 (Fig. 6a); three stretches of pipeline can be seen along the footway south of the tower (the tower is indicated by the arrow in Fig. 6b); on the wall close to the north side of the tower there are imprints of many pipes (Fig. 6c); some of the pipes are preserved at the base of the wall and (Fig. 6d).

[8] On this issue see also Dessales (2013), p. 234-236. Pompeii might be a good example of the application of the Roman law regulating the water concessions '*ex castellis'* introduced in 11 BCE Rome through a *senatus consultum*, as described in *De Aquis*, 106. Regarding pipes branching off from the main pipelines, only one case is known of the large diameter (around 18 cm internal average diameter, measured recently by the authors) stretch of pipeline found at about 130 m distance from the *castellum divisorium*, towards the SE corner of the *insula* VI 16 by Maiuri (1931); he described the smaller branch, of average internal diameter 10 cm, as going towards the *fullonica* at number 4. The *fullonica* water supply is currently being studied by one of the authors in collaboration with other scholars and results will be published in the near future. It can be noted, once again, that the branch does not involve a pipe directed to a fountain and therefore does not exclude the hypothesis of a unique supply pipe, which is at the base of the present study.

[9] These are small fragments, less than 3 cm wide and with a thickness smaller than 4 mm. Although Stanco (2009) suggested that the supply pipes terminated in bronze spouts of smaller diameter for the fountains of Alifae, there is no evidence of the use of similar spouts in Pompeii.

Regarding the lead tanks placed on the top of the towers, only pictures of the tank found on tower 6 in the 1917 excavation records exist (Fig. 7a and b)[10]. The tank, 65 cm wide and 56 cm tall, would have matched the size of the tower cross section as shown in Fig. 7c.

Fig. 7d shows the scheme adopted in this study for the supply pipelines: a unique supply pipeline descends vertically from the water tower top tank, turns horizontal through a large 90° bend, and runs at a short depth below the footway surface, passing under the road at a greater depth and continuing up to the fountain spout with unvaried cross section area.

Regarding the location of the pipelines, in 2015 Olsson proposed the fountain to tower connections shown in Fig. 8a; in this study we will analyse the connections shown in Fig. 8b.

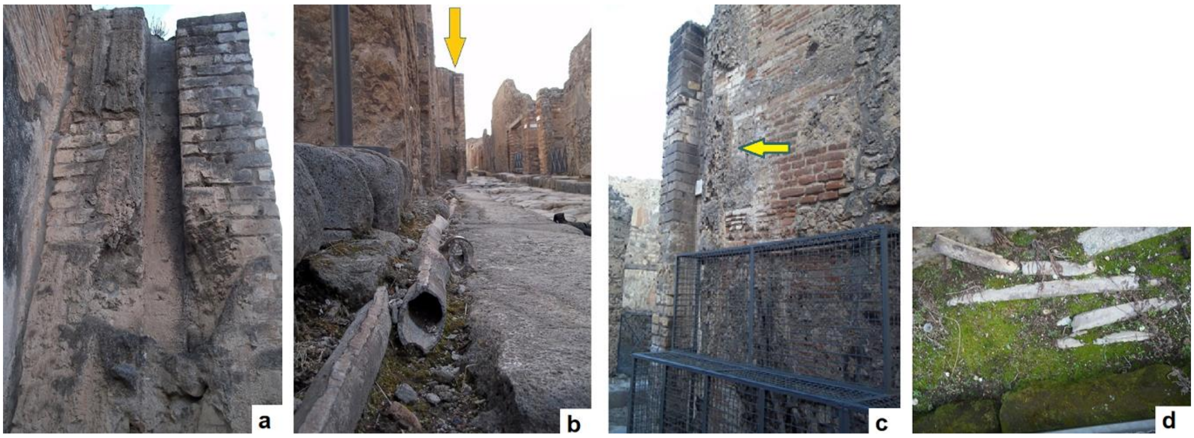

Fig. 6. (a) View of water tower 7 from south, showing sinter deposits with the imprints of many lead pipes; (b) remains of three small diameter pipelines on the footway south of the same tower; (c) view of the sinter deposits and pipe imprints on the wall close to the north side of the tower; (d) stretches of the pipelines preserved near the base of the wall. Photos by the authors.

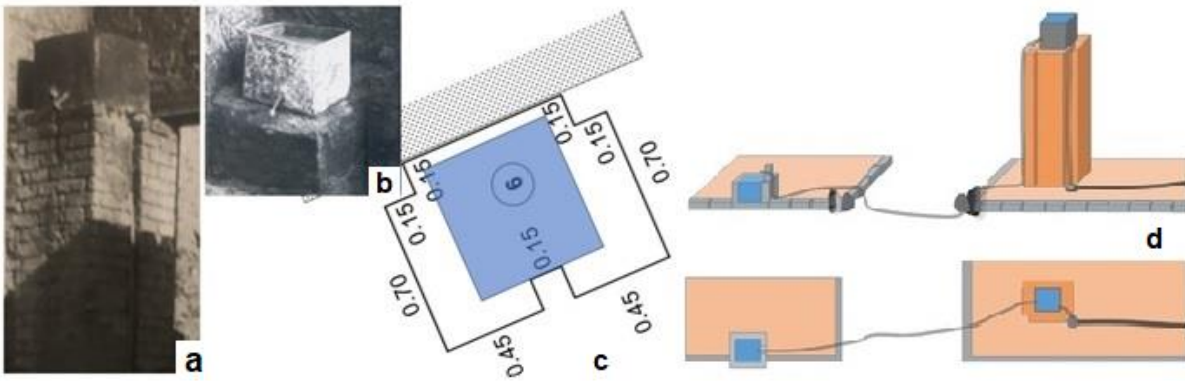

Fig. 7. (a,b) Photos of the lead tank found on top of tower 6 (Spinazzola 1917); (c) size of the tank compared with the cross-section of the tower (data on the tower from Olsson 2015); (d) scheme for the unique fountain-to-tower connection assumed in this study.

[10] Spinazzola (1917), p. 255 and fig. 7. The tank is no longer in place, but the remains of a similar tank are visible in the site stores.

Fig. 8. (a) Connections between fountains and water towers proposed by Olsson (2015); (b) connections between fountains and water towers assumed in this study. When two or more connections are considered, the most probable is identified by a continuous line, the others by a broken line.

When more than one connection is considered for the same fountain, the most probable is shown as a continuous line, the others as dashed lines. For fountain 23, four connections are shown. The connections take into consideration the 14 known water towers; however, some fountains were probably connected to towers that either remain unexcavated or have been destroyed: fountains 26 and 33 to a tower in the southern area of the *Forum*[11]; fountains 2 and 37 to a tower in the unexcavated area of Regio I; fountain 39 and 42 to a tower in the unexcavated area of Regio IX. Although the connection of fountain 10 to tower 14 is shown in Fig. 8, it was not possible to survey this fountain due to building work, and it has not been included in the discharge analysis. Fountains 20 and 21, located in the highest grounds of Via del Vesuvio and Vicolo dei Vettii, could also have been connected directly to the *castellum divisorium*[12]. Fountain 31 has not been included in the

[11] Although Eschebach (1979) mentioned an above ground reservoir at the SE corner of the building of Eumachia and at the SE corner of the Forum Basilica, we could not assess those remains in this study.

[12] It is known from Ohlig (2001) that the *castellum* original layout was different from the visible one, therefore some direct connection to the fountains might have been included.

present analysis, since it presents features similar to fountains 11, 30 and 38, which are of the type without a basin[13].

Hydraulic roughness of the Roman lead pipes

To calculate the discharge of an ancient lead pipeline through a commonly used equation, such as the Darcy-Weisbach equation, a value for the pipe friction factor or absolute roughness is needed. In previously published studies, modern engineers assumed different values for the pipe hydraulic roughness. For example, Lockett (1991) used a value of 0.5 mm for the absolute roughness of the Lyon Gier lead siphons; Burdy in 2002 calculated the discharge of the same lead siphons using the Darcy equation, with values of the coefficients[14] corresponding to an absolute roughness of around 3 mm; Dickers[15] (2002), assumed for the lead pipelines connecting the water towers of Via Stabiana in Pompeii a friction factor of 0.014, corresponding to an absolute roughness lower than 0.1 mm.

Lead pipes were commonly used in European towns for public water distribution in the 1800s; Darcy, in his 1857 experiments[16], tested three lead pipes[17], including more than four joints of the olive type similar to those found in Pompeii (Fig. 9). From his published head loss data, the absolute roughness was in the range 0.045 to 0.098 mm. The pipes were new with no deposits; although the diameter and the type of joints were similar to those of the Pompeian pipes, the pipes did not present the longitudinal soldering bead, so Darcy's range for the absolute roughness should be considered lower than that of the Pompeian pipes.

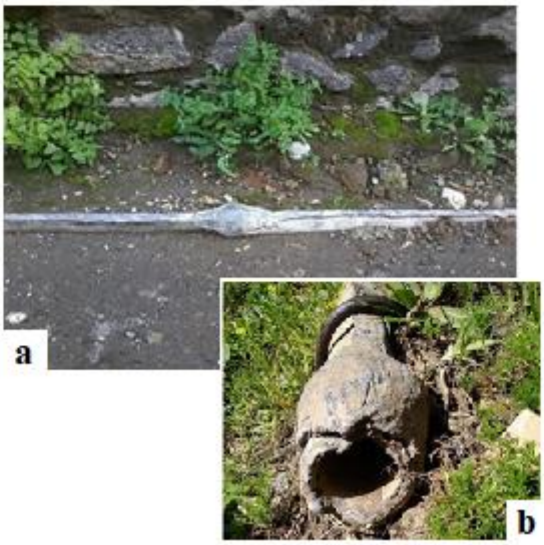

Fig. 9. Joints of the olive type in use in Pompeii: (a) pipeline lying on the eastern footway of Vico della Maschera; (b) remains of a joint of the pipeline located between the reservoir and the *euripus* in the *Praedia* of Julia Felix.

The ancient pipes were formed by soldering the two edges of a lead sheet to form a longitudinal welding bead of various types[18].The irregularities of the soldering bead and joints resulted in values for the frictional head losses higher than those measured for modern lead pipes, industrially manufactured.

[13] These fountains were possibly supplied intermittently with the excess water from a private or public building reservoir. The long distance of these fountains from the visible water towers could support this assumption. Nevertheless, there is not enough information to identify the original source of supply. For the discussion on the fountains without a front basin in Pompeii see also Monteleone *et al.* (2021), note 13.
[14] $A = 0.000507$, $b = 0.00001294\ \text{m}^{-1}$.
[15] In Jansen (2002), Appendix 3.
[16] Darcy (1857), p. 78.
[17] 52.5 m long, with diameters 0.014, 0.027 and 0.041 m.
[18] Cochet and Hansen (1986), pp. 23-34.

An endoscope was used to inspect two pipes from Pompeii[19] and two elements of a pipeline found at the Red House-Beaufront bath house, near Corbridge Roman fort in Northumberland, England (Daniels 1959)[20]. The surface away from the welding bead showed a homogeneous texture, with limited sinter deposits (Fig. 10 a and b). Along the internal surface of the soldering bead, lump masses were unevenly distributed (Fig. 10 a, b and c)[21]; the depth of the protrusions ranged from 3 to 6 mm for the Pompeian pipes and 4 to 9 mm for the Corbridge pipes.

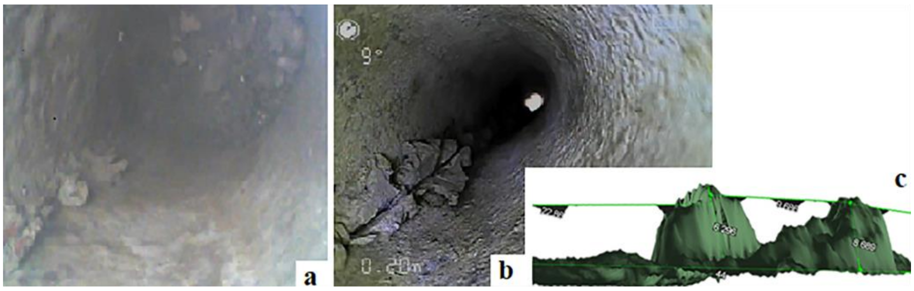

Fig. 10. Endoscopic images of the internal surfaces of Roman lead water pipes: (a) pipe visible close to the reservoir in the garden of the *Praedia* of Julia Felix in Pompeii; (b) stretches preserved from a pipeline found in Corbridge Red House Farm bath house; (c) 3D scan of the replica of a portion of the welding bead surface, showing the lump masses of welding metal.

An area of the surface close to the end cross-section of one of the Corbridge pipes was replicated through dental putty (Fig. 11a, b); the replica was scanned with the Alicona InfiniteFocusG5 3D scanner. The surface scan (Fig. 11c) was analysed through the postprocessing software Alicona MeasureSuite 5.3, to obtain the surface profiles shown in Fig. 11d and e, the first one showing the primary surface profile, the second one the microscopic roughness only. A similar analysis was conducted for the two Pompeian pipes.

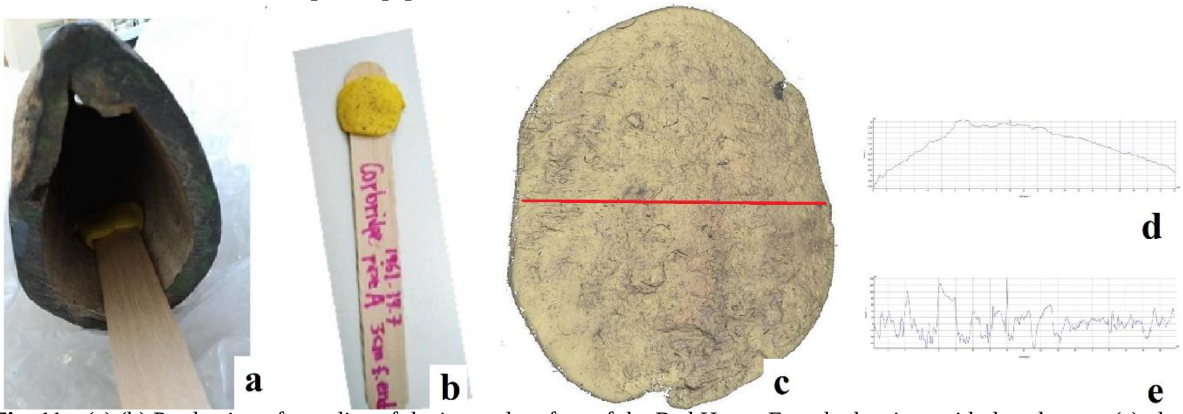

Fig. 11. (a) (b) Production of a replica of the internal surface of the Red House Farm baths pipes with dental putty; (c) photo of the scanned surface replica; (d) surface profile along the marked red line; (e) profile showing the surface microscopic roughness only.

Various statistical surface parameters are identified in the standard ISO 25178-6 to describe the surface roughness; although research is still in progress on this topic, some authors found that the root mean square roughness R_q and the mean peak to valley roughness R_z correspond to the Nikuradse

[19] Pipeline visible on the walkway north of the entrance of Casa dell'Efebo I.VII.12, and pipe visible south of the reservoir in the NW area of the peristyle of the *Praedia* of Julia Felix.
[20] The Society of Antiquaries of Newcastle upon Tyne (UK), which stores in its collections three stretches of the pipeline, kindly allowed the authors to inspect the pipes and conduct experimental work in the hydraulics laboratory of Northumbria University.
[21] In the Pompeian pipes, of average diameter 2.9 to 5.5 cm, the maximum dimension of the masses was on average 4 mm, spaced on average 7 mm apart; in the case of the Corbridge pipe (average diameter 4.5 cm – 4.9 cm), the masses had a height up to 9 mm, and were spaced on average at 2.3 cm.

absolute roughness k_s[22]. To compare the pipe internal surfaces with a reference surface coated with uniform size particles, similar to that identified by Nikuradse (1933), two samples of sandpaper P120 and P240 were also scanned with the same 3D scanner. The average values obtained from eight profiles for each surface sample are shown in Table 2.

Table 2. Values of the surface roughness parameters estimated for Pompeii and Corbridge pipes, and for samples of sandpaper, for the purpose of determining the absolute hydraulic roughness k_s.

Surface parameter	**Average roughness**	**Root mean square roughness**	**Mean peak to valley height**
Profile identification	**mean R_a µm**	**mean R_q µm**	**mean R_z µm**
Pompeii 1	26.638	108.969	158.951
Pompeii 2	26.879	122.100	142.431
Corbridge 1	23.307	158.778	148.555
Sandpaper 120	23.128	122.041	139.110
Sandpaper 240	13.931	65.119	90.048

The values of R_a, R_q and R_z for the two Pompeian pipes and for the Corbridge pipe remain quite similar; the sandpaper P120 surface parameters are closer to the values for the pipes, than the P240 parameters. The P120 particles nominal size is between 115 and 125 µm, or 0.115-0.125 mm.

The effect of the uneven soldering bead on the head losses could be assessed only through hydraulic experiments; for this purpose, some tests were performed in the years 2019- 2020[23], in the hydraulics laboratory of Northumbria University (Crapper *et al.*, 2022). Water was pumped with moderate pressure through two pipe stretches from the Corbridge pipeline, one of which had a sleeve joint at the centre. The head loss between the two ends of the pipe was recorded for various values of the flowrate, estimating a value for the absolute roughness k_s of about 0.9 mm, an order of magnitude higher than the values obtained from the surface scanning, suggesting a significant head loss effect from the soldering bead; the coefficient [24]for the sleeve joint was estimated as 1.159.

The Corbridge pipe joint, soldering bead and general manufacturing were much coarser than those of the Pompeian pipes. On this basis, for the purpose of this study, the range for the Pompeian pipes absolute roughness was assumed to be 0.1 - 0.5 mm. The concentrated head losses at the olive type joints located at intervals of 2.7 – 2.8 m[25] along the pipeline axis were not accounted separately but included as head losses distributed along the pipeline length.

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[22] For a general review on the topic see Flack and Schultz (2010). There is not yet a good agreement on the direct relation between the surface parameters and the hydraulic absolute roughness; various authors have experimented on pipes of different materials, diameter and roughness range, proposing various equations. Farshad *et al.* (2005) and Adams *et al.* (2012) agreed that the very isolated and highest peaks and valleys have the most significant effect on the head losses, therefore the R_q and R_z parameters have been preferred, rather than the arithmetic mean height R_a.
[23] The Society of Antiquaries of Newcastle granted permission to borrow the pipes for two undergraduate dissertation projects, supervised by Prof. M. Crapper.
[24] The coefficients K_i, included in equations 3 and 4, identify the fraction of kinetic energy that is dissipated within the length of the joints and fittings.
[25] The Roman pipes were fabricated in 10 Roman feet lengths, corresponding to 2.944 m (Poehler and Ellis 2014). In Pompeii also the Oscan foot was found in use, corresponding to 27.534 cm (Schoonhoven 2006, 36-

Regarding the fittings present along the pipeline, values for the concentrated head losses coefficients were assumed as those for modern fittings[26], increased by 20% (Table 3). Two conical junction boxes have been found at the base of water towers 4 and 6[27] (Fig. 12); there is evidence that they were placed at the base of the grooves, possibly connecting the pipelines connecting water towers, and not those supplying the street fountains, for which in this study it was assumed that a large radius 90° bend allowed the transition from vertical to horizontal at the tower base. Considering that, to reach the footway on the other side of the road, the pipeline bent down to run at higher depth below the road paving, a 0.80 coefficient was applied for each road underpass.

Table 3. Values of the concentrated head loss coefficients used in this study for the various fittings.

Fitting	Inlet	Short radius bend	Long radius bend	Road underpass
Coefficient	0.65	0.55	0.45	0.80

Fig. 12. Conical junction box at the base of tower 6.

Tabled values of the friction factor (Colebrook-White equation) for Roman lead pipes

The Darcy-Weisbach equation[28] relates the head loss due to friction per unit length of pipeline, or hydraulic slope *S*, to the friction factor λ, the pipe diameter *D* and the mean flow velocity *V*:

$$S = \frac{\lambda V^2}{2gD} \tag{1}$$

38): the pipeline visible on the east pavement of Vico della Maschera is composed of 12 pipes of length between 2.62 m and 2.68 m, close to 10 Oscan feet.

[26] See for example, Kreith 2000, p. 52.

[27] The one below tower 6 is still in place in the pillar recess; the other, described by Lanciani (1975) as found below tower 4, was connected to pipes of diameters 11 and 16 cm, much larger than the diameter of the fountains pipe; this junction box can be seen in the Forum Baths store.

[28] See for example, Chadwick *et al.* (2013), 111.

The friction factor λ is related to the ratio between the absolute roughness and the pipe diameter $\frac{k_s}{D}$, as well as to the average velocity and characteristics of the fluid through the Reynolds number $\mathrm{Re} = \frac{\rho V D}{\mu}$, where ρ and μ are water mass density and dynamic viscosity, respectively. The most used equation for the friction factor is the Colebrook-White equation[29]:

$$\frac{1}{\sqrt{\lambda}} = -2\log\frac{K_s}{3.7D} + \frac{2.51}{Re\sqrt{\lambda}} \qquad (2)$$

The above equation can be calculated for all the diameters of the ancient Roman pipes, defined by Frontinus[30] and summarised by Nir-El (2017). To each value of the absolute roughness K_s between 0.1 and 0.5 mm at 0.1 mm increments, and each value of the flow velocity in the range 0.01 to 1.3 m/s at 0.05 m/s increments, corresponds a value for the friction factor λ calculated from equation (2); from the value λ the hydraulic slope S is calculated through equation (1) while the flowrate is obtained as the product of the average velocity and the cross-sectional area of the pipe. Table 4 shows, for the Roman pipes *digitus* to *denaria*, the values corresponding to an absolute roughness of 0.1 mm and values of the velocity between 0.05 and 0.40 m/s.

In the excel workbook available at the DOI: 10.25398/rd.northumbria.21626414, tables are given for the values of λ and S corresponding to velocities in the range 0.05- 3 m/s, for each value of the absolute roughness K_s between 0.1 and 0.5 mm.

Table 4. Example of the tabled values of the friction factor, head loss per metre and flowrate obtained by applying the Colebrook-White equation to the range of Roman pipes *digitus* to *denaria*, for flow velocities between 0.05 and 0.40 m/s.

e=0.1 mm	e/D	D (m)	λ lim	Velocity (m/s)	0.05	0.10	0.15	0.20
Digitus	0.0054	0.0185	0.0311	Reynolds n.	710.77	1421.54	2132.31	2843.08
				Friction f. $\boldsymbol{\lambda}$	0.0957	0.0634	0.0526	0.0472
				Hyd. Slope $\boldsymbol{S}$ (m/m)	0.0007	0.0017	0.0033	0.0052
				Flowrate (l/s)	0.01	0.03	0.04	0.05
Quinaria	0.0043	0.0230	0.0291	Reynolds n.	884.62	1769.23	2653.85	3538.46
				Friction f. $\boldsymbol{\lambda}$	0.0896	0.0594	0.0493	0.0442
				Hyd. Slope $\boldsymbol{S}$ (m/m)	0.0005	0.0013	0.0025	0.0039
				Flowrate (l/s)	0.02	0.04	0.06	0.08
Senaria	0.0036	0.0277	0.0618	Reynolds n.	1065.38	2130.77	3196.15	4261.54
				Friction f. $\boldsymbol{\lambda}$	0.0849	0.0562	0.0467	0.0419
				Hyd. Slope $\boldsymbol{S}$ (m/m)	0.0004	0.0010	0.0019	0.0031
				Flowrate (l/s)	0.03	0.06	0.09	0.12
Settenaria	0.0031	0.0323	0.0579	Reynolds n.	1242.31	2484.62	3726.92	4969.23
				Friction f. $\boldsymbol{\lambda}$	0.0812	0.0538	0.0447	0.0401
				Hyd. Slope $\boldsymbol{S}$ (m/m)	0.0003	0.0008	0.0016	0.0025
				Flowrate (l/s)	0.04	0.08	0.12	0.16
Ottonaria	0.0027	0.0370	0.0547	Reynolds n.	1423.08	2846.15	4269.23	5692.31
				Friction f. $\boldsymbol{\lambda}$	0.0782	0.0518	0.0430	0.0386
				Hyd. Slope $\boldsymbol{S}$ (m/m)	0.0003	0.0007	0.0013	0.0021
				Flowrate (l/s)	0.05	0.11	0.16	0.21
Denaria	0.0022	0.0462	0.0239	Reynolds n.	1776.92	3553.85	5330.77	7107.69
				Friction f. $\boldsymbol{\lambda}$	0.0736	0.0488	0.0405	0.0363
				Hyd. Slope $\boldsymbol{S}$ (m/m)	0.0002	0.0005	0.0010	0.0016
				Flowrate (l/s)	0.08	0.17	0.25	0.34

[29] Chadwick *et al. Op.Cit.*, 113
[30] See note 6.

Sensitivity analysis - graphical method for estimating discharge

When the values of the head loss per unit length *S* are plotted against the values for the flow velocity, for Roman pipes in the range *digitus* to *denaria,* a curve is obtained for each value of the hydraulic roughness, as shown in Fig. 13a. Corresponding to each value of the velocity, the curves for the discharge of Fig. 13b are obtained. The charts of Fig. 13a and 13b include pipes *digitus* and *quinaria,* and values of the hydraulic slope up to 1.4 m/m; the charts including pipes in the range *senaria* to *denaria* and values of the hydraulic slope up to 0.20 m/m are displayed in appendix, Fig. A1.

The tabled data obtained from the application of the Colebrook-White equation, the charts and the method described in this section are also published as excel worksheets in Northumbria University depository with the DOI: 10.25398/rd.northumbria.21626414

The charts can be used for the determination of the discharge of a pipeline of length *L*, extending between two points of elevation H_1 and H_2. As an example, the discharge for the pipeline connecting fountain 1 to tower 4 will be calculated.

The general energy equation defining the water motion between the two points is

$$S = \frac{(H_2 - H_1) - (1 + \sum_{i=1}^{i} K_i)\frac{V^2}{2g}}{L} \tag{3}$$

where H_2 and H_1 are the physical elevations of water level in the tower tank and of the fountain spout axis, *V* is the mean velocity in the pipe (which depends on diameter and hydraulic roughness) and K_i [31] are the coefficient defined in Table 3[32], depending on the losses at bends and various fittings. The head losses in a pipe for a given flow are thus determined from:

$$\Delta H_1 = (\sum_{i=1}^{n} K_i + 1)\frac{V^2}{2g} \tag{4}$$

When dealing with archaeological remains, uncertainties remain on the original inlet and outlet elevations, as well as on the exact length of the pipeline; therefore, it is preferrable to estimate possible ranges of values, defined by a minimum and maximum value and to calculate the corresponding range for range for the pipeline discharge. The principles of sensitivity analysis[33] have been increasingly applied in archaeology[34] to evaluate the output of a simulation, when parameter values or input data are varied.

In our case, accounting for the uncertainty on the pipeline geometry and elevation data, a range for the slope S between the endpoints is derived; then, with the help of the charts, considering that pipes of various diameter and roughness could have been in place, a range for the pipeline discharge is estimated.

[31] In the case of the connection of fountain 1 to tower 4, one inlet, three long radius bends and two short radius bends are considered.

[32] In the case of the pipelines connected to the water towers, the inlet and two large radius bends ($\sum C_i$= 1.55) will always be present at the departure from the water tower, while two short radius bends ($\sum C_i$=1.1) will always be present when the pipe rises to the fountain spout. Other losses can be added, depending on the path of the pipeline along the footways.

[33] Defined for example in Frey *et al.* (2002) or Brouwer Burg *et al.* (2016).

[34] See, for example, Kanters *et al*., 2021.

In equation 4 the head losses all depend on the flowrate, so an iterative calculation is necessary to determine the velocity and the discharge: after defining the discharge corresponding to the maximum available slope and neglecting the head losses in fittings, the values of the velocity are used to calculate the maximum fitting head losses and a new value for the hydraulic slope range; the procedure is iterated to find new values for the velocity range and recalculating the corrected hydraulic slope, until the range for the velocity converges to two final values.

In the case of the connection between tower 4 to fountain 1 the maximum available range for the hydraulic slope S is 0.550 to 0.605 m/m. These values are identified on the vertical axis of the chart in Fig. 13a as S_{max1}, S_{max2}, and two horizontal lines (orange dashed lines) are drawn, marking the extreme intersection with the curves at points P_1 and P_2. By drawing vertical lines from these points, two values of velocities V_1 and V_2 (1.72 and 2.98 m/s in our example) are found on the horizontal axis. The average value of the velocity $\overline{V_1} = \frac{V_1+V_2}{2}$, is the maximum average velocity that could be reached, when losses in fittings are neglected and K_i=0 in equation 3.

Based on $\overline{V_1}$, the maximum concentrated losses $\Delta H_1 = (\sum_1^n C_i + 1)\frac{\overline{V_1}^2}{2g}$ are calculated and, through equation 3), two new values of the hydraulic slope $S_1 = \frac{H_1-\Delta H_1}{L}$, $S_2 = \frac{H_2-\Delta H_1}{L}$ (0.430 and 0.501 m/m). By drawing the blue dashed horizontal lines, two new values for the velocity V_3 and V_4 (1.67 and 2.64 m/s) are identified. The average value $\overline{V_2} = \frac{V_3+V_4}{2}$ is the minimum average value of velocity, occurring in the pipes when the head losses are at maximum value.

Further iterations are then carried out, until the difference between two consecutive values of the average velocities is very small; in our examples we chose a difference in the average velocities less than 0.05 m/s, corresponding to a variation of the corresponding flowrates below 6%, which generally is obtained after three or four iterations. In the chart in Fig. 13a the red dashed lines identify the final range for the hydraulic slope (0.45 and 0.5 m/m) and the corresponding velocities V_5 and V_6 (1.7 and 2.7 m/s). The corresponding flowrate range is found from the chart in Fig. 13b, intersecting the vertical lines identifying the velocity range with the discharge curves, at points Q_D and Q_Q. Therefore, if tower 4 was connected to fountain 1 with pipes of diameter *digitus* to *quinaria*, with an absolute roughness between 0.1 and 0.5 mm, the flowrate discharged was probably in the region of 0.45 to 1.12 l/s.

The same method is applied to each of the tower-to-fountain connections of Fig. 8. To estimate the initial range for the hydraulic slope *S*, some considerations on the elevation of the water tower tops, the elevation of the fountain spouts axis and the possible length of the pipelines are presented.

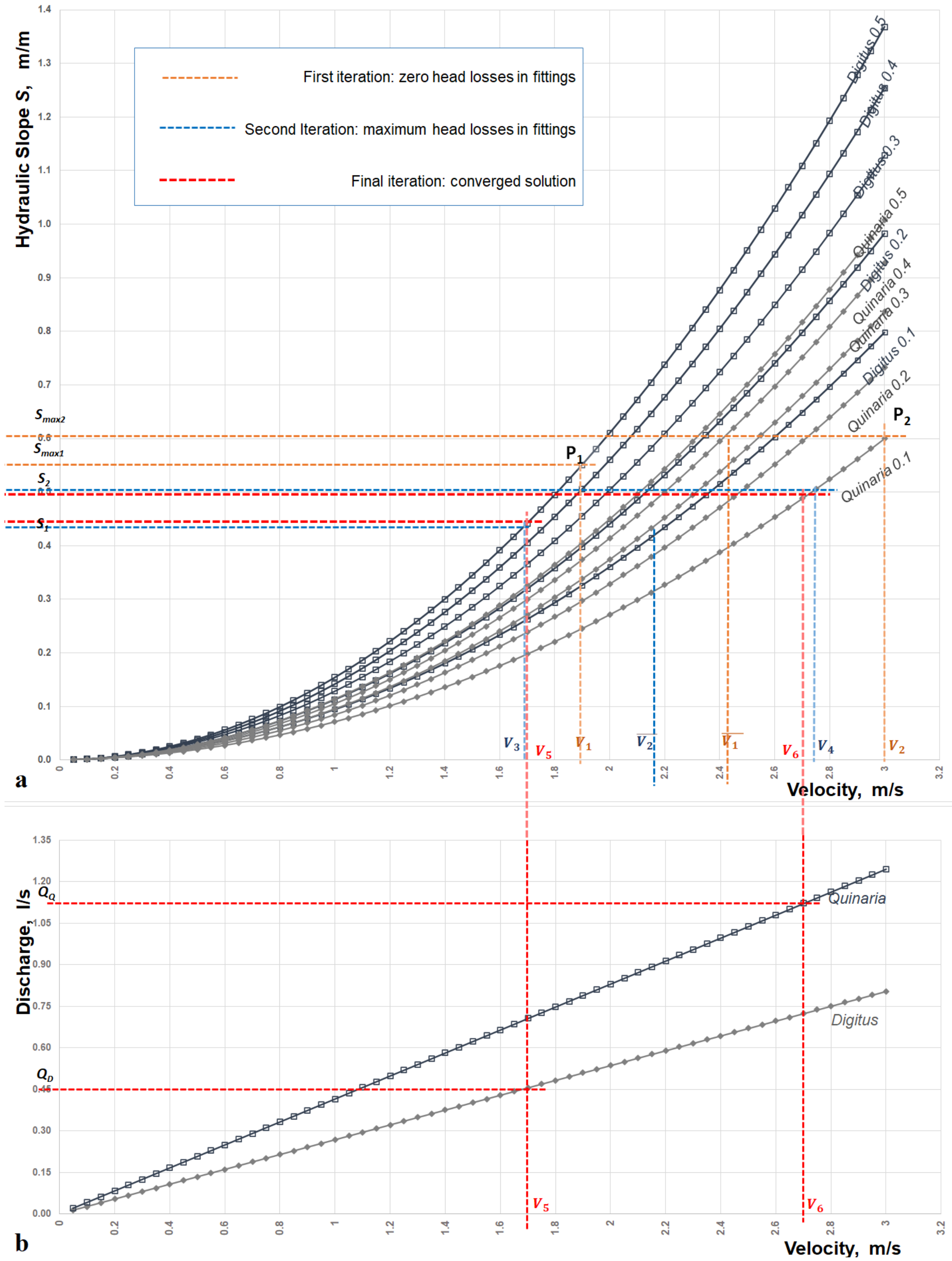

Fig. 13. Method for the graphical calculation of pipeline discharge. Starting from a range for the hydraulic slope *S* on the vertical axis of chart (a), through an iterative procedure, a velocity range is defined, allowing the identification of a range for the flowrate in chart (b). Pipes *digitus* and *quinarian,* hydraulic slope up to 1.4 m/m, absolute roughness k_s in the range 0.1 to 0.5 mm.

Water towers' height – estimates and range of errors

The visible height of the water towers was measured by Dybkjaer-Larsen in 1982, with exclusion of towers 12, 13 and 14 and by Olsson in 2015. Olsson measurements are considered of sufficient accuracy for the purpose of this study, since he made use of triangulation to produce three repeated measurements and gave the values of the standard deviation σ. The values are shown in column 4 of Table 5. The values of the standard error, shown in column 5 of Table 5, were calculated from the standard deviation through the equation $e = \frac{\sigma}{\sqrt{3}}$[35]. The elevation of the water tower top was obtained by adding the tower height to the elevation of a point on the ground, close to the tower base. The elevation of the ground was taken from the data defined in the Piano della Conoscenza di Pompeii (PCP), made available as a GIS shapefile by the officers of Pompeii Beni Culturali SIAV. In a few instances, the point of known elevation was more than 3 m away from the tower, and therefore the estimate of the elevation of the ground at the tower base was less accurate; in other cases, the point was located close to the tower base. To account for this, absolute errors between 0.015 m and 0.1 m, shown in column 3 of Table 5, were assigned to each estimate. The maximum and minimum values for the elevation of the tower top were obtained by adding all the positive errors and the negative errors once each. Moreover, 0.20 m was added to the values obtained, to account for a minimum depth of water in the lead tank on top of the tower. The range of values estimated for the water level at the towers top is displayed in columns 6 and 7 of Table 5.

Table 5. Estimate of the maximum and minimum value for the elevation of the surface of the water contained at the top of each water tower, starting from Olsson (2015)'s measurements and their standard deviation, and accounting for the elevation of the ground.

Tower	Base level		Tower height		Level of water	
	PCP	est. error	Olsson 2015	abs. error	*Hwt max*	*Hwt min*
	m	m	m	m	m	m
1	34.77	0.015	6.67	0.092	41.75	41.53
2	33.02	0.020	6.34	0.196	39.78	39.34
3	29.26	0.020	5.96	0.260	35.70	35.14
4	24.50	0.015	6.15	0.092	30.96	30.75
5	26.00	0.015	3.33	0.040	29.59	29.47
6	23.25	0.025	3.03	0.196	26.70	26.26
7	35.58	0.020	4.37	0.052	40.22	40.08
8	37.71	0.015	2.83	0.052	40.80	40.67
9	32.20	0.020	6.02	0.196	38.64	38.20
10	35.00	0.100	3.15	0.023	38.47	38.23
11	30.54	0.015	5.69	0.035	36.48	36.38
12	39.04	0.030	1.63	0.081	40.98	40.76
13	39.90	0.030	0.80	0.000	40.93	40.87
14	30.40	0.040	5.37	0.092	36.10	35.84

Fountain spout levels – estimates and range of errors

For each fountain, the elevation of the spout was estimated by adding the vertical distance up to the spout orifice, measured during the survey, to the elevation of the footway or road surface. In Fig.14, *hf* indicates the height of the basin top over the footway, *hr* the height of the basin top over the

[35] Everitt and Skrondal (2010), p.409

road surface, and *hs* the height of the spout axis over the basin top. Table 6 contains the values recorded for each fountain.

In those cases when the level of some points on the basin top was available in the GIS shapefile, the elevation of the spout axis could be calculated in multiple ways, and an average value and standard deviation could also be deduced. In other cases, the position of the elevation points was shifted from the object to which they referred, with an offset up to 0.25 m and not in the same direction, which caused further uncertainties. In those cases when a limited number of points on the street or walkway was available, at distances higher than 2.5 m from the basin, a linear interpolation between the values was carried out.

To account for the uncertainty in the estimate of the spout axis elevation, an absolute error (*Ei*) was defined, as reported in the sixth column of Table 6. By summing the errors, the values *Hs min* and *Hs max* were defined (last two columns in Table 6).

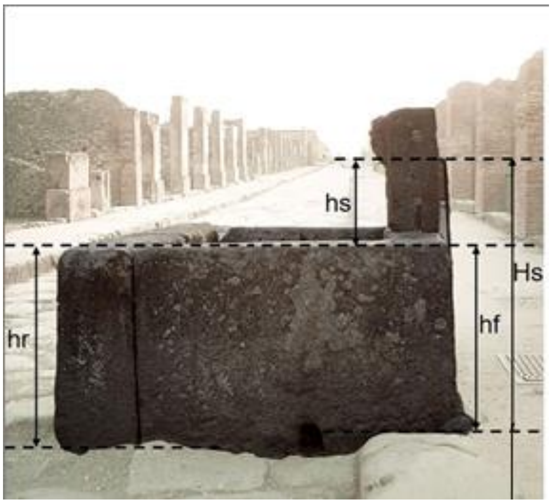

Fig. 14. The vertical distances measured from the basin top and the footway surface (*hf*), the road surface (*hr*), the spout orifice axis (*hs*), and the elevation above sea level *Hs*.

Table 6. Values of the vertical distances measured for each fountain and values of the maximum and minimum spout axis elevation.

Fountain	*hs*, m	*hp*, m	*hr*, m	*Hs*, m	*Ei*,m	*Hs min*, m	*Hs max*, m
1	0.30	0.49	1.05	25.80	0.005	25.795	25.81
2	0.30	0.69	0.77	16.11	0.005	16.105	16.12
3	0.35	0.46	0.76	25.82	0.010	25.810	25.83
4	0.40	0.42	0.80	22.48	0.005	22.475	22.49
5	0.36	0.45	0.73	24.76	0.005	24.755	24.77
6	0.18	0.55	0.80	21.43	0.010	21.420	21.44
7	0.34	0.60	0.83	21.76	0.010	21.750	21.77
8	0.25	0.50	0.78	24.42	0.050	24.370	24.47
9	-	0.56	0.85	23.46	0.020	23.437	23.48
12	0.33	0.78	0.77	40.86	0.040	40.817	40.90
13	0.40	0.61	0.85	39.33	0.040	39.290	39.37
14	0.32	0.40	0.81	39.75	0.020	39.730	39.77
16	0.18	0.43	0.78	33.90	0.060	33.840	33.96
17	-	0.54	0.80	37.32	0.020	37.297	37.34
18	0.28	0.76	-	33.46	0.030	33.430	33.49
19	0.30	0.44	0.84	35.51	0.020	35.490	35.53
20	0.32	0.6	0.84	40.82	0.030	40.790	40.85
21	0.22	0.47	0.80	40.44	0.005	40.435	40.45
22	0.30	0.49	0.87	29.71	0.010	29.700	29.72
23	-	0.63	1.00	34.68	0.050	34.626	34.73
24	0.31	0.86	1.14	37.38	0.050	37.325	37.43
26	0.32	0.62	0.93	30.85	0.020	30.830	30.87
27	0.28	0.65	0.73	31.14	0.020	31.120	31.16
28	0.34	0.62	0.77	26.35	0.005	26.347	26.36
29	0.12	0.70	1.29	34.77	0.005	34.765	34.78
31	0.16	0.40	0.61	34.73	0.100	34.634	34.83
32	0.34	0.67	0.70	32.81	0.050	32.758	32.86
33	0.31	0.40	0.73	33.88	0.050	33.832	33.93
34	0.31	0.56	0.82	25.05	0.080	24.970	25.13
37	0.28	0.70	0.90	10.30	0.080	10.220	10.38
39	0.22	0.44	0.85	29.92	0.060	29.856	29.98
40	0.40	0.56	0.78	32.97	0.005	32.965	32.98
41	0.38	0.60	0.91	31.46	0.010	31.450	31.47
42	0.29	0.59	0.93	27.01	0.005	27.005	27.02

Pipeline length – estimates and range of errors

For each connection of Fig. 8, the length of the pipeline was estimated by assigning a path along the footways in the PCP GIS map and using the ArcGIS-ArcMap tool to determine its length (columns 4 and 5 of Table 7). Since repeated measurements of the same path gave slightly different results, the measurement of the path for three pipelines of 10, 50 and 100 m, was taken 5 times, calculating the standard deviation and standard error. The error increased with the length, therefore the value ±0.005/100 m was adopted. A length of 1.2 to 2cm/100m was added to account for the presence of slope and bends of the original pipelines. For each road crossing 1.5m of length was added. Finally, to obtain the length of the pipeline from the tower top inlet to the fountain spout, the vertical distances *hs* and H_f defined in Fig. 15 were added with their errors, as well as the height of the water towers defined in columns 4 and 5 of Table 5, with their errors. Once again, by adding all the distances and errors, the minimum and maximum pipeline lengths were obtained, as shown in the two last columns of Table 7.

Fig. 15 shows how many fountains are located at various distances from the water towers, based on the values of columns 4 and 5 (distance on plan) in Table 7. Seven fountains remain within 11 m from a water tower while 29 fountains, out of 36, are located within 130 m. If only the most probable connections are considered, the longest distance might have been around 115 m, with 26 fountains lying within this distance.

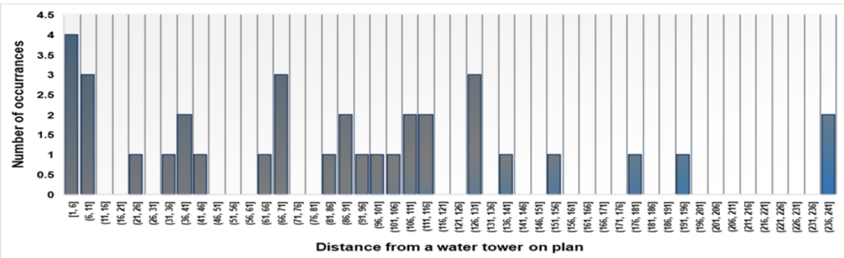

Fig. 15. Distances (in metres) between fountains and water towers, based on the lengths L_{-I} and L_{-II} of table 7

Table 7. Pipeline length estimated for each suggested connection fountain-to-tower.

Fountain	Tower connected		Distance on plan		Pipeline length			
	I	II	L_I	L_{II}	$L_{I\ min}$	$L_{I\ max}$	$L_{II\ min}$	$L_{II\ max}$
1	4		1.75		8.60	8.96		
2	5	4	238.70	249.79	249.13	253.68	262.82	269.42
3	5		86.80		92.99	94.79		
4	5	4	66.60	193.54	72.66	74.09	203.40	208.59
5	6	5	105.03	166.70	112.39	114.83	175.29	179.69
6	6		129.25		138.20	141.08		
7	5	6	192.36	237.77	203.91	207.63	247.53	254.03
8	6		25.60		29.35	30.35		
9	6		70.70		74.87	76.69		
10	14		155.16					
12	13		5.49		7.43	7.67		
13	12	8	110.35	47.64	115.16	117.47	54.42	55.86
14	12		37.74		41.76	42.75		
16	7	2	90.54	88.44	96.09	97.97	96.68	99.44
17	7		35.60		41.00	41.89		
18	2		10.14		17.38	18.10		
19	1		0.83		8.15	8.49		
20	1	*castellum*	100.00	40.00	108.18	110.32	42.89	46.35
21	7		85.83		93.83	96.23		
21	*castellum*		62.00		64.81	68.68		
22	3		10.00		18.05	18.89		
23	2	8	178.76	177.70	188.56	192.34	182.89	187.60
23	3	9	139.22	49.30	149.84	151.98	61.95	62.93
24	8		64.70		70.69	71.90		
26	9		85.30		95.65	97.73		
27	9		43.10		51.64	52.96		
28	4	9	91.70	174.36	102.34	104.33	184.13	189.04
29	10		65.96		73.35	74.74		
32	11		38.21		45.60	46.48		
33	10		126.00		130.70	133.18		
34	11		111.16		117.42	119.62		
37	4		237.19		251.86	256.50		
39	5	4	128.60	128.53	134.93	137.49	138.23	141.79
40	14	2	112.81	155.66	122.82	125.19	170.25	174.69
41	14		2.86		9.19	9.56		
42	5		7.88		13.59	13.96		

Results: discharge of proposed tower to fountain connections

For each connection fountain-to-tower the maximum value of the hydraulic slope to be entered in the charts of Figs. 13 and 14 is calculated from the level differences between the water level in the water tower tank (Table 5) and the spout axis level. The level differences are then divided by the pipeline length in Table 7. The maximum slope value is obtained from the maximum level difference *Hwt max - Hs min* divided by the minimum pipeline length *Lmin*, while the minimum value is obtained from the ratio of the minimum level difference *Hwt min - Hs max* and the maximum pipeline length *Lmax*. The two values obtained are identified on the vertical axis of the chart in Fig.13a (for slope values higher than 0.16 m/m) or 14a (for slope values up to 0.20 m/m) and the graphical method is applied, finally determining the velocity range and pipeline discharge range. When using the chart in Fig. 14a, the method is applied twice, first for the range *senaria* to *denaria* and then restricted to the range *senaria* to *settenaria*. In fact, from the data on the spout diameter of Figure 4, the latter range of pipes were probably supplying 14 out of 19 fountains[36]. Table 8 presents in column 1 and 2 the possible connections, in column 3 the discharge of the *digitus - quinaria* pipes, in column 4 the discharge of the range *senaria – denaria*, and in column 5 the discharge for the range *senaria – settenaria*, as calculated by our graphical method. The grey cells identify the discharge of the pipelines connected to towers 12 and 13, which were not restored to full height after the 62-63 CE earthquake, corresponding to the current measurable height.

Among the 47 examined, only the two connections fountain 21-tower 7 and fountain 39-tower 5 are not possible, being the hydraulic slope negative. Fountain 21 could have been connected to the *castellum divisorium*, fountain 39 to a tower in the unexcavated area of Regio IX .The remaining 45 connections are all viable.[37]

Table 8. Discharge of each fountain-to-tower connection, estimated for various ranges of pipes.

Fountain	**Tower connected**	**Discharge, l/s** *digitus - quinaria*	*senaria- denaria*	*senaria- settenaria*
1	4	0.45 - 1.12		
2	5		0.37 - 2.00	0.37 - 0.77
2	4		0.47 - 2.30	0.47 - 0.9
3	5		0.57 - 3.10	0.60 - 1.20
4	5		0.32 - 1.62	0.32 - 0.62
4	4		0.40 - 1.97	0.40 - 0.76
5	6		0.24 - 1.30	0.24 - 0.48
5	5		0.37 - 1.92	0.37 - 0.75
6	6		0.37 - 1.87	0.37 - 0.75
7	5		0.37 - 1.87	0.37 - 0.75
7	6		0.27 - 0.65	0.27 - 0.50
8	6		0.44 - 2.8	0.44 - 1.05
9	6		0.38 - 2.00	0.38 - 0.77
10	14		0.42 - 2.22	0.70 - 0.86
12	13		0 - 0.65	0 - 0.45
13	12		0.20 - 1.17	0.20 - 0.40

[36] For more details on the supply pipe possible diameter, see Monteleone, Op. cit. fig. 9.
[37] In the present study, we are not commenting further on the simultaneous operation of the fountains, or the availability of flowrate in the public network. See the previous conclusions in Monteleone *et al.* (2021).

13	8		0.28 - 1.50	0.28 - 0.50
14	12		0.25 - 1.67	0.25 - 0.65
16	7		0.50 - 2.55	0.50 - 0.98
17	7		0.47 - 2.32	0.47 - 0.97
18	2	0.37 - 0.91		
19	1	0.52 - 1.26		
20	1		0.17 - 1.15	0.17 - 0.36
20	*castellum*		0.77 - 2.18	0.36 - 0.87
21	7		0.00	0.00
21	*castellum*		0.72 - 2.00	0.35 - 0.77
22	3	0.33 - 0.92		
23	2		0.30 - 1.56	0.30 - 0.62
23	8		0.35 - 1.75	0.35 - 0.70
23	3		0.17 - 0.22	0.10 - 0.17
23	9		0.44 - 2.45	0.44 - 0.92
24	8		0.40 - 2.08	0.40 - 0.82
26	9		0.53 - 2.70	0.53 - 1.05
27	9		0.70 - 3.30	0.70 - 1.47
28	4		0.40 - 2.05	0.40 - 0.77
28	9		0.48 - 2.35	0.48 - 0.95
29	10		0.40 - 2.15	0.40 - 0.82
32	11		0.50 - 2.58	0.50 - 1.05
33	10		0.35 - 1.79	0.35 - 0.70
34	11		0.6 - 2.85	0.60 - 1.15
37	4		0.55 - 2.70	0.55 - 1.12
39	5		0.00	0.00
39	4		0.10 - 0.95	0.10 -0.27
40	14		0.3 - 1.55	0.30 - 0.60
40	2		0.39 - 1.96	0.39 - 0.75
41	14	0.41 - 1.02		
42	5	0.25 - 0.63		

Indicates the discharge of the pipelines connected with the incomplete water towers 12 and 13, calculated with the value of the visible height

Comparison with the basin overflow data

Looking at the various examples of small street *lacus* in the Roman towns[38], their regular operation included a supply through a pipe or channel, and an overflow of the water, once the basin was full. The time interval for which the supply remained continuous and constant is not known, and there might have been times in the day when the flowrate was highly reduced or stopped, with some water remaining in the basin for local use[39]. Certainly, the fact that various fountain basins in Pompeii (fountains 6, 24 and 37) were reduced in volume (two by elevating the basin bed and one by producing an orifice below the regular overflow channel) suggests that there was a time during their lifespan when the distribution system had to cope with a reduced flowrate from the water sources, compared with the initial value[40]. The reduction in the basin volume allowed the water in the basin to overflow more often; previous calculations for the hydraulic retention time in the basins indicated for Pompeii a maximum of 3 hours[41].

To compare the values for the basin inflow obtained in this study with the values of the basin outflow obtained from the 2021 study, the chart of Fig. 16 is presented. For each fountain-to-tower connection, the vertical bars identify the range for the pipeline discharge, for each pipe diameter range. In the case of more than one connection, decimal values in 0.2 intervals are used (e.g for fountain 23 the four connections are represented with the numbers 23.2, 23.4, 23.6 and 23.8). The various diameter ranges are identified by different colours: orange identifies the *digitus* pipe range; green the *quinaria* range, light grey the *ottonaria-denaria* range, and dark grey the *senaria-settenaria* range.

In the same chart three different black markers (square, rhomboid, and triangle), identify the overflow channels discharge values of Table 1.

Fountains 6, 10 and 19 have no markers, since the first had not a proper overflow channel and the other two were not accessible at the time of our previous field survey. The connections identified as 21.0 and 39.0 have no discharge bars, since the connections between fountain 21 and tower 7, and fountain 39 and tower 5 were not possible, as explained before.

The orange circles identify the improbable overflow channel discharges, corresponding to a water jet exceeding the basin width (Monteleone *et al.* 2021).

On the horizontal axis, the fountain-to-tower connections outlined with boxes, are considered less probable, since a connection to a water tower in the unexcavated might rather have been in place.

[38] Agusta-Boularot *Op.cit.*, Schmolder-Veit, *Op.cit.*

[39] The fountain supply could be stopped in the night, to allow the supply of baths and other building cisterns; however, also the supply of private premises could have been stopped at night. There is not yet enough evidence to understand if time limits for the use of water were in place in Pompeii, and more information will be available once the water use in private and public buildings is studied, discussing the water balance for each water tower. For an initial estimate on the quantities of water used in two Pompeian houses, see Monteleone (2020).

[40] On the changes in the layout of the *castellum divisorium* see Ohlig (2001); on the water scarcity in Regio VI see Jones and Robinson (2005); on the reduction of the spring discharge and the adaptation of the town water distribution system, see Keenan-Jones (2015).

[41] Monteleone *et al.*, *Op.cit.*

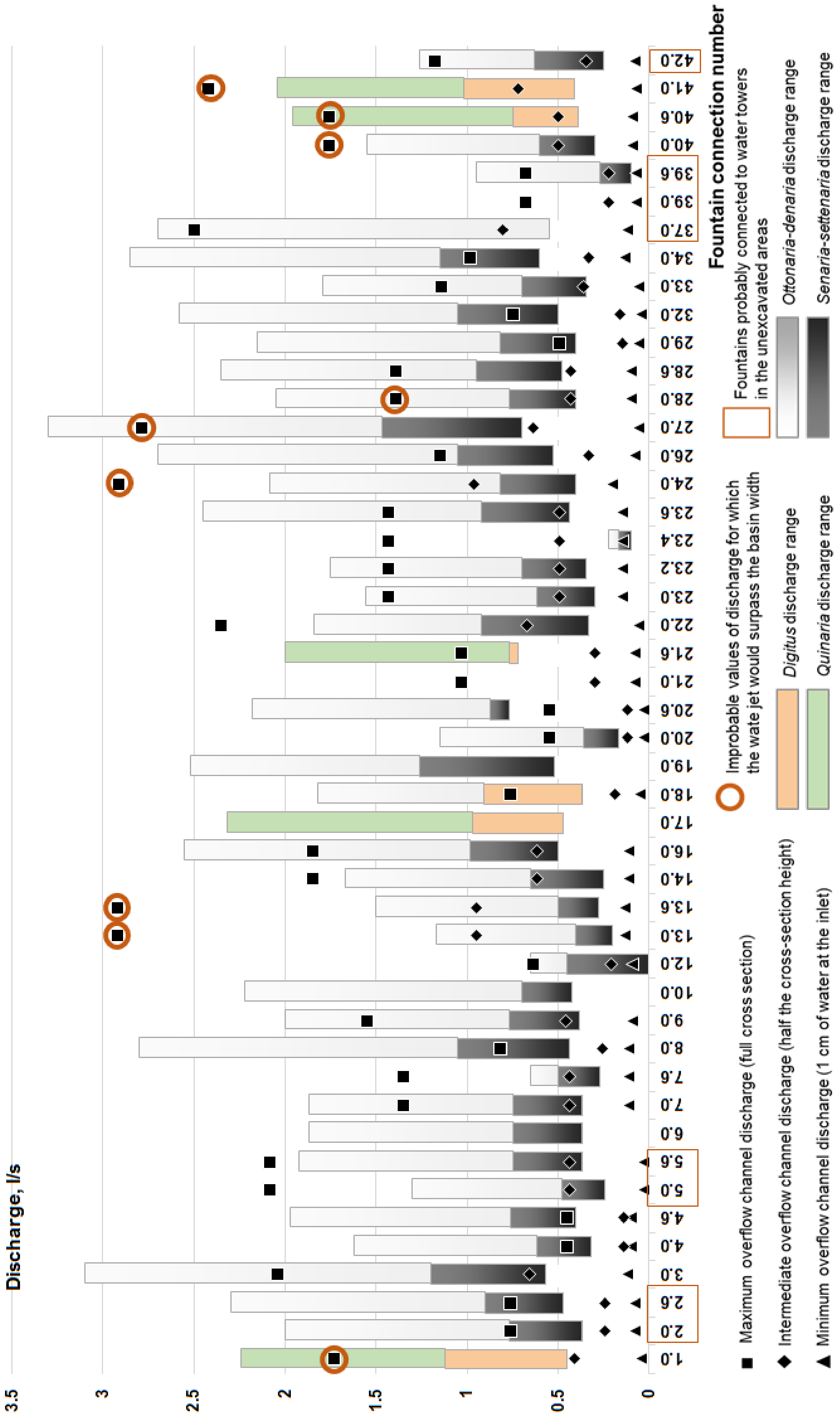

Fig. 16. Chart for the comparison of the discharge range for each fountain-to-tower connection (vertical bars) and the values obtained for the discharge of the fountain basin overflow channels under three water level conditions (black markers).

The agreement of the two sets of results is shown by the position of the black markers with respect to the discharge bars:

- for all the connections examined, the pipeline discharge remains between 0.1 and 3.3 l/s. Looking at the values for the overflow discharge and excluding the improbable circled data, most of the fountains could have been supplied with flowrates in the range 0.2 – 2.5 l/s;
- out of 47 examined connections, 42 values for the lowest overflow channel discharge (black triangles in Fig. 16) remain below the discharge bars, this signifying that possibly the flowrate continuously supplied to the basins was higher than 0.1 l/s, allowing higher levels of water in the overflow channels, or that pipes of diameter smaller than a *senaria* were used[42];
- 30 values for the maximum overflow channel discharge (full channel section, black square markers in Fig. 16) remain within the discharge bars, indicating that it was possible that the fountains operated at the maximum discharge;
- 25 markers for the intermediate overflow channel discharge (black rhomboids in Fig. 16) remain within the discharge bars, generally indicating that the values obtained with the analysis of the overflow channel correspond well with the discharge of the supply pipelines;
- for the fountains close to water towers (1,18,19, 22, 41, 42) the values of the overflow channel discharge confirm the possible use of the *digitus* and *quinaria* as supply pipes; for fountain 1 the discharge had to remain lower than 1.7 l/s to have the water jet contained in the basin width;
- for fountain 20, the overflow channel data confirms the connection to tower 1 rather than to the *castellum divisorium* (20.6). Conversely, fountain 21, since the connection to tower 7 was not possible, might have been supplied directly from the *castellum* (21.6);
- for fountain 23, with four connection options, the markers identify connection 23.4 with tower 3 as less suitable than connections to tower 2, 8 or 9;
- the discharge bars and markers for fountains 12, 13 and 14, connected to fountains 12 and 13, based on the current incomplete height, show that some discharge was possible. However, further consideration of the possible original height of these towers should be made.

Estimate of the original height of towers 12 and 13

Fountain 12 was probably connected to tower 13; the current incomplete height would have caused a discharge up to 0.65 l/s when a *denaria* pipe was used; however, similarly to the other fountains close to a water tower, the original pipes could have been in the range *digitus–quinaria*. In this case, to produce a value for the discharge close to the values obtained for the intermediate and maximum overflow channel discharge (0.21 and 0.68 l/s) the original height would have been close to 6 m.

Information on the original height of tower 13 can be deduced from the discharge of its connections with fountains 12 and 14. For fountain 12 to produce a discharge between 0.43 and 0.61 l/s through a pipe in the range *senaria-settenaria*, the tower height would have been between 4.5 and 6.3 m. If fountain 14 was connected to tower 13 through a *digitus-quinaria* pipeline, a tower height between 3.7 and 6 m would have produced a discharge of 0.21 to 0.5 l/s. To obtain discharge values higher than 0.8 l/s and closer to the maximum overflow channel discharge of 1.85 l/s, a tower height of 7 m would have been needed, which would be unusual for the Pompeian towers. Therefore, for both towers, a maximum height close to 6 m seems likely.

Typical velocity ranges and comments on the *quinaria* unit

Some conclusions can be drawn regarding the values of the average velocity maintained in the pipelines considered. For this purpose, the improbable connections are excluded, leaving 22

[42] The last hypothesis remains less probable, considering that Nappo (Op. Cit.) was tempted to conclude that the pipes directed to the fountains were all close to the *denaria* size.

connections. The range of values for the velocity is shown in Fig. 17a. The 6 fountains close to a water tower present values of average velocity higher than 1.5 m/s, while for 15 connections the average velocity remains below 1.2 m/s. For all the connections the average velocity is above 0.6 m/s.

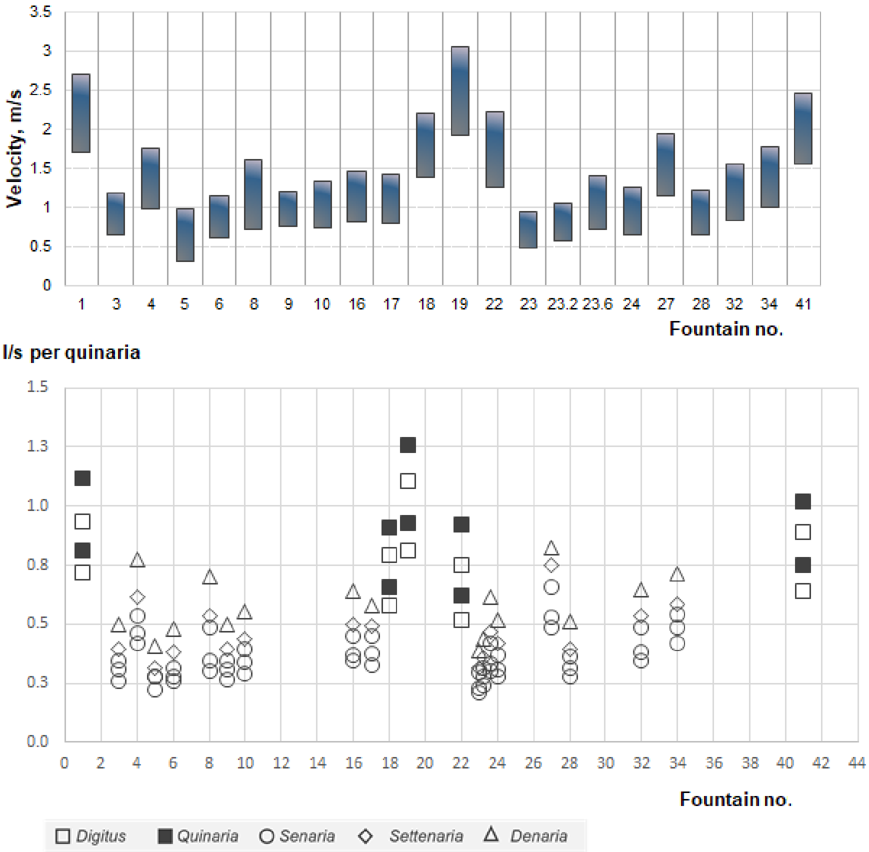

Fig. 17. (a) The range of average velocity in the pipelines for the 22 most probable connections; (b) the value of the unit discharge of the *quinaria* pipe, calculated for the same connections.

Some considerations on the flowrate carried by the ancient *quinaria* unit, as introduced by Frontinus[43], are possible, once the value of the discharge is divided by the number of *quinariae* carried by each pipe[44].

The resulting l/s per *quinaria* are shown in Fig. 17b. The ancient unit, although used as a measure for a flowrate, was in fact a measure of the wet cross-section area. The open debate on the flowrate discharged by an ancient *quinaria* unit[45] was estimated by Di Fenizio (1916), giving a minimum value of 0.47 l/s, and not confirmed by the calculation of the discharge of some sections of the aqueduct channels of Rome completed by Blackman (1978) and Fahlbusch (1982)[46]; the flowrates estimated for some sections of the Anio Novus by Keenan-Jones *et al.* in 2015 correspond to values for the quinaria

[43] Frontinus introduced the *quinaria* as the standard measure for the pipes in 25.4; in 65.3 he describes the measurements taken for a stretch of the Aqua Appia channel, identifying the *quinaria* as a measure for the cross-section occupied by the fluid in motion.
[44] *De Aquis*, 39 - 43. A *digitus* is equivalent to 0.64 *quinariae*; a *quinaria* is obviously 1; a *senaria* 1.44 *quinariae*; a *settenaria* 1.96 *quinariae*; an *ottonaria* 2.56 *quinariae*; a *denaria* 4 *quinariae*.
[45] For a summary of the scholars' contributions to the debate on the *quinaria* unit discharge, see Cioli (2017).
[46] Blackman, although estimating the maximum value for the discharge, found lower values than those corresponding to the DiFenizio *quinaria* discharge for the flowrate of the aqueduct channel; for a discussion on the Blackman and Fahlbusch hypotheses, see Kessener (2013) and Keenan-Jones *et al.* (2015).

flowrate between 0.4 and 0.7 l/s[47]. The values proposed by the various scholars all lie between 0.11 and 0.9 l/s[48]. Although Rodgers (1986) and Bruun (2004) concluded that there would not be an exact value for the *quinaria* discharge, the study of the discharge of existing Roman pipelines and channels does give some hint on the range used by the *aquarii* watermen, who had to choose the appropriate pipe diameter to connect points at known elevation.

In the case of the pipelines supplying the Pompeian fountains, Fig. 17b shows that only the 6 fountains connected to a water tower by means of *digitus* and *quinariae* pipes are associated with discharges for the *quinaria* unit up to 1.2 l/s, while for 80% of the connections the value is lower than 0.70 l/s. 68.5% of the connections present a *quinaria* discharge higher than 0.35 l/s, while for only 57% of the connections does it remain within 0.45- 0.55 l/s, closer to the DiFenizio minimum discharge 0.48 l/s.

Conclusions

The supply of the public *lacus* fountain in Pompeii, connected by means of pressurised lead pipelines to nearby water towers, was estimated on the basis of the hydraulic slope available, with a view to determining which individual tower-to-fountain connections might have been possible. The level differences between the points were estimated from the Piano della Conoscenza di Pompei data available on the site ground elevation, the measurements of the water tower taken by other scholars and the measurement of the spout height above ground, acquired in a dedicated survey. Sensitivity analysis, based on the errors in defining the original pipelines paths and their length, as well as the point elevations, allowed some ranges to be defined for the available hydraulic slope. The calculations were completed considering an absolute roughness of the ancient lead pipes in the range 0.1 to 0.5 mm, supported by the findings from the microscopic surface analysis of some pipes and head loss laboratory experiments.

Two charts, obtained through the application of the Colebrook-White equation for the Roman pipes in the range *digitus* to *denaria*, have been proposed as a basis for a graphical iterative procedure, providing the range of average velocity and corresponding pipelines discharges, when an initial range for the maximum available hydraulic slope is defined.

35 fountain-to-tower connections showed a discharge between 0.1 and 3.3 l/s, while the most probable values remained between 0.2 and 2.5 l/s.

The values previously obtained by Monteleone *et al.* in 2021, for the flowrates overflowing from the fountain basin overflow channels, were compared with the values for the pipelines discharge obtained in the present study, through a bar chart showing their fit, and indicating a general agreement.

The original maximum height of the towers number 12 and 13, not reconstructed after the 62-63 CE earthquake, was estimated as close to 6 m above ground.

By analysing the range of values of the velocity in the pipelines and their cross-section expressed in ancient *quinariae* units, a chart with the estimated values of the *quinaria* discharge showed values between 0.3 and 0.7 l/s as possible.

The results on the ancient Pompeian pipelines discharge are dependent on the values assumed for the pipe absolute roughness; future studies on the hydraulics of the ancient lead pipes will be useful for further validation and development in the study of ancient Roman water networks.

The assumptions made on the supply pipes encased in the fountain spouts will be discussed through the study of the recently excavated fountain and water tower in Regio V, when access to the structure becomes available.

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[47] By using the values for the flowrates estimated for some sections of the Anio Novus by Keenan-Jones et al. when the thickness of the sinter deposit varied from 0 to 0.27 m, estimated in the range 0.35-1.92 m^3/s, and dividing the values by the measure of flow area *(Op.Cit.*, Table 6) expressed in *quinariae* (950-3200 *quinariae*) it is possible to calculate a value for the flowrate through a *quinaria* in the range 0.4-0.7 l/s.
[48] Keenan-Jones *et al.*, Op.cit, fig. 2.

Notations

K_i	Concentrated head loss coefficient for each of the pipe fittings
D	Average internal diameter of the pipeline (cm)
hs	Height of the spout axis over the basin top (m)
hf	Height of the basin top over the footway (m)
hr	Height of the basin top over the road surface (m)
H_1	Elevation of the pipeline inlet cross-section (m asl)
H_2	Elevation of the pipeline outlet cross-section (m asl)
Hs	Elevation of the spout axis (m asl)
$Hs\ min$	Minimum value of the elevation of the spout axis, accounting for possible errors (m asl)
$Hs\ max$	Maximum value of the elevation of the spout axis, accounting for possible errors (m asl)
$Hwt\ max$	Maximum value of the elevation of the water level in the tank of top of a water tower (m asl)
$Hwt\ min$	Minimum value of the elevation of the water level in the tank of top of a water tower (m asl)
L_I	Length of the pipeline on the plan, for the first considered connection (m)
L_{II}	Length of the pipeline on the plan, for the second considered connection (m)
$L_{I\,\mathrm{max}}$	Maximum value of the total length of the pipeline, for the first considered connection (m)
$L_{I\,\mathrm{min}}$	Minimum value of the total length of the pipeline, for the first considered connection (m)
$L_{II\,\mathrm{max}}$	Maximum value of the total length of the pipeline, for the second considered connection (m)
$L_{II\,\mathrm{min}}$	Minimum value of the total length of the pipeline, for the second considered connection (m)
ks	Absolute hydraulic pipe roughness (mm)
S	Hydraulic slope (m/m)
Q_D, Q_Q	Discharge of the *digitus* and *quinaria* pipes, in the example given for the graphic method for the discharge estimate
$Q\ (1\ cm)$	Discharge of the basin overflow channel corresponding to a water height of 1 cm at the inlet cross-section (l/s)
$Q(H/2)$	Discharge of the basin overflow channel corresponding to a water height equal to half the height H of the inlet cross-section (l/s)
$Q(H)$	Discharge of the basin overflow channel corresponding to a water height equal to the height H of the inlet cross-section (l/s)
Re	Reynolds number
R_a	Average surface roughness, according to standard ISO 25178-6 (μm)
R_q	Root mean square roughness, according to standard ISO 25178-6 (μm)
R_z	Max height peak to valley, according to standard ISO 25178-6 (μm)
λ	Friction factor in the Darcy-Weisbach equation
V	Average velocity in a pipe cross-section (m/s)
$\overline{V}_1$, $\overline{V}_2$	Average value of the velocity obtained according to the graphic procedure described (m/s)

Acknowledgments

The authors would like to thank the officers of the Beni Culturali at Pompeii archaeological park: dott.ssa Marialaura Iadanza, for being a reference for general matter and personal support during the survey campaigns, and the archaeologists involved in the restoration of the fountains for sharing the information available on the structures; dott.ssa Grete Stefani for the precious advice and indications regarding the organisation of the work on-site; dott.ssa Annamaria Sodo and dott. Nunzio Vitiello from the SIAV Boscoreale Office for promptly sharing the GIS files with the updated site cartography and elevation records. The support of the staff of the engineering laboratory at Northumbria University was invaluable for the testing of ancient Roman pipes, our thanks go to Craig Dixon, Jonathan Tree, Simon Neville and Adam Cosheril. We would like to thank Dr. Andrew Parkin at the Hancock Museum in Newcastle upon Tyne, for supporting and encouraging the research on the pipes stored in the museum, and the Society of Antiquaries in Newcastle upon Tyne for authorising the experimental work on the pipes in its collections.

References

Adams T, Grant C, Watson H (2012). A Simple Algorithm to Relate Measured Surface Roughness to Equivalent Sand-grain Roughness. International Journal of Mechanical Engineering and Mechatronics 1: 66-71.

Agusta-Boularot S (2008). Le lacus de la rue romaine: un exemple de 'mobilier urbain' antique? In Dieudonne' N, Saliou C (eds) La rue dans l'antiquité': définition, aménagement et devenir. Rennes, Presses Universitaires de Renne: 93–100.

Blackman DR (1978). The volume of water delivered by the four great aqueducts of Rome, PBSR 46: 52-72.

Brouwer Burg M, Peeters H, Lovis WA (Eds.) (2016). Uncertainty and sensitivity analysis in archaeological computational modeling. Springer, Cham.

Bruun C (2004). The impossibility of reaching an exact value for the Roman Quinaria measure, in: Rogers R.H (2004) Frontinus: De Aquaeductu Urbis Romae. Cambridge, 342-346

Burdy J (2002). Les aqueducs Romain de Lyon. Lyon, Presses Universitaire de Lyon.

Campagna L and Scardozzi G (2013). Archeologia delle acque a Hierapolis di Frigia: tematiche principali e metodologie integrate di ricerca. In: L'Anatolie des peuples, des cités et des cultures (IIe millénaire av. J.-C. – Ve siècle ap. J.-C.). Colloque international de Besançon - 26-27 novembre 2010. Volume 2. Besançon : Institut des Sciences et Techniques de l'Antiquité, pp. 197-220.

Chadwick A, Morfett J, Borthwick M (2013). Hydraulics in civil and environmental engineering, 5th edition.

Cioli D (2010). La quinaria di Frontino. Archeologia sotterranea (1): 8-17.

Cochet, A and J Hansen (1986). Conduites et objets de plomb gallo-romains de Vienne (Isère). Paris: Centre national de la recherche scientifique.

Crapper M, Motta D, Sinclair C, Cole D, Monteleone MC, Cosheril A, Tree J, Parkin A (2022). The hydraulic characteristics of Roman lead water pipes: an experimental investigation. The International Journal for the History of Engineering & Technology, DOI: 10.1080/17581206.2022.2054395.

Daniels CM (1959). The Roman Bath House at Red House, Beaufront, Near Corbridge. Archaeologia Aeliana 4 (37): 85-176.

Darcy MH (1857). Recerches experimentales relatives au movement de l'eau dans les tuyaux. Paris, Imprimerie Imperiale.

Dessales H (2013). Le partage de l'eau. Fontaines et distribution hydraulique dans l'habitat urbain de l'Italie romaine. Rome, École française de Rome.

DiFenizio C (1916). Sulla portata degli acquedotti romani e determinazione della quinaria. Giornale del Genio Civile 14: 227-331.

Dybkjaer-Larsen J (1982). The water towers of Pompeii. Analecta Romana Instituti Danici (11): 41-67.

Eschebach H (1979). Probleme der Wasserversorgung Pompejis. Cronache Pompeiane (5): 24 – 60.

Eschebach H and T Schafer (1983). Die offentlichen Laufbrunnen Pompejis. Katalog und Beschreibung. Bollettino del'associazione Internazionale Amici di Pompei 1:11-40.

Everitt, BS and A Skrondal (2010). The Cambridge Dictionary of Statistics. New York, Cambridge Unversity Press.

Fahlbusch H (1982). Vergleich Antiker Griechischer und Romiser Wasserversorgunganlagen. Mitteilungen auf Leichtweiss Institut fur Wasserbau. Technical University of Braunschweig, Germany,Vol. 73.

Farshad FF, Rieke HH, Louisiana U (2005). Technology innovation for determining surface roughness in pipes. Technology today series, Society of Petroleum Engineers (10) 82-86.

Flack, KA and MP Schultz (2010). Review of hydraulic roughness scales in the fully rough regime. Journal of fluids engineering 132. 4, DOI 10.1115/1.4001492.

Frey HC and SR Patil (2002). Identification and review of sensitivity analysis methods. Risk Analysis 22 (3): 553-578.

Haut, B and D Viviers (2007). Analysis of the water supply system of the city of Apamea, using Computational Fluid Dynamics. Hydraulic system in the north-eastern area of the city, in the Byzantine period. Journal of Archaeological Science (34) 415-427.

Jansen GCM (2002). Water in de Romeinse stad: Pompeji, Herculaneum, Ostia. Leuven, Peeters Publishers.

Jones, R and D Robinson (2005). Water, wealth, and social status at Pompeii: The House of the Vestals in the first century. American Journal of Archaeology 109 (4): 695-710.

Kanters H, Brughmans T, Romanowska I (2021). Sensitivity analysis in archaeological simulation: An application to the MERCURY model. Journal of Archaeological Science: Reports: 38 https://doi.org/10.1016/j.jasrep.2021.102974.

Keenan-Jones D (2015). Somma-Vesuvian Ground Movements and the Water Supply of Pompeii and the Bay of Naples. American Journal of Archaeology 119 (2) : 191-215.

Keenan-Jones D, Motta D, Garcia MH, Fouke BW (2015). Travertine-based estimates of the amount of water supplied by ancient Rome's Anio Novus aqueduct. Journal of Archaeological Science: Reports (3): 1-10.

Kessener P (2013). Gedanken über Frontin's quinaria, in: Die Wasserversorgungim antiken Rom. Deutscher Industrie Verlag, München,175-188.

Kreith F (ed) (2000). Fluid Mechanics. United Kingdom, CRC-Press.

Lanciani R (1975). Le acque e gli acquedotti di Roma antica. Roma, Edizioni Quasar, reprint of (1881)Topografia di Roma Antica. I commentari di Frontino intorno le acque e gli acquedotti. Silloge epigrafica aquaria, Atti della R. Accademia dei Lincei. Memorie della Classe di scienze morali, storiche e filologiche, 3 (4) Lockett W G (1991) Roman hydraulic engineering: two Vitruvian problems. Proceedings of the Institution of Civil Engineers 90(1): 123-143.

Maiuri A (1931). Pozzi e condotture d'acqua nell'antica città. Scoperto di un antico pozzo presso Porta Vesuvio. Notizie degli scavi di antichita`: 546-575.

Monteleone MC (2020). Le reti di distribuzione di acqua potabile in epoca romana. Fistule ritrovate e quantità di acqua erogata in due case Pompeiane. In: D’agostino S and D’ambrosio Alfano FR(eds.) Storia dell Ingeneria. Vol I. Proceedings of the 4th AISI International Conference on the History of Engineering, Naples, April 2020. Naples, Cuzzolin Editore, 173-188.

Monteleone MC, Crapper M, Motta D (2021). The supply of the public lacus of Pompeii, estimated from the discharge of their overflow channels', Water History. https://doi.org/10.1007/s12685-021-00281-9.

Nappo, SC (2002). L'impianto idrico a Pompei. I nuovi dati. In Binos Actus Lumina. Rivista di studi e ricerche sull'idraulica storica I: 91-108.

Nikuradse, J (1933). Stromungagesetze in Rauhen Rohren. In V.D.I. Froschungsheft 361:1.

Nir-El Y (2017). Names and sizes of Roman lead pipes for water conduction. In: Wellbrock K (ed) Schriften der Deutschen Wasserhistorischen Gesellschaft, Band 27-2. Siegburg, 645–655.

Ohlig CPJ (2001). De aquis Pompeiorum: das castellum aquae in Pompeii: Herkunft, Zuleitung und Verteilung des Wasser. Circumvesuviana 4, , hg. von J.A.K.E. De Waele und E. M. Moormann, Nijmegen 2001; Herstellung: Books on Demand Norderstedt; ISBN 3-8311-2614-3.

Olsson R (2015). The water supply system in Roman Pompeii. Department of Archaeology and Ancient History. Lund, Lund University.

Ortloff, CR and DP Crouch (2001). The Urban Water Supply istribution system of the Ionian city of Ephesos in the Roman imperial period. J Archaeol Sci 28(8): 843–860.

Pesacreta, C and F Farshad (2003). Coated pipe interior surface roughness as measured by three scanning probe instruments. Anti-corrosion methods and materials 50(1): 6-16.

Pisani Sartorio G, Lombardi L, Rossi Zambotti H (2011). I trofei di Mario, mostra dell’acqua Claudia Anio Nocus: il percorso dell’acqua. Rend.Pont.Acc.Rom. Arch. LXXXIII: 59-89.

Poehler EE and Ellis SJR (2014). The 2013 season of the Pompeii Quadriporticus Project. Final fieldwork and preliminary results. The Journal of Fasti OnLine, www.fastionline.org/docs/FOLDER-it-2014-321.pdf .

Ramona J, Gilles A, Bernot E (2019) Nouvelles données sur les fontaines lyonnaises et l'approvisionnement en eau de la Presqu'île durant l'Antiquité. Revue archéologique de l'Est 68: 191-212.

Rodgers RH (1986). Copia aquarum. Frontinus measurements and the perspective of capacity. Transactions of the American Philological Association 116: 353-360.

Rodgers RH (2004). Frontinus: De aquaeductu urbis Romae. Cambridge classical texts and commentaries 42. Cambridge, Cambridge University Press.

Schmölder-Veit A (2009). Brunnen in den Städten des westlichen Römischen Reichs. Palilia 19, Wiesbaden, Reichert Verlag.

Schoonhoven A (2006). Metrology and meaning in Pompeii. Studi della Soprintendenza archeologica di Pompei : 20. Roma, L'Erma" di Bretschneider.

Spinazzola V (1917). Notizie degli scavi di antichità Vol. 14. Roma, Real Accademia dei Lincei, 247-264.

Stanco EA (2009). Bocche di fontana romane da Alifae e Telesia. Orizzonti, Rassegna di Archeologia X: 121-129.

Tuttahs G (2007). Milet und das Wasser, Schriften der Deuschen Wasserhistorischen Gesellschaft, Sonderband 5.

Vekemans, O and B Haut (2017). Hydraulic analysis of the water supply system of the Roman city of Perge. Journal of Archaeological Science (16): 322–329.

Appendix

The chart of Fig. 13 explained all the steps to be applied for obtaining a range for the discharge of a pipeline, starting from a range of values for the hydraulic slope S. In that case the pipes *quinaria* and *digitus* were considered. The following chart (Fig. A1) can be used for the same purpose, when the lead pipes have diameter in the range *senaria- denaria*.

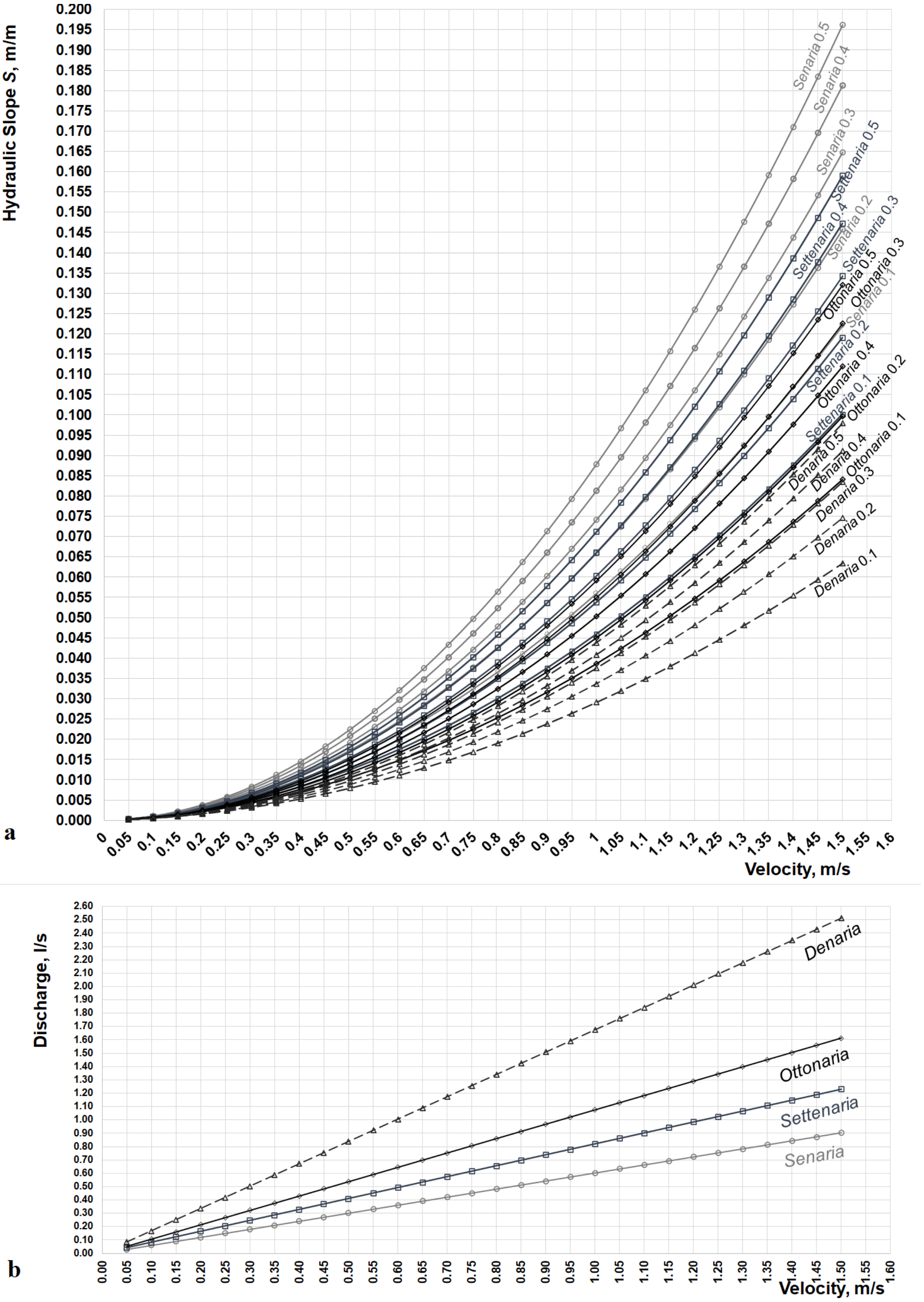

Fig. A1. Charts for the graphical calculation of the pipeline discharge, starting from a range for the hydraulic slope *S*. Pipes *senaria* to *denaria*, hydraulic slope *S* up to 0.20 m/m, absolute roughness k_s in the range 0.1 to 0.5 mm.